T P 10 ej de parcial

Hallar la solución general de la siguiente ecuación diferencial de primer orden no homogénea, y encontrar la ecuación particular $y(1) = 0$.

$$y' + \frac{y}{x} = sen(x)$$

$$y = uv$$

$$y' = u'v + uv'$$

$$u'v + uv' + \frac{uv}{x} = sen(x)$$

$$\underbrace{(u' + \frac{u}{x})}_{0} v + uv' = sen(x)$$

$$u' + \frac{u}{x} = 0$$

$$u' = -\frac{u}{x}$$

$$u' = \frac{du}{dx}$$

$$\frac{du}{dx} = -\frac{u}{x}$$

$$\frac{du}{u} = -\frac{dx}{x}$$

$$\int \frac{du}{u} = -\int \frac{dx}{x}$$

$$\ln(|u|) + c_1 = -\ln(|x|) + c_2$$

$$\ln(|u|) = -\ln(|x|) + c_2 - c_1$$

$$\ln(|u|) = \ln(|x|^{-1}) + \ln(c_3)$$

$$\ln(|u|) = \ln(\frac{c_3}{|x|})$$

$$|u| = \frac{c_3}{|x|}$$

$$|u| \rightarrow \begin{cases} u = \frac{c_3}{|x|} \rightarrow \begin{cases} u = \frac{c_3}{x} \\ u = -\frac{c_3}{x} \end{cases} \\ u = -\frac{c_3}{|x|} \rightarrow \begin{cases} u = -\frac{c_3}{x} \\ u = -\left(-\frac{c_3}{x}\right) \end{cases} \end{cases} \rightarrow u = \frac{k}{x}$$

$$Tomamos\ k = 1$$

$$u = \frac{1}{x}$$

$$\underbrace{(u' + \frac{u}{x})}_{u=\frac{1}{x}} v + uv' = sen(x)$$

$$\frac{v'}{x} = sen(x)$$

$$v' = xsen(x)$$

$$v = \int xsen(x)dx$$

$$u_1 = x \rightarrow du_1 = dx$$

$$dv_1 = sen(x)dx \rightarrow v_1 = -\cos(x)$$

$$v = \int xsen(x)dx = -xcos(x) - \int -\cos(x)\,dx$$

$$v = \int xsen(x)dx = -xcos(x) + \int \cos(x)\,dx$$

$$v = \int xsen(x)dx = -xcos(x) + sen(x) + C$$

$$v = -xcos(x) + sen(x) + C$$

Solución general:

$$y = uv = \frac{1}{x}(-xcos(x) + sen(x) + C)$$

$$y(1) = 0$$

$$0 = \frac{1}{1}(-1cos(1) + sen(1) + C)$$

$$-\left(-1cos(1) + sen(1)\right) = C$$

$$cos(1) - sen(1) = C$$

Solución particular para $y(1) = 0$

$$y = \frac{1}{x}\left(-xcos(x) + sen(x) + cos(1) - sen(1)\right)$$