Resolución TP5:

Ayuda en Ejercicio 5

Tomando $F(x,y,z,u) = 0$ Determinar si define $u = f(x,y,z)$ en $P = (x_0, y_0, z_0, u_0)$ y si es así determinar sus derivadas parciales

Herramientas:

- Se deben formular las 3 condiciones del teorema usando regla de la cadena.

Para empezar:

En este caso podemos considerar la siguiente función compuesta

$H(x,y,z) = F(x,y,z,u = f(x,y,z))$

Derivadas de H:

$H(x,y,z)$ se puede derivar en x, y, z.

$$H_x = \frac{\partial F}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial x}$$

Sabemos que $\frac{\partial x}{\partial x} = 1$, $\frac{\partial y}{\partial x} = 0$, $\frac{\partial y}{\partial z} = 0$ y $\frac{\partial u}{\partial x} = f_x$

$$H_x = F_x + F_u\, f_x$$

Si $F(P) = 0$ entonces $H(x_0, y_0, z_0) = 0$ entonces derivando lado a lado

$$H_x(x_0, y_0, z_0) = 0$$

$$F_x(P) + F_u(P)\, f_x(x_0, y_0, z_0) = 0$$

$$f_x(x_0, y_0, z_0) = -\frac{F_x(P)}{F_u(P)}$$

En Resumen:

$$H_x = \frac{\partial F}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial x} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial x} ==> f_x(x_0, y_0, z_0) = -\frac{F_x(P)}{F_u(P)}$$

$$H_y = \frac{\partial F}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial y} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial y} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial y} ==> f_y(x_0, y_0, z_0) = -\frac{F_y(P)}{F_u(P)}$$

$$H_z = \frac{\partial F}{\partial x}\frac{\partial x}{\partial z} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial z} + \frac{\partial F}{\partial z}\frac{\partial z}{\partial z} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial z} ==> f_z(x_0, y_0, z_0) = -\frac{F_z(P)}{F_u(P)}$$

Sacamos las siguientes condiciones, se cumple TFI en $F(x,y,z,u)=0$ para $u=f(x,y,z)$ Si:

- $P \epsilon F(x,y,z,u)=0$
- Las derivadas F_x F_y F_z y F_u son continuas en el entorno del punto.
- $F_u(P) \neq 0$

si se cumple TFI existe $u=f(x,y,z)$ en P y valen

$$f_x(x_0,y_0,z_0) = -\frac{F_{x(P)}}{F_u(P)}$$

$$f_y(x_0,y_0,z_0) = -\frac{F_{y(P)}}{F_u(P)}$$

$$f_z(x_0,y_0,z_0) = -\frac{F_z(P)}{F_u(P)}$$