$$y' - \frac{1}{x} \cdot y = 0$$

Siendo $P_{(x)} = -\frac{1}{x}$, nos queda que la solución general es

$$y' - \frac{1}{x}y = 0$$

$$y' = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$\ln(|y|) = \ln(|x|) + c_1$$

$$\ln(|y|) = \ln(|x|) + \ln(c_2)$$

$$c_1 = \ln(c_2)$$

$$\ln(|y|) = \ln(|x|c_2)$$

$$|y| = |x|c_2$$

$$c_2 = |c_3|$$

$$|y| = |x||c_3|$$

$$|y| = |xc_3|$$

$$|y| \to y = |xc_3| \to y = xc_3 = c_4x$$

$$|y| \to y = -|xc_3| \to y = -xc_3 = c_4x$$

$$|y| \to y = -|xc_3| \to y = -xc_3 = c_4x$$

$$|y| \to y = -|xc_3| \to y = -xc_3 = c_4x$$

$$|y| \to y = -|xc_3| \to y = -xc_3 = c_4x$$

Solución general : y = kx

Proceso de verificación

$$y' - \frac{1}{x}y = 0$$
$$y = kx$$

$$y' = k$$

$$k - \frac{1}{x}kx = 0$$

$$k - k = 0$$

Solicitamos una solución particular $y\left(\frac{1}{2}\right) = -\frac{1}{2}$

$$y = kx$$

$$-\frac{1}{2} = k\frac{1}{2}$$

$$k = -1$$

Solución particular y = -x para $y\left(\frac{1}{2}\right) = -\frac{1}{2}$

Solicitamos una solución particular $y(2) = -\frac{1}{2}$

$$y = kx$$

$$-\frac{1}{2} = k2$$

$$k = -\frac{1}{4}$$

Solución particular $y = -\frac{1}{4}x$ para $y(2) = -\frac{1}{2}$