Resolución TP8:

Ejercicio 16-d

Verificar que el siguiente campo es conservativo y hallar su función potencial:

$$F(x,y) = (y - \pi y sen(\pi xy)\ , x - \pi x sen(\pi xy))$$

Preparación:

si $F(x,y) = (P(x,y), Q(x,y))$

entonces

$$P(x,y) = y - \pi y sen(\pi xy)$$

$$Q(x,y) = x - \pi x sen(\pi xy)$$

Verificación:

Si Existe $f(x,y)$ tal que $\nabla f(x,y) = F(x,y)$ entonces

$$f_x = P \quad f_y = Q$$

$$f_{xy} = P_y \quad f_{yx} = Q_x$$

Por lo que el teorema de Swarchz aplica de la siguiente manera

$$f_{xy} = f_{yx} \rightarrow P_y = Q_x$$

En este caso:

$$P(x,y) = 2x + 2y \rightarrow P_y = 1 - \pi(sen(\pi xy) + y cos(\pi xy)\pi x)$$

$$Q(x,y) = 2x + 2y \rightarrow Q_x = 1 - \pi(sen(\pi xy) + x cos(\pi xy)\pi y)$$

Se verifica que $\nabla f(x,y) = F(x,y)$

Función Potencial

Método I: $f(x,y) = h(x,y) + \psi(y)\ con \begin{cases} h(x,y) = \int P(x,y)dx \\ \psi'(y) = Q(x,y) - h_y(x,y) \end{cases}$

Método II: $f(x,y) = k(x,y) + \varphi(x)\ con \begin{cases} g(x,y) = \int Q(x,y)dy \\ \varphi'(x) = P(x,y) - g_x(x,y) \end{cases}$

Método III: $f(x,y) = \int P(x,y)dx + \psi(y) = \int Q(x,y)dy + \varphi(x)$

Función Potencial, Método I:

$$h(x,y) = \int P(x,y)dx$$

$$h(x,y) = \int (y - \pi y sen(\pi xy))dx = xy + \cos(\pi xy)$$

$$h_y(x,y) = x - \pi x sen(\pi xy)$$

$$\psi'(y) = Q(x,y) - h_y(x,y)$$

$$\psi'(y) = x - \pi x sen(\pi xy) - (x - \pi x sen(\pi xy)) = 0$$

$$\psi(y) = \int 0 dy = k$$

$$f(x,y) = h(x,y) + \psi(y)$$

$$f(x,y) = xy + \cos(\pi xy) + k$$

Función Potencial, Método II:

$$g(x,y)=\int Q(x,y)dy$$

$$g(x,y)=\int(x-\pi x sen(\pi xy))dy=xy+\cos(\pi xy)$$

$$g_x(x,y)=y-\pi y sen(\pi xy)$$

$$\varphi'(x)=P(x,y)-g_x(x,y)$$

$$\varphi'(x)=y-\pi y sen(\pi xy)-(y-\pi y sen(\pi xy))=0$$

$$\varphi(x)=\int 0dx=k$$

$$f(x,y)=g(x,y)+\varphi(x)$$

$$f(x,y)=xy+\cos(\pi xy)+k$$

Función Potencial, Método III:

$$f(x,y) = \int P(x,y)dx + \psi(y) = \int Q(x,y)dy + \varphi(x)$$

$$\int (y - \pi y sen(\pi xy))dx + \psi(y) = \int (x - \pi x sen(\pi xy))dy + \varphi(x)$$

$$xy + \cos(\pi xy) + k + \psi(y) = xy + \cos(\pi xy) + k + \varphi(x)$$

$$\psi(y) = \varphi(x)$$

$$\psi(y) = k$$

$$\varphi(x) = k$$

$$f(x,y) = \int P(x,y)dx + \psi(y) = \int Q(x,y)dy + \varphi(x)$$

$$f(x,y) = \int P(x,y)dx + k = \int Q(x,y)dy + k$$

$$f(x,y) = xy + \cos(\pi xy) + k + k = xy + \cos(\pi xy) + k + k$$

$$f(x,y) = xy + \cos(\pi xy) + k$$