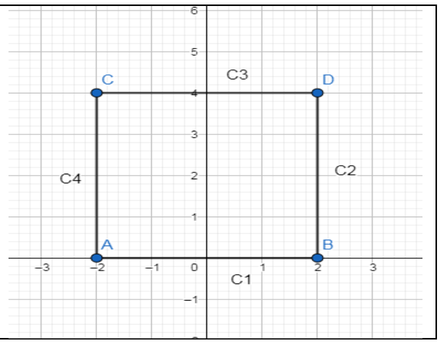
T P 08 Ej. 9-c-Modificado

Calcular la integral de línea para el campo y el camino dado.

$$\int_C \left(\frac{x}{y-1}\right) dx + \left(\frac{y}{x-1}\right) dy$$

C es el cuadrado de vértices $\begin{cases} A = (-2,0) \\ B = (2,0) \\ C = (-2,4) \\ D = (2,4) \end{cases}$



En este ejercicio se puede otro tipo de notación, Traducible como:

$$\int_C P\,dx + Q dy \;\rightarrow F(x,y) = (P,Q) \rightarrow \int_a^b F(\bar{r}(t)) \cdot \bar{r}'(t)\,dt$$

Donde a y b son los límites de variación de la variable t.

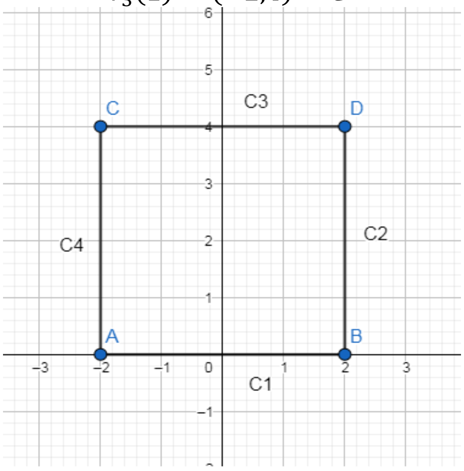
$$\int_C P\,dx + Q dy = \int_C F(\bar{r}(t)) \cdot \bar{r}'(t)\,dt$$

Parametrizando:

$$C3:\begin{cases} r_3(t) = D + t(C - D) \\ r_3(t) = (-4t + 2{,}4) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$$r_3(0) = (2{,}4) = D$$
$$r_3(1) = (-2{,}4) = C$$



$$C4:\begin{cases} r_4(t) = C + t(A - C) \\ r_4(t) = (-2, -4t + 4) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$$r_4(0) = (-2{,}4) = C$$
$$r_4(1) = (-2{,}0) = A$$

$$C2:\begin{cases} r_2(t) = B + t(D - B) \\ r_2(t) = (2{,}4t) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

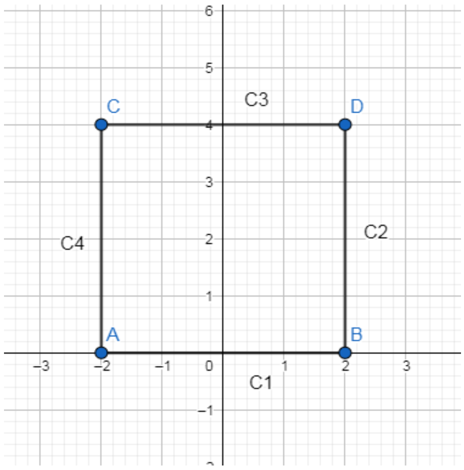
$$r_2(0) = (2{,}0) = B$$
$$r_2(1) = (2{,}4) = D$$

$$C1:\begin{cases} r_1(t) = A + t(B - A) \\ r_1(t) = (4t - 2{,}0) \\ 0 \leq t \leq 1 \\ Sentido + \end{cases}$$

Verificación:

$$r_1(0) = (-2{,}0) = A$$
$$r_1(1) = (2{,}0) = B$$

Construimos entonces cada una de las expresiones que precisamos para la integral.

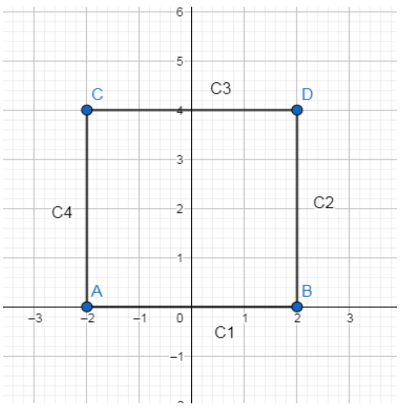


$$C2:\begin{cases} r_2(t) = (2{,}4t) \\ r'(t) = (0{,}4) \\ \\ F\big(r(t)\big) = \left(\dfrac{2}{4t-1}, 4t\right) \\ F\big(r(t)\big)r'^{(t)} = 16[t] \\ \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$C1:\begin{cases} r_1(t) = (4t-2{,}0) \\ r'(t) = (4{,}0) \\ \\ F\big(r(t)\big) = (-4t+2{,}0) \\ F\big(r(t)\big)r'(t) = 8[-2t+1] \\ \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$C3:\begin{cases} r_3(t) = (-4t+2{,}4) \\ r'(t) = (-4{,}0) \\ \\ F\big(r(t)\big) = \left(\dfrac{-4t+2}{3}, \dfrac{4}{-4t+1}\right) \\ F\big(r(t)\big)r'(t) = -\dfrac{8}{3}[-2t+1] \\ \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$

$$C4:\begin{cases} r_4(t) = (-2, -4t+4) \\ r'(t) = (0, -4) \\ \\ F\big(r(t)\big) = \left(\dfrac{-2}{-4t+3}, \dfrac{-4t+4}{-3}\right) \\ F\big(r(t)\big)r'(t) = \dfrac{16}{3}[-t+1] \\ 0 \le t \le 1 \\ Sentido + \end{cases}$$



$$I_1 = 8\int_0^1 [-2t+1]dt = 8[-t^2+t]_0^1 = 0$$

$$I_2 = 16\int_0^1 [t]dt = 16\left[\frac{t^2}{2}\right]_0^1 = 8$$

$$I_3 = -\frac{8}{3}\int_0^1 [-2t+1]dt = -\frac{8}{3}[-t^2+t]_0^1 = 0$$

$$I_4 = \frac{16}{3}\int_0^1 [-t+1]dt = \frac{16}{3}\left[-\frac{t^2}{2}+t\right]_0^1 = \frac{8}{3}$$

Por lo tanto:

$$\int_C F\big(\bar{r}(t)\big) \cdot \bar{r}'(t)\,dt = 0+8+0+\frac{8}{3} = \frac{32}{3}$$