# DIAGONALIZACIÓN DE MATRICES SIMÉTRICAS ($A = A^T$)

$A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$ $\lambda_1 = \lambda_2 = 1$, $m\,a(1) = 2$ $\lambda_3 = 4$, $m\,a(4) = 1$ autovalores

$\lambda_1 = \lambda_2 = 1$ $E_1 = gen\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right\}$ $mg(1) = 2$

$\lambda_3 = 4$ $E_4 = gen\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ $mg(4) = 1$ $B_{E_4} = \left\{ \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \right\}$

$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}^{-1}$ ; A es diagonalizable.

OBSERVACIONES:

$< \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} > = 0$ $< \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} > = 0$ $< \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} > \neq 0$

$v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ ; $v_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} - \frac{< \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} >}{< \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} >} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{pmatrix}$

$B_{E_1} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} ; \begin{pmatrix} -\frac{1}{2} \\ 1 \\ -\frac{1}{2} \end{pmatrix} \right\}$ ; $B_{ON} = \left\{ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} ; \begin{pmatrix} -\frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \end{pmatrix} \right\}$

$\sqrt{\frac{1}{4} + 1 + \frac{1}{4}} = \sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$

$$A = \underbrace{\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}}_{Q \text{ es ortogonal}} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$Q^{-1} = Q^T$

Diagonalización ortogonal.

PROP. 1: Los autov. de una matriz real simétrica son reales.

PROP. 2: Los autovectores de " " " asociados a autovalores distintos son ORTOGONALES

PROP. 3: Las matrices reales simét. son diagonalizables siempre y admiten la diagonalización ortogonalmente.