T P 04 Ej. 21-c

Obtener las ecuaciones del plano tangente y la recta normal a la gráfica de la función en el punto dado.

<u>SUGERENCIA:</u> Dada una función $z = f(x,y)$ definida en forma explícita, podemos redefinir a la misma como $F(x,y\,,z) = f(x,y) - z = 0$ (forma implícita) y luego proceder como en el ejercicio N°19 de este T.P.

Entonces:

<u>Ecuación del Plano Tangente:</u>

$\pi: \vec{\nabla}F(\vec{P}_0) \circ (\vec{P} - \vec{P}_0) = 0$

<u>Ecuación de la Recta Normal:</u>

$\mathbb{L}: \vec{\nabla}F(\vec{P}_0).t + \vec{P}_0$

$$\boldsymbol{z = z = f(x,y) = \frac{\cos(x) + e^{x.y}}{x^2 + y^2} \quad en \quad \vec{p}_0 = (\pi, 1)}$$

$$F(x,y,z) = f(x,y) - z = \frac{\cos(x) + e^{x.y}}{x^2 + y^2} - z = 0$$

$$\vec{P}_0 = (x_0\,, y_0\,, f(x_0, y_0)) = (\pi\,, 1\,, f(\pi\,,1)) = \left(\pi\,; 1\,; \frac{\cos(\pi) + e^{\pi.1}}{\pi^2 + 1^2}\right)$$

$$\boldsymbol{\vec{P}_0 = (\pi\,, 1\,, f(\pi\,, 1)) = \left(\pi\,; 1\,; \frac{e^{\pi} - 1}{\pi^2 + 1}\right)}$$

$$\vec{\nabla}F(x,y,z) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) \rightarrow$$

Por una cuestión de espacio tenemos:

$$\boldsymbol{\frac{\partial F}{\partial x}(x,y,z) = \frac{(-\sin x + y.e^{x.y}).(x^2 + y^2) - (\cos x + e^{x.y}).2x}{(x^2 + y^2)^2}}$$

$$\boldsymbol{\frac{\partial F}{\partial y}(x,y,z) = \frac{x.e^{x.y}.(x^2 + y^2) - (\cos x + e^{x.y}).2y}{(x^2 + y^2)^2}}$$

$$\frac{\partial F}{\partial z}(x,y,z) = -1$$

Evaluando en $\vec{P}_0$:

$$\frac{\partial F}{\partial x}\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right) = \frac{e^{\pi}.(\pi-1)^2+2\pi}{(\pi^2+1)^2} = \alpha$$

$$\frac{\partial F}{\partial y}\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right) = \frac{e^{\pi}.(\pi^3+\pi-2)+2}{(\pi^2+1)^2} = \beta$$

$$\frac{\partial F}{\partial z}\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right) = -1$$

$$\vec{\nabla}F\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right) = (\alpha\,;\ \beta\,;\ -1)$$

Ecuación del Plano Tangente:

$$\pi: \vec{\nabla}F(\vec{P}_0)\circ(\vec{P}-\vec{P}_0) = 0 \rightarrow$$

$$\pi: \vec{\nabla}F\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right)\circ\left[(x\,,y\,,z)-\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right)\right] = 0 \rightarrow$$

$$\pi: (\alpha\,;\ \beta\,;\ -1)\circ\left(x-\pi\,;y-1\,;z-\frac{e^{\pi}-1}{\pi^2+1}\right) = 0 \rightarrow$$

$$\pi: \alpha.(x-\pi)+\beta.(y-1)-1.\left(z-\frac{e^{\pi}-1}{\pi^2+1}\right) = 0 \rightarrow$$

$$\pi: \alpha.x-\pi.\alpha+\beta.y-\beta-z+\frac{e^{\pi}-1}{\pi^2+1} = 0 \rightarrow$$

$$\boldsymbol{\pi: \alpha.x+\beta.y-z = \pi.\alpha+\beta-\frac{e^{\pi}-1}{\pi^2+1}}$$

Ecuación de la Recta Normal:

$$\mathbb{L}: \vec{\nabla}F(\vec{P}_0).t+\ \vec{P}_0 \rightarrow$$

$$\mathbb{L}: \vec{\nabla}F\left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right).t+\ \left(\pi\,;1\,;\,\frac{e^{\pi}-1}{\pi^2+1}\right) \rightarrow$$

$$\boldsymbol{\mathbb{L}: (\alpha\,;\ \beta\,;\ -1).t+\left(\pi\,;1\,;\,\tfrac{e^{\pi}-1}{\pi^2+1}\right)}$$