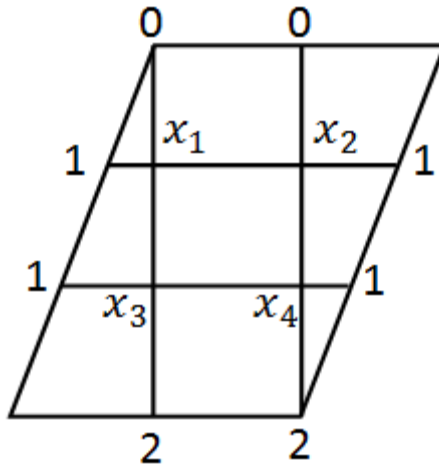


EJERCICIO RESUELTO DE TRASMISIÓN DE CALOR

Resuelto por la Profesora Julieta Matteucci

Calcular las temperaturas x_1 , x_2 , x_3 y x_4 en la placa metálica del siguiente esquema utilizando la propiedad de la temperatura media.



Resolución:

Armamos las ecuaciones:

$$\begin{aligned}x_1 &= \frac{1 + 0 + x_2 + x_3}{4} && \rightarrow 4x_1 - x_2 - x_3 = 1 \\x_2 &= \frac{1 + 0 + x_1 + x_4}{4} && \rightarrow 4x_2 - x_1 - x_4 = 1 \\x_3 &= \frac{1 + 2 + x_1 + x_4}{4} && \rightarrow 4x_3 - x_1 - x_4 = 3 \\x_4 &= \frac{1 + 2 + x_2 + x_3}{4} && \rightarrow 4x_4 - x_2 - x_3 = 3\end{aligned}$$

Resolvemos por el método de Gauss:

$$\left(\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 1 \\ -1 & 4 & 0 & -1 & 1 \\ -1 & 0 & 4 & -1 & 3 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right)$$

$$\begin{aligned}
& \left(\begin{array}{cccc|c} 4 & -1 & -1 & 0 & 1 \\ -1 & 4 & 0 & -1 & 1 \\ -1 & 0 & 4 & -1 & 3 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right) \rightarrow F_1 + 3F_2 \left(\begin{array}{cccc|c} 1 & 11 & -1 & -3 & 4 \\ -1 & 4 & 0 & -1 & 1 \\ -1 & 0 & 4 & -1 & 3 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right) \\
& \rightarrow \begin{matrix} F_1 + F_2 \\ F_1 + F_3 \end{matrix} \left(\begin{array}{cccc|c} 1 & 11 & -1 & -3 & 4 \\ 0 & 15 & -1 & -4 & 5 \\ 0 & 11 & 3 & -4 & 7 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right) \rightarrow \begin{matrix} 15F_4 + F_2 \\ 11F_4 + F_3 \end{matrix} \left(\begin{array}{cccc|c} 1 & 11 & -1 & -3 & 4 \\ 0 & 0 & -16 & 56 & 50 \\ 0 & 0 & -8 & 40 & 40 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right) \\
& \rightarrow F_2 - 2F_3 \left(\begin{array}{cccc|c} 1 & 11 & -1 & -3 & 4 \\ 0 & 0 & 0 & -24 & -30 \\ 0 & 0 & -8 & 40 & 40 \\ 0 & -1 & -1 & 4 & 3 \end{array} \right)
\end{aligned}$$

Rearmamos las ecuaciones y despejamos:

$$\begin{aligned}
& \left\{ \begin{array}{l} x_1 + 11x_2 - x_3 - 3x_4 = 4 \\ -24x_4 = -30 \rightarrow x_4 = \frac{5}{4} \\ -8x_3 + 40x_4 = 40 \\ -x_2 - x_3 + 4x_4 = 3 \end{array} \right. \rightarrow \left\{ \begin{array}{l} x_1 + 11x_2 - x_3 - 3\frac{5}{4} = 4 \\ x_4 = \frac{5}{4} \\ -8x_3 + 40\frac{5}{4} = 40 \rightarrow x_3 = \frac{5}{4} \\ -x_2 - x_3 + 4\frac{5}{4} = 3 \end{array} \right. \\
& \rightarrow \left\{ \begin{array}{l} x_1 + 11x_2 - \frac{5}{4} - \frac{15}{4} = 4 \\ x_4 = \frac{5}{4} \\ x_3 = \frac{5}{4} \\ -x_2 - \frac{5}{4} + 5 = 3 \rightarrow x_2 = \frac{3}{4} \end{array} \right. \rightarrow \left\{ \begin{array}{l} x_1 = \frac{3}{4} \\ x_4 = \frac{5}{4} \\ x_3 = \frac{5}{4} \\ x_2 = \frac{3}{4} \end{array} \right. \rightarrow \left\{ \begin{array}{l} x_1 = \frac{3}{4} \\ x_2 = \frac{3}{4} \\ x_3 = \frac{5}{4} \\ x_4 = \frac{5}{4} \end{array} \right.
\end{aligned}$$

Entonces:

$$x_1 = \frac{3}{4} \circ C, x_2 = \frac{3}{4} \circ C, x_3 = \frac{5}{4} \circ C, x_4 = \frac{5}{4} \circ C$$