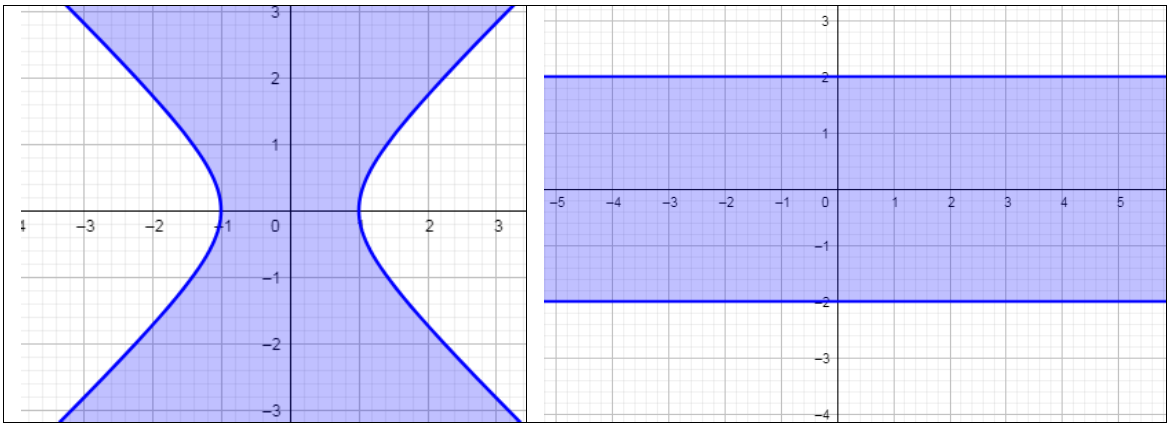
Resolución TP7:

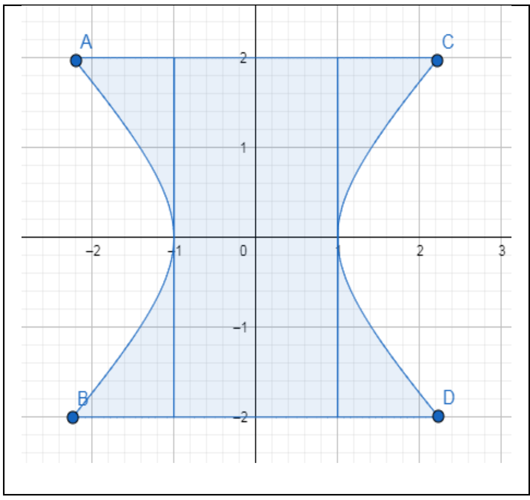
Ejercicio 12 - d

Calcular el área de la región de R por medio de integrales.

$$R: \{(x, y) \epsilon R^2 / \ x^2 - y^2 \leq 1 \ \wedge \ -2 \leq y \leq 2\}$$

Resolviendo:





$$x^2 - y^2 \leq 1$$
$$x^2 \leq 1 + y^2$$
$$-\sqrt{1 + y^2} \leq x \leq \sqrt{1 + y^2}$$

$$R: \left\{ \begin{array}{c} (x, y) \epsilon R^2 / \\ -\sqrt{1 + y^2} \leq x \leq \sqrt{1 + y^2} \\ -2 \leq y \leq 2 \end{array} \right\}$$

A modo informativo, en A y B, se resuelve que $x = -\sqrt{5}$ y en C y D, se resuelve que $x = \sqrt{5}$

Si R es una región del plano, se proporciona su área mediante la integral

$$I = \iint_R 1 dx dy$$

$$I = \int_{2}^{-2} \int_{-\sqrt{1+y^2}}^{\sqrt{1+y^2}} dx\, dy$$

$$I = \int_{2}^{-2} \left(\sqrt{1+y^2}\right) - \left(-\sqrt{1+y^2}\right) dy$$

$$I = 2 \int_{2}^{-2} \sqrt{1+y^2} dy$$

ver cálculos auxiliares

$$I = 2 \left[\frac{1}{4}\sinh(2arcsenh(y)) + \frac{1}{2} arcsenh(y)\right]_{y=-2}^{y=2}$$

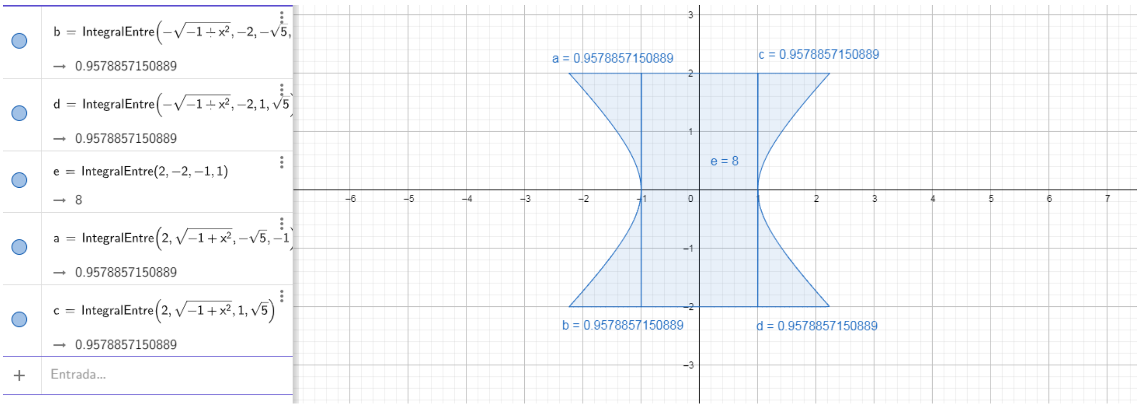
ver cálculos auxiliares

$$I = 2 \left(2\left(\frac{1}{4}\sinh(2arcsenh(2)) + \frac{1}{2} arcsenh(2)\right)\right)$$

$$I = \sinh(2arcsenh(2)) + 2arcsenh(2)$$

$$I \simeq 11.83$$

Verificamos el resultado con GeoGebra:



8+4(0.95788) = 11.83152

Ejercicio propuesto: Verificar el resultado con el teorema de Fubbini

C/A

Usando sustitución en base a:

$$\cosh^2(t) - senh^2(t) = 1 \rightarrow \cosh^2(t) = 1 + senh^2(t)$$

$$y = senh(t) \rightarrow \begin{cases} \sqrt{1+t^2} = \cosh(t) \\ dy = cosh(t)dt \end{cases}$$

$$\sqrt{1+y^2}dy = \cosh^2(t)\,dt$$

$$P(y) = \int \sqrt{1+y^2}dy$$

$$P(y) = \int \cosh^2(t)\,dt$$

Dado $\cosh^2(t) = \frac{e^t+e^{-t}}{2}$

$$P(y) = \int \left(\frac{e^t + e^{-t}}{2}\right)^2 dt$$

$$P(y) = \frac{1}{4}\int e^{2t} + 2 + e^{-2t}dt$$

$$P(y) = \frac{1}{4}\left(\frac{e^{2t}}{2} + 2t - \frac{e^{-2t}}{2}\right)$$

$$P(y) = \frac{1}{4}\left(\frac{e^{2t} - e^{-2t}}{2} + 2t\right)$$

$$P(y) = \frac{1}{4}\sinh(2t) + \frac{1}{2}t$$

$$P(y) = \frac{1}{4}\sinh(2arcsenh(y)) + \frac{1}{2}arcsenh(y)$$

C/A Barrow

$$P(2) = \frac{1}{4}\sinh(2arcsenh(2)) + \frac{1}{2}arcsenh(2)$$

$$P(-2) = \frac{1}{4}\sinh(2arcsenh(-2)) + \frac{1}{2}arcsenh(-2)$$

Si $arcsenh(2) = -arcsenh(-2) \rightarrow arcsenh(-2) = -arcsenh(2)$

$$P(-2) = \frac{1}{4}\sinh(-2arcsenh(2)) - \frac{1}{2}arcsenh(2)$$

$$P(-2) = -\frac{1}{4}\sinh(2arcsenh(2)) - \frac{1}{2}arcsenh(2) = -P(2)$$

$$P(2) - P(-2) = 2P(2)$$

C/A

Demostracion $arcsenh(2) = -arcsenh(-2)$

Tomando la siguiente logica:

$$Senh(x) = \frac{e^{x} - e^{-x}}{2}$$

$$Senh(-x) = \frac{e^{-x} - e^{-(-x)}}{2} = \frac{e^{-x} - e^{x}}{2} = -\frac{e^{x} - e^{-x}}{2} = -Senh(x)$$

Entonces:

$$arcsenh(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$arcsenh(-x) = \ln\left(-x + \sqrt{(-x)^2 + 1}\right) = \ln\left(-x + \sqrt{x^2 + 1}\right)$$

$$\ln\left(\frac{\left(-x + \sqrt{x^2 + 1}\right)\left(x + \sqrt{x^2 + 1}\right)}{\left(x + \sqrt{x^2 + 1}\right)}\right) = \ln\left(\frac{1}{\left(x + \sqrt{x^2 + 1}\right)}\right)$$

$$\ln(1) - \ln\left(x + \sqrt{x^2 + 1}\right) = -\ln\left(x + \sqrt{x^2 + 1}\right) = -arcsenh(x)$$