En el espacio euclídeo $\left(R^{2\times 2}, \langle A;B\rangle\right)$, se define

$S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}^{2\times 2} \;/\; a = b = c + d \right\}$ $\qquad \begin{pmatrix} a & a \\ a-d & d \end{pmatrix}$ $\qquad c+d = a$, $c = a-d$

$\langle A;B\rangle = Tr(A \cdot B^T)$

a) hallar una base ortonormal de S

$B_S = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \right\}$; $\dim S = 2$

$\left\langle \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} ; \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \right\rangle = -1 \neq 0$, B_S no es una base ortogonal. Hay que ORTOGONALIZAR. (G-S)

$B' = \{ w_1 ; w_2 \}$ $\qquad w_1 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

$$w_2 = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} - \frac{\left\langle \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\rangle} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} - \left(\frac{-1}{3}\right) \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$w_2 = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix}$$

$B' = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} ; \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix} \right\}$ base ortogonal

$$\left\| \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\| = \sqrt{\left\langle \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} ; \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\rangle} = \sqrt{3}$$

$$\left\| \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix} \right\| = \sqrt{\left\langle \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix} ; \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix} \right\rangle} = \frac{\sqrt{15}}{3} \;/\; \sqrt{\frac{5}{3}}$$

$$\sqrt{\frac{1}{9} + \frac{1}{9} + \frac{4}{9} + \frac{9}{9}} = \sqrt{\frac{15}{9}} = \frac{\sqrt{15}}{3} = \sqrt{\frac{5}{3}}$$

$\langle kv, w\rangle = k\langle v, w\rangle$

$$\sqrt{\left\langle \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} ; \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} \right\rangle} = \sqrt{\frac{1}{3} \cdot \frac{1}{3} \left\langle \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} ; \begin{pmatrix} 1 & 1 \\ -2 & 3 \end{pmatrix} \right\rangle} = \sqrt{\frac{1}{9} \cdot 15} = \frac{\sqrt{15}}{3}$$

$$B'' = \left\{ \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} ; \frac{3}{\sqrt{15}} \cdot \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & 1 \end{pmatrix} \right\} = \left\{ \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & 0 \end{pmatrix} ; \begin{pmatrix} \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} & \frac{3}{\sqrt{15}} \end{pmatrix} \right\}$$ base ortonormal.

$\frac{1}{3} : \frac{\sqrt{15}}{3} = \frac{1}{\sqrt{15}}$ $\qquad 1 : \frac{\sqrt{15}}{3} = \frac{3}{\sqrt{15}}$

$-\frac{2}{3} : \frac{\sqrt{15}}{3} = -\frac{2}{\sqrt{15}}$

$$\left\| \begin{pmatrix} \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} & \frac{3}{\sqrt{15}} \end{pmatrix} \right\| = \sqrt{\left\langle \begin{pmatrix} \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} & \frac{3}{\sqrt{15}} \end{pmatrix} ; \begin{pmatrix} \frac{1}{\sqrt{15}} & \frac{1}{\sqrt{15}} \\ -\frac{2}{\sqrt{15}} & \frac{3}{\sqrt{15}} \end{pmatrix} \right\rangle} = \sqrt{\frac{1}{15} + \frac{1}{15} + \frac{4}{15} + \frac{9}{15}} = \sqrt{\frac{15}{15}} = 1$$

c) hallar la proyección de $M = \begin{pmatrix} 2 & -1 \\ -2 & 1 \end{pmatrix}$ sobre el subespacio S..

d) hallar la distancia de M al subespacio S.

b) hallar el complemento ortogonal de S ($S^\perp$)

$$S^\perp = \left\{ \begin{pmatrix} x & y \\ z & t \end{pmatrix} \in \mathbb{R}^{2\times 2} \;/\; \left\langle \begin{pmatrix} x & y \\ z & t \end{pmatrix} ; \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \right\rangle = 0 \;\wedge\; \left\langle \begin{pmatrix} x & y \\ z & t \end{pmatrix} ; \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \right\rangle = 0 \right\}$$

$\begin{cases} x + y + z = 0 \\ -z + t = 0 \end{cases}$ $\quad x + y + z = 0 \rightarrow x = -y - z$; $\quad t = z$ $\qquad \begin{pmatrix} -y-z & y \\ z & z \end{pmatrix} \in S^\perp$

$$tr(A \cdot B^T) = tr\left[\begin{pmatrix} x & y \\ z & t \end{pmatrix} \cdot \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \right] = tr \begin{pmatrix} 0 & -x+y \\ 0 & -z+t \end{pmatrix} = 0 - z + t = \underline{-z+t}$$

$S^\perp = \left\{ \begin{pmatrix} x & y \\ z & t \end{pmatrix} \in \mathbb{R}^{2\times 2} \;/\; x = -y - z \;\wedge\; t = z \right\}$

$B_{S^\perp} = \left\{ \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} ; \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \right\}$; $\dim S^\perp = 2$