Resolución TP7:

Ejercicio 11 - c -Modificado

Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

$$R5: \{(x,y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge y \geq \sqrt{3}|x|\}$$

$$R6: \{(x,y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge y \geq \sqrt{3}x\}$$

$$R7: \{(x,y) \epsilon R^2 / \ 1 \ \leq (x-2)^2 + (y+3)^2 \leq 4\}$$

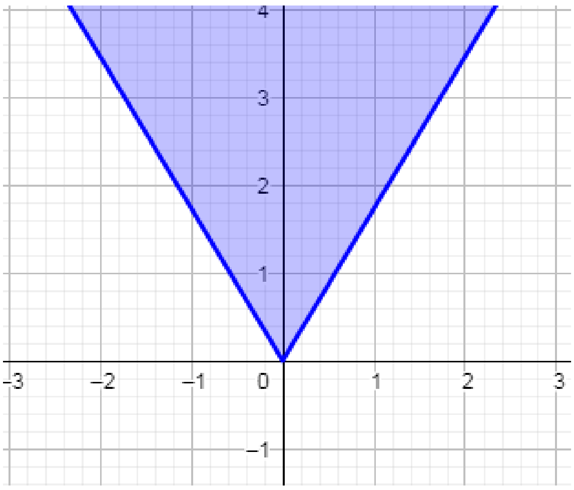
$$R8: \{(x,y) \epsilon R^2 / \ \frac{x^2}{4} + \frac{y^2}{16} \leq 1 \}$$

$$R9: \{(x,y) \epsilon R^2 / \ \frac{(x-1)^2}{9} + \frac{(y-5)^2}{4} \leq 1 \}$$

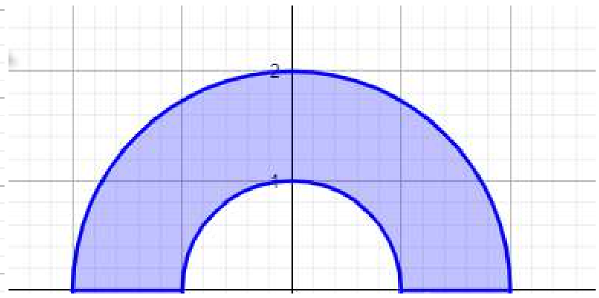
5- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + ydxdy$$

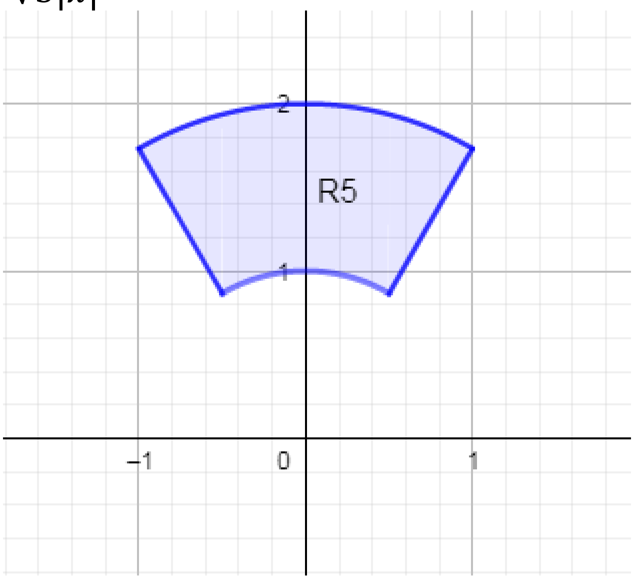
$$R5: \{(x,y)\epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge y \geq \sqrt{3}|x|\}$$

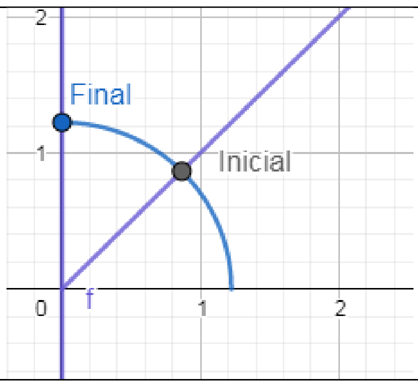


$$y \geq \sqrt{3}|x|$$



$$1 \ \leq x^2 + y^2 \leq 4$$
$$0 \ \leq y$$





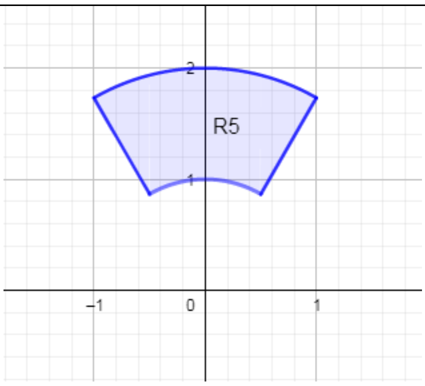
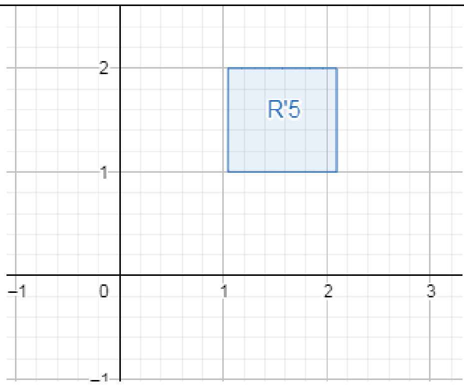
Para αFinal =>

$$\alpha\text{Final} = \pi - \alpha\text{Inicial}$$

Para αInicial=>

$$y = \sqrt{3}x$$

$$\tan(\alpha\text{Inicial}) = \frac{y}{x} = \sqrt{3} => \ \alpha\text{Inicial} = \frac{\pi}{3}$$

 $R5 = \left\{ \begin{matrix} (x,y)\epsilon R^2 / \\ 1 \ \leq x^2 + y^2 \leq 4 \\ \wedge \\ y \geq \sqrt{3}\lvert x \rvert \end{matrix} \right\}$	=>	 $R'5\colon \left\{ \begin{matrix} (r,\alpha)\epsilon R^2 / \\ 1 \ \leq r \leq 2 \\ \wedge \\ \frac{\pi}{3} \leq \alpha \leq \frac{2\pi}{3} \end{matrix} \right\}$

$$I = \iint\limits_R x + ydxdy = \int_1^2 \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (cos\alpha + sen\alpha)\, d\alpha dr$$

$$I = \int_1^2 r^2 \, [sen\alpha - cos\alpha]_{\frac{\pi}{3}}^{\frac{2\pi}{3}} dr$$

$$I = \int_1^2 r^2 \, [\left(sen\frac{2\pi}{3} - cos\frac{2\pi}{3}\right) - (sen\frac{\pi}{3} - cos\frac{\pi}{3})]dr$$

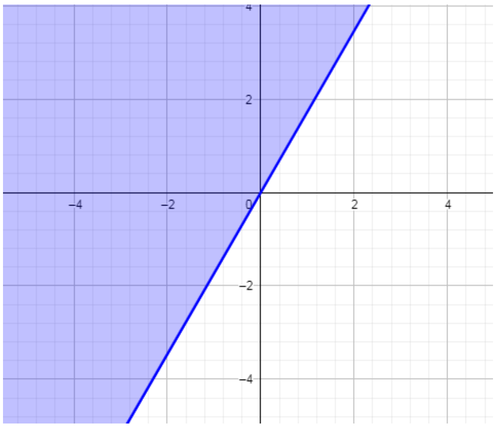
$$I = \int_1^2 r^2 \, [\left(\frac{\sqrt{3}}{2} - (-\frac{1}{2})\right) - (\frac{\sqrt{3}}{2} - \frac{1}{2})]dr$$

$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8-1}{3} = \frac{7}{3}$$

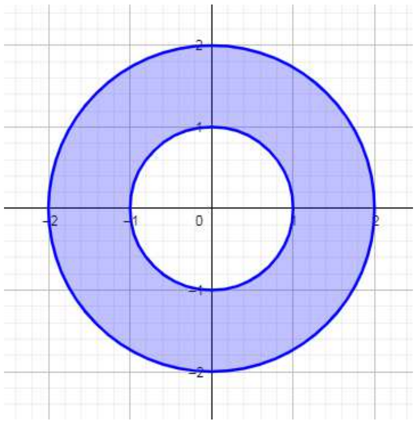
6- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

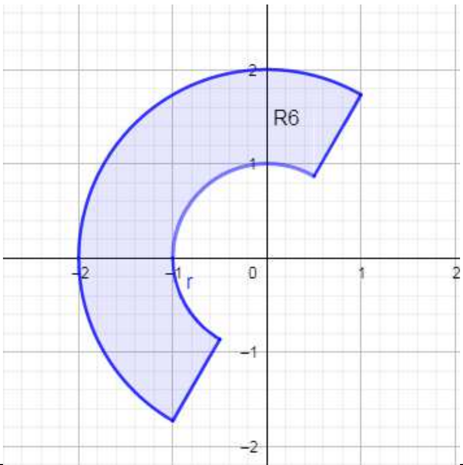
$$R6: \{(x, y)\epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge y \geq \sqrt{3}x\}$$

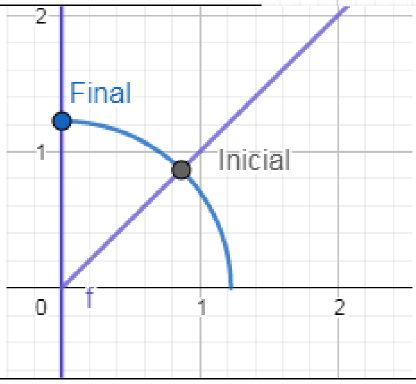


$y \geq \sqrt{3}x$



$1 \ \leq x^2 + y^2 \leq 4$





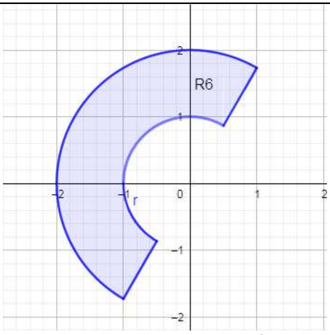
Para αFinal =>

$$\alpha\text{Final} = \pi + \alpha\text{Inicial}$$

Para αInicial=>

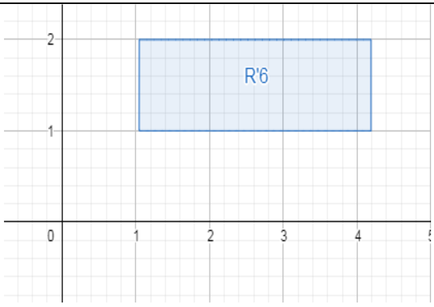
$$y = \sqrt{3}x$$

$$\tan(\alpha\text{Inicial}) = \frac{y}{x} = \sqrt{3} \Rightarrow \ \alpha\text{Inicial} = \frac{\pi}{3}$$



$$R6 = \left\{ \begin{matrix} (x, y)\epsilon R^2 / \\ 1 \; \leq x^2 + y^2 \leq 4 \\ \wedge \\ y \geq \sqrt{3}x \end{matrix} \right\}$$

=>



$$R'6\!: \left\{ \begin{matrix} (r, \alpha)\epsilon R^2 / \\ 1 \; \leq r \leq 2 \\ \wedge \\ \frac{1}{3}\pi \leq \alpha \leq \frac{4}{3}\pi \end{matrix} \right\}$$

$$I = \iint_R x + y dxdy = \int_1^2 \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} (cos\alpha + sen\alpha)\, d\alpha dr$$

$$I = \int_1^2 r^2 \, [sen\alpha - cos\alpha]_{\frac{1}{3}\pi}^{\frac{4}{3}\pi} dr$$

$$I = \int_1^2 r^2 \, [\left(sen\frac{4}{3}\pi - cos\frac{4}{3}\pi\right) - (sen\frac{1}{3}\pi - cos\frac{1}{3}\pi)]dr$$

$$I = \int_1^2 r^2 \, [\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}\right) - (\frac{\sqrt{3}}{2} - \frac{1}{2})]dr$$

$$I = \int_1^2 r^2 \, [\, 1 + \sqrt{3}]dr$$

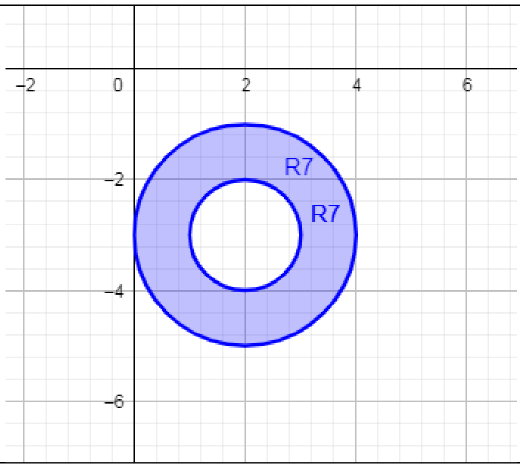
$$I = \; (1 + \sqrt{3}) \int_1^2 r^2 dr = (1 + \sqrt{3}) \left[\frac{r^3}{3}\right]_1^2 = (1 + \sqrt{3}) \frac{8 - 1}{3} = \frac{7(1 + \sqrt{3})}{3}$$

7- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

$$R7: \{(x,y)\epsilon R^2 / \ 1 \ \leq (x-2)^2 + (y+3)^2 \leq 4\}$$

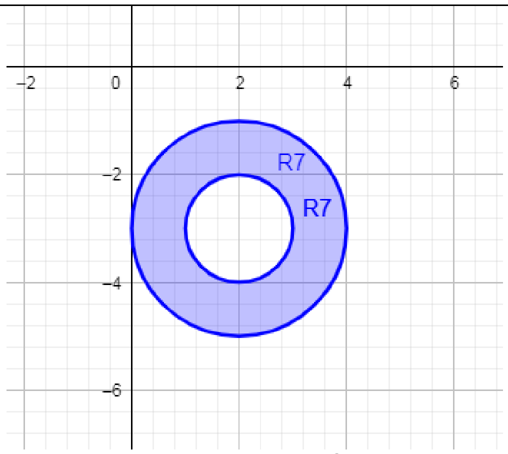
$$(x-2)^2 + (y+3)^2 = \cos^2\alpha + sen^2\alpha = 1$$
$$(x-2)^2 + (y+3)^2 = 4\cos^2\alpha + 4sen^2\alpha = 4$$



$$(x-2)^2 = r^2\cos^2\alpha$$
$$x - 2 = rcos\,\alpha$$
$$x = r\cos\alpha + 2$$

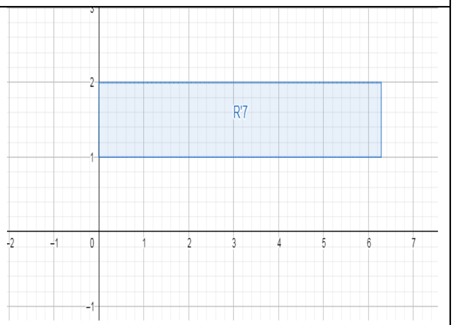
$$(y+3)^2 = r^2\operatorname{sen}^2\alpha$$
$$y + 3 = rsen\,\alpha$$
$$y = r\,sen\,\alpha - 3$$

$$|J| = r$$



$$R7 = \left\{ \begin{matrix} (x,y)\epsilon R^2 / \\ 1 \ \leq (x-2)^2 + (y+3)^2 \leq 4 \end{matrix} \right\}$$

=>



$$R'7: \left\{ \begin{matrix} (r,\alpha)\epsilon R^2 / \\ 1 \ \leq r \leq 2 \\ \wedge \\ 0 \leq \alpha \leq 2\pi \end{matrix} \right\}$$

$$I = \iint_R x + y dxdy = \int_1^2 \int_0^{2\pi} (rcos\alpha + 2 + rsen\alpha - 3) r \, d\alpha dr$$

$$I = \int_1^2 r^2 \int_0^{2\pi} (cos\alpha + sen\alpha)\, d\alpha dr - \int_1^2 r \int_0^{2\pi} d\alpha\, dr$$

$$I = \int_1^2 r^2\, [sen\alpha - cos\alpha]_0^{2\pi} dr - \int_1^2 r 2\pi dr$$

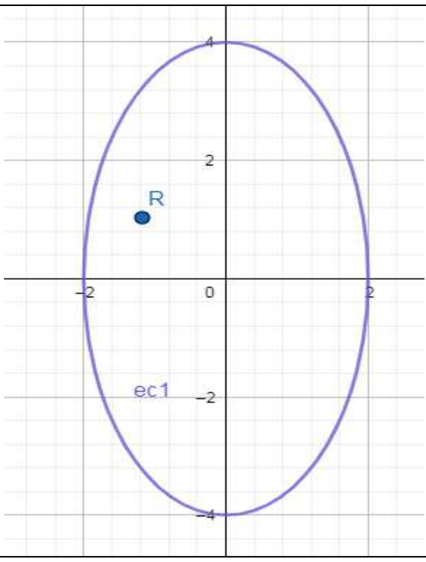
$$I = 0 - 2\pi \int_1^2 rdr$$

$$I = -\pi [r^2]_1^2 = -3\pi$$

8- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + ydxdy$$

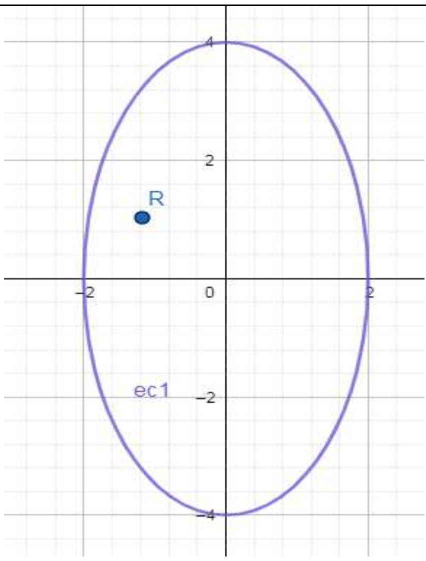
$$R8\colon \{(x,y)\epsilon R^2 / \ \frac{x^2}{4} + \frac{y^2}{16} \leq 1\ \}$$



$$\frac{x^2}{4} = r^2 \cos^2 \alpha$$
$$\frac{x}{2} = r cos\ \alpha$$
$$x = 2r\ cos\ \alpha$$

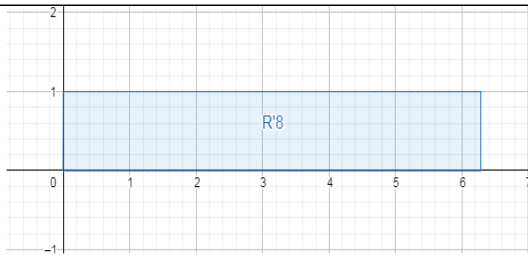
$$\frac{y^2}{16} = r^2 \operatorname{sen}^2 \alpha$$
$$\frac{y}{4} = r sen\ \alpha$$
$$y = 4r\ sen\ \alpha$$

$$|J| = abr = 8r$$



$$R8 = \left\{ \begin{matrix} (x,y)\epsilon R^2 / \\ \frac{x^2}{4} + \frac{y^2}{16} \leq 1 \end{matrix} \right\}$$

=>



$$R'8\colon \left\{ \begin{matrix} (r,\alpha)\epsilon R^2 / \\ 0 \leq r \leq 1 \\ \wedge \\ 0 \leq \alpha \leq 2\pi \end{matrix} \right\}$$

$$I = \iint_R x + y dx dy = \int_0^1 \int_0^{2\pi} (2r cos\alpha + 4r sen\alpha\ ) 8r\ d\alpha dr$$

$$I = \ 8 \int_0^1 r^2 \int_0^{2\pi} (2cos\alpha + 4sen\alpha)\ d\alpha dr$$

$$I = \ 8 \int_0^1 r^2\ [2sen\alpha - 4cos\alpha]_0^{2\pi} dr$$

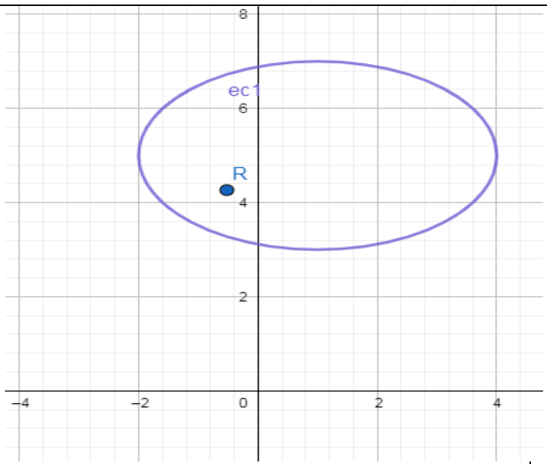
$$I = \ 8 * 0$$

$$I = 0$$

5- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + ydxdy$$

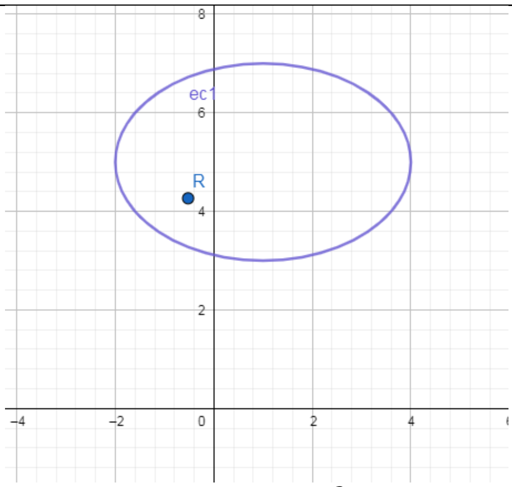
$$R9: \{(x,y)\epsilon R^2 / \ \frac{(x-1)^2}{9} + \frac{(y-5)^2}{4} \leq 1 \}$$



$$\frac{(x-1)^2}{9} = r^2 \cos^2 \alpha$$
$$\frac{x-1}{3} = rcos\ \alpha$$
$$x = 3r\,cos\ \alpha + 1$$

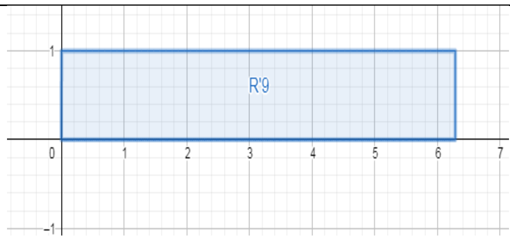
$$\frac{(y-5)^2}{4} = r^2 \operatorname{sen}^2 \alpha$$
$$\frac{y-5}{2} = rsen\ \alpha$$
$$y = 2r\,sen\ \alpha + 5$$

$$|J| = abr = 6r$$



$$R9 = \left\{ \begin{matrix} (x,y)\epsilon R^2 / \\ \frac{(x-1)^2}{9} + \frac{(y-5)^2}{4} \leq 1 \end{matrix} \right\}$$

=>



$$R'9: \left\{ \begin{matrix} (r,\alpha)\epsilon R^2 / \\ 0 \leq r \leq 1 \\ \wedge \\ 0 \leq \alpha \leq 2\pi \end{matrix} \right\}$$

$$I = \iint_R x + ydxdy = \int_0^1 \int_0^{2\pi} (3rcos\alpha + 1 + 2rsen\alpha + 5\ )6r\, d\alpha dr$$

$$I = \ 6\int_0^1 r^2 \int_0^{2\pi} (3cos\alpha + 2sen\alpha)\, d\alpha dr - 36\int_1^2 r \int_0^{2\pi} d\alpha\, dr$$

$$I = \ 0 - 36 * 2\pi * 7 = 504\pi$$