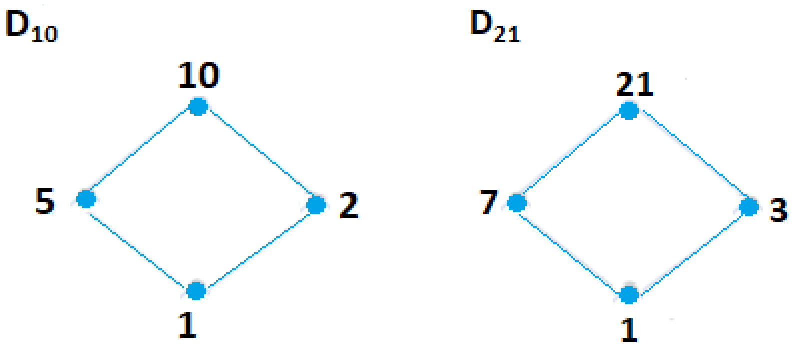
Sean (A; ∨; ∧) y (B; ∨'; ∧') dos Álgebras de Boole.

Una función f: A → B se dice **homomorfismo** si verifica las siguientes condiciones:

- $\forall a \in A: f(\bar{a}) = \overline{f(a)}$
- $\forall a \in A, \forall b \in A: f(a \vee b) = f(a) \vee' f(b)$
- $\forall a \in A, \forall b \in A: f(a \wedge b) = f(a) \wedge' f(b)$
- $f(0_A) = 0_b$
- $f(1_A) = 1_b$

Ejemplo: Sean dos conjuntos ordenados (D_{10}, |) y (D_{21}, |), cuyo diagrama Hasse son:



Como 10 = 5 . 2 y 21 =7 . 3 (D_{10}, |) y (D_{21}, |), son Algebras de Boole

Probemos que es un homomorfismo

En D_{10} $\bar{1} = 10$ $\quad \overline{10} = 1$ $\quad \bar{2} = 5$ $\quad \bar{5} = 2$

En D_{21} $\bar{1} = 21$ $\quad \overline{21} = 1$ $\quad \bar{3} = 7$ $\quad \bar{7} = 3$

1. $f(\bar{a}) = \overline{f(a)}$

$f(\bar{2}) = f(5) = 7 \qquad \overline{f(2)} = \bar{3} = 7 \qquad f(\bar{1}) = f(10) = 21 \qquad \overline{f(1)} = \bar{1} = 21$

$f(\bar{5}) = f(2) = 3 \qquad \overline{f(5)} = \bar{7} = 3 \qquad f(\overline{10}) = f(1) = 1 \qquad \overline{f(10)} = \overline{21} = 1$

2. $f\ (a \vee b)\ =\ f\ (a)\ \vee'\ f\ (b)$

$f\ (1 \vee 2)\ = f\ (2) = 3 \qquad f(1) \vee' f(2) = 1 \vee 3 = 3$

$f\ (1 \vee 5)\ = f\ (5) = 7 \qquad f(1) \vee' f(5) = 1 \vee 7 = 7$

$f\ (1 \vee 10)\ = f\ (10) = 21 \qquad f(1) \vee' f(10) = 1 \vee 21 = 21$

$f\ (2 \vee 5)\ = f\ (10) = 21 \qquad f(2) \vee' f(5) = 3 \vee 7 = 21$

$f\ (2 \vee 10)\ = f\ (10) = 21 \qquad f(2) \vee' f(10) = 3 \vee 21 = 21$

$f\ (5 \vee 10)\ = f\ (10) = 21 \qquad f(5) \vee' f(10) = 7 \vee 21 = 21$

3. $f\ (a \wedge b)\ =\ f\ (a)\ \wedge'\ f\ (b)$

$f\ (1 \wedge\ 2)\ \ = f\ (1) = 1 \qquad f(1) \wedge\ 'f(2) = 1 \wedge 3 = 1$

$f\ (1 \wedge\ 5)\ \ = f\ (1) = 1 \qquad f(1) \wedge\ 'f(5) = 1 \wedge 7 = 1$

$f\ (1 \wedge 10)\ \ = f\ (1) = 1 \qquad f(1) \wedge 'f(10) = 1 \vee 21 = 1$

$f\ (2 \wedge\ 5)\ \ = f\ (1) = 1 \qquad f(2) \wedge 'f(5) = 3 \wedge 7 = 1$

$f\ (2 \wedge\ 10) \ = f\ (2) = 3 \quad f(2) \wedge{}' f(10) = 3 \wedge 21 = 3$

$f\ (5 \wedge 10) \ = f\ (5) = 7 \quad f(5) \wedge{}' f(10) = 7 \wedge 21 = 7$

4. $f(0_A) = 0_B \qquad f(1) = 1 = 0_B$
5. $f(1_A) = 1_B \qquad f(10) = 21 = 1_B$

Como por definición es biyectiva, resulta ser un isomorfismo.