UNIDAD 5. Continuación.

Complemento ortogonal.

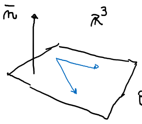
Hallar el complemento ortogonal del subespacio W, esto es $W^{\perp}$.

Sea, $B_W = \{(1,2,0);(0,3,1)\}$. y $\langle u, v \rangle$ = producto interno usual.

II) $W^{\perp} = \{(x,y,z) \in \mathbb{R}^3 / \langle (x,y,z);(1,2,0)\rangle = 0 \wedge \langle (x,y,z)(0,3,1)\rangle = 0\}$

$\begin{cases} x.1 + y.2 + z.0 = 0 \\ x.0 + y.3 + z.1 = 0 \end{cases}$; $\begin{cases} x + 2y = 0 \rightarrow x = -2y \\ 3y + z = 0 \rightarrow z = -3y \end{cases}$ $\forall y \in \mathbb{R}$

$W^{\perp} = \{(x,y,z) \in \mathbb{R}^3 / x = -2y \wedge z = -3y\}$



$(-2y, y, -3y) \rightarrow W^{\perp} = \{(-2y, y, -3y) ; y \in \mathbb{R}\}$

$\dim W^{\perp} = 1$

$B_{W^{\perp}} = \{(-2,1,-3)\}$

$2x - y + 3z = 0$ plano que contiene a $(0,0,0)$

$\bar{n} = (2,-1,3)$

$(-2,1,-3) \parallel \bar{n}$

$W \cap W^{\perp} = \{(0,0,0)\}$

$\dim W + \dim W^{\perp} = 2 + 1 = 3 = \dim \mathbb{R}^3 = \dim E$

$B_W \cup B_{W^{\perp}} = \{(1,2,0);(0,3,1);(-2,1,-3)\}$

3 vectores l.i. en $\mathbb{R}^3$ forman una base de $\mathbb{R}^3$

$B_W \cup B_{W^{\perp}} = B_E$

EJERCICIO

En $(P_2[\mathbb{R}], \langle p,q \rangle = p(-1).q(-1) + p(0).q(0) + p(1)q(1))$ hallar el complemento ortogonal $(S^{\perp})$ de $S = \text{gen}\{x^2 - 1\}$.

$S^{\perp} = \{p \in P_2[\mathbb{R}] / \langle p ; x^2 - 1 \rangle = 0\}$

$p = ax^2 + bx + c$

$\langle ax^2 + bx + c, x^2 - 1 \rangle = 0$

$(a - b + c) \cdot 0 + c(-1) + (a + b + c).0 = 0$

$-c = 0 \rightarrow \boxed{p = ax^2 + bx}$

$B_{P_2} = \{x^2 - 1, x^2, x\}$ base de $P_2[\mathbb{R}]$

$B_S = \{x^2 - 1\}$ $\dim S = 1$

$S^{\perp} = \{p \in P_2[\mathbb{R}] / p = ax^2 + bx ; a \in \mathbb{R} ; b \in \mathbb{R}\}$

$B_{S^{\perp}} = \{x^2, x\}$; $\dim S^{\perp} = 2$

$S^{\perp} = \text{gen}\{x^2, x\}$

$S^{\perp} = \{ax^2 + bx, a \in \mathbb{R} ; b \in \mathbb{R}\}$

$a(-1)^2 + b(-1) + c = a - b + c$

$a.0 + b.0 + c = c$

$(-1)^2 - 1 = 0$

$(0)^2 - 1 = -1$

$a.1 + b.1 + c = a + b + c$

$1^2 - 1 = 0$