Resolución TP7:

Ejercicio 26- i

Calcular el volumen dentro de $2z = x^2 + y^2 + z^2$ con $z \geq \sqrt{x^2 + y^2}$

C/A

$$2z = x^2 + y^2 + z^2$$

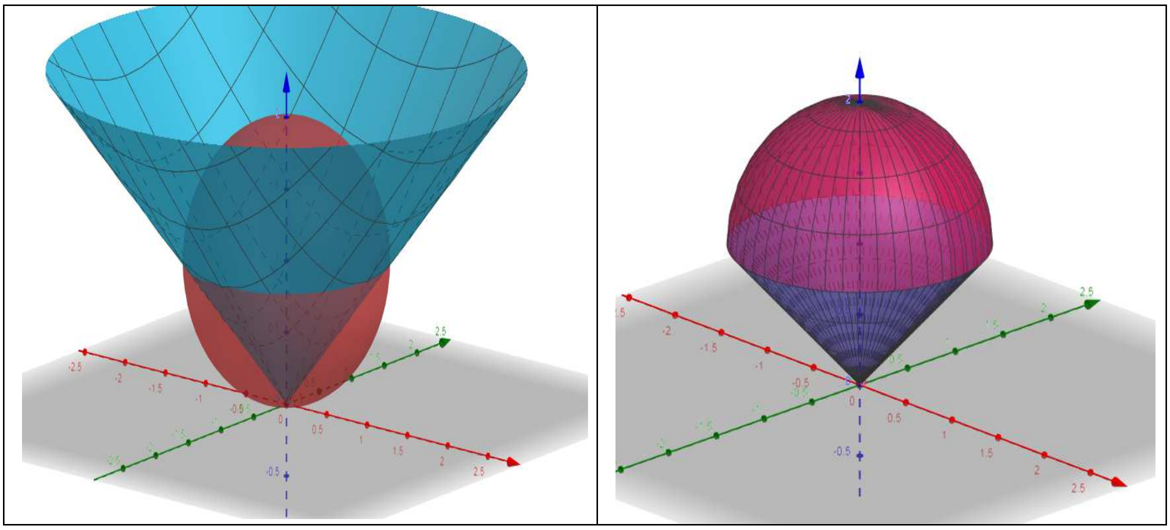
$$x^2 + y^2 + z^2 - 2z = 0$$

$$x^2 + y^2 + z^2 + 2(z)(-1) = 0$$

$$x^2 + y^2 + z^2 - 2z + 1 - 1 = 0$$

$$x^2 + y^2 + (z-1)^2 - 1 = 0$$

$$V: \begin{cases} x^2 + y^2 + (z-1)^2 \leq 1 \\ z \geq \sqrt{x^2 + y^2} \end{cases}$$



Calculamos las intersecciones para saber el límite del cono

$$\begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z = \sqrt{x^2 + y^2} \end{cases} \rightarrow \begin{cases} x^2 + y^2 + (z-1)^2 = 1 \\ z = x^2 + y^2 \end{cases} \rightarrow z^2 + (z-1)^2 = 1$$

$z^2 + (z-1)^2 = 1$ --> $z^2 + z^2 - 2z + 1 = 1$ --> $2z^2 - 2z = 0$ --> $z = 0 \ \vee \ z = \mathbf{1}$

Como se puede ver en la imagen el cono llega hasta z=1 donde se genera la curva interseccion

$$\sqrt{x^2 + y^2} \leq 1 \rightarrow 0 \leq x^2 + y^2 \leq 1$$

Calculamos los limites para z

$$x^2+y^2+(z-1)^2 \leq 1 \to 1-\sqrt{1-(x^2+y^2)} \leq z \leq 1+\sqrt{1-(x^2+y^2)}$$

$$si\; z \leq 1 \to z \geq \sqrt{x^2+y^2}$$

$$si\; z \geq 1 \to z \leq 1+\sqrt{1-(x^2+y^2)}$$

por lo tanto

$$\sqrt{x^2+y^2} \leq z \leq 1+\sqrt{1-(x^2+y^2)}$$

$$V:\begin{cases} x^2+y^2+(z-1)^2 \leq 1 \\ z \geq \sqrt{x^2+y^2} \end{cases} \to \begin{cases} \sqrt{x^2+y^2} \leq z \leq 1+\sqrt{1-(x^2+y^2)} \\ 0 \leq x^2+y^2 \leq 1 \end{cases}$$

Aplicando transformaciones cilíndricas

$$V:\begin{cases} x = rcos\theta \\ y = rsen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases} r \leq z \leq 1+\sqrt{1-r^2} \\ 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \end{cases} \end{cases}$$

$$V = \iiint_V 1dxdydz = \int_0^{2\pi}\int_0^1\int_r^{1+\sqrt{1-r^2}} r\; dzdrd\theta$$

$$V = \int_0^{2\pi}\int_0^1 r\left(1+\sqrt{1-r^2}-r\right)drd\theta$$

$$V = \int_0^{2\pi}\int_0^1 \left(r+r\sqrt{1-r^2}-r^2\right)drd\theta = \int_0^{2\pi} d\theta \int_0^1 \left(r+r\sqrt{1-r^2}-r^2\right)dr$$

$$c/a \int r\sqrt{1-r^2}dr \overset{\substack{1-r^2=t \\ -2rdr=dt \\ rdr=-\frac{dt}{2}}}{\cong} -\frac{1}{2}\int \sqrt{t}dt = -\frac{1}{3}\sqrt{t^3} \overset{1-r^2=t}{\cong} -\frac{1}{3}\sqrt{(1-r^2)^3}$$

$$V = \int_0^{2\pi} d\theta \left[\frac{r^2}{2}-\frac{1}{3}\sqrt{(1-r^2)^3}-\frac{r^3}{3}\right]_0^1 = 2\pi\left[\left(\frac{1}{2}-\frac{1}{3}(0)-\frac{1}{3}\right)-\left(0-\frac{1}{3}-0\right)\right] = \pi$$