Resolución TP7:

Ejercicio 23-c-modificado

Resolver la integral triple I con él recinto V.

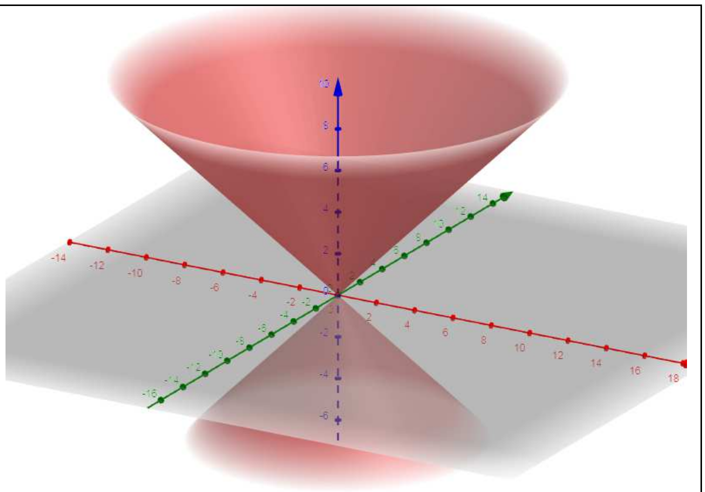
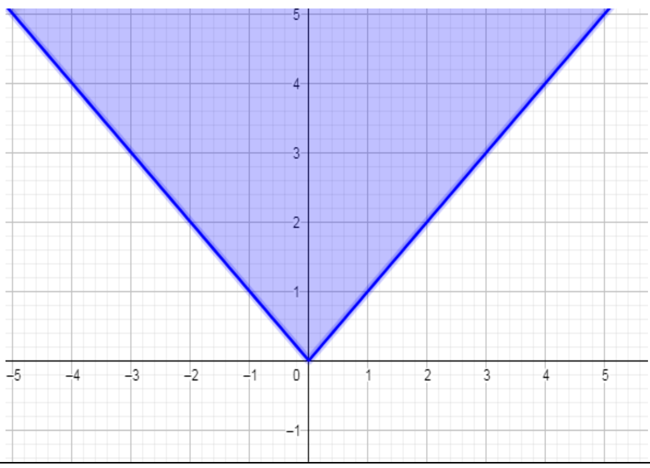
$V\colon \{(x, y, z) \epsilon R^3 / \ x^2 + y^2 \leq z^2 \ \wedge 0 \leq z \leq 4\}$

$$I = \iiint_V 1 + (x^2 + y^2)^2 dxdydz$$

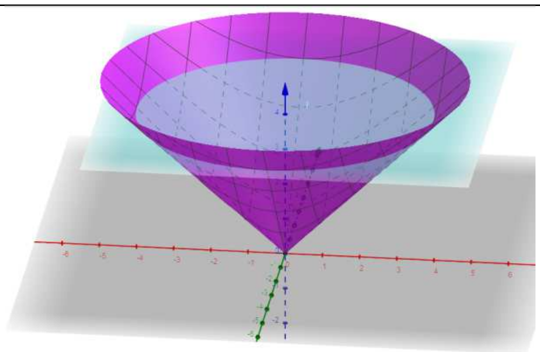
$$x^2 + y^2 \leq z^2 \ \wedge 0 \leq z \leq 4$$

$$x^2 + y^2 \leq z^2$$

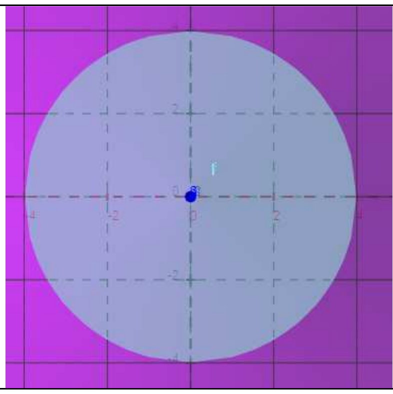
$$\sqrt{x^2 + y^2} \leq |z|$$

Dos Conos $\sqrt{x^2 + y^2} \leq \lvert z\rvert$ $z > 0$ $\sqrt{x^2 + y^2} \leq z$	
$x = 0$ $\sqrt{0^2 + y^2} \leq z$ $\sqrt{y^2} \leq z$ $\lvert y\rvert \leq z$	

Si $z > 0$	Si $z < 0$
$\sqrt{x^2+y^2} \le z$ Junto $0 \le z \le 4$ Por transitividad $0 \le \sqrt{x^2+y^2} \le z \le 4$	$\sqrt{x^2+y^2} \le -z$ $z \le -\sqrt{x^2+y^2}$ No coincide en $0 \le z \le 4$

$\sqrt{x^2+y^2} \le z \le 4$	

Buscamos limites de x e y:

Por transitividad $0 \le \sqrt{x^2+y^2} \le z \le 4$ $\sqrt{x^2+y^2} \le 4$ $x^2+y^2 \le 16$ La proyeccion resulta ser la enunciada $x^2+y^2 \le 16$	

En resumen

$$V:\begin{cases}\sqrt{x^2+y^2} \le z \le 4 \\ x^2+y^2 \le 16\end{cases}$$

$$V:\begin{cases} x = r\cos\theta \\ y = r sen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases}\sqrt{r^2} \le z \le 4 \\ r^2 \le 16\end{cases}\end{cases}$$

$$V:\begin{cases} x = rcos\theta \\ y = rsen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases} r \leq z \leq 4 \\ 0 \leq \ r \leq 4 \\ 0 \leq \theta \leq 2\pi \end{cases} \end{cases}$$

$$I = \iiint_V 1 + (x^2 + y^2)^2 dxdydz = \iiint_{V\prime} (1 + r^4)r\ dzdrd\theta$$

$$I = \int_0^{2\pi} \int_0^4 \int_r^4 (r + r^5)\ dzdrd\theta$$

$$I = \int_0^{2\pi} \int_0^4 (r + r^5)(4 - r)drd\theta$$

$$I = \int_0^{2\pi} \int_0^4 (4r + 4r^5 - r^2 - r^6)drd\theta$$

$$I = \int_0^{2\pi} \left[2r^2 + \frac{2r^6}{3} - \frac{r^3}{3} - \frac{r^7}{7}\right]_0^4 d\theta$$

$$I = \int_0^{2\pi} \left[\left(2(4)^2 + \frac{2(4)^6}{3} - \frac{(4)^3}{3} - \frac{(4)^7}{7}\right) - (0)\right] d\theta$$

$$I = \int_0^{2\pi} \left[\frac{8416}{21}\right] d\theta$$

$$I = \frac{8416}{21} 2\pi$$

$$I = \frac{16832}{21}\pi$$