# Resolución TP10:

## Ejercicio 6 - b

Dado el campo vectorial F y la superficie S, calcular el flujo saliente.

$$F(x, y, z) = (x, y, z)$$

S: $x^2 + y^2 + z^2 = 1$

Resolviendo:

$$I = \iint_S F \cdot dS = \iint_{R_\Phi} F(\Phi) \cdot (\Phi_u X \Phi_v) du dv$$

Considerando que se trata de resolver con cartesianas, la superficie se dividirá en 2:

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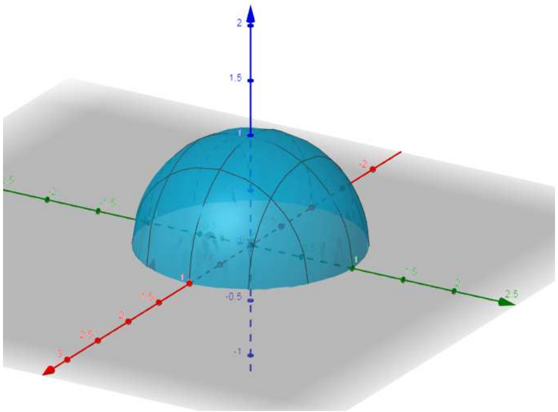

$$S_1: \begin{cases} \Phi(x, y) = \left(x, y, \sqrt{1 - x^2 - y^2}\right) \\ R_\Phi: \{x^2 + y^2 \leq 1 \end{cases}$$

$$\Phi_x = (1,0, -\frac{x}{\sqrt{1 - x^2 - y^2}})$$

$$\Phi_y = (0,1, -\frac{y}{\sqrt{1 - x^2 - y^2}})$$

$$\Phi_x X \Phi_y = \begin{bmatrix} i & j & k \\ 1 & 0 & -\frac{x}{\sqrt{1 - x^2 - y^2}} \\ 0 & 1 & -\frac{y}{\sqrt{1 - x^2 - y^2}} \end{bmatrix}$$

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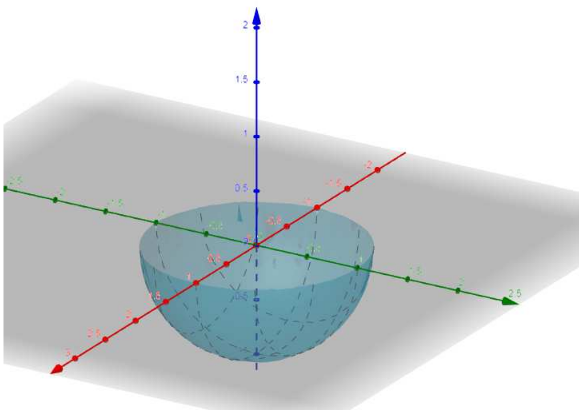

$$S_2: \begin{cases} \Phi(x, y) = \left(x, y, -\sqrt{1 - x^2 - y^2}\right) \\ R_\Phi: \{x^2 + y^2 \leq 1 \end{cases}$$

$$\Phi_x = (1,0, \frac{x}{\sqrt{1 - x^2 - y^2}})$$

$$\Phi_y = (0,1, \frac{y}{\sqrt{1 - x^2 - y^2}})$$

$$\Phi_x X \Phi_y = \begin{bmatrix} i & j & k \\ 1 & 0 & \frac{x}{\sqrt{1 - x^2 - y^2}} \\ 0 & 1 & \frac{y}{\sqrt{1 - x^2 - y^2}} \end{bmatrix}$$

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$$i = \begin{vmatrix} 0 & -\frac{x}{\sqrt{1-x^2-y^2}} \\ 1 & -\frac{y}{\sqrt{1-x^2-y^2}} \end{vmatrix}$$

$$i = \frac{x}{\sqrt{1-x^2-y^2}}$$

$$j = -\begin{vmatrix} 1 & -\frac{x}{\sqrt{1-x^2-y^2}} \\ 0 & -\frac{y}{\sqrt{1-x^2-y^2}} \end{vmatrix}$$

$$j = \frac{y}{\sqrt{1-x^2-y^2}}$$

$$k = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\Phi_x X \Phi_y = \left(\frac{x}{\sqrt{1-x^2-y^2}}, \frac{y}{\sqrt{1-x^2-y^2}}, 1\right)$$

$$F(\Phi) = \Phi(x, y)$$

$$F(\Phi) = \left(x, y, \sqrt{1-x^2-y^2}\right)$$

$$F(\Phi) \cdot \left(\Phi_x X \Phi_y\right) =$$

$$\frac{x^2}{\sqrt{1-x^2-y^2}} + \frac{y^2}{\sqrt{1-x^2-y^2}} + \sqrt{1-x^2-y^2} =$$

$$si\ \sqrt{1-x^2-y^2}\,\frac{\sqrt{1-x^2-y^2}}{\sqrt{1-x^2-y^2}} = \frac{1-x^2-y^2}{\sqrt{1-x^2-y^2}}$$

$$\frac{x^2}{\sqrt{1-x^2-y^2}} + \frac{y^2}{\sqrt{1-x^2-y^2}} + \frac{1-x^2-y^2}{\sqrt{1-x^2-y^2}} =$$

$$\frac{x^2+y^2+1-x^2-y^2}{\sqrt{1-x^2-y^2}} =$$

$$F(\Phi) \cdot \left(\Phi_x X \Phi_y\right) = \frac{1}{\sqrt{1-x^2-y^2}}$$

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$$i = \begin{vmatrix} 0 & \frac{x}{\sqrt{1-x^2-y^2}} \\ 1 & \frac{y}{\sqrt{1-x^2-y^2}} \end{vmatrix}$$

$$i = -\frac{x}{\sqrt{1-x^2-y^2}}$$

$$j = -\begin{vmatrix} 1 & \frac{x}{\sqrt{1-x^2-y^2}} \\ 0 & \frac{y}{\sqrt{1-x^2-y^2}} \end{vmatrix}$$

$$j = -\frac{y}{\sqrt{1-x^2-y^2}}$$

$$k = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$\Phi_x X \Phi_y = \left(-\frac{x}{\sqrt{1-x^2-y^2}}, -\frac{y}{\sqrt{1-x^2-y^2}}, 1\right)$$

Normal contraria

$$F(\Phi) = \Phi(x, y)$$

$$F(\Phi) = \left(x, y, -\sqrt{1-x^2-y^2}\right)$$

$$\frac{x^2}{\sqrt{1-x^2-y^2}} + \frac{y^2}{\sqrt{1-x^2-y^2}} + (-1)(-1\sqrt{1-x^2-y^2}) =$$

$$\frac{x^2}{\sqrt{1-x^2-y^2}} + \frac{y^2}{\sqrt{1-x^2-y^2}} + \sqrt{1-x^2-y^2} =$$

$$\frac{x^2+y^2+1-x^2-y^2}{\sqrt{1-x^2-y^2}} =$$

$$F(\Phi) \cdot \left(\Phi_x X \Phi_y\right) = \frac{1}{\sqrt{1-x^2-y^2}}$$

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$$I = \iint_S F \cdot dS = \iint_{S_1} F \cdot dS_1 + \iint_{S_2} F \cdot dS_2$$

$$I = \iint_{x^2+y^2 \le 1} \frac{1}{\sqrt{1-x^2-y^2}}\,dxdy + \iint_{x^2+y^2 \le 1} \frac{1}{\sqrt{1-x^2-y^2}}\,dxdy$$

$$I = 2 \iint_{x^2+y^2 \le 1} \frac{1}{\sqrt{1-x^2-y^2}}\,dxdy$$

$$I \overset{\substack{Trans\\Polares}}{\cong} 2 \iint\limits_{\substack{0\le r\le 1\\0\le\alpha\le 2\pi}} \frac{r}{\sqrt{1-r^2}}\,drd\alpha$$

$$I = 2\int_0^1\int_0^{2\pi}\frac{r}{\sqrt{1-r^2}}\,drd\alpha$$

$$I = 2\int_0^1\frac{r}{\sqrt{1-r^2}}\,dr\int_0^{2\pi}d\alpha$$

$$I = 4\pi\int_0^1\frac{r}{\sqrt{1-r^2}}\,dr$$

sustitucion $t = 1-r^2 \to dt = -2rdr \to \; rdr = \frac{dt}{-2}$

$$I = 4\pi\left(-\frac{1}{2}\right)\int_{r=0}^{r=1}\frac{1}{\sqrt{t}}\,dt$$

$$I = 4\pi\left(-\frac{1}{2}\right)\left[2\sqrt{t}\right]_{r=0}^{r=1}$$

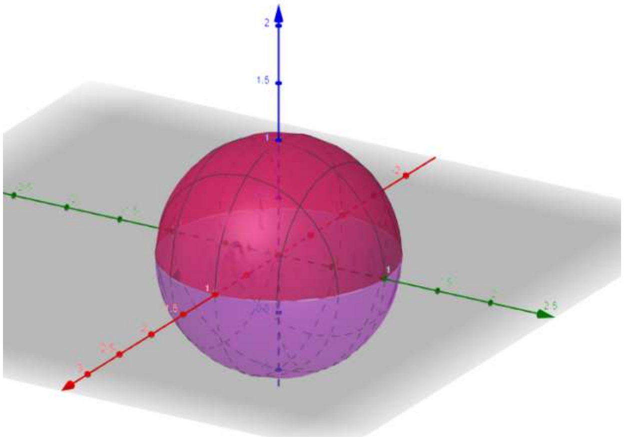
$$I = 4\pi\left(-\frac{1}{2}\right)\left[2\sqrt{1-r^2}\right]_{r=0}^{r=1}$$

$$I = -4\pi\left[\sqrt{1-r^2}\right]_{r=0}^{r=1}$$

$$I = -4\pi\left(\sqrt{0}-\sqrt{1}\right)$$

$$I = 4\pi$$

Considerando que se trata de resolver con coordenadas esféricas de radio fijo:



$$S:\begin{cases}\Phi(\theta,\varphi) = (\cos(\theta)\,sen(\varphi), \operatorname{sen}(\theta)\,sen(\varphi), cos(\varphi)) \\ R_{\Phi}:\begin{cases}0 \leq \theta \leq 2\pi \\ 0 \leq \varphi \leq \pi\end{cases}\end{cases}$$

$$\Phi_{\theta} = (-\operatorname{sen}(\theta)\,sen(\varphi), \cos(\theta)\,sen(\varphi), 0)$$

$$\Phi_{\varphi} = (\cos(\theta)\,cos(\varphi), \operatorname{sen}(\theta)\,cos(\varphi), -sen(\varphi))$$

$$\Phi_{\theta} X \Phi_{\varphi} = \begin{bmatrix} i & j & k \\ -\operatorname{sen}(\theta)\,sen(\varphi) & \cos(\theta)\,sen(\varphi) & 0 \\ \cos(\theta)\,cos(\varphi) & \operatorname{sen}(\theta)\,cos(\varphi) & -sen(\varphi)\end{bmatrix}$$

$$\Phi_{\theta} X \Phi_{\varphi} = \left(-\cos(\theta)\,sen^2(\varphi), -\operatorname{sen}(\theta)\,sen^2(\varphi), -\operatorname{sen}(\varphi)\,cos(\varphi)\right)$$

$$¿\,Que\ direccion\ posee?$$

$$si:\begin{cases}\theta = 0 \\ \varphi = 0\end{cases} \rightarrow \Phi_{\theta} X \Phi_{\varphi} = (0,0,0)\,no\ sirve\ para\ determinar$$

Pero

$$\Phi_{\theta} X \Phi_{\varphi} = \operatorname{sen}(\varphi)\underbrace{\left(-\cos(\theta)\operatorname{sen}(\varphi), -\operatorname{sen}(\theta)\operatorname{sen}(\varphi), -cos(\varphi)\right)}_{v}$$

$$si:\begin{cases}\theta = 0 \\ \varphi = 0\end{cases} \rightarrow v = (0,0,-1)\ direccion\ entrante$$

$$N = -(\Phi_{\theta} X \Phi_{\varphi}) = \left(\cos(\theta)\,sen^2(\varphi), \operatorname{sen}(\theta)\,sen^2(\varphi), \operatorname{sen}(\varphi)\,cos(\varphi)\right)$$

N es la dirección buscada

$$F(\Phi) = \Phi(\theta, \varphi) = (\cos(\theta)\, sen(\varphi), \mathrm{sen}(\theta)\, sen(\varphi), cos(\varphi))$$

$$F(\Phi) \cdot N = \cos^2(\theta)\, sen^3(\varphi) + \mathrm{sen}^2(\theta)\, sen^3(\varphi) + \mathrm{sen}(\varphi) \cos^2(\varphi)$$

$$F(\Phi) \cdot N = sen^3(\varphi) + \mathrm{sen}(\varphi) \cos^2(\varphi)$$

$$F(\Phi) \cdot N = \mathrm{sen}(\varphi)\, sen^2(\varphi) + \mathrm{sen}(\varphi) \cos^2(\varphi)$$

$$F(\Phi) \cdot N = \mathrm{sen}(\varphi)$$

$$I = \int_0^{2\pi} \int_0^{\pi} \mathrm{sen}(\varphi)\, \mathrm{d}\varphi \mathrm{d}\theta$$

$$I = \int_0^{2\pi} [-\cos(\varphi)]_0^{\pi}\, \mathrm{d}\theta$$

$$I = \int_0^{2\pi} (-\cos(\pi)) - (-\cos(0))\, \mathrm{d}\theta$$

$$I = \int_0^{2\pi} (-(-1)) - (-1)\, \mathrm{d}\theta$$

$$I = \int_0^{2\pi} 2\, \mathrm{d}\theta$$

$$I = 4\pi$$

Corolario:

Si se dispone el trabajo por cartesianas y en algún momento se aplica transformaciones no se debe olvidar el jacobiano.

Si se dispone el trabajo con esféricas desde un inicio este no necesita la utilización del jacobino YA que no se estaría realizando un cambio de variable o transformación