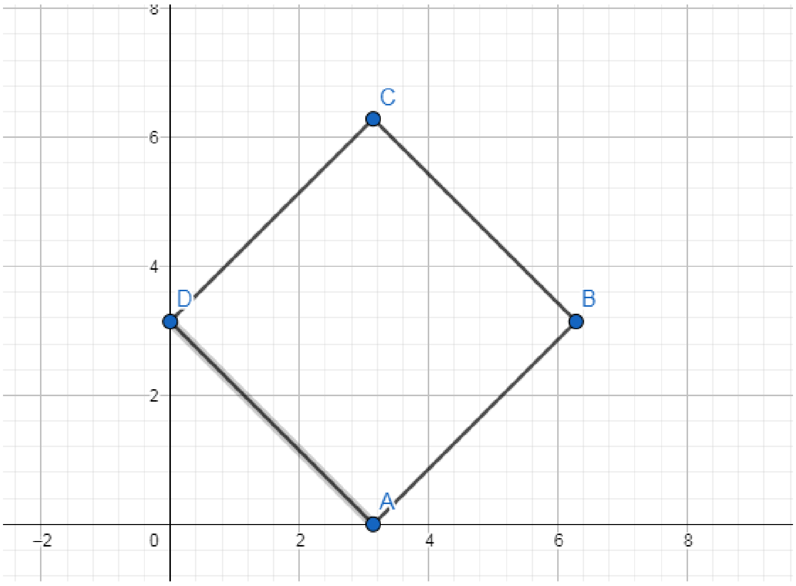
Resolución TP7:

Ejercicio 4 - d

Graficar la región de integración R y resolver la integral I.

$$R: \begin{cases} es\ el\ paralelogramo\ de\ vertices \\ A = (\pi, 0)\ B = (2\pi, \pi) \\ C = (\pi, 2\pi), D = (0, \pi) \end{cases}$$

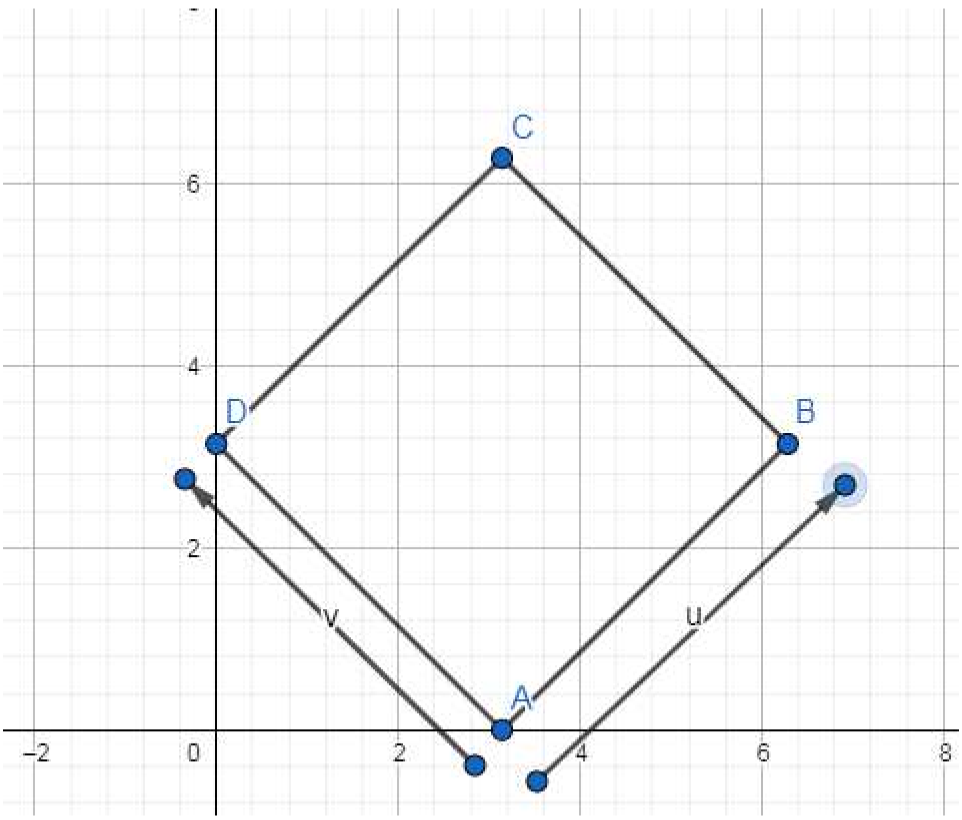
$$I = \iint_R (x-y)^2 sen^2(x+y) dxdy$$



$$R: \begin{cases} A = (\pi, 0) \\ B = (2\pi, \pi) \\ C = (\pi, 2\pi) \\ D = (0, \pi) \end{cases}$$

Aplicando Teorema de TLA, Método II (TLAII).

Buscamos dos direcciones que acompañen las rectas del grafico de R:



Estos vectores son la diferencia entre los puntos extremo e inicial:

$$\vec{u} = \overrightarrow{w_1} = \overrightarrow{B - A} = (\pi, \pi)$$

$$\vec{v} = \overrightarrow{w_2} = \overrightarrow{D - A} = (-\pi, \pi)$$

Entonces los podemos asociar a parametros u y v de manera parametrica, con origen en A:

$$(x, y) = T(u, v) = A + u\overrightarrow{w_1} + v\overrightarrow{w_2}$$

$$(x, y) = T(u, v) = (\pi\ ,0) + u(\pi, \pi) + v(-\pi, \pi)$$

$$(x, y) = T(u, v) = (\pi u - \pi v + \pi, \pi u + \pi v)$$

$$x = \pi u - \pi v + \pi$$

$$y = \pi u + \pi v$$

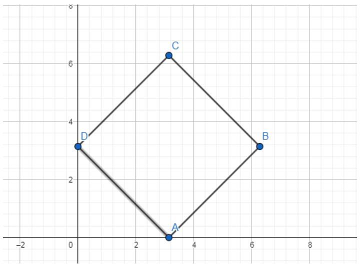
$$(u, v) = (0,0) \rightarrow T(0,0) = (\pi, 0) = A$$

$$(u, v) = (1,0) \rightarrow T(1,0) = (2\pi, \pi) = B$$

$$(u,v) = (0,1) \to T(0,1) = (-\pi + \pi, \pi) = (0,\pi) = D$$

$$(u,v) = (1,1) \to T(1,1) = (\pi - \pi + \pi, \pi + \pi) = (\pi, 2\pi) = C$$

$$0 \le u \le 1 \;\; 0 \le v \le 1$$



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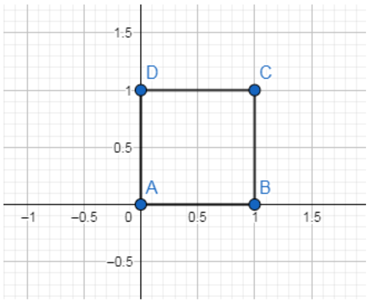

$$=> \; R': \begin{cases} 0 \le u \le 1 \\ 0 \le v \le 1 \end{cases}$$

Hallando el Jacobino:

Dado:

$$(x,y) = T(u,v) = (\pi u - \pi v + \pi, \pi u + \pi v)$$

Entonces:

$$|J| = \left\| \begin{matrix} x_u & x_v \\ y_u & y_v \end{matrix} \right\| = \left\| \begin{matrix} \pi & -\pi \\ \pi & \pi \end{matrix} \right\| = |(\pi)(\pi) - (-\pi)(\pi)|$$

$$|J| = |\pi^2 + \pi^2| = |2\pi^2| = 2\pi^2$$

Aplicando la transformacion al argumento de la integral $(x-y)sen^2(x+y)$

$$x - y = \pi u - \pi v + \pi - \pi u - \pi v = -2\pi v + \pi$$

$$x + y = \pi u - \pi v + \pi + \pi u + \pi v = 2\pi u + \pi$$

Armado de la integral:

$$I = \iint_R (x-y)sen(x+y)dxdy = \int_{v=0}^{v=1} \int_{u=0}^{u=1} (-2\pi v + \pi)sen^2(2\pi u + \pi)(2\pi^2)du\,dv$$

$$I = 2\pi^2 \int_{v=0}^{v=1} (-2\pi v + \pi)dv \int_{u=0}^{u=1} sen^2(2\pi u + \pi)du$$

De la manera rapida:

$$I = 2\pi^2 \underbrace{\underbrace{[-\pi v^2 + \pi v]_0^1}_{(-\pi+\pi)-(-0+0)}}_{0} \int_{u=0}^{u=1} sen^2(2\pi u + \pi)du = 0$$

De manera de resolver el seno

-----------C/A--

$$\int sen^2(2\pi u+\pi)du \overset{2\pi u+\pi=w}{\cong} \int \frac{sen^2(w)dw}{2\pi} = \frac{1}{2\pi}\int \frac{1-cos(2w)dw}{2} =$$

$$\int sen^2(2\pi u+\pi)du \overset{2\pi u+\pi=w}{\cong} \frac{1}{4\pi}\int 1-cos(2w)dw = \frac{1}{4\pi}\left(w+\frac{sen(2w)}{2}\right)$$

$$\int sen^2(2\pi u+\pi)du = \frac{1}{4\pi}\left(2\pi u+\pi+\frac{sen(2(2\pi u+\pi))}{2}\right)$$

$$\int sen^2(2\pi u+\pi)du = \frac{2\pi u}{4\pi}+\frac{\pi}{4\pi}+\frac{sen(2(2\pi u+\pi))}{8\pi}$$

$$\int sen^2(2\pi u+\pi)du = \frac{u}{2}+\frac{1}{4}+\frac{sen(4\pi u+2\pi)}{8\pi}$$

$$I = 2\pi^2\int_{v=0}^{v=1}(-2\pi v+\pi)dv\int_{u=0}^{u=1}sen^2(2\pi u+\pi)du$$

$$I = 2\pi^2[-\pi v^2+\pi v]_0^1\left[\frac{u}{2}+\frac{1}{4}+\frac{sen(4\pi u+2\pi)}{8\pi}\right]_0^1$$

$$I = 2\pi^2[(-\pi+\pi)-(-0+0)]\left[\left(\frac{1}{2}+\frac{1}{4}+\frac{sen(4\pi+2\pi)}{8\pi}\right)-\left(0+\frac{1}{4}+\frac{sen(2\pi)}{8\pi}\right)\right]$$

$$I = 2\pi^2[0-0]\left[\left(\frac{3}{4}+\frac{0}{8\pi}\right)-\left(\frac{1}{4}+\frac{0}{8\pi}\right)\right]$$

$$I = 2\pi^2[0-0]\left[\frac{3}{4}-\frac{1}{4}\right]$$

$$I = 0$$