Resolución TP5:

Ayuda en Ejercicio 10

Tomando el sistema conformado por:

$$\begin{cases} F(x, y, u, v) = 0 \\ G(x, y, u, v) = 0 \end{cases}$$

Determinar si definen $u = f(x, y)\ y\ \ v = g(x, y)\ en\ P$ y si es así determinar sus derivadas

Herramientas:

- Se deben formular las 3 condiciones del teorema usando regla de la cadena.

Para empezar:

En este caso podemos componer:

$$H(x, y) = F(x, y, u = f(x, y), v = g(x, y)))$$

$$I(x, y) = G(x, y, u = f(x, y), v = g(x, y)))$$

Derivadas de las composiciones:

$H(x, y)$ e $I(x, y)$ se pueden derivar en x, y.

$$H_x = \frac{\partial F}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial F}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial F}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial F}{\partial v}\frac{\partial v}{\partial x}$$

$$I_x = \frac{\partial G}{\partial x}\frac{\partial x}{\partial x} + \frac{\partial G}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial G}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial G}{\partial v}\frac{\partial v}{\partial x}$$

Sabemos que $\frac{\partial x}{\partial x} = 1$, $\frac{\partial y}{\partial x} = 0$, $\frac{\partial u}{\partial z} = u_x$ y $\frac{\partial v}{\partial x} = g_x$

$$H_x = F_x + F_u\ f_x + F_v\ g_x$$

$$I_x = G_x + G_u\ f_x + G_v\ g_x$$

Si $F(P) = 0$ entonces $H(x_0, y_0) = 0$ entonces derivando lado a lado

$$F_x(P) + F_u(P)\ f_x(x_0, y_0) + F_v(P)\ g_x(x_0, y_0) = 0$$

Si $G(P) = 0$ entonces $I(x_0, y_0) = 0$ entonces derivando lado a lado

$$G_{x(P)} + G_u(P)\ f_x(x_0, y_0) + G_v(P)\ g_x(x_0, y_0) = 0$$

Se debe resolver el sistema donde $f_x(x_0, y_0)$ y $g_x(x_0, y_0)$ son las incógnitas:

$$F_{x(P)} + F_u(P)f_x(x_0, y_0) + F_v(P)g_x(x_0, y_0) = 0$$

$$G_{x(P)} + G_u(P)f_x(x_0, y_0) + G_v(P)g_x(x_0, y_0) = 0$$

Resolviéndolo por determinantes:

$$u_x(x_0, y_0) = f_x(x_0, y_0) = -\frac{\begin{vmatrix} F_x(P) & F_v(P) \\ G_x(P) & G_v(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

$$v_x(x_0, y_0) = g_x(x_0, y_0) = -\frac{\begin{vmatrix} F_u(P) & F_x(P) \\ G_u(P) & G_x(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

Ahora derivamos respecto a y:

$$H_y = \frac{\partial \mathrm{F}}{\partial \mathrm{x}}\frac{\partial \mathrm{x}}{\partial \mathrm{y}} + \frac{\partial \mathrm{F}}{\partial \mathrm{y}}\frac{\partial \mathrm{y}}{\partial \mathrm{y}} + \frac{\partial \mathrm{F}}{\partial \mathrm{u}}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} + \frac{\partial \mathrm{F}}{\partial \mathrm{v}}\frac{\partial \mathrm{v}}{\partial \mathrm{y}}$$

$$I_y = \frac{\partial \mathrm{G}}{\partial \mathrm{x}}\frac{\partial \mathrm{x}}{\partial \mathrm{y}} + \frac{\partial \mathrm{G}}{\partial \mathrm{y}}\frac{\partial \mathrm{y}}{\partial \mathrm{y}} + \frac{\partial \mathrm{G}}{\partial \mathrm{u}}\frac{\partial \mathrm{u}}{\partial \mathrm{y}} + \frac{\partial \mathrm{G}}{\partial \mathrm{v}}\frac{\partial \mathrm{y}}{\partial \mathrm{y}}$$

Sabemos que $\frac{\partial \mathrm{x}}{\partial \mathrm{y}} = 0$, $\frac{\partial \mathrm{y}}{\partial \mathrm{y}} = 1$, $\frac{\partial \mathrm{u}}{\partial \mathrm{y}} = u_y$ y $\frac{\partial \mathrm{v}}{\partial \mathrm{y}} = g_y$

$$H_y = F_y + F_u f_y + F_v g_y$$

$$I_y = G_y + G_u f_y + G_v g_y$$

Si $F(P) = 0$ entonces $H(x_0, y_0) = 0$ entonces derivando lado a lado

$$F_{y(P)} + F_u(P)f_y(x_0, y_0) + F_y(P)g_x(x_0, y_0) = 0$$

Si $G(P) = 0$ entonces $I(x_0, y_0) = 0$ entonces derivando lado a lado

$$G_{y(P)} + G_u(P)f_y(x_0, y_0) + G_v(P)g_y(x_0, y_0) = 0$$

Se debe resolver el sistema donde $f_x(x_0, y_0)$ y $g_x(x_0, y_0)$ son las incógnitas:

$$F_{y(P)} + F_u(P)f_y(x_0, y_0) + F_y(P)g_x(x_0, y_0) = 0$$

$$G_{y(P)} + G_u(P)f_y(x_0, y_0) + G_v(P)g_y(x_0, y_0) = 0$$

Resolviéndolo por determinantes:

$$u_y(x_0, y_0) = f_y(x_0, y_0) = -\frac{\begin{vmatrix} F_y(P) & F_v(P) \\ G_y(P) & G_v(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

$$v_y(x_0, y_0) = g_y(x_0, y_0) = -\frac{\begin{vmatrix} F_u(P) & F_y(P) \\ G_u(P) & G_y(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

Veamos las siguientes condiciones, se cumple TFI en un sistema $F(x, y, u, v) = 0,\ y\ G(x, y, u, v) = 0$ para $u = f(x, y)$ y $v = g(x, y)$ Si:

- $F(P) = 0,\ G(P) = 0$
- Las derivadas $F_x\ F_y\ F_v\ \ F_u$ y $G_x\ G_y\ G_v\ \ G_u$, continuas en un entorno del punto P.
- $\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix} \neq 0$

si se cumple TFI existen $u = f(x, y)$ y $v = g(x, y)$ en P y valen

$$u_x(x_0, y_0) = f_x(x_0, y_0) = -\frac{\begin{vmatrix} F_x(P) & F_v(P) \\ G_x(P) & G_v(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

$$v_x(x_0, y_0) = g_x(x_0, y_0) = -\frac{\begin{vmatrix} F_u(P) & F_x(P) \\ G_u(P) & G_x(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

$$u_y(x_0, y_0) = f_y(x_0, y_0) = -\frac{\begin{vmatrix} F_y(P) & F_v(P) \\ G_y(P) & G_v(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$

$$v_y(x_0, y_0) = g_y(x_0, y_0) = -\frac{\begin{vmatrix} F_u(P) & F_y(P) \\ G_u(P) & G_y(P) \end{vmatrix}}{\begin{vmatrix} F_u(P) & F_v(P) \\ G_u(P) & G_v(P) \end{vmatrix}}$$