Resolución TP7:

Ejercicio adicional

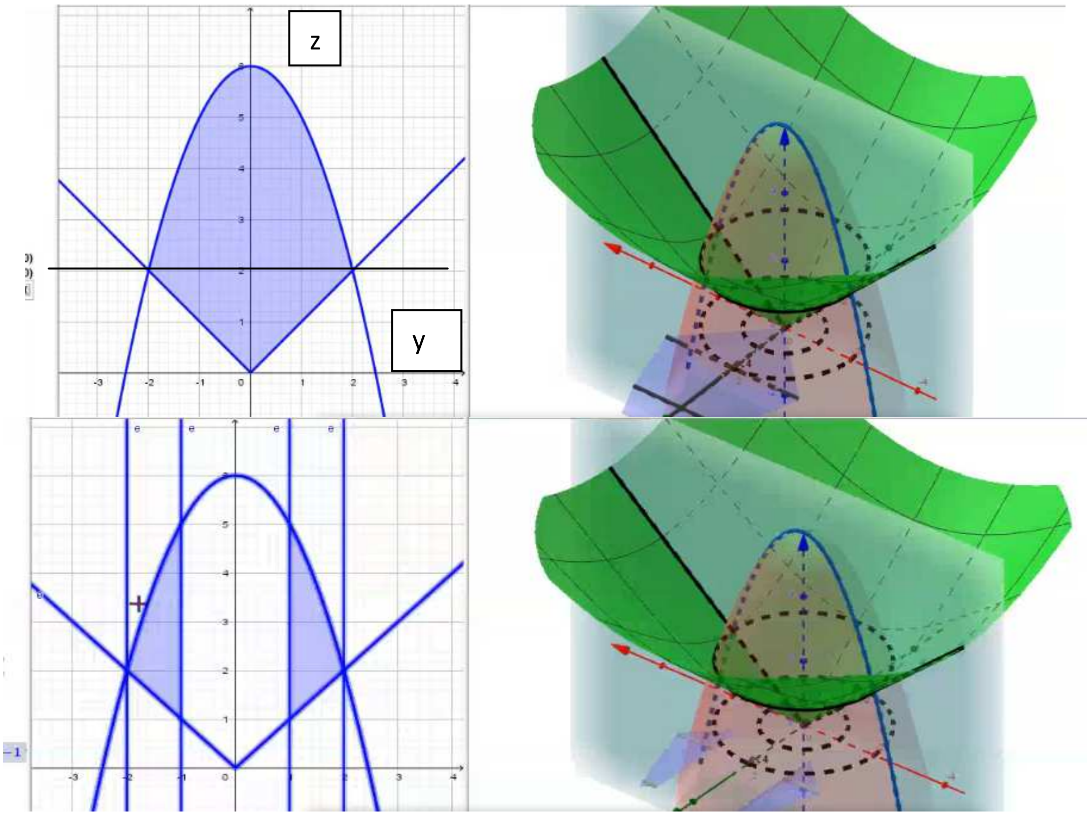
Resolver la integral triple I con él recinto V.

$$V\colon \{(x,y,z)\epsilon R^3 \colon z \geq \sqrt{x^2+y^2} \ \wedge \ z \leq 6 - x^2 - y^2 \wedge x^2 + y^2 \geq 1\,\}$$

$$I = \iint T(x,y) - P(x,y)dxdy$$

$$x = 0 \rightarrow z \geq \sqrt{0^2+y^2} \ \wedge \ z \leq 6 - 0^2 - y^2$$

$$x = 0 \rightarrow z \geq |y| \ \wedge \ z \leq 6 - y^2$$



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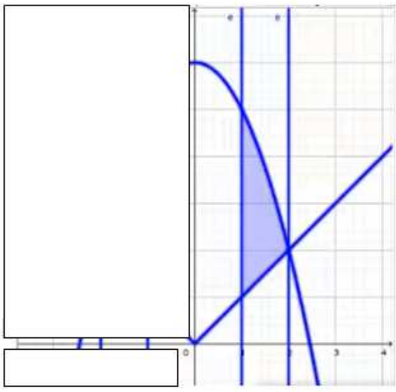
<td></td>
<td>Si se recorre como solido de revolucion gracias a
$0 \leq \theta \leq 2\pi$
Cubre toda la superficie entre el paraboloide y el cono.</td>
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Buscando Limites para x e y:

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<td>
$z = \sqrt{x^2+y^2} \wedge z = 6 - x^2 - y^2$

$z = z$
$\sqrt{x^2+y^2} = 6 - x^2 - y^2$

$z^2 = x^2 + y^2 \ \wedge$
$z = 6 - (x^2 + y^2)$

$z = 6 - z^2$
$z^2 + z - 6 = 0$
$z = \dfrac{-1 \pm \sqrt{1-4(-6)}}{2(1)}$
$z = \dfrac{-1 \pm \sqrt{25}}{2} = -3 \ o \ 2$
Tomando la interseccion:
$x^2 + y^2 = 4$
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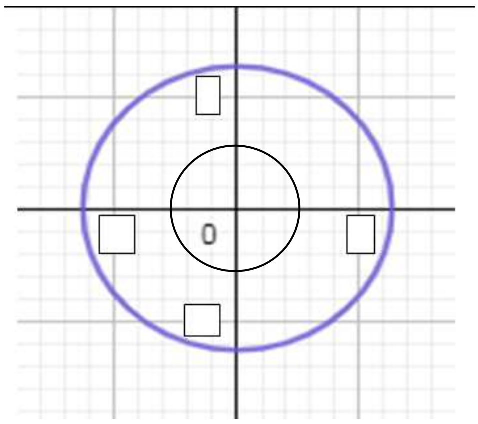
<td>
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En resumen

$$V: \begin{cases} \sqrt{x^2+y^2} \leq z \leq 6 - x^2 - y^2 \\ 1 \leq x^2 + y^2 \leq 4 \end{cases}$$

$$I = \iint T - P dxdy$$

$$I = \iint_{1 \leq x^2+y^2 \leq 4} \overbrace{6 - x^2 - y^2}^{techo} - \overbrace{\sqrt{x^2+y^2}}^{piso} dxdy$$

$$R:\begin{cases} x = r\cos\theta \\ y = r\,sen\theta \\ x^2 + y^2 = r^2 \\ |J| = r \\ R' = \begin{cases} 1 \leq r \leq 2 \\ 0 \leq \theta \leq 2\pi \end{cases} \end{cases}$$

$$I = \int_0^{2\pi}\int_1^2 \left(\overbrace{6 - r^2}^{techo} - \overbrace{\sqrt{r^2}}^{piso}\right) r\,dr\,d\theta$$

$$I = \int_0^{2\pi}\int_1^2 (6 - r^2 - r) r\,dr\,d\theta$$

$$I = \int_0^{2\pi}\int_1^2 (6r - r^3 - r^2)\,dr\,d\theta$$

$$I = \int_0^{2\pi} \left[3r^2 - \frac{r^4}{4} - \frac{r^3}{3}\right]_1^2 d\theta$$

$$I = 2\pi\left[\left(3 \cdot 4 - 4 - \frac{8}{3}\right) - \left(3 - \frac{1}{4} - \frac{1}{3}\right)\right] = \frac{35}{6}\pi$$

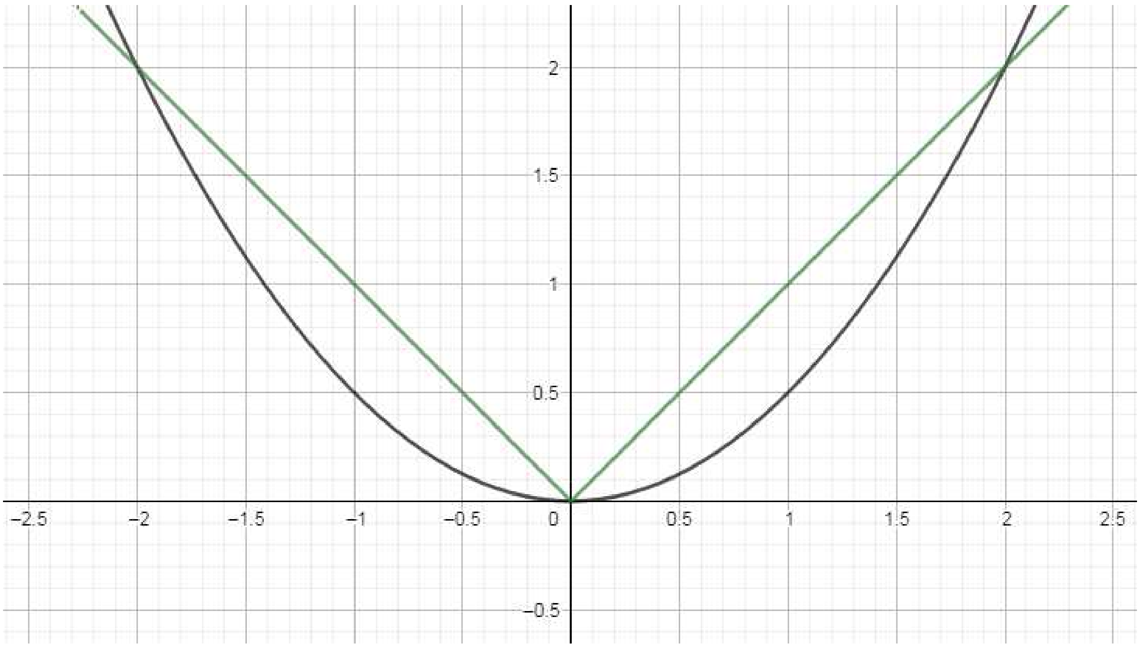
Resolver la integral triple I con él recinto V.

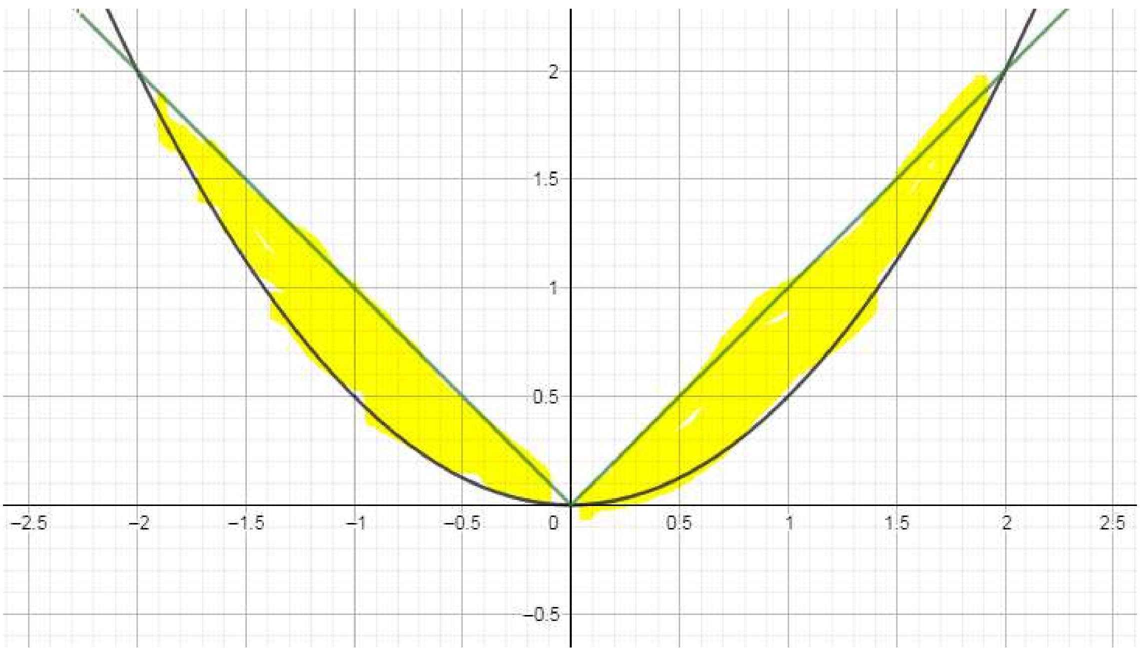
$$V:\{(x,y,z)\epsilon R^3:\ figura\ entre\ \ z=\sqrt{x^2+y^2}\ \wedge\ 2z=x^2+y^2\ \}$$

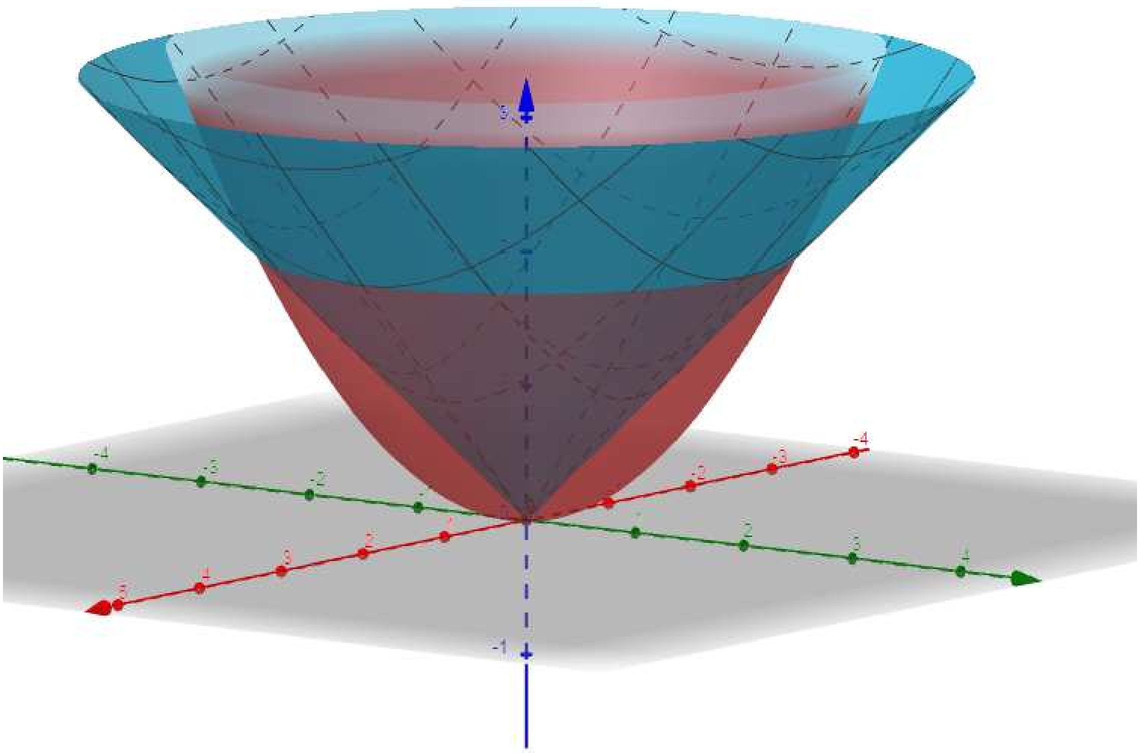
$$I=\ \iint T(x,y)-P(x,y)dxdy$$

$$x=0\rightarrow z=\sqrt{0^2+y^2}\ \wedge\ z=\frac{y^2}{2}$$

$$x=0\rightarrow z=|y|\wedge\ z=\frac{y^2}{2}$$







$$V: \{(x,y,z)\epsilon R^3 : \ z \leq \sqrt{x^2+y^2} \ \wedge \ z \geq \frac{x^2}{2}+\frac{y^2}{2}\}$$

$$V: \{(x,y,z)\epsilon R^3 : \frac{x^2}{2}+\frac{y^2}{2} \leq z \leq \sqrt{x^2+y^2} \ \}$$

$$\color{red}{\frac{x^2}{2}+\frac{y^2}{2} = \sqrt{x^2+y^2}}$$

$$\underbrace{\frac{x^2+y^2}{2} = z \qquad \overbrace{z = \sqrt{x^2+y^2}}^{z^2=x^2+y^2}}_{\frac{z^2}{2}=z}$$

$$\frac{z^2}{2} = z \rightarrow \color{red}{z=0} \ o \ \color{green}{z=2} \rightarrow x^2+y^2 \leq 4$$

$$V:\{(x,y,z)\epsilon R^3: \frac{x^2}{2}+\frac{y^2}{2}\leq z\leq\sqrt{x^2+y^2}\ \ \wedge\ \ x^2+y^2\leq 4\,\}$$

$$\iint\limits_{x^2+y^2\leq 4}\sqrt{x^2+y^2}-\frac{x^2+y^2}{2}dxdy$$

Transformacion de Coordenas polares

$$\int_0^2\int_0^{2\pi}\left(r-\frac{r^2}{2}\right)\overbrace{r}^{|J|}\,d\alpha dr$$

$$\int_0^2\int_0^{2\pi}\left(r^2-\frac{r^3}{2}\right)d\alpha dr$$

$$2\pi\int_0^2\left(r^2-\frac{r^3}{2}\right)dr$$

$$2\pi\left[\frac{r^3}{3}-\frac{r^4}{8}\right]_0^2=\frac{4}{3}\pi$$