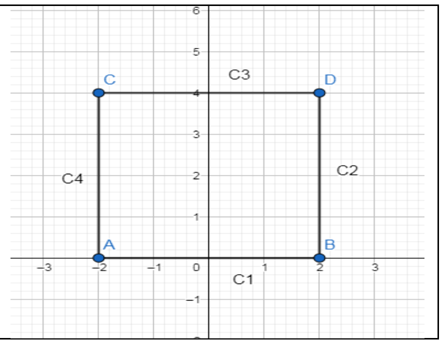
T P 08 Ej. 9-c-Modificado

Calcular la integral de línea para el campo y el camino dado.

$$\int_C \left(\frac{x}{y-1}\right) dx + \left(\frac{y}{x-1}\right) dy$$

C es el cuadrado de vértices $\begin{cases} A = (-2,0) \\ B = (2,0) \\ C = (-2,4) \\ D = (2,4) \end{cases}$



En este ejercicio se puede otro tipo de notación, Traducible como:

$$\int_C P\,dx + Q dy \;\rightarrow F(x,y) = (P,Q) \rightarrow \int_a^b F(\bar{r}(t)) \cdot \bar{r}'(t)\,dt$$

Donde a y b son los límites de variación de la variable t.

Para este ejercicio vamos a conservar la siguiente notación:

$$\int_C P\,dx + Q dy = \int_a^b P(\bar{r}(t))x'(t) + Q(\bar{r}(t))y'(t)$$

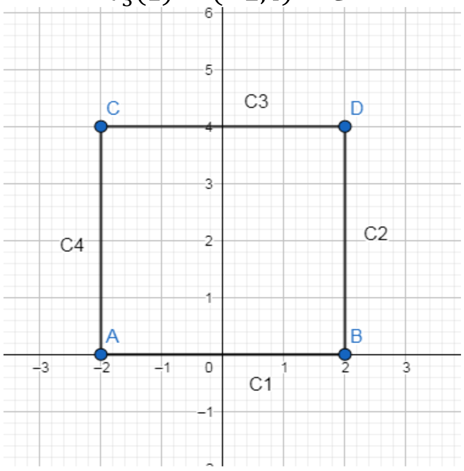
Parametrizando:

$$C3:\begin{cases} r_3(t) = D + t(C - D) \\ r_3(t) = (-4t + 2,4) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$r_3(0) = (2,4) = D$

$r_3(1) = (-2,4) = C$



$$C4:\begin{cases} r_4(t) = C + t(A - C) \\ r_4(t) = (-2, -4t + 4) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$r_4(0) = (-2,4) = C$

$r_4(1) = (-2,0) = A$

$$C2:\begin{cases} r_2(t) = B + t(D - B) \\ r_2(t) = (2,4t) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$r_2(0) = (2,0) = B$

$r_2(1) = (2,4) = D$

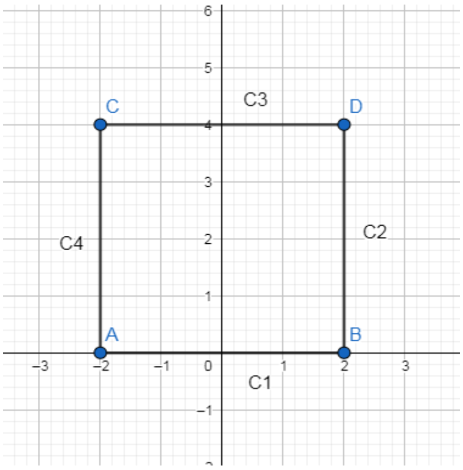
$$C1:\begin{cases} r_1(t) = A + t(B - A) \\ r_1(t) = (4t - 2,0) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$r_1(0) = (-2,0) = A$

$r_1(1) = (2,0) = B$

Construimos entonces cada una de las expresiones que precisamos para la integral.



$$C2:\begin{cases} r_2(t) = (2{,}4t) \\ x(t) = 2 \rightarrow x'(t) = 0dt = 0 \\ y(t) = 4t \rightarrow y'(t) = 4dt \\ P(x,y) = \dfrac{x}{y-1} \rightarrow P(r(t)) = \dfrac{2}{4t-1} \\ Q(x,y) = \dfrac{y}{x-1} \rightarrow Q(r(t)) = 4t \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$C1:\begin{cases} r_1(t) = (4t-2{,}0) \\ x(t) = 4t-2 \rightarrow x'(t) = 4dt \\ y(t) = 0 \rightarrow y'(t) = 0dt = 0 \\ P(x,y) = \dfrac{x}{y-1} \rightarrow P(r(t)) = -4t+2 \\ Q(x,y) = \dfrac{y}{x-1} \rightarrow Q(r(t)) = 0 \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$I_1 = \int_0^1 [(-4t+2)4dt + (0)(0)]$$

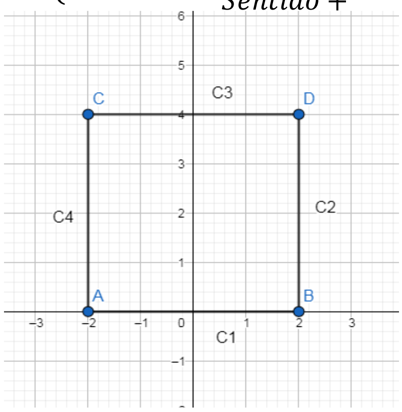
$$I_2 = \int_0^1 \left[\left(\frac{2}{4t-1}\right)(0) + (4t)4dt\right]$$

$$I_3 = \int_0^1 \left[\left(\frac{-4t+2}{3}\right)(-4)dt + \left(\frac{4}{-4t+1}\right)(0)\right]$$

$$I_4 = \int_0^1 \left[\left(\frac{-2}{-4t+3}\right)(0) + \left(\frac{-4t+4}{-3}\right)(-4)dt\right]$$

$$C3:\begin{cases} r_3(t) = (-4t+2{,}4) \\ x(t) = -4t+2 \rightarrow x'(t) = -4dt \\ y(t) = 4 \rightarrow y'(t) = 0 \\ P(x,y) = \dfrac{x}{y-1} \rightarrow P(r(t)) = \dfrac{-4t+2}{3} \\ Q(x,y) = \dfrac{y}{x-1} \rightarrow Q(r(t)) = \dfrac{4}{-4t+1} \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$C4:\begin{cases} r_4(t) = (-2,-4t+4) \\ x(t) = -2 \rightarrow x'(t) = 0 \\ y(t) = -4t+4 \rightarrow y'(t) = -4 \\ P(x,y) = \dfrac{x}{y-1} \rightarrow P(r(t)) = \dfrac{-2}{-4t+3} \\ Q(x,y) = \dfrac{y}{x-1} \rightarrow Q(r(t)) = \dfrac{-4t+4}{-3} \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$



$$I_1 = \int_0^1 [(-4t+2)4dt + (0)(0)] = 8\int_0^1 [-2t+1]dt = 8[-t^2+t]_0^1 = 0$$

$$I_2 = \int_0^1 \left[\left(\frac{2}{4t-1}\right)(0) + (4t)4dt\right] = 16\int_0^1 [t]dt = 16\left[\frac{t^2}{2}\right]_0^1 = 8$$

$$I_3 = \int_0^1 \left[\left(\frac{-4t+2}{3}\right)(-4)dt + \left(\frac{4}{-4t+1}\right)(0)\right] = -\frac{8}{3}\int_0^1 [-2t+1]dt = -\frac{8}{3}[-t^2+t]_0^1 = 0$$

$$I_4 = \int_0^1 \left[\left(\frac{-2}{-4t+3}\right)(0) + \left(\frac{-4t+4}{-3}\right)(-4)dt\right] = \frac{16}{3}\int_0^1 [-t+1]dt = \frac{16}{3}\left[-\frac{t^2}{2}+t\right]_0^1 = \frac{8}{3}$$

Por lo tanto:

$$\int_C \left(\frac{x}{y-1}\right) dx + \left(\frac{y}{x-1}\right) dy = 0 + 8 + 0 + \frac{8}{3} = \frac{32}{3}$$