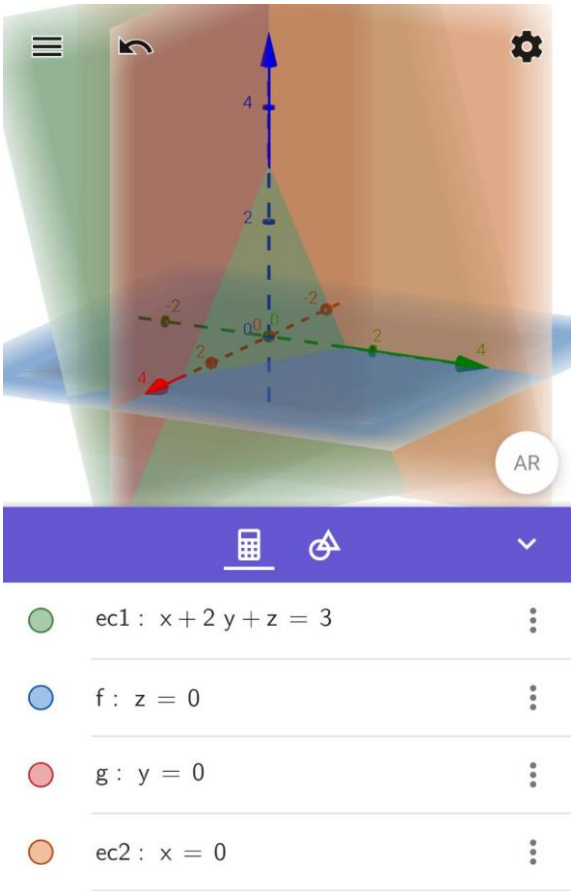
19.- Calcular:

$a)$ $\iiint (x + y + z)\, dx\, dy\, dz$

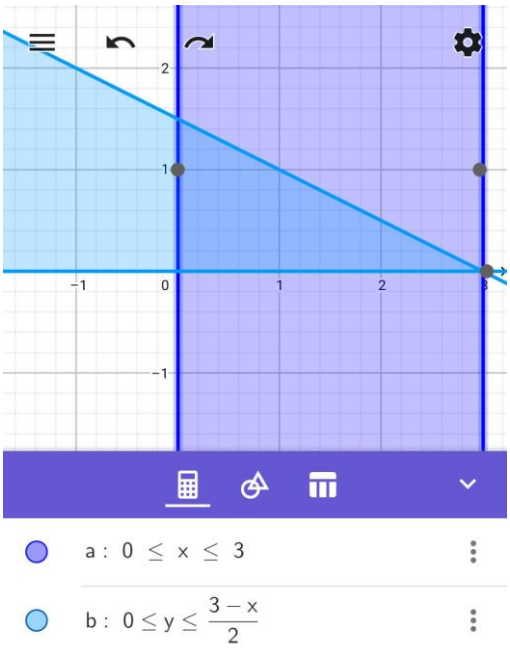
S es el tetraedro limitado por los planos coordenados y el plano de ecuación $x + 2y + z = 3$.

Graficamos el plano $x + 2y + z = 3$

$\rightarrow \frac{x}{3} + \frac{y}{\frac{3}{2}} + z = 1$



$0 \leq x \leq 3$ $\qquad$ $0 \leq y \leq \frac{3-x}{2}$



$$I = \int_0^3 dx \int_0^{\frac{3-x}{2}} dy \int_0^{3-x-2y} (x+y+z)\,dz =$$

$$I = \int_0^3 dx \int_0^{\frac{3-x}{2}} (-3y - \frac{1}{2}x^2 - xy + \frac{9}{2})\,dy =$$

$$= \int_0^3 (-\frac{3}{2}y^2 - \frac{1}{2}x^2y - \frac{1}{2}xy^2 + \frac{9}{2}y)\Big|_0^{\frac{3-x}{2}} dx =$$

$$I = \int_0^3 (-\frac{3}{8}(3-x)^2 + \frac{1}{8}x^3 - \frac{27}{8}x + \frac{27}{4})\,dx =$$

$$I = \frac{135}{32}$$