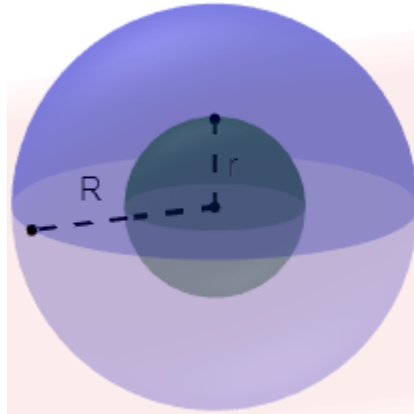TP 7, ejercicio 25 b

b) $\iiint_V \frac{1}{\sqrt{x^2+y^2+z^2}}\, dx\, dy\, dz$

$$V = \{(x, y, z) \in \mathbb{R}^3 : 1 \le x^2 + y^2 + z^2 \le 4\}$$

Utilizando coordenadas esféricas.

$$\begin{cases} x = \rho \cdot \cos(\theta) \cdot sen(\phi) \\ y = \rho \cdot sen(\theta) \cdot sen(\phi) \\ z = \rho \cdot \cos(\phi) \end{cases}$$

Siendo

$$J = \frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)} = \rho^2 \cdot sen(\phi)$$

$$x^2 + y^2 + z^2 = \rho^2 \rightarrow \sqrt{x^2 + y^2 + z^2} = \rho$$

$$1 \le x^2 + y^2 + z^2 \le 4$$

$$0 \le \theta \le 2\pi$$
$$0 \le \phi \le \pi$$
$$1 \le \rho \le 2$$

$$I = \int\limits_{0}^{2\pi} d\theta \int\limits_{0}^{\pi} d\phi \int\limits_{1}^{2} \frac{\rho^2 \sin(\phi)}{\rho} \, d\rho$$

$$I = \int\limits_{0}^{2\pi} d\theta \int\limits_{0}^{\pi} \sin(\phi) \, d\phi \left(\left. \frac{\rho^2}{2} \right|_{1}^{2} \right)$$

$$I = \frac{3}{2} \int\limits_{0}^{2\pi} d\theta \left(-\cos(\phi) |_{0}^{\pi} \right)$$

$$I = 6\pi$$