Calcular las direcciones para que la derivada direccional de la función f en el punto indicado de como resultado el valor k

a) $f(x,y) = -\mathrm{e}^{-\mathrm{x}+2\mathrm{y}} + \mathrm{e}^{\mathrm{xy}}, P = (1,1), k = 0$

b) $f(x,y) = \frac{1}{x^2+y^2}, P = (1,2), k = \frac{7}{10}$

c) $f(x,y) = \mathrm{x}^{\mathrm{y}}, P = (1,5), k = \frac{1}{4}$

d) $f(x,y) = \ln(2\mathrm{x} - \mathrm{y}^3), P = (2,2), k = 0$

e) $f(x,y) = 1 + \frac{2\mathrm{xy}}{\mathrm{x}^2+\mathrm{y}^2}, P = (-1,1), k = \sqrt{2}$

f) $f(x,y) = 5\mathrm{x}^3 + 2\mathrm{y}^4 - \mathrm{xy}^3, P = (0,2), k = \frac{1}{3}\ max$

g) $f(x,y) = \mathrm{e}^{\mathrm{x}-\mathrm{y}} + \mathrm{e}^{\mathrm{xy}}, P = (3,-1). k = 100.\ ¿es\ posible?\ Justificar\ sin\ contemplar\ v$

h) $f(x,y) = 4\mathrm{x}^2 - 2\mathrm{e}^{\mathrm{x}+\mathrm{y}-2}, P = (1,2)\ , k = \frac{\mathrm{kmin} + (kmax - kmin)}{2}$

i)$f(x,y) = 4\mathrm{x}^2 - 2\mathrm{e}^{\mathrm{x}+\mathrm{y}-2}, P = (1,2)\ , k = 3(kmax)\ ¿esposible?\ Justificarsinv$

j) $f(x,y) = 4\mathrm{x}^2 - 2\mathrm{e}^{\mathrm{x}+\mathrm{y}-2}, P = (1,2)\ , k = (\ kmax)\left(\cos\left(\frac{\pi}{4}\right)\right)$

Ejemplo 1:

$f(x,y) = \ln(x+y^2)\, e, P = (1,1), k = 0$

$f_x(x,y) = \dfrac{e}{x+y^2} \to f_x(P) = \dfrac{e}{2}$

$f_y(x,y) = \dfrac{2ye}{x+y^2} \to f_y(P) = e$

Como las derivadas son continuas en $U = \{(x,y)\epsilon\mathbb{R}^2 / \, x+y^2 > 0$ entonces (por teorema de Cauchy) $f(x,y)$ es diferenciable en U, lo cual incluye a P.

Entonces podemos hacer uso de la siguiente propiedad para todo $\vec{w} = (a,b)$ genérico, con $|\vec{w}| = 1$:

- $f_w(P) = \nabla f(P) \cdot \vec{w} = \boldsymbol{k}$ convertida en ecuación

si $\nabla f(x,y) = \left(\frac{e}{x+y^2}, \frac{2ye}{x+y^2}\right) \to \nabla f(P) = \left(\frac{e}{2}, e\right)$

$$\nabla f(P) \cdot (a,b) = \frac{e}{2}a + eb = \mathbf{0}$$

$$b = -\frac{e}{2}\frac{1}{e}\,a = -\frac{1}{2}a$$

$$|w| = 1 \to a^2 + b^2 = 1$$

$$a^2 + \left(-\frac{1}{2}a\right)^2 = 1$$

$$\frac{5}{4}a^2 = 1$$

$$a = \pm\frac{2}{\sqrt{5}} \to b = -\left(\pm\frac{1}{\sqrt{5}}\right)$$

$\pm$No es una combinatoria

$$v_1 = \left(\tfrac{2}{\sqrt{5}}, -\tfrac{1}{\sqrt{5}}\right) v_2 = \left(-\tfrac{2}{\sqrt{5}}, \tfrac{1}{\sqrt{5}}\right)$$

Comprobación:

$$\left(\frac{e}{2}, e\right)\left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right) = \frac{e}{\sqrt{5}} - \frac{e}{\sqrt{5}} = 0$$

Ejemplo 2:

$f(x,y) = \ln(x+y^2)\, e, P = (1,1), k = 1$

$f_w(P) = \nabla f(P) \cdot w = \mathbf{1} = \boldsymbol{k}$

$\left(\frac{e}{2}, e\right)(a,b) = \frac{e}{2}a + eb = 1$

$b = 1 - \frac{ea}{2} = \frac{1}{e} - \frac{1}{2}a$

$|v| = 1 \rightarrow a^2 + b^2 = 1$

$a^2 + \left(\frac{1}{e} - \frac{1}{2}a\right)^2 = 1$

$\frac{5}{4}a^2 - \frac{a}{e} + \frac{1}{e^2} - 1 = 0 \quad usar\ c = \frac{1}{e^2} - 1$

$$a = \frac{\frac{1}{e} \pm \sqrt{\frac{1}{\mathrm{e}^2} - 4\left(\frac{5}{4}\right)\left(\frac{1}{e^2} - 1\right)}}{2\left(\frac{5}{4}\right)}$$

---c/a

$$\frac{1}{\mathrm{e}^2} - 4\left(\frac{5}{4}\right)\left(\frac{1}{e^2} - 1\right)$$

$$\frac{1}{\mathrm{e}^2} - 5\left(\frac{1}{e^2} - 1\right)$$

$$\frac{1}{\mathrm{e}^2} - 5\frac{1}{e^2} + 5$$

$$-\frac{4}{e^2} + 5 \simeq 4.458658867$$

$$\mathrm{a} = \frac{\frac{1}{e} \pm \sqrt{-\frac{4}{e^2} + 5}}{\frac{5}{2}} = \frac{2}{5e} \pm \frac{2}{5}\sqrt{-\frac{4}{e^2} + 5}$$

$$a = \frac{2}{5e} \pm \frac{2}{5}\sqrt{-\frac{4}{e^2} + 5}$$

$$b = \frac{1}{e} - \frac{1}{2}\left(\frac{2}{5e} \pm \frac{2}{5}\sqrt{-\frac{4}{e^2} + 5}\right) = \frac{1}{e} - \frac{1}{5e} - \left(\pm\frac{1}{5}\sqrt{-\frac{4}{e^2} + 5}\right)$$

$$b = \frac{4}{5e} - \left(\pm \frac{1}{5} \sqrt{-\frac{4}{e^2} + 5} \right)$$

$$v_1 = \left(\frac{2}{5e} + \frac{2}{5} \sqrt{-\frac{4}{e^2} + 5}, \frac{4}{5e} - \frac{1}{5} \sqrt{-\frac{4}{e^2} + 5} \right)$$

$$\left(\frac{e}{2}, e\right) \left(\frac{2}{5e} + \frac{2}{5} \sqrt{-\frac{4}{e^2} + 5}, \frac{4}{5e} - \frac{1}{5} \sqrt{-\frac{4}{e^2} + 5} \right)$$

$$\frac{1}{5} + \frac{e}{5} \sqrt{-\frac{4}{e^2} + 5} + \frac{4}{5} - \frac{1}{5} e \sqrt{-\frac{4}{e^2} + 5}$$

$$f_w = \frac{1}{5} + \frac{4}{5} = 1$$

$$v_2 = \left(\frac{2}{5e} - \frac{2}{5} \sqrt{-\frac{4}{e^2} + 5}, \frac{4}{5e} + \frac{1}{5} \sqrt{-\frac{4}{e^2} + 5} \right)$$

Ejemplo 3:

$f(x,y) = \ln(x + y^2)\, e, P = (1,1), k = \frac{1}{2} max$

Sugerencia:

$f_w(P) = \nabla f(P) \cdot w = \frac{\mathbf{1}}{\mathbf{2}} \boldsymbol{max} = \frac{|\nabla f(P)|}{\mathbf{2}} = \frac{\sqrt{5}e}{2}$

es decir $k = \frac{\sqrt{5}e}{2}$

$(e, 2e)(a,b) = ea + 2eb = \frac{\sqrt{5}e}{2}$