EJERCICIOS RESUELTOS DE DETERMINANTES

Resuelto por la Profesora Julieta Matteucci

1) Sean $A = \begin{pmatrix} 2 & 2 & 3 \\ 3 & 5 & 5 \\ 6 & 7 & 9 \end{pmatrix}$ y $B \in \mathbb{R}^{3x3}$, $\det(B) = 2$, calcular:

a) $det\ (A - I)$, $det(A)$ usando la regla de Laplace

b) $det\ (A^T.B^3)$

c) Sabiendo que $det(B + I) = 3$, hallar $k \in \mathbb{R}$ para que el $det\ (kB^tB + kB^t) = 48$

Resolución:

a) Calculamos el $det\ (A - I)$, usando la regla de Laplace.

$$A - I = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix}$$

$$det(A - I) = det\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 6 & 7 & 8 \end{pmatrix} = 1.det\begin{pmatrix} 4 & 5 \\ 7 & 8 \end{pmatrix} - 2det\begin{pmatrix} 3 & 5 \\ 6 & 8 \end{pmatrix} + 3det\begin{pmatrix} 3 & 4 \\ 6 & 7 \end{pmatrix}$$

$$det(A - I) = 1.(4 * 8 - 7 * 5) - 2(3 * 8 - 6 * 5) + 3(3 * 7 - 6 * 4) = -3 + 12 - 9 = 0$$

$$\boxed{det(A - I) = 0}$$

Ahora calculamos el det (A)

$$\det(A) = det\begin{pmatrix} 2 & 2 & 3 \\ 3 & 5 & 5 \\ 6 & 7 & 9 \end{pmatrix} = 2.det\begin{pmatrix} 5 & 5 \\ 7 & 9 \end{pmatrix} - 2det\begin{pmatrix} 3 & 5 \\ 6 & 9 \end{pmatrix} + 3det\begin{pmatrix} 3 & 5 \\ 6 & 7 \end{pmatrix} = -1$$

$$\boxed{det(A) = -1}$$

b) $det\ (A^T.B^3)$

$$\det(A^T.B^3)$$

$$det\ (A^T).\det\ (B^3)$$

$$\det(A)\ .\ (\det(B))^3$$

$$-1.(2)^3$$

$$det\ (A^T.B^3) = -8$$

c) Sabiendo que $det(B+I) = 3$, hallar $k \in \mathbb{R}$ para que el $det\ (kB^tB + kB^t) = 48$

$$\det(kB^tB + kB^t) =$$

$$det\left(kB^t(B+I)\right) = k^3.\det(B^t).det(B+I) = k^3.2.3 = 48$$

$$k^3.6 = 48 \quad \rightarrow k^3 = 8 \quad \rightarrow \boxed{k=2}$$

2) Sean $A \in \mathbb{R}^{4x4}, A = \begin{pmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{pmatrix}$ y $\det(A) = 2$, $C = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 3 & 1 \\ 2 & -1 & -2 \end{pmatrix}$ calcular:

a) $\det(B)$, $\det\ (2B^tA^3)$ si $B = \begin{pmatrix} 2F_4 \\ F_2 + F_1 \\ -F_1 \\ F_3 \end{pmatrix}$

b) $det(C)$ usando la regla de Laplace.

Resolución:

a) $|B| = \begin{vmatrix} 2F_4 \\ F_2 + F_1 \\ -F_1 \\ F_3 \end{vmatrix} = -2 \begin{vmatrix} F_4 \\ F_2 + F_1 \\ F_1 \\ F_3 \end{vmatrix} = -2 \left(\begin{vmatrix} F_4 \\ F_2 \\ F_1 \\ F_3 \end{vmatrix} + \begin{vmatrix} F_4 \\ F_1 \\ F_1 \\ F_3 \end{vmatrix} \right) = -2 \left(\begin{vmatrix} F_4 \\ F_2 \\ F_1 \\ F_3 \end{vmatrix} + 0 \right) = -2 \begin{vmatrix} F_4 \\ F_2 \\ F_1 \\ F_3 \end{vmatrix} =$

$-2(-1) \begin{vmatrix} F_1 \\ F_2 \\ F_4 \\ F_3 \end{vmatrix} = 2(-1) \begin{vmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{vmatrix} = -2\det(A) \rightarrow \boxed{\det(B) = -4}$

$$\det(2B^tA^3) = 2^4 \det(B^t)\det(A^3) = 16.\det(B).(\det\ (A))^3 = 16.(-4).8 = -512$$

b) $C = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 3 & 1 \\ 2 & -1 & -2 \end{pmatrix} \rightarrow \det(C) = 1 \begin{vmatrix} 3 & 1 \\ -1 & -2 \end{vmatrix} - 1(-1) \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} + 2 \begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix}$

$$\det(C) = 1(-6+1) + 1(-2-2) + 2(-1-6) = -23$$