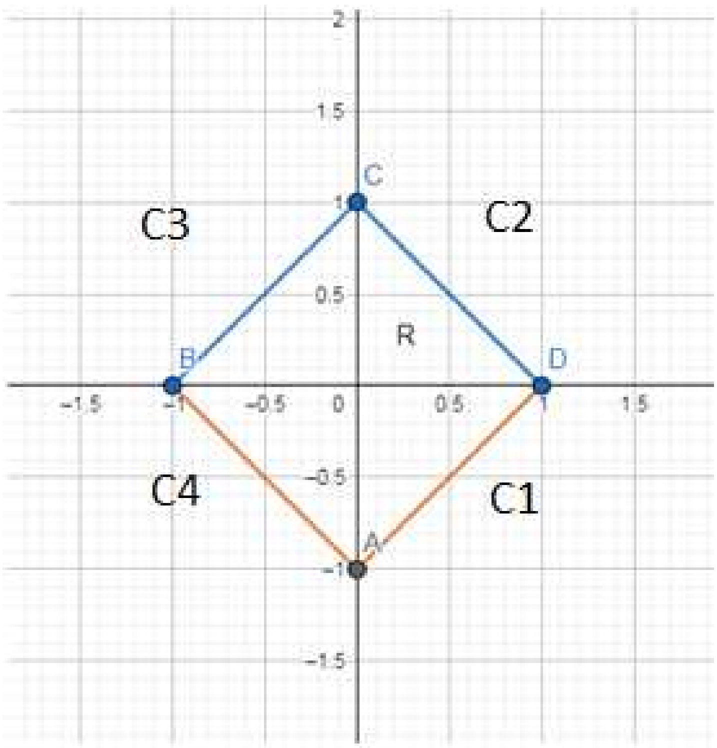
Resolución TP8:

Ejercicio 6-c, Mod. Arco de 3-c

Calcular la integral de campo escalar de la curva definida por:

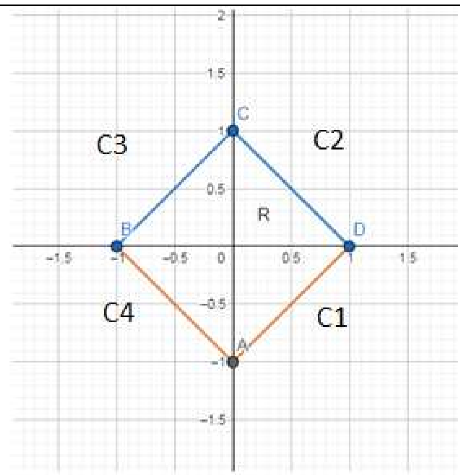
$$C: \{(x, y) \epsilon R^2 / \ |x| + |y| = 1\}$$
$$f(x, y) = x^2 + y^2$$

Resolución:



$A = (0, -1)$
$B = (-1,0)$
$C = (0,1)$
$D = (1,0)$

<table>
<tr>
<td>$C3: \begin{cases} r_3(t) = C + t(B - C) \\ r_3(t) = (-t, 1 - t) \\ 0 \le t \le 1 \\ Sentido + \end{cases}$

$C4: \begin{cases} r_4(t) = B + t(A - B) \\ r_4(t) = (-1 + t, -t) \\ 0 \le t \le 1 \\ Sentido + \end{cases}$</td>
<td>

<table><tr><td>$A = (0, -1)$
$B = (-1,0)$</td><td>$C = (0,1)$
$D = (1,0)$</td></tr></table></td>
<td>$C2: \begin{cases} r_2(t) = D + t(C - D) \\ r_2(t) = (1 - t, t) \\ 0 \le t \le 1 \\ Sentido + \end{cases}$

$C1: \begin{cases} r_1(t) = A + t(D - A) \\ r_1(t) = (t, t - 1) \\ 0 \le t \le 1 \\ Sentido + \end{cases}$</td>
</tr>
</table>

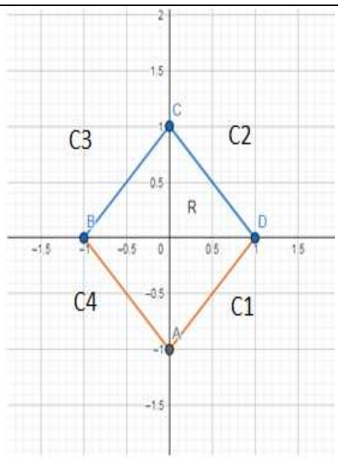
Para la integral de campo escalar de una curva a partir de su parametrización, utilizamos:

$$E_i(C) = \int_a^b f(r_i(t))\|\overline{r_i}'(t)\|\,dt = \int_a^b f(x_i(t), y_i(t))\sqrt{(x_i'(t))^2 + (y_i'(t))^2}\,dt$$

$$E(C) = E_i(C) + E_i(C) + E_i(C) + E_i(C)$$

$$C3: \begin{cases} r_3(t) = (-t, 1-t) \\ r'_3(t) = (-1,-1) \\ |r'_3(t)| = \sqrt{2} \\ f(r_3(t)) = (-t)^2 + (1-t)^2 \\ f(r_3(t)) = 2t^2 - 2t + 1 \\ 0 \le t \le 1 \end{cases}$$

$$C4: \begin{cases} r_4(t) = (-1+t, -t) \\ r'_4(t) = (1,-1) \\ |r'_4(t)| = \sqrt{2} \\ f(r_4(t)) = (-1+t)^2 + (-t)^2 \\ f(r_4(t)) = 2t^2 - 2t + 1 \\ 0 \le t \le 1 \end{cases}$$



$A = (0,-1)$	$C = (0,1)$
$B = (-1,0)$	$D = (1,0)$

$$C2: \begin{cases} r_2(t) = (1-t, t) \\ r'_2(t) = (-1,1) \\ |r'_2(t)| = \sqrt{2} \\ f(r_2(t)) = (1-t)^2 + t^2 \\ f(r_2(t)) = 2t^2 - 2t + 1 \\ 0 \le t \le 1 \end{cases}$$

$$C1: \begin{cases} r_1(t) = (t, t-1) \\ r'_1(t) = (1,1) \\ |r'_1(t)| = \sqrt{2} \\ f(x,y) = x^2 + y^2 \\ f(r_1(t)) = t^2 + (t-1)^2 \\ f(r_1(t)) = 2t^2 - 2t + 1 \\ 0 \le t \le 1 \end{cases}$$

$$E(C) = \int_0^1 (2t^2 - 2t + 1)\sqrt{2}\,dt + \int_0^1 (2t^2 - 2t + 1)\sqrt{2}\,dt + \int_0^1 (2t^2 - 2t + 1)\sqrt{2}\,dt + \int_0^1 (2t^2 - 2t + 1)\sqrt{2}\,dt$$

$$E(C) = 4\int_0^1 (2t^2 - 2t + 1)\sqrt{2}\,dt$$

$$E(C) = 4\sqrt{2}\int_0^1 (2t^2 - 2t + 1)\,dt$$

$$E(C) = 4\sqrt{2}\left[\frac{2}{3}t^3 - t^2 + t\right]_0^1$$

$$E(C) = 4\sqrt{2}\left[\left(\frac{2}{3} - 1 + 1\right) - (0 - 0 + 0)\right]$$

$$E(C) = 4\sqrt{2}\left[\frac{2}{3}\right]$$

$$E(C) = \frac{8}{3}\sqrt{2}$$