Actividad integradora.

1) Sea $f: \mathbb{R}^3 \to \mathbb{R}^3$ tal que $Nu(f) = \{(x_1, x_2, x_3) / x_1 - x_2 - 2x_3 = 0\}$

a) $x_1 = x_2 + 2x_3$

$(x_2 + 2x_3, x_2, x_3) \in Nuf$, $\dim Nuf = 2$

$\dim Imf = \underset{\sim}{1}$ porque $2 + 1 = 3$

$\dim Nuf + \dim Imf = \dim \mathbb{R}^3$

b) $f(1,1,0) = (0,0,0)$

$f(2,0,1) = (0,0,0)$

TEO. FUNDAMENTAL

$f(0,1,0) = (1,2,2)$

$B = \{(1,1,0); (2,0,1); (0,1,0)\}$

base de $\mathbb{R}^3$, esp. vect. de salida

card. de $B = \underset{\sim}{\dim \mathbb{R}^3} = 3$

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & 2 \end{pmatrix} \therefore B \text{ es l.i.}$$

La t.l. no es única. Agregamos $f(0,1,0) = (1,2,2)$ y buscaremos la forma explícita de ésta t.l. en particular

$(x,y,z) = \alpha(1,1,0) + \beta(2,0,1) + \gamma(0,1,0)$

$$\begin{cases} \alpha + 2\beta = x & \longrightarrow \alpha = x - 2z \\ \alpha + \gamma = y & \longrightarrow \gamma = y - \alpha = y - x + 2z \\ \beta = z \end{cases}$$

$f(x,y,z) = f[(x-2z)(1,1,0) + z(2,0,1) + (-x+y+2z)(0,1,0)]$

$f(x,y,z) = f(x-2z)(1,1,0) + f\,z(2,0,1) + f(-x+y+2z)(0,1,0)$ 1º AX.

$f(x,y,z) = (x-2z) \cdot f(1,1,0) + z \cdot f(2,0,1) + (-x+y+2z) f(0,1,0)$ 2º AX.

$f(x,y,z) = (0,0,0) + (0,0,0) + (-x+y+2z)(1,2,2)$

$f(x,y,z) = (-x+y+2z, -2x+2y+4z, -2x+2y+4z)$ FORMA EXPLÍCITA.

c) Mf_{BB} : $f(1,1,0) = (0,0,0) \to [(0,0,0)]_B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$f(2,0,1) = (0,0,0)$

$f(0,1,0) = (1,2,2) \to [(1,2,2)]_B = \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$

$$Mf_{BB} = \begin{pmatrix} 0 & 0 & -3 \\ 0 & 0 & 2 \\ 0 & 0 & 5 \end{pmatrix}$$

$(1,2,2) = \alpha(1,1,0) + \beta(2,0,1) + \gamma(0,1,0)$

d) $f(v_1) = \lambda \cdot v_1$; $v_1 = ?$

$f(v_2) = \lambda\, v_2$; $v_2 = ?$

$f(1,1,0) = (0,0,0) = 0(1,1,0)$ ✓

$f(2,0,1) = (0,0,0) = 0(2,0,1)$ ✓

$v_1 = (1,1,0)$

$v_2 = (2,0,1)$

$u = (2,-1,-1) \to f(u) = (-5,-10,-10)$

$(-5,-10,-10) \overset{?}{=} \lambda(2,-1,-1)$ ¿existe λ? NO

$(-5,-10,-10) = (2\lambda, -\lambda, -\lambda)$

$-5 = 2\lambda$

$-\frac{5}{2} = \lambda$

$\therefore u$ no es respuesta

2) $B = \{v_1, v_2, v_3\}$

$B' = \{w_1, w_2, w_3\}$

$$Mf_{BB'} = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$

$f: V \to W$

BASE DEL NÚCLEO

$$\left(\begin{array}{ccc|c} -1 & -2 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \to \begin{matrix} a + \frac{2}{3}c = 0 \\ b + \frac{1}{3}c = 0 \end{matrix} \to \begin{pmatrix} -\frac{2}{3}c \\ -\frac{1}{3}c \\ c \end{pmatrix} = c \begin{pmatrix} -\frac{2}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix} ; \dim Nuf = 1$$

$g.l = 3 - 2 = 1$

$-\frac{2}{3}v_1 - \frac{1}{3}v_2 + v_3$; $Nuf = gen\{-\frac{2}{3}v_1 - \frac{1}{3}v_2 + v_3\}$; $B_{Nuf} = \{-\frac{2}{3}v_1 - \frac{1}{3}v_2 + v_3\}$

BASE DE IMAGEN

$\dim V = 3$

$\dim W = 3$

; $\dim Imf = 2$

$Imf = gen\{-w_1 + w_2, 2w_1 + w_2 + 3w_3\}$; $B_{Imf} = \{-w_1 + w_2, 2w_1 + w_2 + 3w_3\}$

¿$2w_2 + 2w_3 \in Im\,f$?

$2w_2 + 2w_3 = \alpha(-w_1 + w_2) + \beta(2w_1 + w_2 + 3w_3) = (-\alpha + 2\beta)w_1 + \underbrace{(\alpha+\beta)}w_2 + \underbrace{3\beta}w_3$

$$\begin{cases} 2 = \alpha + \beta \longrightarrow 2 - \frac{2}{3} = \alpha \; ; \; \frac{4}{3} = \alpha \\ 2 = 3\beta \longrightarrow \frac{2}{3} = \beta \end{cases}$$

Además $-\alpha + 2\beta = 0$ VERIFICO $-\frac{4}{3} + 2 \cdot \frac{2}{3} = 0$ ✓

Luego $2w_2 + 2w_3 \in Imf$

OTRO PROCEDIMIENTO

$$\begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

$Mf_{BB'} \cdot [v]_B \quad [fv]_{B'}$

¿Existe $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$?

$$\begin{cases} -a + 2b = 0 \\ a + b + c = 2 \\ 3b + c = 2 \end{cases}$$

$$\left(\begin{array}{ccc|c} -1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 1 & 1 & 1 & 2 \\ 0 & 3 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 3 & 1 & 2 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & 0 \\ 0 & 3 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$r(A) = r(A') = 2$

$g.l: 3 - 2 = 1$ $\underset{\sim}{SCI}$

Existe $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, luego $2w_2 + 2w_3 \in Imf$

c) $B'' = \{u_1, u_2, u_3\}$

$$\begin{array}{c|c} V & W \\ \hline B & B' \\ B'' & \end{array}$$

$C_{B''B}$

$$Mf_{B''B'} = Mf_{BB'} \cdot C_{B''B} = \begin{pmatrix} -1 & 2 & 0 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$

$Mf_{B''B'}$

$$C_{BB''} = \begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix} ; \left(C_{BB''}\right)^{-1} = \frac{1}{-5}\begin{pmatrix} -2 & & \\ & & \\ & & \end{pmatrix}^T = \begin{pmatrix} \frac{2}{5} & -\frac{1}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{3}{5} \\ \frac{3}{5} & \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$

$\det(C_{BB''}) = 1 \cdot (-2) + 1(-3) = -5$