LaTex Sample (STAT 564 hw 2)

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These are my answers to a class assignment, but I include this as an example of my use of LaTex and not for the math itself.

Problems

1. (a) $\dfrac{(335)_{25}}{(365)_{25}} = 0.1084563$

   (b) $\dfrac{\binom{30}{3}\binom{335}{22}}{\binom{365}{25}} = .20008$

   (c) $\dfrac{\binom{30}{x}\binom{335}{25-x}}{\binom{365}{25}} \quad 0 \leq x \leq 25$

2. (a) $\dfrac{4}{13} \cdot \dfrac{3}{12} \cdot \dfrac{6}{11} \cdot \dfrac{5}{10} \cdot \dfrac{3}{9} \cdot \dfrac{2}{8} = \dfrac{1}{572} = .001748$

   (b) $\dfrac{\binom{4}{2}\binom{6}{3}\binom{3}{2}}{\binom{13}{6}} = .157343$

   (c) i. $\left(\dfrac{4}{13}\right)^2 \cdot \left(\dfrac{6}{13}\right)^2 \cdot \left(\dfrac{3}{13}\right)^2 = \dfrac{5184}{4826809} = .001074$

   ii. $\dfrac{(4)_2 \cdot (6)_2 \cdot (3)_2}{((13)_2)^3} = .000569$

3. Event A: testing positive

   Event C: having dengue

   $P(A|C^C) = .1$

   $P(A^C|C) = .01$

   (a) $P(A|C) = .99$

   (b) $P(C) = .01$

   $P(A) = P(A|C)P(C) + P(A|C^C)P(C^C)$

   $= (.99)(.01) + (.1)(.99)$

   $= .1089$

   (c) $P(C|A) = \dfrac{P(A \cap C)}{P(A)} = \dfrac{P(A|C)P(C)}{P(A)}$

   $= \dfrac{(.99)(.01)}{.1089} = .09091$

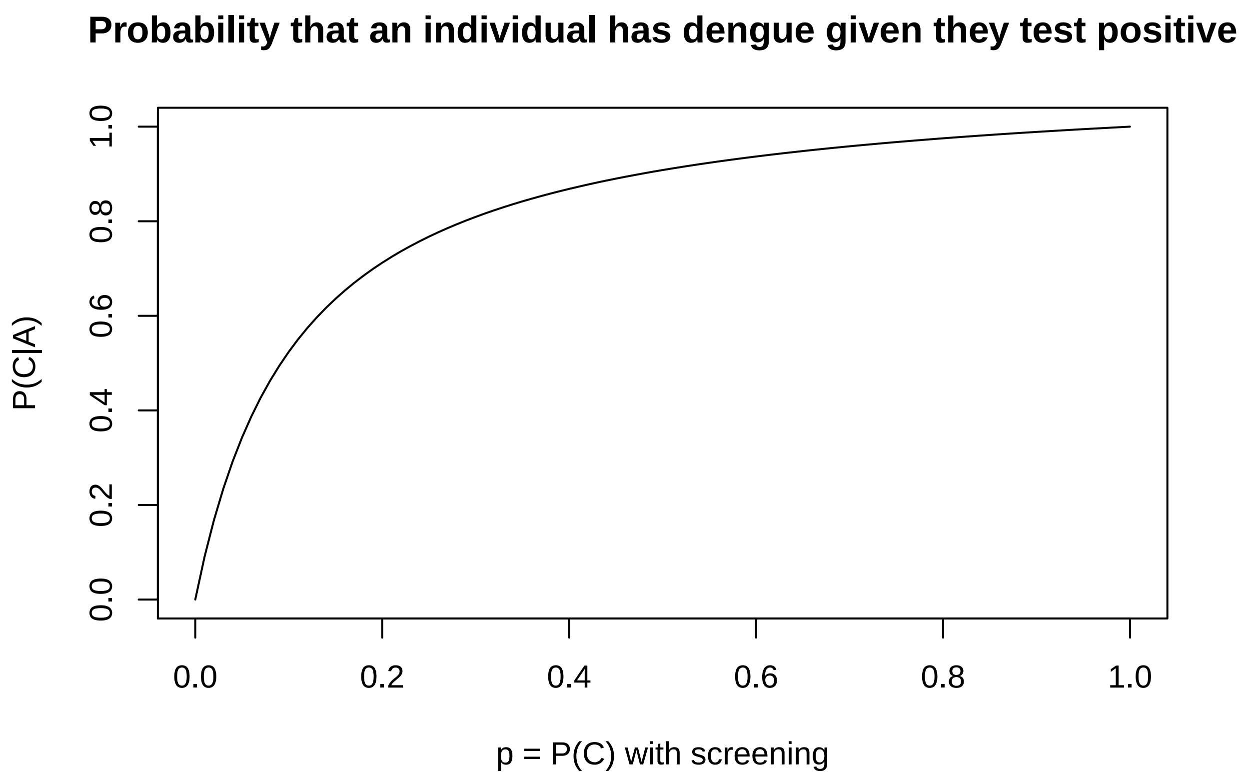
(d) $P(C) = .5$ now

$$P(C|A) = \frac{P(A|C)P(C)}{P(A|C)P(C) + P(A|C^C)P(C^C)}$$

$$= \frac{(.99)(.5)}{(.99)(.5) + (.1)(.5)}$$

$$= .908257$$

(e)



Challenging Problems

1. (a) i. Let A be the event Player A wins.

$$P(A) = \lim_{n\to\infty} \tfrac{1}{2} + \left(\tfrac{1}{2}\right)^4 + \left(\tfrac{1}{2}\right)^7 + ... + \left(\tfrac{1}{2}\right)^{3n-2}$$

Since this is a geometric sum with $a_1 = \frac{1}{2}$ and $r = \frac{1}{8}$, the sum

$$S = \frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}} = \frac{4}{7}$$

Therefore $P(A) = \dfrac{4}{7}$

ii. $P(B)$ is $\frac{1}{2}P(A)$ since every term is multiplied by another $\frac{1}{2}$ since player A goes first (another flip before B), so

$$P(B) = \frac{2}{7}$$

iii. For the same reasons, $P(C) = \frac{1}{2}P(B)$, so

$P(C) = \dfrac{1}{7}$

(b) i. $P(A) = \lim_{n\to\infty} p + p(1-p)^3 + p(1-p)^6 + ... + p(1-p)^{3n}$

This is a geometric sum with $a_1 = p$ and $r = (1-p)^3$, the sum

$S = P(A) = \dfrac{a_1}{1-r} = \dfrac{p}{1-(1-p)^3} = \dfrac{p}{p^3 - 3p^2 + 3p}$

$= \dfrac{1}{p^2 - 3p + 3}$

ii. $P(B)$ is $(1-p)P(A)$ since every term is multiplied by another $(1-p)$ since player A goes first (another tails flip before B), so

$P(B) = \dfrac{1-p}{p^2 - 3p + 3}$

iii. For the same reasons, $P(C) = (1-p)P(B)$, so

$P(C) = \dfrac{(1-p)^2}{p^2 - 3p + 3}$

(c) As $p \to 0$,

$P(A) \to \frac{1}{3}$

$P(B) \to \frac{1}{3}$

$P(C) \to \frac{1}{3}$

2. (a) $f_Y(y) = f_x(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$

where $g(x) = x^p$ for $p > 0$.

$g^{-1}(x) = x^{\frac{1}{p}}$

$f_Y(y) = \dfrac{\beta}{y^{\frac{\beta+1}{p}}} \left(\dfrac{1}{p} y^{\frac{1}{p}-1} \right) = \dfrac{\beta}{py^{\frac{\beta+p}{p}}}, \quad p > 0$

$$f_Y(y) = \begin{cases} 0 & x < 1 \\ \dfrac{\beta}{py^{\frac{\beta+p}{p}}} & x \geq 1 \end{cases}$$

$P\{Y > 2\} = \int_2^\infty f_Y(y) = \dfrac{\beta}{p} \displaystyle\int_2^\infty y^{-\frac{\beta+p}{p}}$

$= \lim_{a\to\infty} -y^{-\frac{\beta}{p}} \Big|_2^a$

$= \lim_{a\to\infty} -\dfrac{1}{a^{\frac{\beta}{p}}} + 2^{-\frac{\beta}{p}}$

$= 2^{-\frac{\beta}{p}}$

(b)

x		$P\{X=x\}$
1	$\dfrac{3!}{(\alpha+3)_3}$	$\dfrac{6}{(\alpha+3)_3}$
2	$\dfrac{\alpha(2\cdot 1+3\cdot 1+2\cdot 3)}{(\alpha+3)_3}$	$\dfrac{11\alpha}{(\alpha+3)_3}$
3	$\dfrac{\alpha^2(1+2+3)}{(\alpha+3)_3}$	$\dfrac{6\alpha^2}{(\alpha+3)_3}$
4	$\dfrac{\alpha^3}{(\alpha+3)_3}$	$\dfrac{\alpha^3}{(\alpha+3)_3}$

$$EX_4 = 1\frac{6}{(\alpha+3)_3} + 2\frac{11\alpha}{(\alpha+3)_3} + 3\frac{6\alpha^2}{(\alpha+3)_3} + 4\frac{\alpha^3}{(\alpha+3)_3}$$

$$= \frac{4\alpha^3 + 18\alpha^2 + 22\alpha + 6}{(\alpha+3)_3}$$