Resolución TP7:

Ejercicio 26- i

Calcular el volumen dentro de $2z = x^2 + y^2 + z^2$ con $z \geq \sqrt{x^2 + y^2}$

C/A

$$2z = x^2 + y^2 + z^2$$

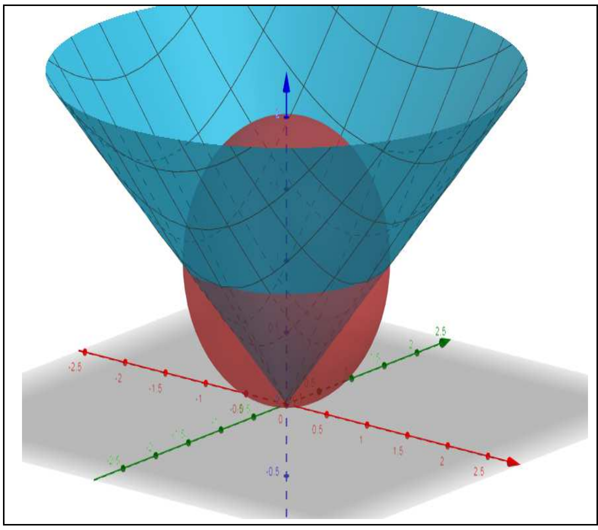
$$x^2 + y^2 + z^2 - 2z = 0$$

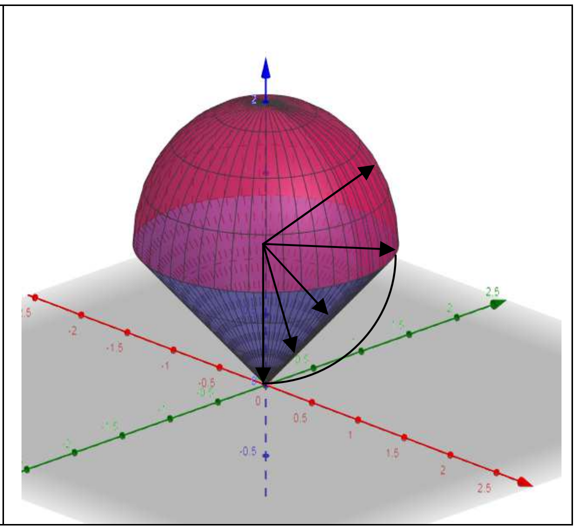
$$x^2 + y^2 + z^2 + 2(z)(-1) + 0 = 0$$

$$x^2 + y^2 + z^2 - 2z + 1 - 1 = 0$$

$$x^2 + y^2 + (z-1)^2 - 1 = 0$$

$$V: \begin{cases} x^2 + y^2 + (z-1)^2 \leq 1 \\ z \geq \sqrt{x^2 + y^2} \end{cases}$$





Calculamos las intersecciones para saber el límite del cono

$$\begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z = \sqrt{x^2 + y^2} \end{cases} \rightarrow \begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z^2 = x^2 + y^2 \\ z \geq 0 \end{cases} \rightarrow z^2 + (z-1)^2 = 1$$

$$z^2 + z^2 - 2z + 1 = 1$$

$$2z^2 - 2z = 0$$

$$2z(z-1) = 0$$

$$z = 0 \lor z = \mathbf{1}$$

Como se puede ver en la imagen el cono llega hasta z=1 donde se genera la curva interseccion

$$\sqrt{x^2+y^2} \le 1 \rightarrow 0 \le x^2+y^2 \le 1$$

Calculamos los limites para z

$$x^2+y^2+(z-1)^2 \le 1$$

$$(z-1)^2 \le 1-(x^2+y^2)$$

$$|z-1| \le \sqrt{1-(x^2+y^2)}$$

$$-\sqrt{1-(x^2+y^2)} \le z-1 \le \sqrt{1-(x^2+y^2)}$$

$$1-\sqrt{1-(x^2+y^2)} \le z \le 1+\sqrt{1-(x^2+y^2)}$$

$$si\ z \le 1 \rightarrow z \ge \sqrt{x^2+y^2}$$

$$si\ z \ge 1 \rightarrow z \le 1+\sqrt{1-(x^2+y^2)}$$

Por lo tanto

$$\sqrt{x^2+y^2} \le z \le 1+\sqrt{1-(x^2+y^2)}$$

$$V:\begin{cases} x^2+y^2+z^2 \le 2z \\ z \ge \sqrt{x^2+y^2} \end{cases} \rightarrow \begin{cases} \sqrt{x^2+y^2} \le z \le 1+\sqrt{1-(x^2+y^2)} \\ 0 \le x^2+y^2 \le 1 \end{cases}$$

Aplicando transformaciones cilíndricas

$$V:\begin{cases} x = r\cos\theta \\ y = r sen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases} r \le z \le 1+\sqrt{1-r^2} \\ 0 \le r \le 1 \\ 0 \le \theta \le 2\pi \end{cases} \end{cases}$$

$$V = \iiint_V 1dxdydz = \int_0^{2\pi}\int_0^1\int_r^{1+\sqrt{1-r^2}} 1\cdot r\ dzdrd\theta$$

$$V = \int_0^{2\pi}\int_0^1 r\left(1+\sqrt{1-r^2}-r\right)drd\theta$$

$$V = \int_0^{2\pi}\int_0^1 \left(r+r\sqrt{1-r^2}-r^2\right)drd\theta = \int_0^{2\pi} d\theta \int_0^1 \left(r+r\sqrt{1-r^2}-r^2\right)dr$$

$$c/a\int r\sqrt{1-r^2}dr \overset{\substack{1-r^2=t\\ -2rdr=dt\\ rdr=-\frac{dt}{2}}}{\triangleq} -\frac{1}{2}\int \sqrt{t}dt = -\frac{1}{3}\sqrt{t^3} \overset{1-r^2=t}{\triangleq} -\frac{1}{3}\sqrt{(1-r^2)^3}$$

$$V = \int_0^{2\pi} d\theta \left[\frac{r^2}{2} - \frac{1}{3}\sqrt{(1-r^2)^3} - \frac{r^3}{3}\right]_0^1$$

$$V = 2\pi\left[\left(\frac{1}{2} - \frac{1}{3}(0) - \frac{1}{3}\right) - \left(0 - \frac{1}{3} - 0\right)\right]$$

$$V = 2\pi\left[\frac{1}{2}\right] = \pi$$