Resolución TP7:

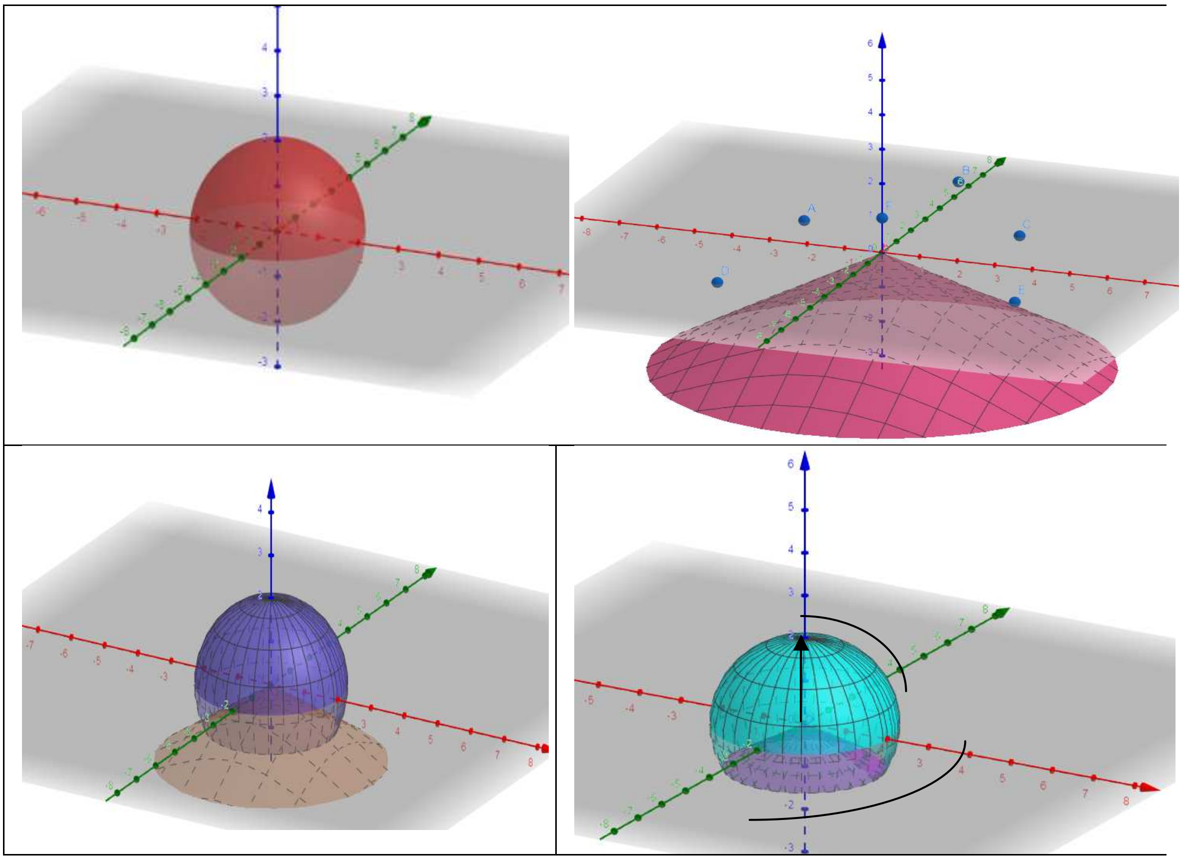
Resolver I usando V

$$V:\{(x,y,z)\epsilon R^3/\ x^2+y^2+z^2 \leq 4\ \wedge z \geq -\frac{\sqrt{3}}{3}\sqrt{x^2+y^2}\}$$

$$z^2 = x^2+y^2 \rightarrow cono\ doble$$

$$z \quad = -\sqrt{x^2+y^2} \rightarrow cono\ negativo$$

$$I = \iiint\limits_V \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}}\,dxdydz$$



Con coordenadas Esfericas

$$V:\begin{cases} x^2+y^2+z^2 \le 4 \\ z \ge -\frac{\sqrt{3}}{3}\sqrt{x^2+y^2} \end{cases}$$

$$V:\begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} 0 \le r \le 2 \\ 0 \le \varphi \le ? \\ 0 \le \theta \le 2\pi \end{cases} \end{cases}$$

$$I = \iiint_V \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}} dxdydz$$

$$I = \iiint_{V'} \frac{e^{x(r,\theta,\varphi)^2+y(r,\theta,\varphi)^2+z(r,\theta,\varphi)^2}}{\sqrt{x(r,\theta,\varphi)^2+y(r,\theta,\varphi)^2+z(r,\theta,\varphi)^2}} |J(r,\theta,\varphi)| drd\theta d\varphi$$

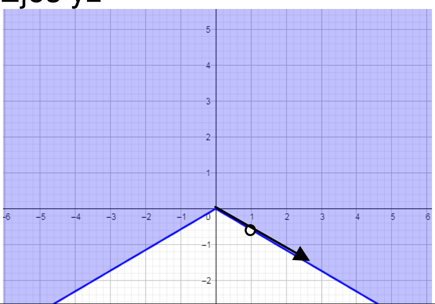
$$I = \iiint_{V'} \frac{e^{r^2}}{\sqrt{r^2}} \overbrace{r^2 sen(\varphi)}^{|J|} drd\theta d\varphi$$

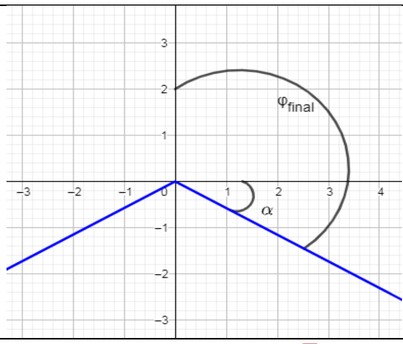
$$I = \iiint_{V'} e^{r^2} r sen(\varphi) drd\theta d\varphi$$

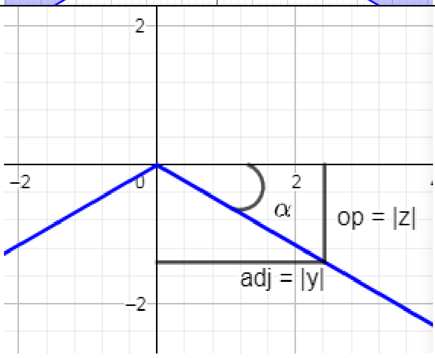
si $x = 0 \rightarrow z \ge -\frac{\sqrt{3}}{3}\sqrt{0^2+y^2} \rightarrow z \ge -\frac{\sqrt{3}}{3}|y|$

$$y = 1 \rightarrow z = -\frac{\sqrt{3}}{3} \; aprox \; z = -\frac{1.7}{3} = 0.5$$

Ejes yz







$$\varphi_{final} = \frac{\pi}{2} + \alpha$$

$$tg(\alpha) = \frac{|z|}{|y|} = \frac{\frac{\sqrt{3}}{3}|-|y||}{|y|} = \frac{\sqrt{3}}{3}$$

$$\alpha = arctg\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\varphi_{final} = \frac{2}{3}\pi$$

si $z = 0 \rightarrow x^2+y^2+0^2 \le 4 \rightarrow x^2+y^2 \le 4 \rightarrow r \le 2 \; 0 \le \theta \le 2\pi$

Con coordenadas Esfericas

$$V:\begin{cases} x^2+y^2+z^2 \le 4 \\ z \ge -\frac{\sqrt{3}}{3}\sqrt{x^2+y^2} \end{cases}$$

$$I = \iiint_{V'} e^{r^2} r sen(\varphi) drd\theta d\varphi$$

$$I = \int_0^{\frac{2}{3}\pi}\int_0^{2\pi}\int_0^{2} e^{r^2} r sen(\varphi) drd\theta d\varphi$$

$V: \begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ \lvert J \rvert = r^2 sen(\varphi) \\ V' = \begin{cases} 0 \leq r \leq 2 \\ 0 \leq \varphi \leq \frac{2}{3}\pi \\ 0 \leq \theta \leq 2\pi \end{cases} \end{cases}$	

$$I = \int_0^{\frac{2}{3}\pi} \int_0^{2\pi} \int_0^{2} e^{r^2} r\, sen(\varphi) drd\theta d\varphi = \int_0^{\frac{2}{3}\pi} sen(\varphi) d\varphi \int_0^{2\pi} d\theta \underbrace{\int_0^{2} e^{r^2} rdr}_{\substack{sustitucion\, r^2=t \\ rdr=\frac{dt}{2}}}$$

$$I = [-\cos(\varphi)]_0^{\frac{2}{3}\pi} [\theta]_0^{2\pi} \left[\frac{e^{r^2}}{2}\right]_0^2$$

$$I = \left[\left(-(-\frac{1}{2})\right) - (-1)\right] [2\pi - 0] \left[\frac{e^4}{2} - \frac{e^0}{2}\right]$$

$$I = \left[\frac{1}{2} + 1\right] [2\pi] \left[\frac{e^4}{2} - \frac{1}{2}\right]$$

$$I = 3\pi \left[\frac{e^4}{2} - \frac{1}{2}\right] = \frac{3}{2}\pi[e^4 - 1]$$

C/A

$$\int e^{r^2} rdr \overset{\substack{r^2=t \\ 2rdr=dt \\ rdr=\frac{dt}{2}}}{\triangleq} \int \frac{e^t dt}{2} = \frac{e^t}{2} \overset{r^2=t}{\triangleq} \frac{e^{r^2}}{2}$$