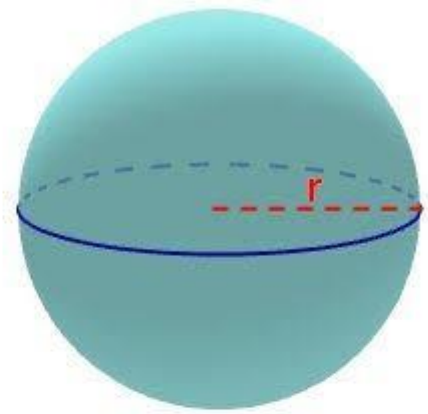
TP 7, ej 25-a

a) $\iiint_V \frac{1}{\sqrt{x^2+y^2+z^2}}\, dx\, dy\, dz$

$$V = \{(x, y, z) \in \mathbb{R}^3 :\ x^2 + y^2 + z^2 \leq 4\}$$



Utilizando coordenadas esféricas.

$$\begin{cases} x = \rho \cdot \cos(\theta) \cdot sen(\phi) \\ y = \rho \cdot sen(\theta) \cdot sen(\phi) \\ z = \rho \cdot \cos(\phi) \end{cases}$$

Siendo

$$J = \frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)} = \rho^2 \cdot sen(\phi)$$

$$0 \leq \theta \leq 2\pi$$

$$0 \leq \phi \leq \pi$$

$$0 \leq \rho \leq 2$$

Observar que $\quad x^2 + y^2 + z^2 = \rho^2 \rightarrow \sqrt{x^2 + y^2 + z^2} = \rho$

$$I = \int_0^{2\pi} d\theta \int_0^{\pi} d\phi \int_0^{2} \frac{\rho^2 \sin(\phi)}{\rho}\, d\rho$$

$$I = \int_0^{2\pi} d\theta \int_0^{\pi} \sin(\phi)\, d\phi \left(\frac{\rho^2}{2}\Big|_0^2 \right)$$

$$I = 2\int_{0}^{2\pi} d\theta \left(-cos(\phi)\right|_{0}^{\pi})$$

$$I = 8\pi$$