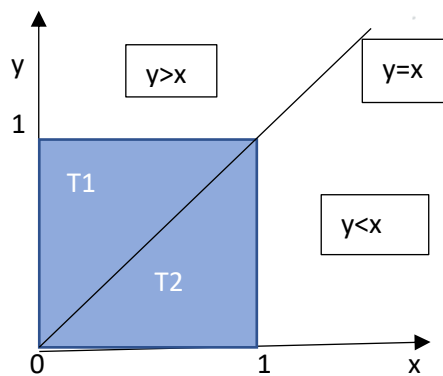


Ejercicio 2-TP 7

Calcular las siguientes integrales:

- Integrando primero respecto de x
- Integrando primero respecto de y

$$d) \iint_R |x - y| \, dx \, dy \quad R = [0,1] \times [0,1]$$



$$|x - y| = \begin{cases} x - y & \text{si } x - y \geq 0 \\ -x + y & \text{si } x - y < 0 \end{cases}$$

$$\rightarrow |x - y| = \begin{cases} x - y & \text{si } x \geq y \\ -x + y & \text{si } x < y \end{cases}$$

o sea que en la zona T1 ($y > x$) $\rightarrow |x - y| = -x + y$

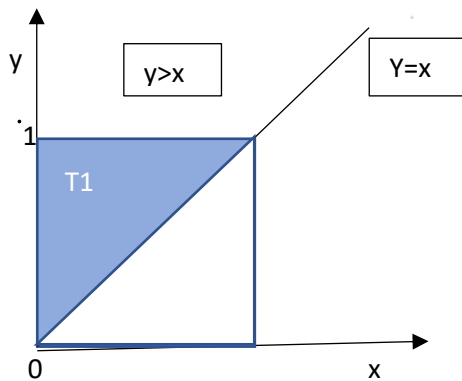
y en la zona T2 ($y < x$) $\rightarrow |x - y| = x - y$

Entonces por propiedad de la integral doble

$$\iint_R f(x, y) \, dx \, dy = \iint_{T1 \cup T2} f(x, y) \, dx \, dy = \iint_{T1} f(x, y) \, dx \, dy + \iint_{T2} f(x, y) \, dx \, dy = I1 + I2$$

$$I1 = \iint_{T1} f(x,y) dx dy = \iint_{T1} |x - y| dx dy = \iint_{T1} (-x + y) dx dy$$

a) Integramos primero en x



$$I1 = \int_0^1 dy \int_0^y (-x + y) dx = \int_0^1 \left(-\frac{x^2}{2} + yx \Big|_0^y \right) dy = \int_0^1 \left(-\frac{y^2}{2} + y^2 \right) dy$$

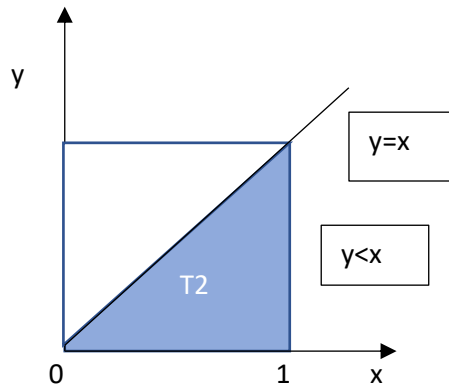
$$I1 = \int_0^1 \frac{1}{2} y^2 dy = \frac{y^3}{6} \Big|_0^1 = \frac{1}{6}$$

b) Integramos primero en y

$$I1 = \int_0^1 dx \int_x^1 (-x + y) dy = \int_0^1 \left(-xy + \frac{y^2}{2} \Big|_x^1 \right) dx = \int_0^1 \left(-x + \frac{1}{2} + x^2 - \frac{x^2}{2} \right) dx$$

$$I1 = \int_0^1 \left(-x + \frac{1}{2} + \frac{x^2}{2} \right) dx = -\frac{x^2}{2} + \frac{1}{2}x + \frac{x^3}{6} \Big|_0^1 = \frac{1}{6}$$

Calculamos I2



$$I2 = \iint_{T2} f(x, y) \, dx \, dy = \iint_{T2} |x - y| \, dx \, dy = \iint_{T2} (x - y) \, dx \, dy$$

a) Integramos primero en x

$$I2 = \int_0^1 dy \int_y^1 (x - y) \, dx = \int_0^1 \left(\frac{x^2}{2} - yx \right) \Big|_y^1 dy = \int_0^1 \left(\frac{1}{2} - y - \frac{y^2}{2} + y^2 \right) dy$$

$$I2 = \int_0^1 \left(\frac{1}{2} - y + \frac{1}{2}y^2 \right) dy = \frac{1}{2}y - \frac{y^2}{2} + \frac{y^3}{6} \Big|_0^1 = \frac{1}{6}$$

b) Integramos primero en y

$$I2 = \int_0^1 dx \int_0^x (x - y) \, dy = \int_0^1 \left(xy - \frac{y^2}{2} \right) \Big|_0^x dx = \int_0^1 \left(x^2 - \frac{x^2}{2} \right) dx$$

$$I_2 = \int_0^1 \left(\frac{1}{2}x^2\right) dx = \frac{x^3}{6} \Big|_0^1 = \frac{1}{6}$$

$$\iint_R |x - y| \, dx \, dy = I_1 + I_2 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$