Resolución TP10:

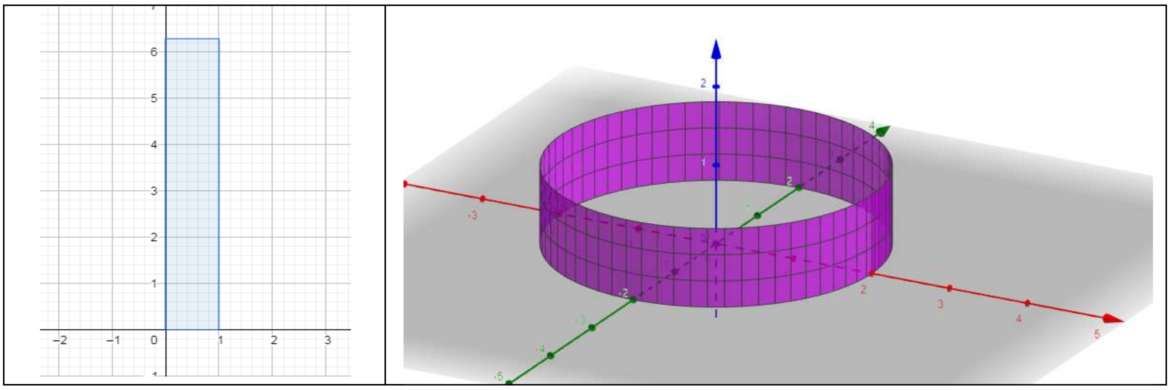
Ejercicio 4 - d

Calcular el área la superficie de la grafica del cilindro $x^2 + y^2 = 4$, limitado por $z = 0$ y $z = 1$

Resolviendo:

En el caso de coordenadas polares/cilíndricas:

$$S: \begin{cases} \Phi(\alpha, z) = (2cos(\alpha), 2sen(\alpha), z) \\ Dom\Phi = [0,2\pi]X[0,1] \end{cases} \rightarrow Area(S) = \iint_{[0,2\pi]X[0,1]} ||\Phi_\alpha X \Phi_z|| drd\alpha$$



$$\Phi(\alpha, z) = (2cos(\alpha), 2sen(\alpha), z)$$

$$\Phi_\alpha(\alpha, z) = (-2sen(\alpha), 2\cos(\alpha), 0)$$

$$\Phi_z(\alpha, z) = (0,0,1)$$

$$|\Phi_\alpha X \Phi_z| = \begin{bmatrix} i & j & k \\ -2\,\mathrm{sen}(\alpha) & 2\cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|\Phi_\alpha X \Phi_z| = \left(\begin{bmatrix} 2\cos(\alpha) & 0 \\ 0 & 1 \end{bmatrix}, -\begin{bmatrix} -2\,\mathrm{sen}(\alpha) & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -2\,\mathrm{sen}(\alpha) & 2\cos(\alpha) \\ 0 & 0 \end{bmatrix} \right)$$

$$|\Phi_\alpha X \Phi_z| = (2\cos(\alpha), 2\,\mathrm{sen}(\alpha), 0)$$

$$||\Phi_\alpha X \Phi_z|| = \sqrt{(2\cos(\alpha))^2 + (2\,\mathrm{sen}(\alpha))^2 + 0^2} = 2$$

$$Area(S) = \iint_{[0,2\pi]X[0,1]} ||\Phi_\alpha X \Phi_z|| drd\alpha = 2\int_0^{2\pi} d\alpha \int_0^1 dz = 4\pi$$

Verificación:

$$Area(Cilindro) = long * Perimetro(Circulo\, radio\ 2) = (1-0)(2\pi(2)) = 4\pi$$