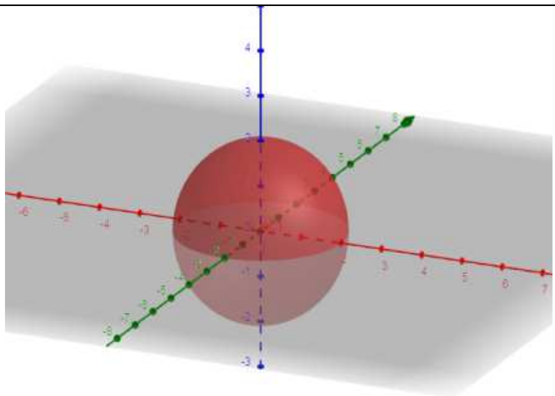
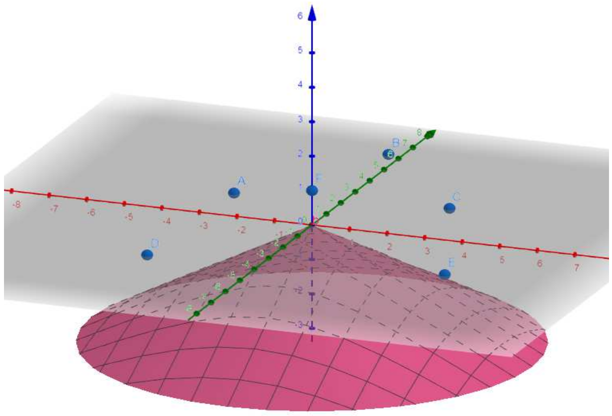
Resolución TP7:

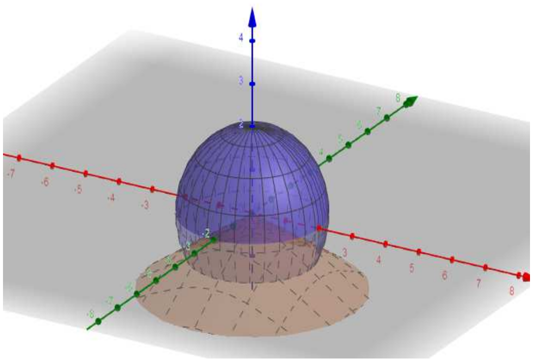
Resolver I usando V

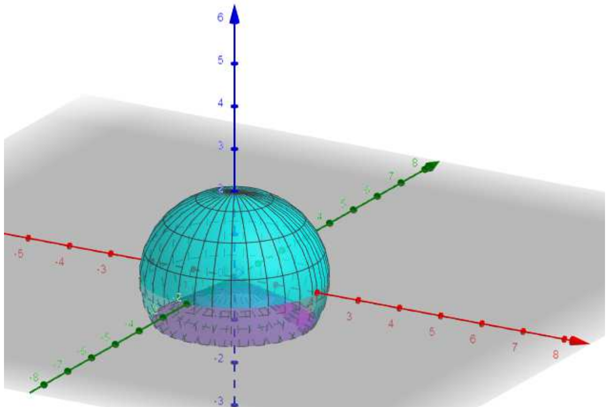
$V: \{(x,y,z) \epsilon R^3 / \ x^2+y^2+z^2 \leq 4 \ \wedge z \geq -\sqrt{x^2+y^2} \wedge x \geq 0 \ \wedge y \geq \sqrt{3}x\}$

$$Vol(V) = \iiint_V 1dxdydz$$

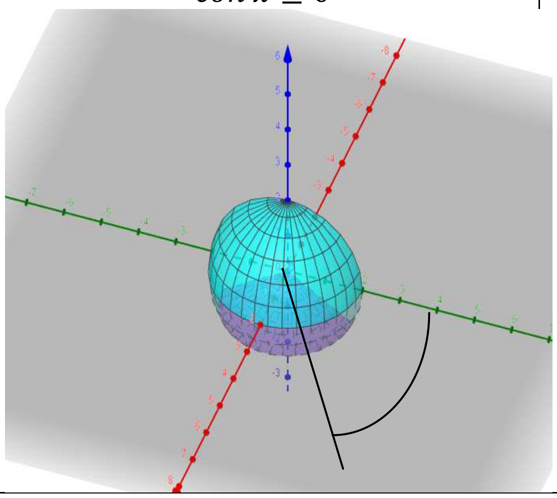




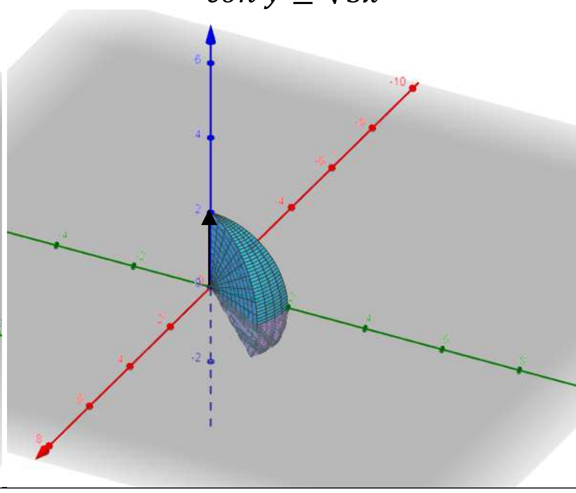




$con\ x \geq 0$



$con\ y \geq \sqrt{3}x$



Con coordenadas Esfericas

$$V:\begin{cases} x^2+y^2+z^2\le 4 \\ z\ge -\sqrt{x^2+y^2} \\ x\ge 0 \\ y\ge \sqrt{3}x \end{cases}$$

$$V:\begin{cases} x = r\cos\theta sen(\varphi) \\ y = r sen\theta sen(\varphi) \\ z = r\cos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} ?\le r\le ? \\ ?\le \varphi\le ? \\ ?\le \theta\le ? \end{cases} \end{cases}$$

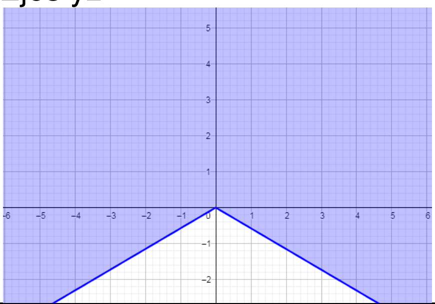
$$I = \iiint_V 1dxdydz$$

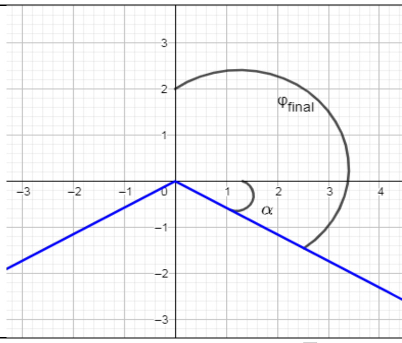
$$I = \iiint_{V'} |J(r,\theta,\varphi)|drd\theta d\varphi$$

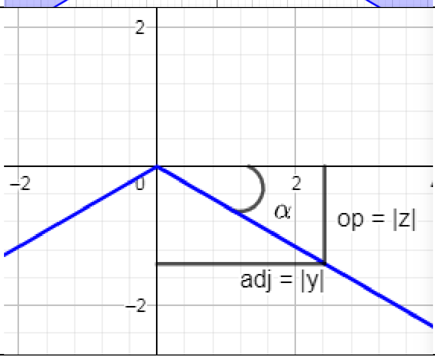
$$I = \iiint_{V'} r^2 sen(\varphi)drd\theta d\varphi$$

si $x = 0 \rightarrow z \ge -|y|$

Ejes yz







$$\varphi_{final} = \frac{\pi}{2} + \alpha$$

$$x = 0 \rightarrow z = -|y|$$

$$y = 1 \rightarrow z = -1$$

$$\alpha = arctg\left(\frac{|-1|}{1}\right) = \frac{\pi}{4}$$

$$\varphi_{final} = \frac{3}{4}\pi$$

Si $z = 0 \rightarrow x^2 + y^2 + 0^2 \leq 4$

$$\begin{cases} x^2 + y^2 \leq 4 \\ x \geq 0 \\ y \geq \sqrt{3}x \end{cases}$$

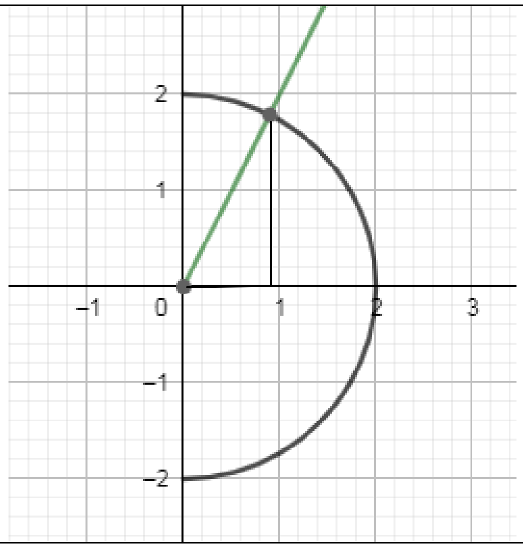
$$tg(\theta_i) = \frac{|y|}{|x|} = \frac{|\sqrt{3}x|}{|x|} = \sqrt{3}$$

$$\theta_i = arctg(\sqrt{3}) = \frac{\pi}{3}$$

Entonces

$$r \leq 2$$

$$\frac{\pi}{3} \leq \theta \leq \frac{\pi}{2}$$



Con coordenadas esfericas

$$V: \begin{cases} x^2 + y^2 + z^2 \leq 4 \\ z \geq -\sqrt{x^2 + y^2} \\ x \geq 0 \\ y \geq \sqrt{3}x \end{cases}$$

$$V: \begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} 0 \leq r \leq 2 \\ 0 \leq \varphi \leq \frac{3}{4}\pi \\ \frac{\pi}{3} \leq \theta \leq \frac{\pi}{2} \end{cases} \end{cases}$$

$$I = \iiint_V 1dxdydz$$

$$I = \iiint_{V'} r^2 sen(\varphi) dr d\theta d\varphi$$

$$I = \int_0^{\frac{3}{4}\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^2 r^2 sen(\varphi) dr d\theta d\varphi$$

$$I = \int_0^{\frac{3}{4}\pi} sen(\varphi) d\varphi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d\theta \int_0^2 r^2 dr$$

$$I = [-\cos(\varphi)]_0^{\frac{3}{4}\pi} [\theta]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \left[\frac{r^3}{3}\right]_0^2$$

$$I = \left[\left(-\left(-\frac{\sqrt{2}}{2}\right)\right) - (-1)\right]\left[\frac{\pi}{2} - \left(\frac{\pi}{3}\right)\right]\left[\frac{8}{3} - 0\right]$$

$$I = \left[\frac{\sqrt{2}}{2} + 1\right]\left[\frac{\pi}{6}\right]\left[\frac{8}{3}\right] = \frac{4}{9}\pi\left[\frac{\sqrt{2}}{2} + 1\right]$$