T P 10 ej de parcial

Hallar la solución general de la siguiente ecuación diferencial de primer orden no homogénea, y encontrar la ecuación particular $y(1) = 1$.

$$xy' + y = e^{2x-1}$$

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$$y' + \frac{y}{x} = \frac{e^{2x-1}}{x}$$

$$y = uv$$

$$y' = u'v + uv'$$

$$u'v + uv' + \frac{uv}{x} = \frac{e^{2x-1}}{x}$$

$$\underbrace{\left(u' + \frac{u}{x}\right)}_{0} v + uv' = \frac{e^{2x-1}}{x}$$

$$u' + \frac{u}{x} = 0$$

$$u' = -\frac{u}{x}$$

$$u' = \frac{du}{dx}$$

$$\frac{du}{dx} = -\frac{u}{x}$$

$$\frac{du}{u} = -\frac{dx}{x}$$

$$\int \frac{du}{u} = -\int \frac{dx}{x}$$

$$\ln(|u|) + c_1 = -\ln(|x|) + c_2$$

$$\ln(|u|) = -\ln(|x|) + c_2 - c_1$$

$$\ln(|u|) = -\ln(|x| \quad) + \ln(c_3)$$

$$\ln(|u|) = \ln\left(\frac{c_3}{|x|}\right)$$

$$|u| = \frac{c_3}{|x|}$$

$$|u| \to \begin{cases} u = \dfrac{c_3}{|x|} \to \begin{cases} u = \dfrac{c_3}{x} \\ u = -\dfrac{c_3}{x} \end{cases} \\ u = -\dfrac{c_3}{|x|} \to \begin{cases} u = -\dfrac{c_3}{x} \\ u = -\left(-\dfrac{c_3}{x}\right) \end{cases} \end{cases} \to u = \frac{k}{x}$$

$$Tomamos\ k = 1$$

$$u = \frac{1}{x}$$

$$\underbrace{\left(u' + \frac{u}{x}\right)}_{u=\frac{1}{x}} v + uv' = \frac{e^{2x-1}}{x}$$

$$uv' = \frac{e^{2x-1}}{x}$$

$$v' = e^{2x-1}$$

$$v = \int e^{2x-1} dx$$

$$v = \frac{e^{2x-1}}{2} + C$$

Solución general:

$$y = uv = \frac{1}{x}\left(\frac{e^{2x-1}}{2} + C\right)$$

Verificación:

$$y' = -\frac{1}{x^2}\left(\frac{e^{2x-1}}{2} + C\right) + \frac{1}{x}e^{2x-1}$$

$$xy' + y = e^{2x-1}$$

$$x\left(-\frac{1}{x^2}\left(\frac{e^{2x-1}}{2} + C\right) + \frac{1}{x}e^{2x-1}\right) + \frac{1}{x}\left(\frac{e^{2x-1}}{2} + C\right) = e^{2x-1}$$

$$y(1) = 1$$

$$1 = \frac{1}{1}\left(\frac{e^{2-1}}{2} + C\right)$$

$$1 - \frac{e}{2} = C$$

Solución particular para $y(1) = 1$

$$y = \frac{1}{x}\left(\frac{e^{2x-1} - e}{2} + 1\right)$$