Resolución TP8:

Ejercicio 21 - a -Modificado

Aplicar la extensión del teorema de Green para el campo y el camino dado.

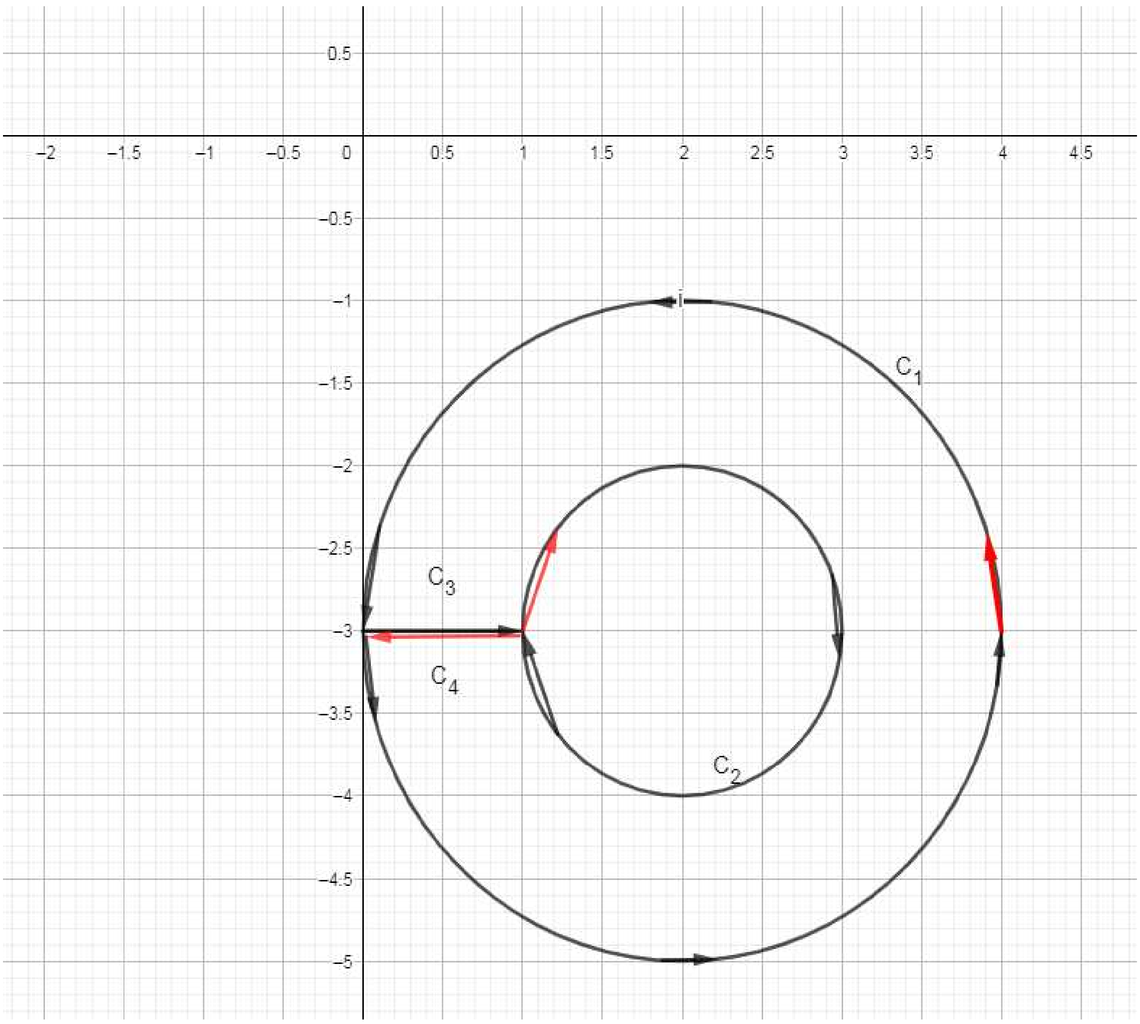
$$F(x, y) = (-xy, xy)$$

$$C\ es\ la\ curva\ definida\ por:$$

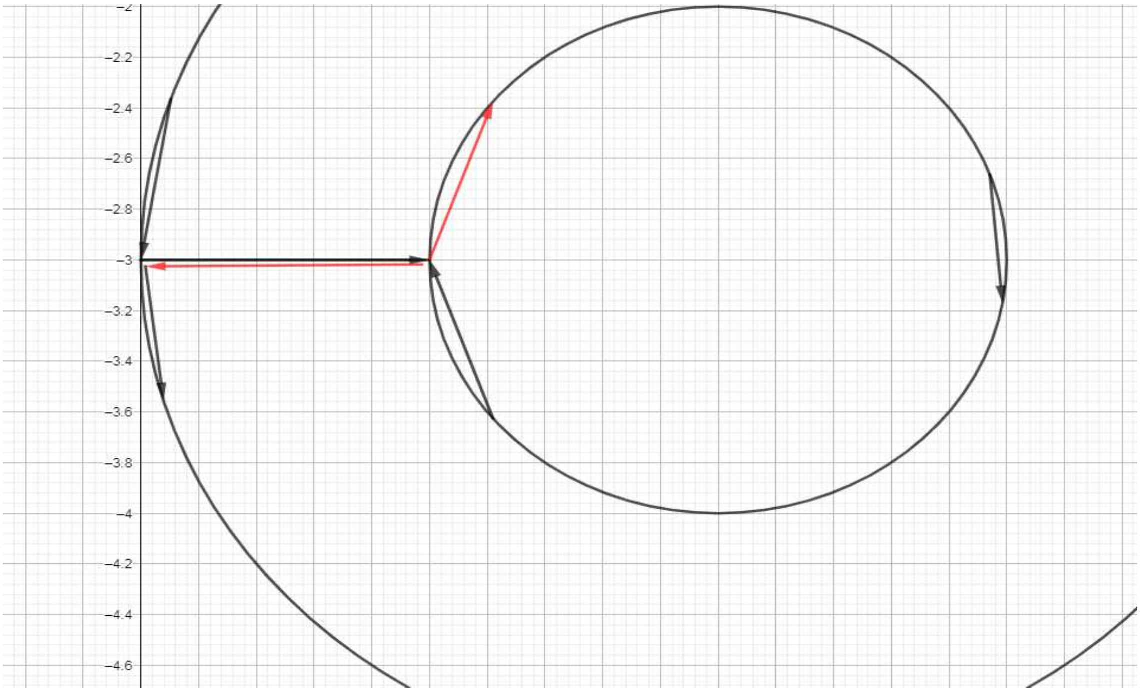
C_1 está definida por $\{(x, y)\epsilon R^2 / \ (x-2)^2 + (y+3)^2 = 4\}$ recorrida en sentido positivo

C_2 está definida por $\{(x, y)\epsilon R^2 / \ (x-2)^2 + (y+3)^2 = 1\}$ recorrida en sentido negativo.

$C_3\ y\ C_4\ un\ trayecto\ que\ los\ conecte$



Si observamos más de cerca podemos ver que $C_4\ y\ C_3$ se cancelan



Resolviendo la integral:

$$I = \int_C F \cdot dC = I_{C_1} + \underbrace{\left(-I_{C_2}\right)}_{\substack{Sentido \\ negativo}} + \underbrace{I_{C_3} + \underbrace{\left(-I_{C_4}\right)}_{\substack{Sentido \\ negativo}}}_{Se\ cancelan}$$

$$I = I_{C_1} - I_{C_2}$$

$$I_{C_1} = \oint_{C_1} P\,dx + Q dy \overset{\substack{Teorema \\ Green}}{\cong} \iint_{R_1} \left[Q_x - P_y\right] dx\,dy = \iint_{R_1} (y - (-x))dxdy$$

$$I_{C_1} = \iint_{(x-2)^2+(y+3)^2 \leq 4} (y + x)dxdy$$

$$I_{C_2} = \oint_{C_2} P\,dx + Q dy \overset{\substack{Teorema \\ Green}}{\cong} \iint_{R_2} \left[Q_x - P_y\right] dx\,dy = \iint_{R_2} (y - (-x))dxdy$$

$$I_{C_2} = \iint_{(x-2)^2+(y+3)^2 \leq 1} (y + x)dxdy$$

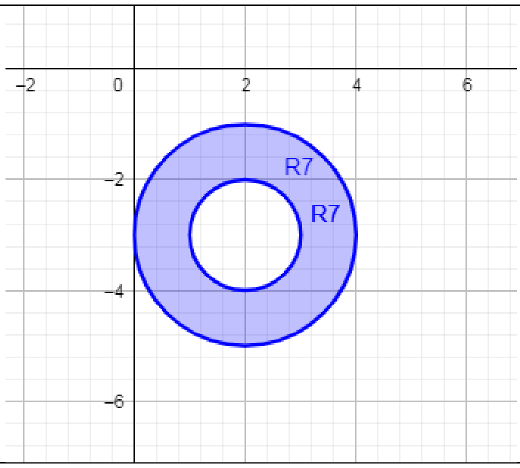
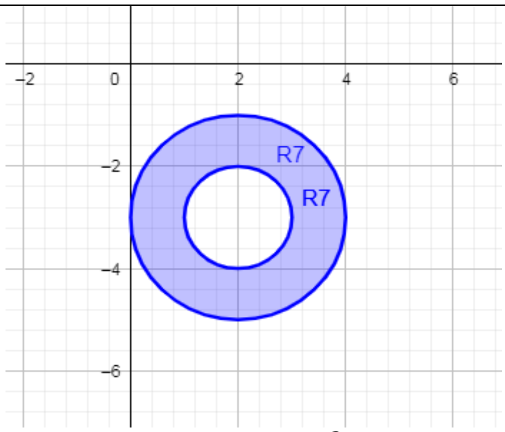
$$I = I_{C_1} - I_{C_2} = \iint\limits_{(x-2)^2+(y+3)^2\leq 4} (y+x)dxdy - \iint\limits_{(x-2)^2+(y+3)^2\leq 1} (y+x)dxdy$$

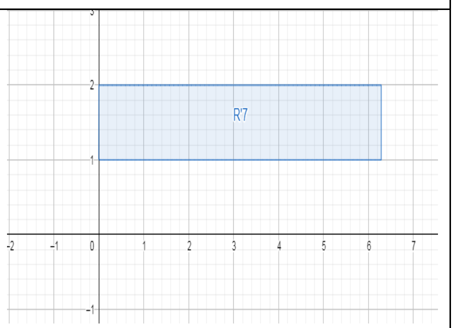
$$I = \iint\limits_{1\leq (x-2)^2+(y+3)^2\leq 4} (x+y)dxdy$$

Aplicando Transformaciones Polares Particulares

$$R: \{(x,y)\epsilon R^2 /\ 1 \leq (x-2)^2 + (y+3)^2 \leq 4\}$$

$$(x-2)^2 + (y+3)^2 = \cos^2\alpha + sen^2\alpha = 1$$
$$(x-2)^2 + (y+3)^2 = 4\cos^2\alpha + 4sen^2\alpha = 4$$

<table>
<tr>
<td>
</td>
<td></td>
<td>$$(x-2)^2 = r^2\cos^2\alpha$$ $$x-2 = rcos\,\alpha$$ $$x = r\cos\alpha + 2$$ $$(y+3)^2 = r^2\,\text{sen}^2\,\alpha$$ $$y+3 = rsen\,\alpha$$ $$y = r\,sen\,\alpha - 3$$ $$|J| = r$$</td>
</tr>
<tr><td></td><td></td><td></td></tr>
<tr>
<td>

$$R7 = \left\{\begin{matrix}(x,y)\epsilon R^2/\\ 1 \leq (x-2)^2 + (y+3)^2 \leq 4\end{matrix}\right\}$$</td>
<td>=></td>
<td>

$$R'7: \left\{\begin{matrix}(r,\alpha)\epsilon R^2/\\ 1 \leq r \leq 2\\ \wedge\\ 0 \leq \alpha \leq 2\pi\end{matrix}\right\}$$</td>
</tr>
</table>

$$I = \iint_R x + ydxdy = \int_1^2 \int_0^{2\pi} (rcos\alpha + 2 + rsen\alpha - 3)r \, d\alpha dr$$

$$I = \int_1^2 r^2 \int_0^{2\pi} (cos\alpha + sen\alpha) \, d\alpha dr - \int_1^2 r \int_0^{2\pi} d\alpha \, dr$$

$$I = \int_1^2 r^2 \, [sen\alpha - cos\alpha]_0^{2\pi} dr - \int_1^2 r 2\pi dr$$

$$I = 0 - 2\pi \int_1^2 rdr$$

$$I = -\pi[r^2]_1^2 = -3\pi$$

Finalmente

$$I = \int_C F \cdot dC = -3\pi$$