Resolución TP7:

Ejercicio 4 - c

Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dx dy$$

$$R1: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq 0 \ \wedge y \geq |x|\}$$

$$R2: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq 0 \ \wedge y \leq |x|\}$$

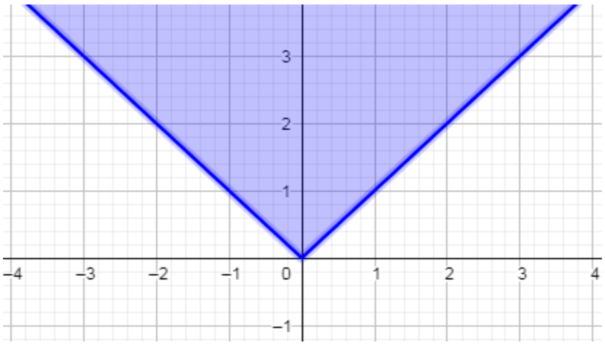
$$R3: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq |y| \ \wedge y \geq 0\}$$

$$R4: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \leq |y| \ \wedge y \geq 0\}$$

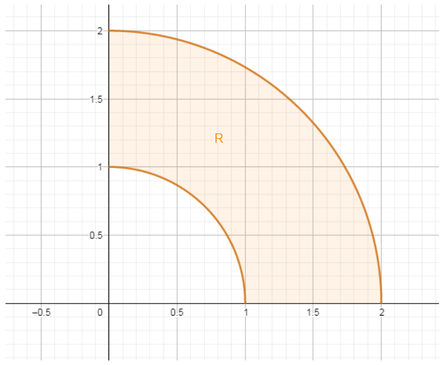
1- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

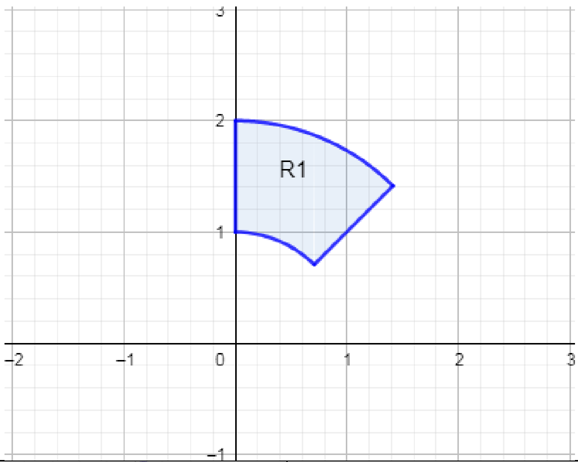
$$R1: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq 0 \ \wedge y \geq |x|\}$$

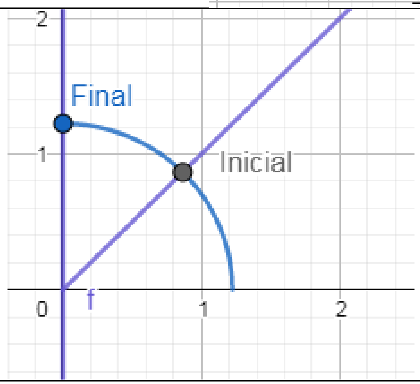


$y \geq |x|$



$$1 \ \leq x^2 + y^2 \leq 4$$
$$0 \ \leq x$$
$$0 \ \leq y$$





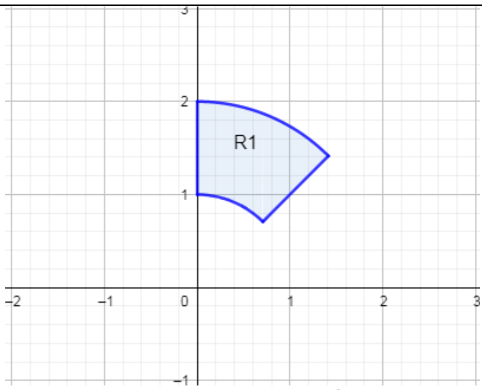
Para obtener αFinal e αInicial se usan las funciones Trigonometricas. Las reglas SOH CAH TOA segun corresponda

Para αFinal => x=0

$$\cos(\alpha Final\,) = \frac{x}{H} = 0 ==> \ \alpha\text{Final} = \frac{\pi}{2}$$

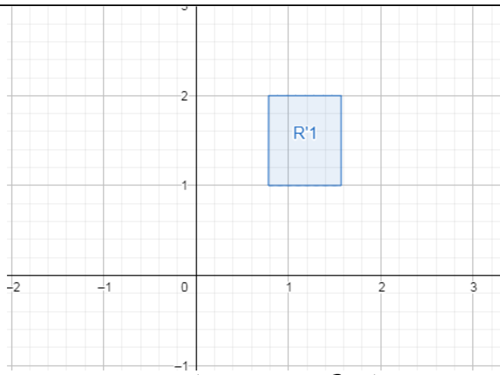
Para αInicial=> x=y

$$\tan(\alpha\text{Inicial}\,) = \frac{y}{x} = 1 ==> \ \alpha\text{Inicial} = \frac{\pi}{4}$$



$$R1 = \left\{ \begin{matrix} (x, y)\epsilon R^2/ \\ 1 \ \le x^2 + y^2 \le 4 \\ \wedge \\ x \ \ge 0 \ \wedge y \ge |x| \end{matrix} \right\}$$

=>



$$R'1: \left\{ \begin{matrix} (r, \alpha)\epsilon R^2/ \\ 1 \ \le r \le 2 \\ \wedge \\ \frac{\pi}{4} \le \alpha \le \frac{\pi}{2} \end{matrix} \right\}$$

$$I = \iint_R x + ydxdy = \int_1^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (cos\alpha + sen\alpha)\, d\alpha dr$$

$$I = \int_1^2 r^2\, [sen\alpha - cos\alpha]_{\frac{\pi}{4}}^{\frac{\pi}{2}} dr$$

$$I = \int_1^2 r^2\, [\left(sen\frac{\pi}{2} - cos\frac{\pi}{2}\right) - (sen\frac{\pi}{4} - cos\frac{\pi}{4})]dr$$

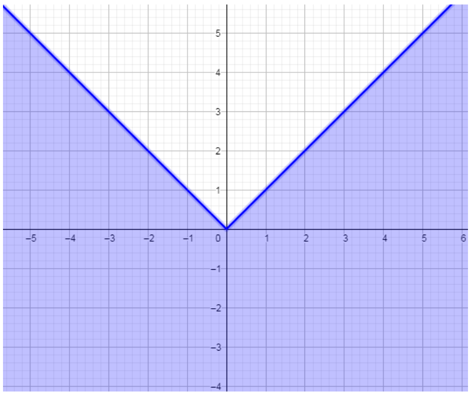
$$I = \int_1^2 r^2\, [(1 - 0) - (\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2})]dr$$

$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8 - 1}{3} = \frac{7}{3}$$

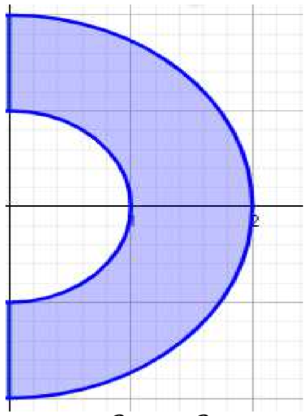
2- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_{R} x + ydxdy$$

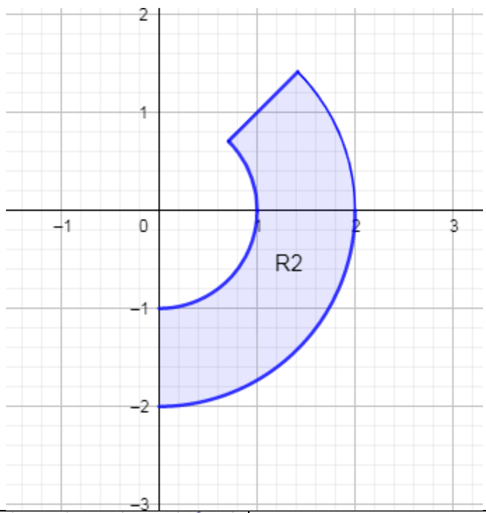
$$R2: \{(x, y)\epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq 0 \ \wedge y \leq |x|\}$$

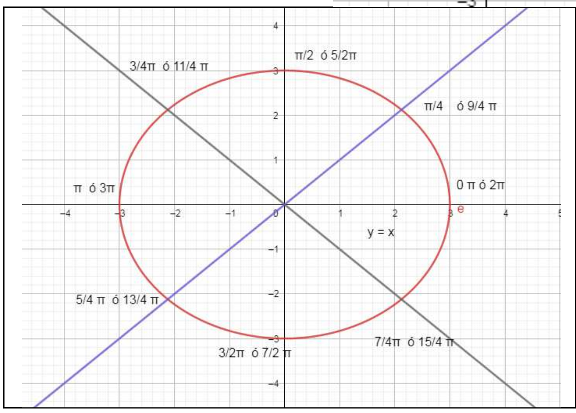


$$y \leq |x|$$



$$1 \ \leq x^2 + y^2 \leq 4$$
$$0 \ \leq x$$





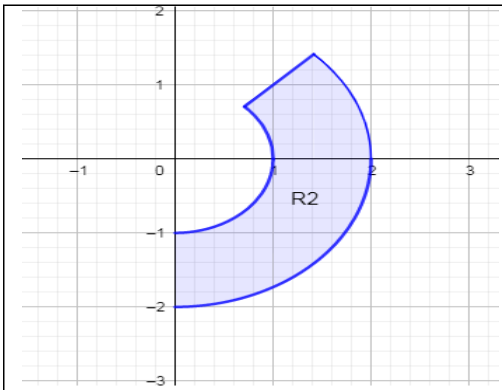
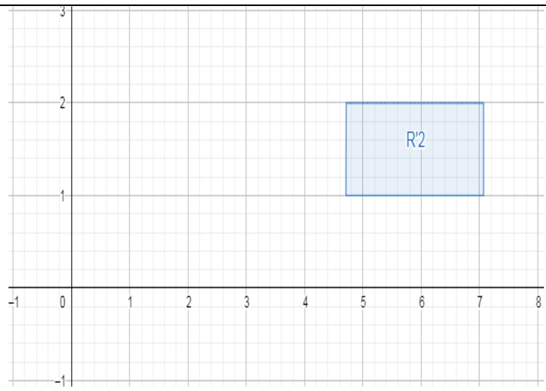
Para obtener αFinal e αInicial se usan las funciones Trigonometricas. Las reglas SOH CAH TOA segun corresponda AL CUADRANTE y la continuidad DEL GIRO

Para αFinal => x=y

$$\tan(\alpha Final) = \frac{y}{x} = 1 ==> \ \alpha\text{Final} = \frac{9}{4}\pi$$

Para αInicial=> x=0

$$\cos(\alpha\text{Inicial}) = \frac{x}{H} = 0 => \alpha\text{Inicial} = \frac{3}{2}\pi$$

 $R2 = \left\{ \begin{matrix} (x,y)\epsilon R^2/ \\ 1 \ \leq x^2 + y^2 \leq 4 \\ \wedge \\ x \ \geq 0 \ \wedge y \leq \lvert x \rvert \end{matrix} \right\}$	=>	 $R'2\colon \left\{ \begin{matrix} (r,\alpha)\epsilon R^2/ \\ 1 \ \leq r \leq 2 \\ \wedge \\ \frac{3}{2}\pi \leq \alpha \leq \frac{9}{4}\pi \end{matrix} \right\}$

$$I = \iint_R x + ydxdy = \int_1^2 \int_{\frac{3}{2}\pi}^{\frac{9}{4}\pi} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_{\frac{3}{2}\pi}^{\frac{9}{4}\pi} (cos\alpha + sen\alpha)\, d\alpha dr$$

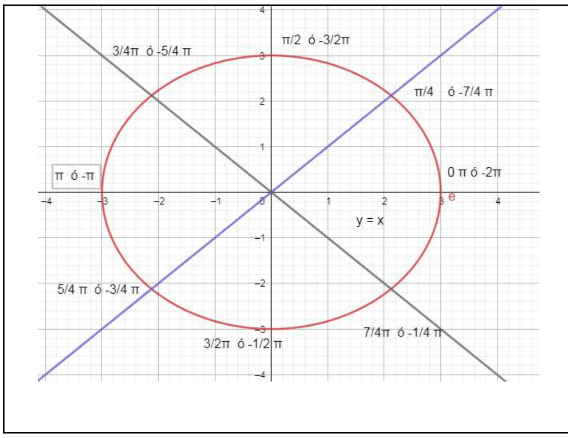
$$I = \int_1^2 r^2 \, [sen\alpha - cos\alpha]_{\frac{3}{2}\pi}^{\frac{9}{4}\pi} dr$$

$$I = \int_1^2 r^2 \, [\left(sen\frac{9}{4}\pi - cos\frac{9}{4}\pi\right) - (sen\frac{3}{2}\pi - cos\frac{3}{2}\pi)]dr$$

$$I = \int_1^2 r^2 \, [\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - ((-1) - 0)]dr$$

$$I = \int_1^2 r^2 \, [\,(0) - (-1)]dr$$

$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8-1}{3} = \frac{7}{3}$$



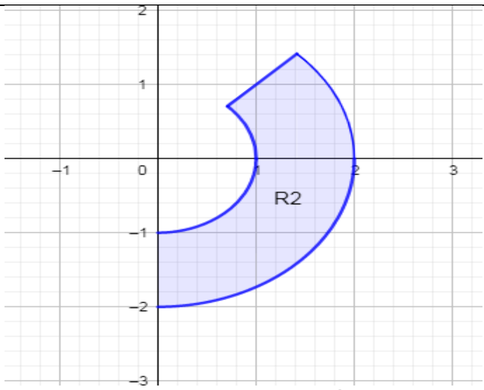
Para obtener αFinal e αInicial se usan las funciones Trigonometricas. Las reglas SOH CAH TOA segun corresponda AL CUADRANTE y la continuidad DEL GIRO empezando en valores negativos

Para αFinal => x=y

$$\tan(\alpha Final) = \frac{y}{x} = 1 ==> \ \text{αFinal} = \frac{1}{4}\pi$$

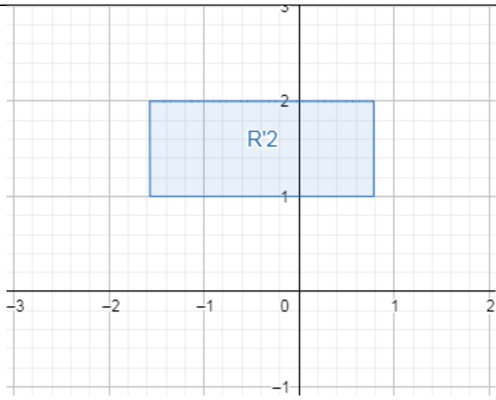
Para αInicial=> x=0

$$\cos(\alpha\text{Inicial}) = \frac{x}{H} = 0 => \text{αInicial} = -\frac{1}{2}\pi$$



$$R2 = \left\{ \begin{array}{c} (x,y)\epsilon R^2/ \\ 1 \ \leq x^2 + y^2 \leq 4 \\ \wedge \\ x \ \geq 0 \ \wedge y \leq |x| \end{array} \right\}$$

=>



$$R'2: \left\{ \begin{array}{c} (r,\alpha)\epsilon R^2/ \\ 1 \ \leq r \leq 2 \\ \wedge \\ -\frac{1}{2}\pi \leq \alpha \leq \frac{1}{4}\pi \end{array} \right\}$$

$$I = \iint_R x + ydxdy = \int_1^2 \int_{\frac{-1}{2}\pi}^{\frac{1}{4}\pi} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \,[sen\alpha - cos\alpha]_{\frac{-1}{2}\pi}^{\frac{1}{4}\pi} dr$$

$$I = \int_1^2 r^2 \,[\left(sen\frac{1}{4}\pi - cos\frac{1}{4}\pi\right) - (sen\frac{-1}{2}\pi - cos\frac{-1}{2}\pi)]dr$$

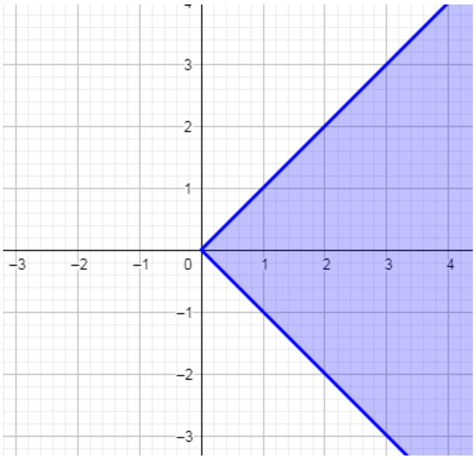
$$I = \int_1^2 r^2 \,[\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - ((-1) - 0)]dr$$

$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8-1}{3} = \frac{7}{3}$$

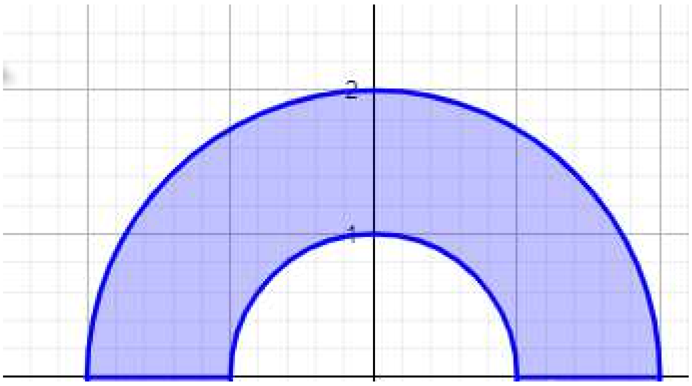
3- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

$$R3: \{(x, y)\epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq |y| \ \wedge y \geq 0\}$$

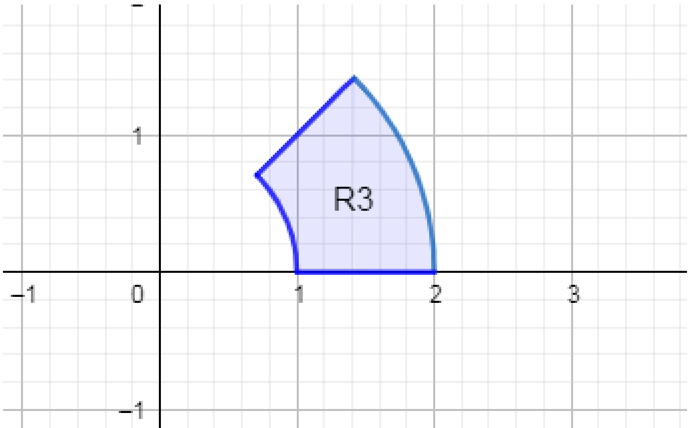


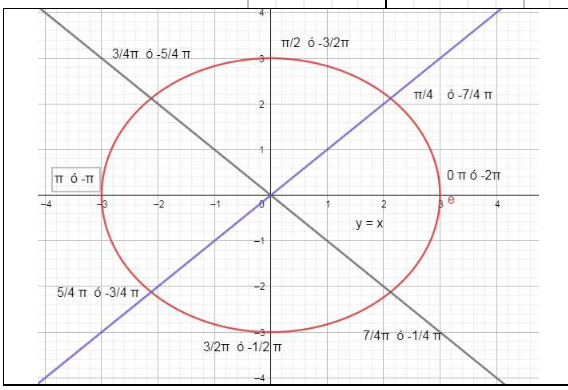
$x \geq |y|$



$1 \ \leq x^2 + y^2 \leq 4$

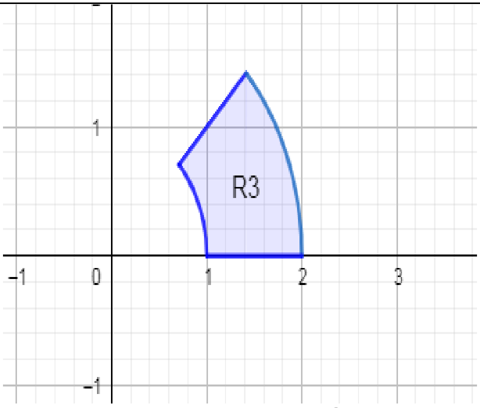
$0 \ \leq y$





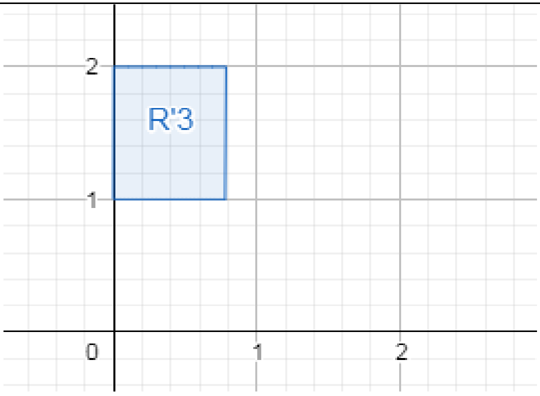
$$\alpha\text{Final} = \frac{\pi}{4}$$

$$\alpha\text{Inicial} = 0$$



$$R3 = \left\{ \begin{matrix} (x,y)\epsilon R^2/ \\ 1 \ \leq x^2 + y^2 \leq 4 \\ \wedge \\ x \geq |y| \ \wedge y \geq 0 \end{matrix} \right\}$$

=>



$$R'1{:} \left\{ \begin{matrix} (r,\alpha)\epsilon R^2/ \\ 1 \ \leq r \leq 2 \\ \wedge \\ 0 \leq \alpha \leq \frac{\pi}{4} \end{matrix} \right\}$$

$$I = \iint_R x + ydxdy = \int_1^2 \int_0^{\frac{\pi}{4}} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_0^{\frac{\pi}{4}} (cos\alpha + sen\alpha)\, d\alpha dr$$

$$I = \int_1^2 r^2\, [sen\alpha - cos\alpha]_0^{\frac{\pi}{4}} dr$$

$$I = \int_1^2 r^2\, [\left(sen\frac{\pi}{4} - cos\frac{\pi}{4}\right) - (sen\ 0 - \cos 0)]dr$$

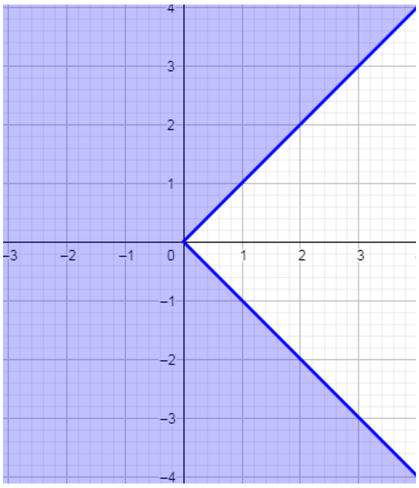
$$I = \int_1^2 r^2\, [\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) - (0 - 1)]dr$$

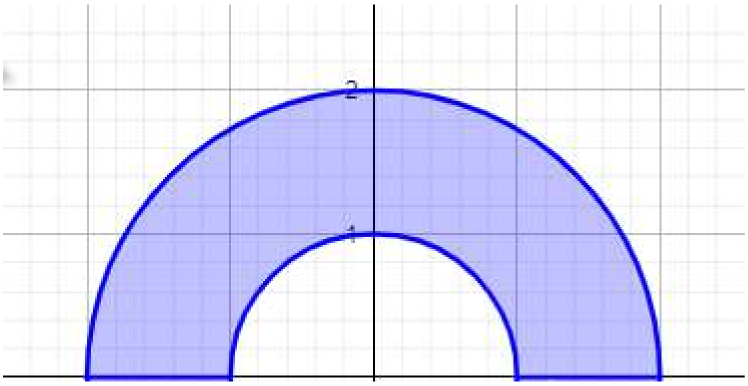
$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8-1}{3} = \frac{7}{3}$$

4- Graficar las regiones de integración dados y resolver la integral I.

$$I = \iint_R x + y dxdy$$

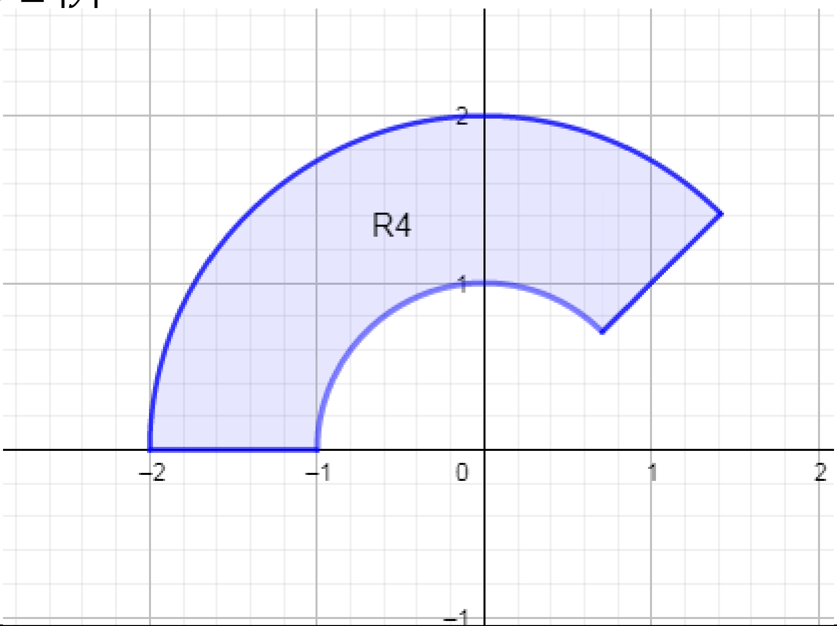
$$R4: \{(x, y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \leq |y| \ \wedge y \geq 0\}$$

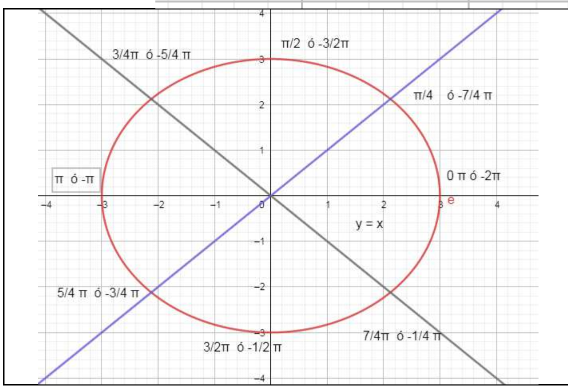




$$1 \ \leq x^2 + y^2 \leq 4$$
$$0 \ \leq y$$

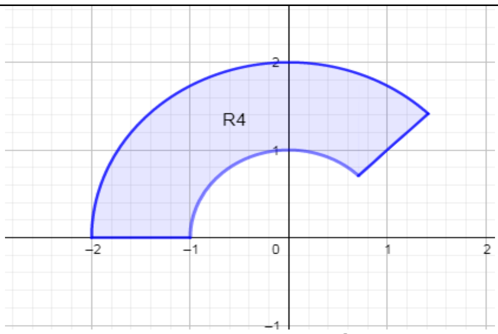
$x \leq |y|$





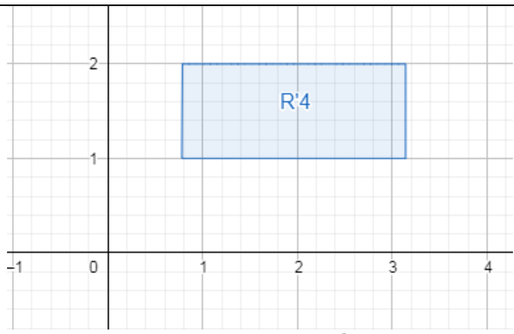
$$\alpha\text{Final} = \pi$$

$$\alpha\text{Inicial} = \frac{\pi}{4}$$



$$R1 = \left\{ \begin{matrix} (x,y)\epsilon R^2/ \\ 1 \ \leq x^2 + y^2 \leq 4 \\ \wedge \\ x \leq |y| \ \wedge y \geq 0 \end{matrix} \right\}$$

=>



$$R'1\colon \left\{ \begin{matrix} (r,\alpha)\epsilon R^2/ \\ 1 \ \leq r \leq 2 \\ \wedge \\ \frac{\pi}{4} \leq \ \alpha \ \leq \pi \end{matrix} \right\}$$

$$I = \iint_R x + ydxdy = \int_1^2 \int_{\frac{\pi}{4}}^{\pi} (r^2(cos\alpha + sen\alpha))\, d\alpha dr$$

$$I = \int_1^2 r^2 \int_{\frac{\pi}{4}}^{\pi} (cos\alpha + sen\alpha)\, d\alpha dr$$

$$I = \int_1^2 r^2 \, [sen\alpha - cos\alpha]_{\frac{\pi}{4}}^{\pi} dr$$

$$I = \int_1^2 r^2 \, [(sen\pi - cos\pi) - (sen\frac{\pi}{4} - \cos\frac{\pi}{4})]dr$$

$$I = \int_1^2 r^2 \, [(0 - (-1)) - \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)]dr$$

$$I = \int_1^2 r^2 dr = [\frac{r^3}{3}]_1^2 = \frac{8-1}{3} = \frac{7}{3}$$