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Mine Crafting Report

I. Introduction

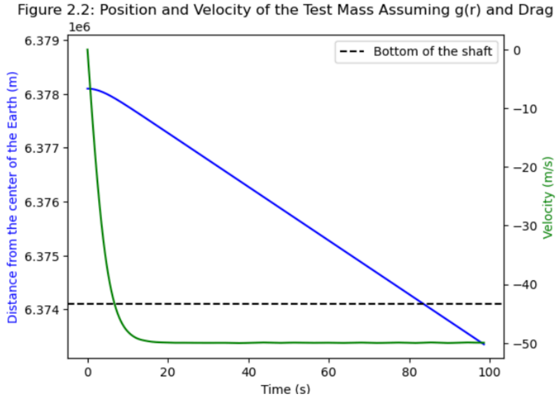
As a scientist working for a mining company operating a mine at Earth's equator, I have been tasked with determining the depth of the mine using a simple physical experiment. The proposed method involves dropping a 1 kg test mass from rest at the top of the shaft and measuring the time it takes to reach the bottom. The shaft is approximately 4 km deep, making it one of the deepest mines on Earth. This report presents a quantitative analysis of the feasibility and accuracy of this timing-based approach to depth measurement. In this investigation, we use Python and its libraries such as NumPy, Matplotlib, and SciPy, which contain computing functions. We model the motion of the falling mass under the influence of gravity, accounting for the variation of gravitational acceleration with depth, and considering possible real-world complications such as drag and the Earth's rotation.

II. Calculation of Fall Time

To begin the investigation, we used a simple free-fall calculation to find the time it would take for the test mass to fall to the bottom of the shaft, assuming ideal free-fall conditions. Treating gravitational acceleration as a constant, g=9.81 m/s², we found that it would take 28.6 s to reach the bottom. Next, we turned g into a function dependent distance to the center of the Earth, and we reduced the equation on the right into a system of coupled first order differential equations by setting the velocity equal to the change in the distance to the center of the Earth with respect to time. Still neglecting drag (α), we used the solve_ivp function of Python to evaluate the equation. This produced a time very close to the time calculated previously, ultimately being 28.6 s. We concluded that turning the gravitational acceleration into a function dependent on the distance from the center of the Earth did not make a significant difference, as 4 km is an extremely short distance compared to the radius of the Earth. Next, we wanted to see the effect that drag would have on the fall time. We calculated the drag using the terminal velocity of the mass, which is 50 m/s. We plotted the position and velocity of the mass using Matplotlib's plotting functions, which is modeled by the graph in Figure 2.2. The graph shows that the velocity plummets in the first few seconds, then plateaus to the terminal velocity at around the 15 s mark. Once again, we employed solve_ivp's events detection capability to find that the falling time of the test mass is 83.5 s when accounting for a height-dependent g as well as drag. This is

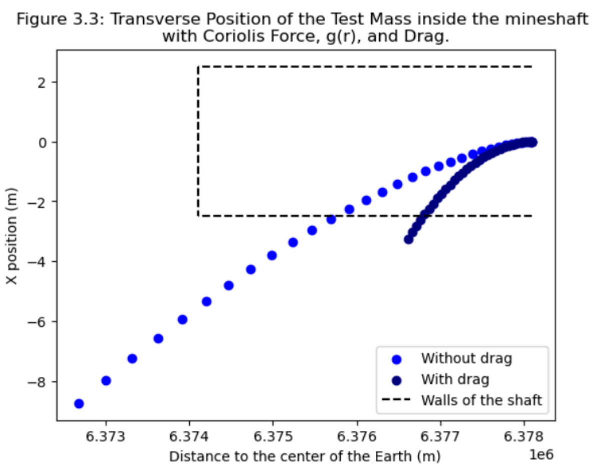
$$\frac{d^2y}{dt^2} = -g + \alpha\left|\frac{dy}{dt}\right|^{\gamma}$$

Figure 2.2: Position and Velocity of the Test Mass Assuming g(r) and Drag



because the drag pushes back up on the object as it is falling. When an object reaches terminal velocity, it is no longer accelerating. When we neglect drag, we treat the mass as though it continues accelerating forever, which would mean that it falls much faster than if drag were affecting it and halting its acceleration after reaching its terminal velocity.

III. Feasibility of Depth Measurement Including Coriolis Forces

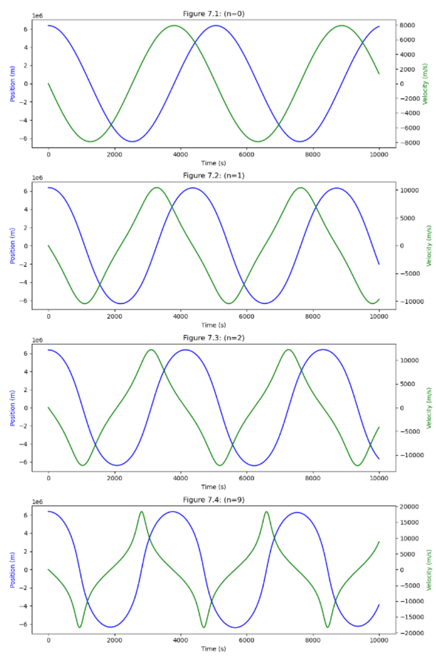


Next, we accounted for the Coriolis force, which affects the movement of an object that is inside another rotating object, such as the Earth. In order to do this, we defined a vectorized function dependent on time. We then modeled the transverse trajectory of the test mass, as though we were viewing the shaft on its side. As you can see in the graph of Figure 3.3, if the mass is dropped from the top center of the mineshaft, the Coriolis force would cause the object to hit the fall before it reaches the bottom of the mineshaft, whether drag is accounted for or not. In fact, it seems that drag makes the object hit the wall closer to the height from which it was dropped, but it takes a longer amount of time. When neglecting drag, the test mass hits the wall of the shaft at 21.9 s, after falling 2,353.9 m. When drag is included in the calculation, the test mass hits the wall after falling 1,302.5 m. However, it takes the mass 29.6 s to fall this distance. This is due to the fact that drag slows the mass down. Because of this, we advise to either use a different strategy for measuring the depth of the shaft. However, there are possible ways of overcoming this complication. One option could be to make further calculations to account for the mass hitting the shaft wall. Another option is to make the shaft much wider so there is no possibility of the mass hitting the wall. One thing to consider is that if the shaft were theoretically at one of the Earth's poles, a test mass would not be affected by the Coriolis force, as the Earth would be revolving around the shaft like an axis. However, because the shaft is at the equator, the Coriolis force has the greatest effect.

IV. Calculation of Crossing Times for Homogeneous and Non-Homogeneous Earth

The next thing we calculated was the time it would take for an object to pass through the center of the Earth, known as the crossing time, if it was dropped down a theoretical mine that went through the entire diameter of the Earth. We did this for both a "homogeneous" Earth, which assumes the Earth's density is constant throughout; as well as a non-homogeneous Earth, which assumes the Earth has different densities at different depths. For a "homogenous" Earth scenario, we referred back to the function we defined to calculate fall time, and defined some event functions. These event functions would tell us when the object reaches the center of the Earth, and what the velocity of the object is at that point. We found that, for a “homogeneous” Earth, the crossing time of the object would be 1,266.5 s, with a velocity of 7,910.8 m/s. Then, using the orbital speed of the Earth, we calculated that the orbital period is 5,069.4 s, which is approximately 4 times the crossing time. Moving on to the crossing time of the non-homogeneous Earth, we evaluated the distance of an object to the center of the Earth, as well as the velocity, versus time as it falls through an infinite mine. To do this, we defined four different functions based on the equation to the right, at four different density concentrations (n). Figures 7.1-7.4 to the left show the plots of the position and velocity versus time of an object falling through the Earth at n=0, n=1, n=2, and n=9 respectively. Then, using the solve_ivp function, we had Python calculate the crossing times of the object for each n. We found that, as n increases, the crossing times decrease, with n=0 yielding 1,267.2 s, and n=9 yielding 943.9 s. This demonstrates an inversely proportional relationship between n and crossing time. To reinforce this relationship, we calculated the crossing time of an object falling in an infinitely long mineshaft through the moon. The moon has a density of 3,341.8 kg/m^3, which is 2,153.1 kg/m^3 less than the Earth’s density. We defined a function for the moon’s gravitational acceleration, then we plugged it into a function for evaluating the crossing time and velocity of an object falling through the moon. Python calculated a crossing time of 1,624.9 s. The crossing time of an object through the moon is much larger than the crossing time of an object through the much denser Earth, thus reinforcing the inverse relationship.

$$\rho(r) = \rho_n \left(1 - \frac{r^2}{R_\oplus^2}\right)^n$$



V. Discussion and Future Work

This investigation explored the feasibility and accuracy of using a timing-based experiment to determine the depth of a mine shaft near Earth’s equator. We concluded that, due to the Coriolis force, this method of measuring the depth of the shaft would need some fine-tuning. We realize there are many factors to consider when making these calculations, such as a height-dependent gravitational acceleration, the effect of the Earth’s non-homogeneous nature on the test mass’s motion. In the future, we will also have to account for the Earth’s unevenness in sphericality in order to make our calculations more accurate.