Hallar la ecuación canónica correspondiente a la siguiente cónica.

$$2x^2 - 4xy - y^2 - 4x + 10y - 13 = 0$$

Hallamos la matriz S, sus autovalores y autovectores.

$S = \begin{pmatrix} 2 & -2 \\ -2 & -1 \end{pmatrix}$; $\det(S - \lambda I) = \begin{vmatrix} 2-\lambda & -2 \\ -2 & -1-\lambda \end{vmatrix} = 0$ $\lambda_1 = 3$; $\lambda_2 = -2$

$\lambda_1 = 3$

$\begin{pmatrix} -1 & -2 \\ -2 & -4 \end{pmatrix} \sim \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix} \to -a - 2b = 0$, $a = -2b$; $E_3 = \text{gen}\{(2,-1)\}$ $\|(2,-1)\| = \sqrt{5}$

$\lambda_2 = -2$

$\begin{pmatrix} 4 & -2 \\ -2 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 \\ -2 & 1 \end{pmatrix} \to -2a + b = 0$; $a = \frac{1}{2}b$; $E_{-2} = \text{gen}\{(1,2)\}$ $\|(1,2)\| = \sqrt{5}$

$C = \frac{1}{\sqrt{5}} \begin{pmatrix} +2 & 1 \\ -1 & 2 \end{pmatrix}$

C.A.

$(-4 \quad 10) \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$

$\frac{1}{\sqrt{5}} (-18 \quad 16) \begin{pmatrix} x' \\ y' \end{pmatrix}$

Ecuación transformada

$3x'^2 - 2y'^2 - \frac{18}{\sqrt{5}}x' + \frac{16}{\sqrt{5}}y' = 13$ en $Ox'y'$

$3\left(x'^2 - \frac{6}{\sqrt{5}}x'\right) - 2\left(y'^2 + \frac{8}{\sqrt{5}}y'\right) = 13$

$3\left(x'^2 - \frac{6x'}{\sqrt{5}} + \left(\frac{3}{\sqrt{5}}\right)^2\right) - 2\left(y'^2 - \frac{8}{\sqrt{5}}y' + \left(\frac{4}{\sqrt{5}}\right)^2\right) = 13 + 3 \cdot \frac{9}{5} - 2 \cdot \frac{16}{5}$

$3\left(x' - \frac{3}{\sqrt{5}}\right)^2 - 2\left(y' - \frac{4}{\sqrt{5}}\right)^2 = 12$

$$\frac{\left(x' - \frac{3}{\sqrt{5}}\right)^2}{4} - \frac{\left(y' - \frac{4}{\sqrt{5}}\right)^2}{6} = 1$$

Centro $C\left(\frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}\right)$ $a = 2$, $b = \sqrt{6}$

Eje focal o principal $\to$ paralelo a x'