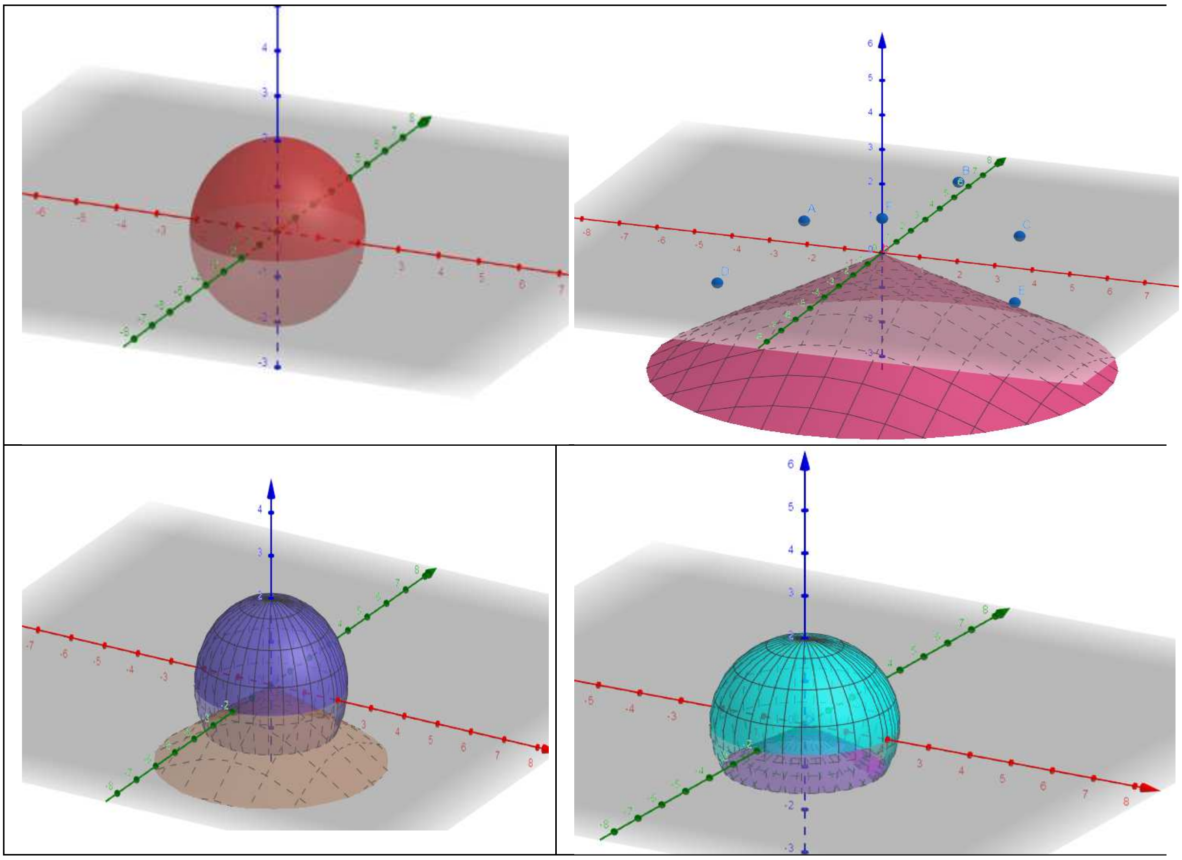
Resolución TP7:

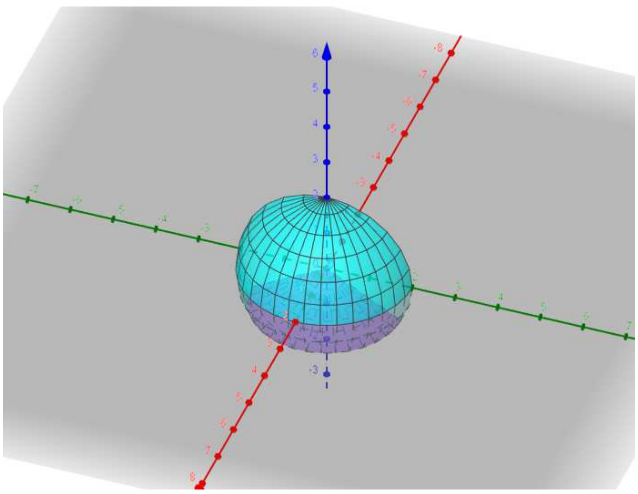
Resolver I usando V

$V\colon \{(x,y,z)\epsilon R^3/\ x^2+y^2+z^2 \leq 4\ \wedge z \geq -\sqrt{x^2+y^2} \wedge x \geq 0\}$

$$I = \iiint\limits_V e^{\sqrt{x^2+y^2+z^2}}\,dxdydz$$



$con\ x \geq 0$



Con coordenadas Esfericas

$$V: \begin{cases} x^2 + y^2 + z^2 \leq 4 \\ z \geq -\sqrt{x^2 + y^2} \\ x \geq 0 \end{cases}$$

$$V: \begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} ? \leq r \leq ? \\ ? \leq \varphi \leq ? \\ ? \leq \theta \leq ? \end{cases} \end{cases}$$

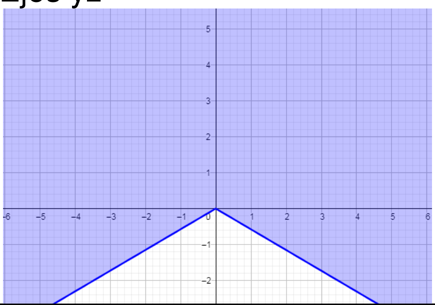
$$I = \iiint_V e^{\sqrt{x^2+y^2+z^2}} dxdydz$$

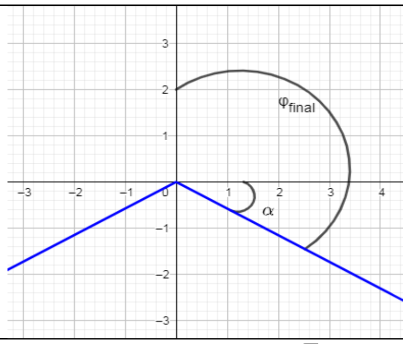
$$I = \iiint_{V'} e^{\sqrt{x(r,\theta,\varphi)^2+y(r,\theta,\varphi)^2+z(r,\theta,\varphi)^2}} |J(r,\theta,\varphi)| drd\theta d\varphi$$

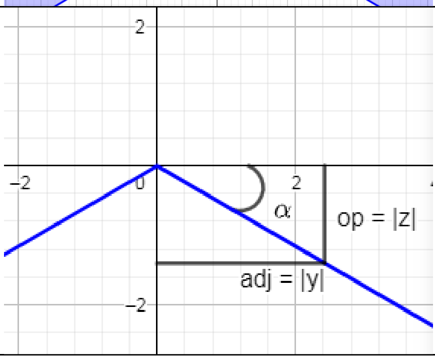
$$I = \iiint_{V'} e^r r^2 sen(\varphi) drd\theta d\varphi$$

si $x = 0 \rightarrow z \geq -|y|$

Ejes yz







$$\varphi_{final} = \frac{\pi}{2} + \alpha$$

$$tg(\alpha) = \frac{|z|}{|y|} = \frac{|-|y||}{|y|} = 1$$

$$\alpha = arctg(1) = \frac{\pi}{4}$$

$$\varphi_{final} = \frac{3}{4}\pi$$

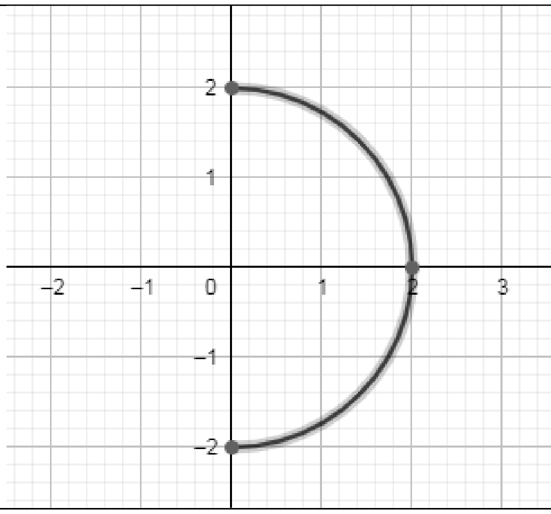
Si $z = 0 \rightarrow x^2 + y^2 + 0^2 \leq 4$

$$\begin{cases} x^2 + y^2 \leq 4 \\ x \geq 0 \end{cases}$$

Entonces

$$r \leq 2$$

$$-\frac{1}{2}\pi \leq \theta \leq \frac{\pi}{2} \text{ ó } \frac{3}{2}\pi \leq \theta \leq \frac{5}{2}\pi$$



Con coordenadas Esfericas	
$V: \begin{cases} x^2+y^2+z^2 \leq 4 \\ z \geq -\sqrt{x^2+y^2} \\ x \geq 0 \end{cases}$ $V: \begin{cases} x = r\cos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = r\cos(\varphi) \\ \lvert J \rvert = r^2 sen(\varphi) \\ V' = \begin{cases} 0 \leq r \leq 2 \\ 0 \leq \varphi \leq \frac{3}{4}\pi \\ -\frac{1}{2}\pi \leq \theta \leq \frac{\pi}{2} \end{cases} \end{cases}$	$I = \iiint_{V'} e^r r^2 sen(\varphi) dr d\theta d\varphi$ $I = \int_0^{\frac{3}{4}\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 e^r r^2 sen(\varphi) dr d\theta d\varphi$

$$I = \int_0^{\frac{3}{4}\pi} sen(\varphi) d\varphi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^2 e^r r^2 dr$$

$$I = [-\cos(\varphi)]_0^{\frac{3}{4}\pi} [\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [r^2 e^r - 2re^r + 2e^r]_0^2$$

$$I = \left[\left(-(-\frac{\sqrt{2}}{2})\right) - (-1)\right]\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right][(4e^2 - 4e^2 + 2e^2) - (0 - 0 + 2e^0)]$$

$$I = \left[\frac{\sqrt{2}}{2} + 1\right][\pi][2e^2 - 2]$$

C/A

$$\int \underbrace{r^2}_{u} \underbrace{e^r dr}_{dv} = r^2 e^r - 2\int \underbrace{r}_{u} \underbrace{e^r dr}_{dv} = r^2 e^r - 2\left[re^r - \int e^r dr\right]$$

$$\int r^2 e^r dr = r^2 e^r - 2re^r + 2e^r$$