Resolución TP10:

Ayuda en Ejercicio 4 - f

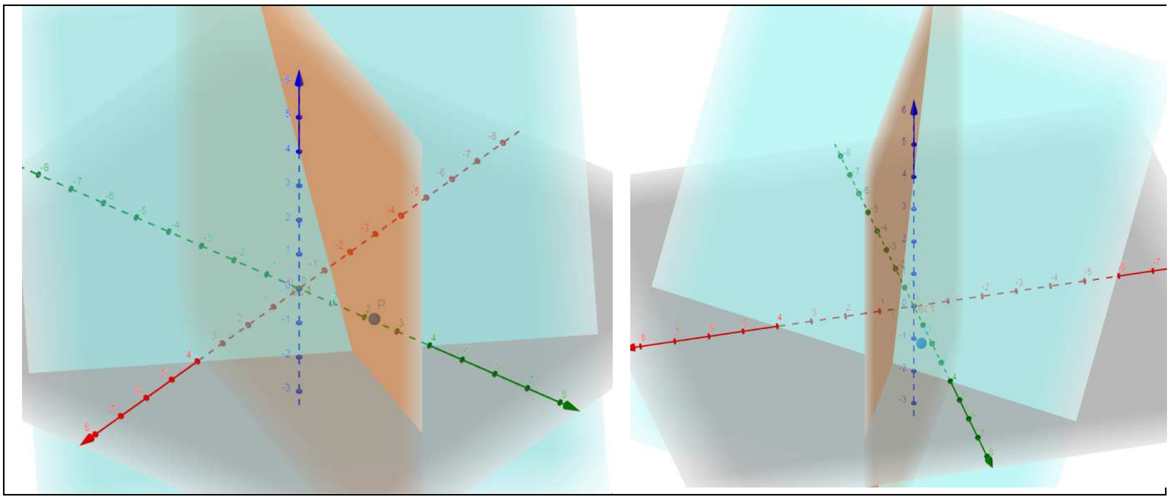
Parametrizar el área de la frontera del cuerpo definido por

$x + y + z \leq 4$ con $y \geq 2x$ en el primer octante

Resolviendo:

Considerando que se trata de un volumen se puede nombrar la siguiente descripción:

$$V: \begin{cases} x + y + z \leq 4 \\ y \geq 2x \\ x \geq 0 \quad y \geq 0 \quad z \geq 0 \end{cases}$$

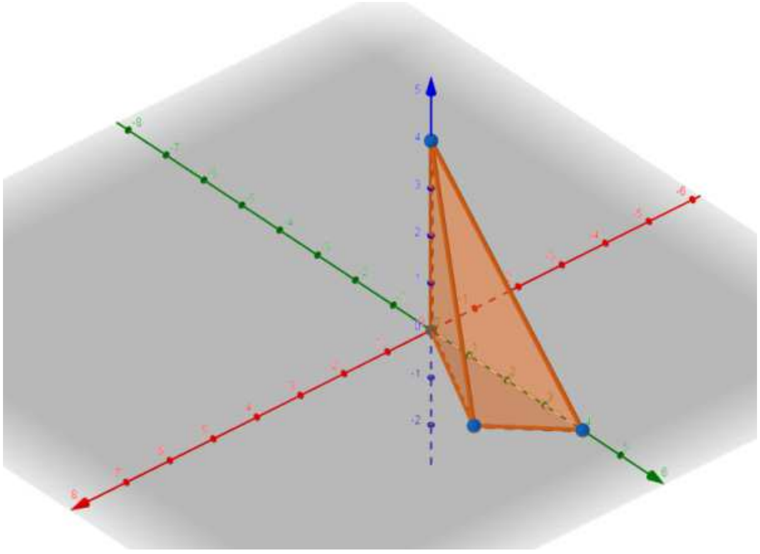


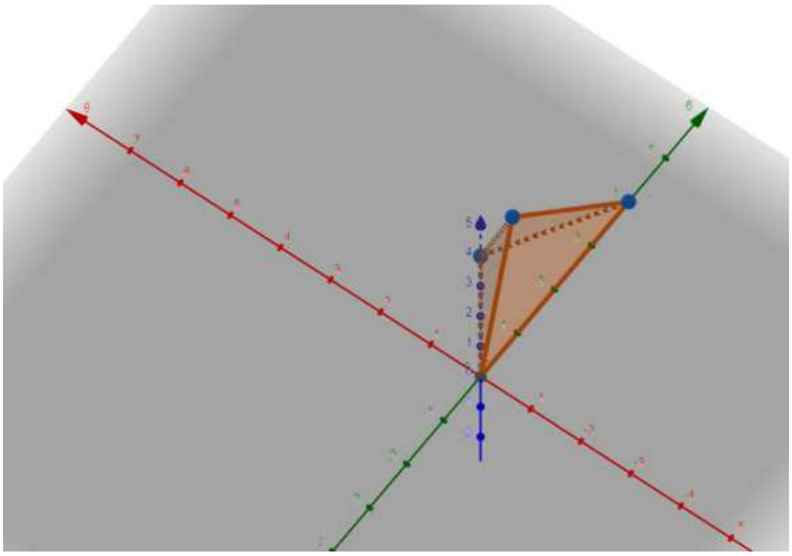
En la figura se prueba con el punto $P = (\frac{1}{2}, 2{,}1)$ el cual fue elejido por cumplir las 3 condiciones

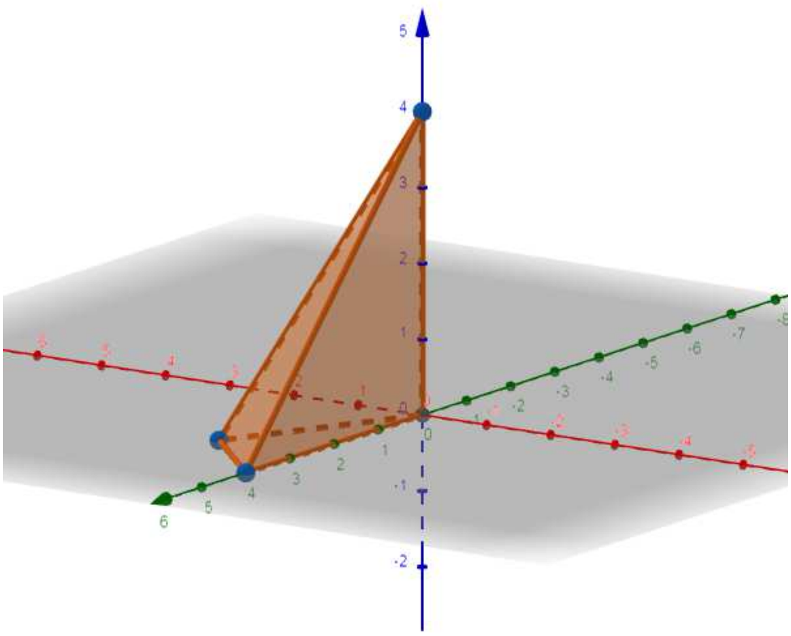
$$\frac{1}{2} + 2 + 1 = \frac{7}{2} \leq 4$$

$$2 \geq \frac{1}{2}$$

$$\frac{1}{2} \geq 0\,, 2 \geq 0, 1 \geq 0$$

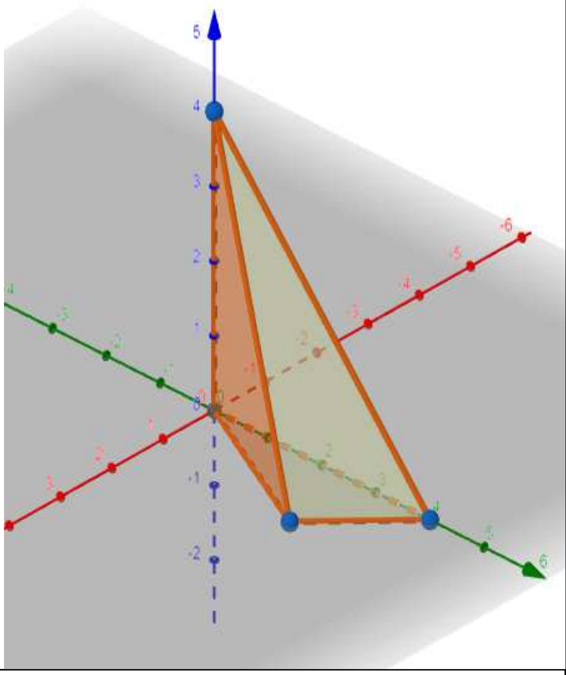




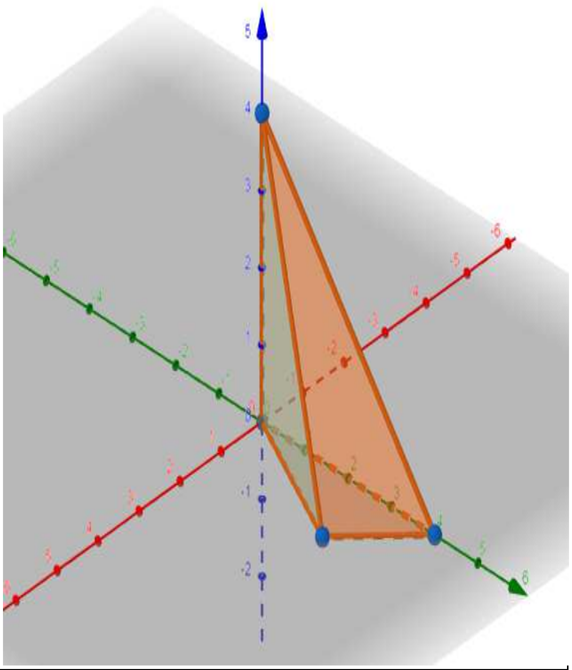


Nombramos las superficies

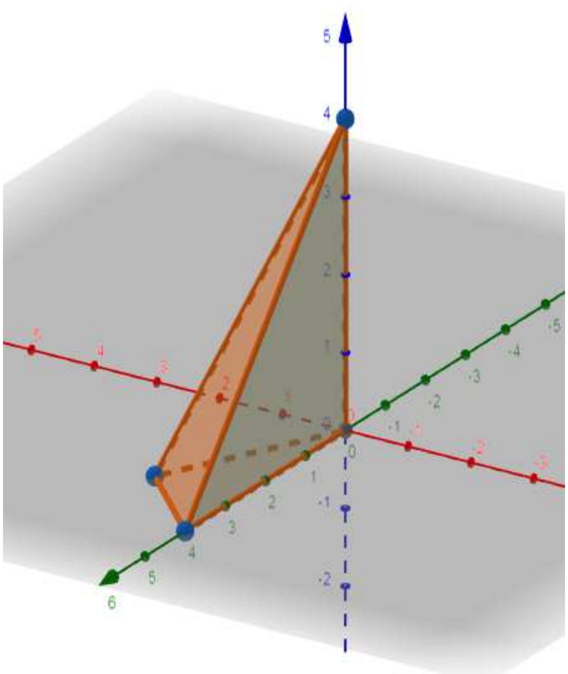
$$S_1:\begin{cases} x+y+z=4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$



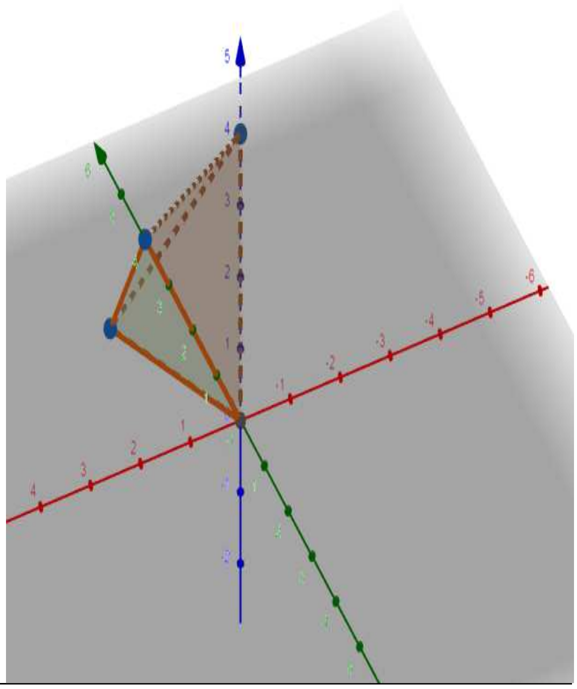
$$S_2:\begin{cases} x+y+z\leq 4 \\ y=2x \\ x\geq 0 \quad z\geq 0 \end{cases}$$



$$S_3:\begin{cases} x+y+z\leq 4 \\ y\geq 2x \\ x=0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$



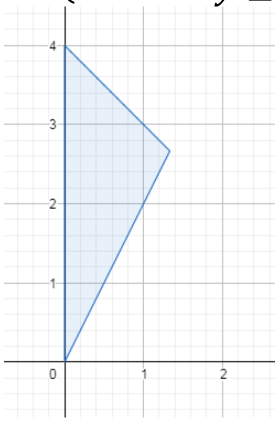
$$S_4:\begin{cases} x+y+z\leq 4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \quad z=0 \end{cases}$$



$$S_1:\begin{cases} x+y+z=4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$

$$S_1:\begin{cases} z=4-x-y \\ y\geq 2x \\ x\geq 0 \\ y\geq 0 \\ 4-x-y\geq 0 \end{cases}$$

$$\begin{cases} \Phi_1(x,y)=(x,y,4-x-y) \\ R_1:\begin{cases} y\geq 2x \\ x\geq 0 \\ y\geq 0 \\ 4-x-y\geq 0 \end{cases} \end{cases}$$



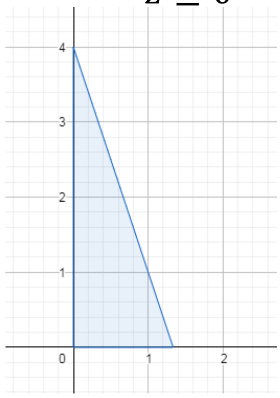
Conviene evaluar la región como tipo2

$$\begin{cases} \Phi_1(x,y)=(x,y,4-x-y) \\ R_1:\begin{cases} 2x\leq y\leq 4-x \\ 0\leq x\leq \dfrac{4}{3} \end{cases} \end{cases}$$

$$S_2:\begin{cases} x+y+z\leq 4 \\ y=2x \\ x\geq 0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$

$$S_2:\begin{cases} 3x+z\leq 4 \\ y=2x \\ x\geq 0 \quad z\geq 0 \end{cases}$$

$$\begin{cases} \Phi_2(x,z)=(x,2x,z) \\ R_2:\begin{cases} 3x+z\leq 4 \\ x\geq 0 \\ z\geq 0 \end{cases} \end{cases}$$

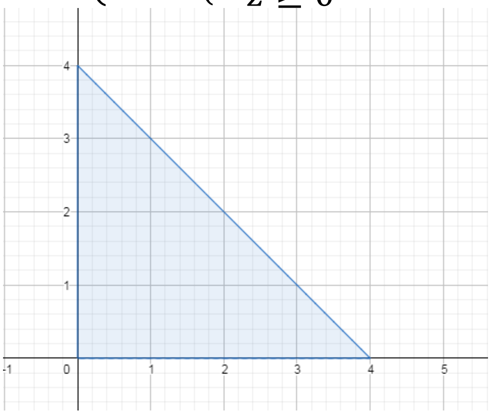


$$\begin{cases} \Phi_2(x,z)=(x,2x,z) \\ R_2:\begin{cases} 0\leq z\leq 4-3x \\ 0\leq x\leq \dfrac{4}{3} \end{cases} \end{cases}$$

$$S_3:\begin{cases} x+y+z\leq 4 \\ y\geq 2x \\ x=0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$

$$S_3:\begin{cases} 0+y+z\leq 4 \\ y\geq 2(0) \\ x=0 \quad y\geq 0 \quad z\geq 0 \end{cases}$$

$$\begin{cases} \Phi_3(y,z)=(0,y,z) \\ R_3:\begin{cases} y+z\leq 4 \\ y\geq 0 \\ z\geq 0 \end{cases} \end{cases}$$

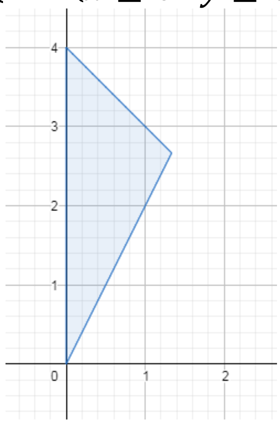


$$\begin{cases} \Phi_3(y,z)=(0,y,z) \\ R_3:\begin{cases} 0\leq z\leq 4-y \\ 0\leq y\leq 4 \end{cases} \end{cases}$$

$$S_4:\begin{cases} x+y+z\leq 4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \quad z=0 \end{cases}$$

$$S_4:\begin{cases} x+y\leq 4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \quad z=0 \end{cases}$$

$$\begin{cases} \Phi(x,y)=(x,y,0) \\ R_4:\begin{cases} x+y\leq 4 \\ y\geq 2x \\ x\geq 0 \quad y\geq 0 \end{cases} \end{cases}$$



Conviene evaluar la región como tipo2

$$\begin{cases} \Phi_4(x,y)=(x,y,0) \\ R_4:\begin{cases} 2x\leq y\leq 4-x \\ 0\leq x\leq \dfrac{4}{3} \end{cases} \end{cases}$$