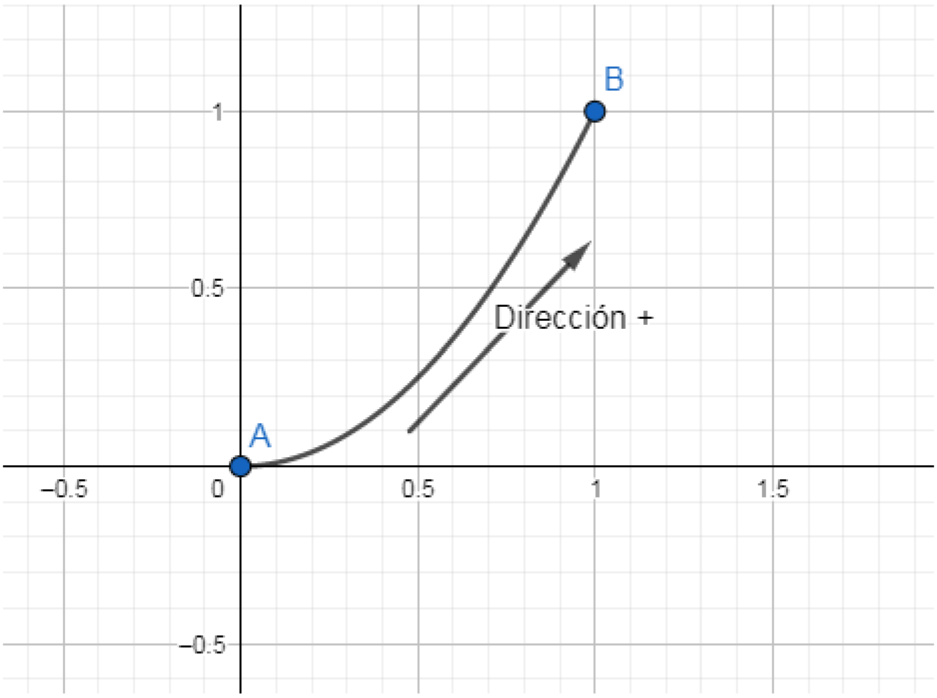
Resolución TP8:

Ejercicio 17-A-Modificado

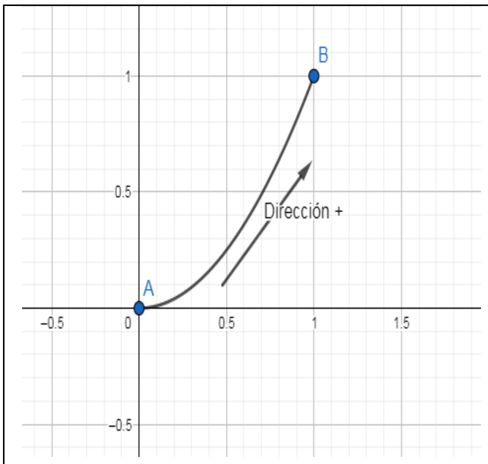
Sobre los campos conservativos (16-a, 16-d) verificar el resultado para la integral de línea para el recorrido:

$$\mathfrak{C}: \begin{cases} r(t) = (t, t^2) \\ 0 \leq t \leq 1 \end{cases}$$



Metodo I (Integración usual) -15-a

$$I=\int_{\mathfrak{C}} F d\mathfrak{C}=\int_{t_0}^{t_f} F\big(r(t)\big)\cdot r'(t)dt$$



$$\mathfrak{C}:\begin{cases} r(t)=(t,t^2) \\ 0\leq t\leq 1\end{cases}$$

$$F(x,y)=(2x+2y\,,2x+2y)$$
$$F\big(r(t)\big)=(2t+2t^2,2t+2t^2)$$

$$r'(t)=(1{,}2t)$$

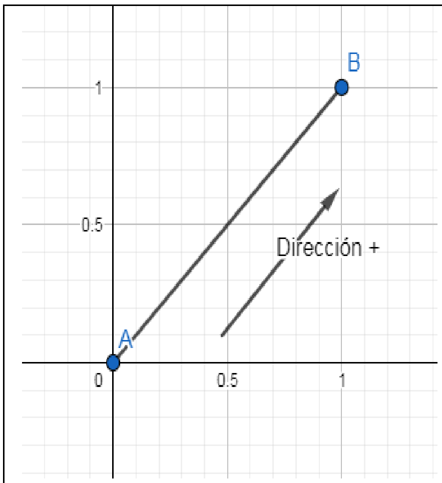
$$F\big(r(t)\big)\cdot r'(t)=2t+2t^2+4t^2+4t^3$$
$$F\big(r(t)\big)\cdot r'(t)=2t+6t^2+4t^3$$

$$I=\int_{\mathfrak{C}} F d\mathfrak{C}=\int_{t_0}^{t_f} F\big(r(t)\big)\cdot r'(t)dt=\int_0^1(2t+6t^2+4t^3)dt$$

$$I=\int_0^1(2t+6t^2+4t^3)dt=[t^2+2t^3+t^4]_0^1=4$$

Método II (Redefinir camino en un campo conservativo) -15-a

$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F(r(t)) \cdot r'(t) dt$$



$$\mathfrak{C}: \begin{cases} r(t) = (t, t) \\ 0 \leq t \leq 1 \end{cases}$$

$$F(x, y) = (2x + 2y\,, 2x + 2y)$$

$$F(r(t)) = (2t + 2t, 2t + 2t) = (4t, 4t)$$

$$r'(t) = (1,1)$$

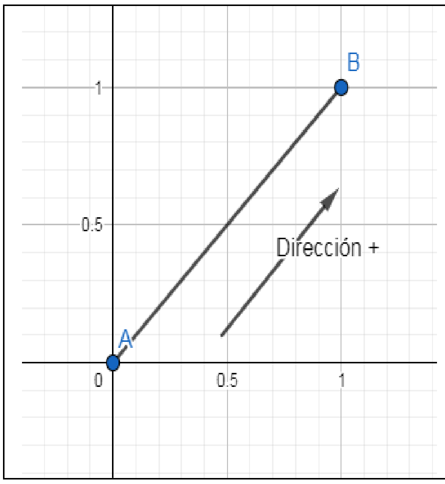
$$F(r(t)) \cdot r'(t) = 4t + 4t = 8t$$

$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F(r(t)) \cdot r'(t) dt = \int_0^1 (8t) dt$$

$$I = \int_0^1 (8t) dt = [4t^2]_0^1 = 4$$

Método III (Método de función potencial) -15-a

$$F(x,y) = (2x + 2y\,, 2x + 2y) \to f(x,y) = x^2 + 2xy + y^2 + k$$



$$f(B) = f(1{,}1) = 1 \cdot 1^2 + 2 \cdot 1 \cdot 1 + 1 \cdot 1^2$$
$$f(B) = 4$$

$$f(A) = f(0{,}0) = 1 \cdot 0^2 + 2 \cdot 0 \cdot 0 + 1 \cdot 0^2$$
$$f(A) = 0$$

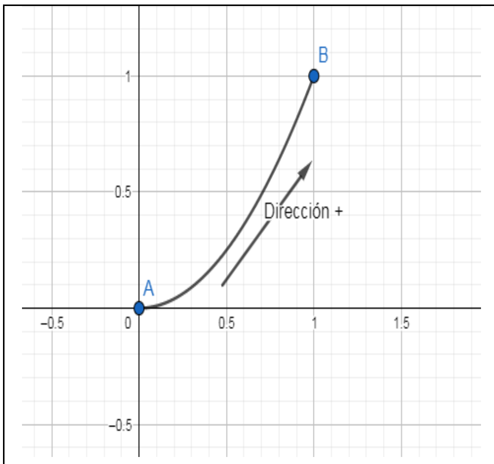
$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = f(B) - f(A)$$

$$I = 4 - 0$$

$$I = 4$$

Metodo I (Integración usual) -15-d

$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F\big(r(t)\big) \cdot r'(t) dt$$

	$\mathfrak{C}: \begin{cases} r(t) = (t, t^2) \\ 0 \leq t \leq 1 \end{cases}$ $F(x,y) = (y - \pi y sen(\pi xy)\ , x - \pi x sen(\pi xy))$ $F(r(t)) = (t^2 - \pi t^2 sen(\pi t^3)\ , t - \pi t sen(\pi t^3))$ $r'(t) = (1{,}2t)$ $F\big(r(t)\big) \cdot r'(t) = t^2 - \pi t^2 sen(\pi t^3) + 2t^2 - 2\pi t^2 sen(\pi t^3)$ $F\big(r(t)\big) \cdot r'(t) = 3t^2 - 3\pi t^2 sen(\pi t^3)$

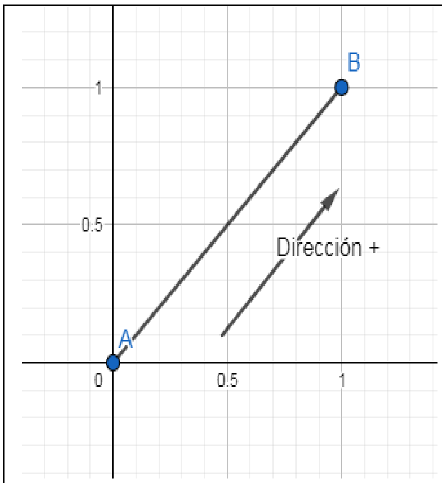
$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F\big(r(t)\big) \cdot r'(t) dt = \int_0^1 \big(3t^2 - 3\pi t^2 sen(\pi t^3)\big) dt$$

$$I = \int_0^1 \big(3t^2 - 3\pi t^2 sen(\pi t^3)\big) dt = [t^3 + cos(\pi t^3)]_0^1$$

$$I = [t^3 + cos(\pi t^3)]_0^1 = \left(1 + \underbrace{\cos(\pi)}_{-1}\right) - \left(0 + \underbrace{\cos(0)}_{1}\right) = -1$$

Método II (Redefinir camino en un campo conservativo) -15-d

$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F\big(r(t)\big) \cdot r'(t)dt$$



$$\mathfrak{C}: \begin{cases} r(t) = (t,t) \\ 0 \le t \le 1 \end{cases}$$

$$F(x,y) = (y - \pi y sen(\pi xy)\ , x - \pi x sen(\pi xy))$$
$$F(r(t)) = (t - \pi t sen(\pi t^2)\ , t - \pi t sen(\pi t^2))$$

$$r'(t) = (1{,}1)$$

$$F\big(r(t)\big) \cdot r'(t) = t - \pi t sen(\pi t^2) + t - \pi t sen(\pi t^2)$$
$$F\big(r(t)\big) \cdot r'(t) = 2t - 2\pi t sen(\pi t^2)$$

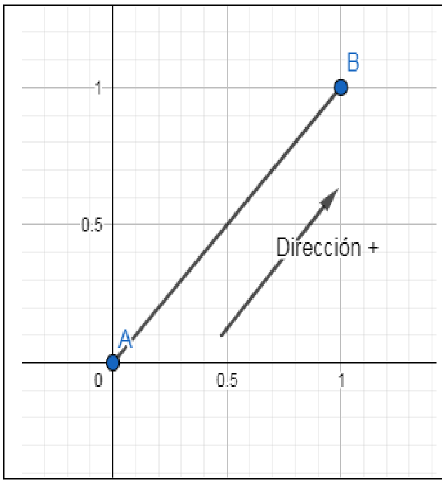
$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = \int_{t_0}^{t_f} F\big(r(t)\big) \cdot r'(t)dt = \int_0^1 \big(2t - 2\pi t sen(\pi t^2)\big)dt$$

$$I = \int_0^1 \big(2t - 2\pi t sen(\pi t^2)\big)dt = [t^2 + \cos(\pi t^2)]_0^1$$

$$I = [t^2 + \cos(\pi t^2)]_0^1 = \Bigg(1 + \underbrace{\cos(\pi)}_{-1}\Bigg) - \Bigg(0 + \underbrace{\cos(0)}_{1}\Bigg) = -1$$

Método III (Método de función potencial) -15-d

$$F(x,y) = (y - \pi y sen(\pi xy)\,, x - \pi x sen(\pi xy)) \rightarrow f(x,y) = xy + \cos(\pi xy) + k$$

	$f(B) = f(1{,}1) = 1 \cdot 1 + \cos\,(\pi \cdot 1 \cdot 1)$ $f(B) = 1 - 1 = 0$ $f(A) = f(0{,}0) = 0 \cdot 0 + \cos\,(\pi \cdot 0 \cdot 0)$ $f(A) = 0 + 1 = 1$

$$I = \int_{\mathfrak{C}} F d\mathfrak{C} = f(B) - f(A)$$

$$I = 0 - 1$$

$$I = -1$$