T P 10Ej4

b)

$$y' + \big(tg(x)\big)y = cos(x)$$

$$y' + P(x)y = Q(x)$$

Planteamos el producto:

$$y = uv = u_{(x)}v_{(x)}$$

$$y' = u'v + uv'$$

$$u'v + uv' + \big(tg(x)\big)uv = cos(x)$$

$$u'v + \big(tg(x)\big)uv \ + uv' = cos(x)$$

$$(u' + \big(tg(x)\big)u)v \ + uv' = cos(x)$$

$$\begin{cases} u' + \big(tg(x)\big)u = 0 \\ \quad uv' = cos(x) \end{cases} \rightarrow y = uv$$

Teniendo en cuenta el procedimiento establecido anteriormente, procedemos de la misma manera, es decir, tenemos:

$$P_{(x)} = tg(x)$$

$$\mu_{(x)} = e^{-\int tg(x)dx}$$

Sabemos que:

$$\int tg(x)dx = -\ln|cos(x)| + k_1$$

$$\mu_{(x)} = e^{\ln|cos(x)|+k_1}$$

$$\mu_{(x)} = e^{\ln|cos(x)|}e^{k_1} \quad k_1\epsilon\mathbb{R}$$

$$e^{k_1} = k_2$$

$$\mu_{(x)} = e^{\ln|cos(x)|}k_2 \quad k_2\epsilon\mathbb{R}$$

$$\mu_{(x)} = k|cos(x)|$$

$$\mu_{(x)} = k|cos(x)| = \begin{cases} kcos(x) & si \ \cos x \geq 0 \\ -kcos(x) & si \ \cos x < 0 \end{cases}$$

$$\mu_{(x)} = k \cos x \, , ahora \; se \; toma \; k = 1$$

$$\mu_{(x)} = cos(x) \;\; \cos x \geq 0$$

$$\begin{cases} \mu_{(x)} = \cos(x) \\ uv' = cos(x) \end{cases} \rightarrow y = \cos(x) v$$

$$uv' = cos(x)$$

$$uv' = Q(x)$$

$$v' = \frac{Q(x)}{u_{(x)}}$$

$$v = \int \frac{Q(x)}{u_{(x)}} dx$$

$$v = \int \frac{cos(x)}{\cos(x)} dx = \int 1 dx = x + C$$

$$\begin{cases} \mu_{(x)} = \cos(x) \\ v_{(x)} = x + C \end{cases} \rightarrow y = \cos(x)(x + C)$$

La solución general del sistema : $y = \cos(x)(x + C)$ para cuando $\cos(x) \geq 0$

Verificación:

$$y' + (tg(x))y = cos(x)$$

$$y = \cos(x)(x + C)$$

$$y' = -sen(x)(x + C) + \cos(x)$$

$$y' + \big(tg(x)\big)y = -sen(x)(x + C) + \cos(x) + tg(x)\cos(x)\,(x + C)$$

$$tg(x) = \frac{sen(x)}{\cos(x)}$$

$$tg(x)\cos(x) = sen(x)$$

$$y' + \big(tg(x)\big)y = -sen(x)(x + C) + \cos(x) + sen(x)(x + C)$$

$$y' + \big(tg(x)\big)y = \cos(x)$$

Como llegamos a la ecuación diferencial original, se verifica el resultado!

Una verificación más simple:

$$y' + \big(tg(x)\big)y = cos(x)$$

$$y = \cos(x)\,(x + C)$$

$$Si\ C = 0$$

$$y = \cos(x)\,x \to y' = -sen(x)x + \cos\,(x)$$

$$y' + \big(tg(x)\big)y = -sen(x)x + \cos(x) + tg(x)\cos(x)\,x = \cos\,(x)$$

Hallar la solución particular para $y(0) = 1$

$$1 = \cos(0)\,(0 + C)$$

$$1 = C$$

La solución particular para $y(0) = 1$ es :

$$y = \cos(x)\,(x + 1)$$

Hallar la solución particular para $y\left(\frac{\pi}{2}\right)=1$

$$y=\cos(x)\,(x+C) \text{ para cuando } \cos(x)\geq\ 0$$

$$\cos\left(\frac{\pi}{2}\right)\geq\ 0\rightarrow 0\geq 0$$

$$1=\cos\left(\frac{\pi}{2}\right)\left(\frac{\pi}{2}+C\right)$$

$$1=0\rightarrow no\ existe\ una\ solucion\ particular\ y\left(\frac{\pi}{2}\right)=1$$

Hallar la solución particular para $y(\pi)=1$

Tenemos que buscar la solución general acorde a $x=\pi$

$$\mu_{(x)}=k|cos(x)|=\begin{cases} kcos(x) & si\ \cos x\geq 0 \\ -kcos(x) & si\ \cos x<0 \end{cases}$$

$$\mu_{(x)}=-k\cos x\,, ahora\ se\ toma\ k=1$$

$$\mu_{(x)}=-cos(x)$$

$$v=\int-\frac{cos(x)}{\cos\,(x)}dx=\int-1dx=-x+C$$

$$\begin{cases}\mu_{(x)}=-\cos\,(x) \\ v_{(x)}=-x+C\end{cases}\rightarrow y=-\cos(x)\,(-x+C)\ para\cos(x)<0$$

$$y(\pi)=1$$

$$1=-\cos(\pi)\,(-\pi+C)$$

$$1=1(-\pi+C)$$

$$C=1+\pi$$

La solución particular para $y(\pi)=1$ es:

$$y=-\cos(x)\,(-x+1+\pi)$$