Resolución TP10:

Ejercicio 6 - b - Aplicando Divergencia

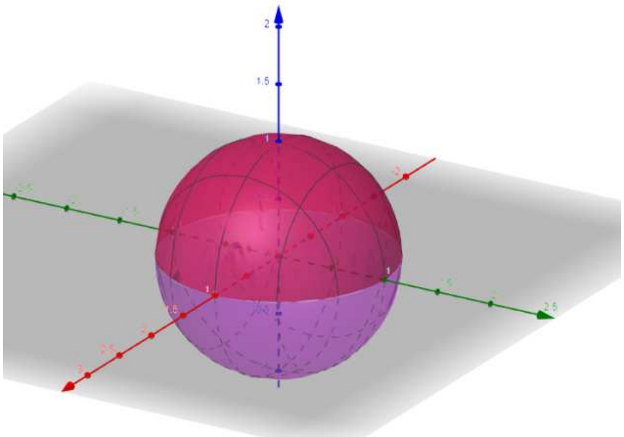
Dado el campo vectorial F y la superficie S, calcular el flujo saliente.

$$F(x, y, z) = (x, y, z)$$

S: $x^2 + y^2 + z^2 = 1$

Resolviendo:

$$I = \iint_S F \cdot dS = \iint_{R_\Phi} F(\Phi) \cdot (\Phi_u X \Phi_v) du dv = \iiint_{V_S} Div(F) dV_S$$



$$Div(F) = 3$$

$$I = \iint_S F \cdot dS = \iiint_{V_S} Div(F) dV_S$$

$$I = \iiint_{V_S} 3 dx dy dz$$

V: $x^2 + y^2 + z^2 \leq 1$

Aplicando Transformación esférica

$$I = \iiint_{V_S} 3 dx dy dz = \iiint_{V'} 3|J| d\rho d\theta d\varphi$$

$$V:\begin{cases} TL(\rho,\theta,\varphi) = (\rho\cos(\theta)\,sen(\varphi), \rho\,\mathrm{sen}(\theta)\,sen(\varphi), \rho\,cos(\varphi)) \\ V':\begin{cases} 0 \leq \theta < 2\pi \\ 0 \leq \varphi < \pi \\ 0 \leq \rho \leq 1 \end{cases} \\ |J| = \rho^2 sen\varphi \end{cases}$$

$$I = 3\int_0^{\pi}\int_0^{2\pi}\int_0^{1}\rho^2 sen\varphi\; d\rho d\theta d\varphi$$

$$I = 3\int_0^{\pi} sen\varphi d\varphi \int_0^{2\pi} d\theta \int_0^{1}\rho^2\, d\rho$$

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$$\int_0^{\pi} sen\varphi d\varphi = [-cos\varphi]_0^{\pi} = 2$$

$$\int_0^{1}\rho^2\, d\rho = \left[\frac{\rho^3}{3}\right]_0^1 = \frac{1}{3}$$

$$\int_0^{2\pi} d\theta = 2\pi$$

$$I = 3\int_0^{\pi} sen\varphi d\varphi \int_0^{2\pi} d\theta \int_0^{1}\rho^2\, d\rho = 3(2)(2\pi)\left(\frac{1}{3}\right) = 4\pi$$