T P 04 Ej. 12-c

Determinar el vector gradiente de la siguiente función en los puntos indicados.

$$\boldsymbol{f(x,y) = \sin(\sin(x).\cos(y))} \qquad \boldsymbol{en} \quad \left(\frac{\pi}{4}, \frac{\pi}{6}\right)$$

Sabiendo que: $\vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

Derivando con respecto a "x"

$$\frac{\partial f}{\partial x}(x,y) = \cos[\sin(x).\cos(y)].[\cos(x)\cos(y) + \sin(x).0] \quad \rightarrow$$

$$\boldsymbol{\frac{\partial f}{\partial x}(x,y) = \cos(x).\cos(y).\cos[\sin(x).\cos(y)]}$$

Derivando con respecto a "y", tenemos:

$$\frac{\partial f}{\partial y}(x,y) = \cos[\sin(x).\cos(y)].[0.\cos(y) - \sin(x).\sin(y)] \quad \rightarrow$$

$$\boldsymbol{\frac{\partial f}{\partial y}(x,y) = -\sin(x).\sin(y).\cos[\sin(x).\cos(y)]}$$

Evaluando las derivadas parciales en $\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$, tenemos:

$$\frac{\partial f}{\partial x}\left(\frac{\pi}{4}, \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{4}\right).\cos\left(\frac{\pi}{6}\right).\cos\left[\sin\left(\frac{\pi}{4}\right).\cos\left(\frac{\pi}{6}\right)\right] \rightarrow$$

$$\frac{\partial f}{\partial x}\left(\frac{\pi}{4}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}\right).\cos\left(\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}\right) \rightarrow$$

$$\boldsymbol{\frac{\partial f}{\partial x}\left(\frac{\pi}{4}, \frac{\pi}{6}\right) = \left(\frac{\sqrt{6}}{4}\right).\cos\left(\frac{\sqrt{6}}{4}\right)}$$

$$\frac{\partial f}{\partial y}\left(\frac{\pi}{4}, \frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{4}\right).\sin\left(\frac{\pi}{6}\right)\cos\left[\sin\left(\frac{\pi}{4}\right).\cos\left(\frac{\pi}{6}\right)\right] \rightarrow$$

$$\frac{\partial f}{\partial y}\left(\frac{\pi}{4},\frac{\pi}{6}\right) = \left(-\frac{\sqrt{2}}{2}.\frac{1}{2}\right).\cos\left(\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}\right) \rightarrow$$

$$\boldsymbol{\frac{\partial f}{\partial y}\left(\frac{\pi}{4},\frac{\pi}{6}\right) = \left(-\frac{\sqrt{2}}{4}\right).\cos\left(\frac{\sqrt{6}}{4}\right)}$$

Finalmente:

$$\boldsymbol{\vec{\nabla} f\left(\frac{\pi}{4},\frac{\pi}{6}\right) = \left(\frac{\sqrt{6}}{4}.\cos\left(\frac{\sqrt{6}}{4}\right)\ ;\ -\frac{\sqrt{2}}{4}.\cos\left(\frac{\sqrt{6}}{4}\right)\right)}$$