T P 04 Ej. 12-b

Determinar el vector gradiente de la siguiente función en los puntos indicados.

$$\boldsymbol{f(x,y) = arc\tan\left(\frac{x+y}{x-y}\right) \qquad en \quad (1,0)}$$

Sabiendo que: $\vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)$

Sí $f(x) = arc\tan[u(x)] \to \dot{f}(x) = \frac{\dot{u}}{1+u^2}$

Derivando con respecto a "x"

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{1+\left(\frac{x+y}{x-y}\right)^2} \cdot \frac{[1.(x-y)-(x+y).1]}{(x-y)^2} \quad \to$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{1+\frac{(x+y)^2}{(x-y)^2}} \cdot \frac{(\cancel{x}-y-\cancel{x}-y)}{(x-y)^2} \quad \to$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{1}{\frac{(x-y)^2+(x+y)^2}{(x-y)^2}} \cdot \frac{(-2.y)}{(x-y)^2} \quad \to$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{\cancel{(x-y)^2}}{(x-y)^2+(x+y)^2} \cdot \frac{(-2.y)}{\cancel{(x-y)^2}} \quad \to$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{-2.y}{x^2-\cancel{2.x.y}+y^2+x^2+\cancel{2.x.y}+y^2} \quad \to$$

$$\frac{\partial f}{\partial x}(x,y) = \frac{-\cancel{2}.y}{\cancel{2}.(x^2+y^2)} \quad \to$$

$$\boldsymbol{\frac{\partial f}{\partial x}(x,y) = \frac{-y}{x^2+y^2}}$$

Derivando con respecto a “y”, tenemos:

$$\frac{\partial f}{\partial y}=\frac{1}{1+\left(\frac{x+y}{x-y}\right)^2}.\frac{[1.(x-y)-(x+y).(-1)]}{(x-y)^2} \quad \rightarrow$$

$$\frac{\partial f}{\partial y}(x,y)=\frac{1}{1+\frac{(x+y)^2}{(x-y)^2}}.\frac{(x-\cancel{y}+x+\cancel{y})}{(x-y)^2} \quad \rightarrow$$

$$\frac{\partial f}{\partial y}(x,y)=\frac{1}{\frac{(x-y)^2+(x+y)^2}{(x-y)^2}}.\frac{(2.x)}{(x-y)^2} \quad \rightarrow$$

$$\frac{\partial f}{\partial y}(x,y)=\frac{\cancel{(x-y)^2}}{(x-y)^2+(x+y)^2}.\frac{(2.x)}{\cancel{(x-y)^2}} \quad \rightarrow$$

$$\frac{\partial f}{\partial y}(x,y)=\frac{2.x}{x^2-\cancel{2.x.y}+y^2+x^2+\cancel{2.x.y}+y^2} \quad \rightarrow$$

$$\frac{\partial f}{\partial x}(x,y)=\frac{\cancel{2}.x}{\cancel{2}.(x^2+y^2)} \quad \rightarrow$$

$$\boldsymbol{\frac{\partial f}{\partial y}(x,y)=\frac{x}{x^2+y^2}}$$

Finalmente:

$$\boldsymbol{\vec{\nabla} f(x,y)=\left(\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right)=\left(\frac{-y}{x^2+y^2}\ \ ;\ \ \frac{x}{x^2+y^2}\right)}$$

$$\boldsymbol{\vec{\nabla} f(1,0)=\left(\frac{\partial f}{\partial x}(1,0),\frac{\partial f}{\partial y}(1,0)\right)=\left(\frac{-0}{1^2+0^2},\frac{1}{1^2+0^2}\right)=\ (0\ ,1)}$$