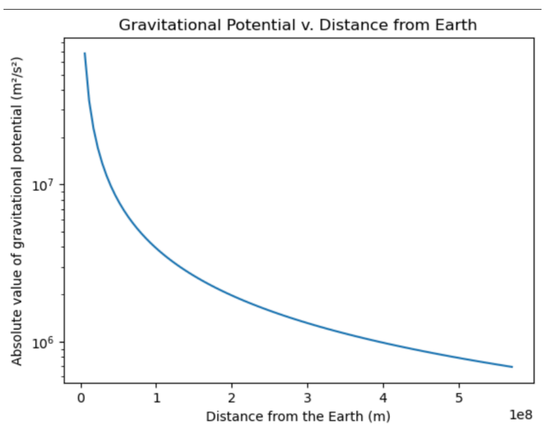
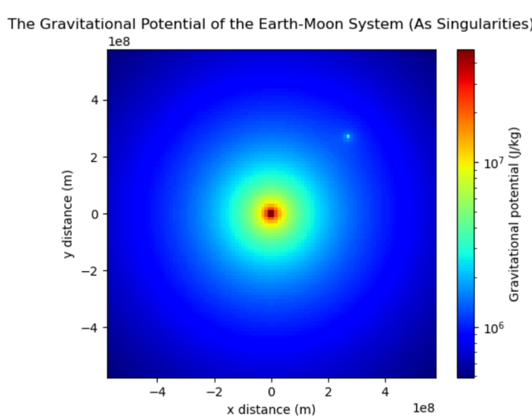
I. Introduction

As you may know, the mission of NASA's Apollo program is to send men to the moon. As an engineer working on this mission, my fellow colleagues and I have conducted thorough investigations to quantify the performance of the new rocket that will carry the capsule: the Saturn V. We've employed the use of the Python programming language to model our mathematical findings. Through our rigorous efforts in creating precise models, the Apollo program will achieve a feat of mankind that was once thought to be impossible. This report concisely explains our understanding of the gravitational potential and forces this mission will experience, including visual representations we've created to aid in our investigations. We are excited to show how we can push the boundaries of what humanity can achieve.

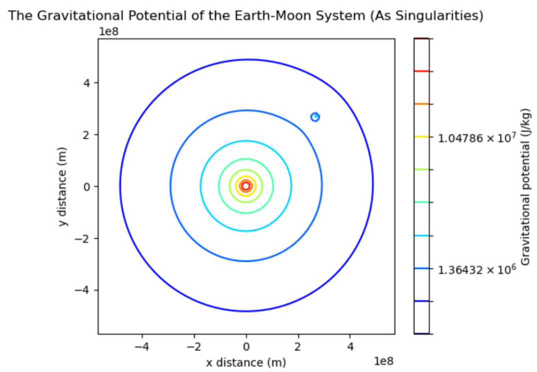
II. The Gravitational Potential of the Earth-Moon System



The gravitational potential of the Earth is defined as the amount of work needed to move a mass in the Earth's gravitational field. The equation we used to represent the gravitational potential is $V = -GM/r$, where G is the gravitational constant of a large celestial body, M is the mass of the body, and r is the distance between the body and some reference point. To illustrate the relationship between gravitational potential and distance, we imported Python libraries to our code, such as numpy and matplotlib which carry functions that aid in computing and plotting data. First, we defined a general function using the equation for gravitational potential. We plotted the potential over a distance from zero to 1.5 times the distance from the Earth to the Moon. The figure to the left is our model of the exponential relationship between the strength of Earth's gravitational potential versus distance from the Earth. We used logarithmic scaling in all of our figures, as the values are extremely large. There is a steep drop in gravitational potential as one gets further from the Earth's surface.



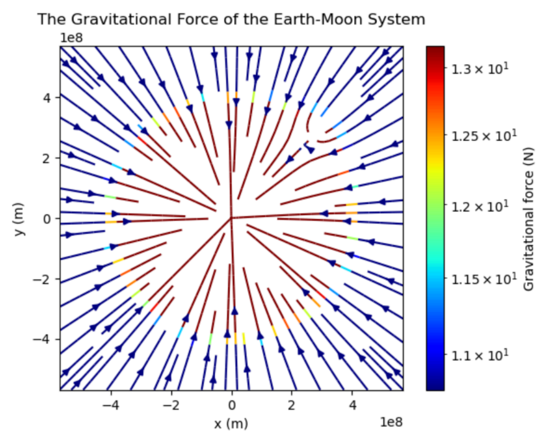
Next, we created a 2D colormap of the gravitational potential of the Earth as well as the Moon . It is important to note that, in this model, we didn't account for the volume of the Earth or Moon, so they are represented as singularities where the mass is concentrated at one point. However, this does not affect the accuracy of our calculations. For this model, we created two separate functions of gravitational potential for the Earth and the Moon, and we added them together, setting the moon's position as 1.5 times the actual distance from the



Earth to the Moon. We did this to be consistent with our calculations, and so that there is a stronger visual distinction between the Earth's gravitational potential and the Moon's. We also created a 2D contour map of the Earth and Moon's gravitational potential to get a better distinction of energy at certain distances. With these models and calculations, we will be able to plan the trajectory of the mission.

III. The Gravitational Force of the Earth-Moon System

In order to model the gravitational force of the Earth and the Moon, we defined a function in our code using the equation for gravitational force vector, represented by $F = (G*M1*m2)/r^2$. We had the function give us the x and y components of the force, which ultimately gives us the magnitude and the direction of the force at a given point. Using this function, we created a visual model that represents the gravitational forces surrounding the Earth. As you can see, the arrows are all pointing towards the center, showing that the forces are all pointing towards the Earth. The arrows are bending in the upper right corner, showing that the gravitational force is affected by the moon's presence. Taking a look at the colorbar as reference, you can see that the forces get stronger as you get closer to the Earth. In order for us to successfully send our rocket to the, we must understand that the rocket needs to overcome the gravitational force and gravitational potential of the Earth, and take advantage of the gravitational force and potential of the Moon.



IV. Projected Performance of the Saturn V Stage 1

Finally, we had to run some calculations for the amount of time it takes for all the fuel to burn out during Stage 1 of the rocket launch. First, we defined the equation for the fuel burn time as T = (m0-mf)/m where m0 is the wet fuel mass, mf is the dry fuel mass, and m is the rate at which the material burns. We found that it takes 158 seconds for all the fuel to burn up in Stage 1 of the Saturn V.

Next, we had to figure out the rocket's change in velocity at a given time during Stage 1. However, we had to account for when the fuel runs out, so we defined a function for the change in velocity where, when the time reaches 158 seconds, the change in velocity becomes zero. If we didn't do this, the function would tell us that the rocket is still having a positive change in velocity, even though there is no more fuel, which would not be accurate. After running the function, we found that the change in velocity right before reaching 158 seconds is 219 meters per second. Using this function, we moved onto the last set of calculations, where we imported the ‘quad’ function from scipy.integrate, which is another python library. Using the quad function, we integrated the change in velocity function from 0 to 158 seconds, which gave us the final altitude of the rocket when the fuel ran out as 74,000 meters above the Earth’s surface. These calculations are crucial for our engineers to know when to initiate Stage 2 of Saturn V.

V. Discussion and Future Work

In summary, our team was able to model the gravitational potential of the Earth and the Moon, the gravitational force of the Earth and the Moon, and we calculated the fuel burn time during Stage 1, the change in velocity at a given time during Stage 1, and the final altitude of Stage 1 once the fuel runs out. Last week, NASA released their official findings of the fuel burn time and the final altitude of Stage 1 of Saturn V. Our findings ended up being fairly close to NASA’s, which yielded 160 seconds for the fuel burn time, and 70,000 meters as the final altitude. Our final altitude ended up being an overestimate, as we neglected to account for factors such as air drag, which will have a significant impact on the velocity of the rocket. In the future, we will calculate these confounding errors to yield more accurate results.