Resolución TP10:

Ejercicio 6 - a - Modificado

Dado el campo vectorial F y la superficie S, calcular el flujo saliente.

$$F(x,y,z) = (x^2y, xz, y^2z)$$

S: es la superficie del Volumen determinado por la intersección del plano de ecuación $x + y + z = 1$ y los planos coordenados.

Resolviendo:

$$I = \iint_S F \cdot dS = \sum_i \iint_{R_{\Phi_i}} F(\Phi_i) \cdot (\Phi_{iu} X \Phi_{iv}) dudv$$

Considerando que se trata solo del triangulo podemos nombrar a la superficie con la siguiente descripción:

$$V: \begin{cases} x + y + z \leq 1 \\ y \geq 0 \\ x \geq 0 \\ z \geq 0 \end{cases}$$

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Dibujamos calculando las trazas y los vertices:
$S_1 \overset{x=0}{\rightleftarrows} \begin{cases} y + z = 1 \\ y \geq 0 \\ z \geq 0 \end{cases} \overset{y=0}{\rightleftarrows} \begin{cases} z = 1 \\ A = (0,0,1) \end{cases}$
$S_1 \overset{y=0}{\rightleftarrows} \begin{cases} x + z = 1 \\ x \geq 0 \\ z \geq 0 \end{cases} \overset{z=0}{\rightleftarrows} \begin{cases} x = 1 \\ B = (1,0,0) \end{cases}$
$S_1 \overset{z=0}{\rightleftarrows} \begin{cases} x + y = 1 \\ x \geq 0 \\ y \geq 0 \end{cases} \overset{x=0}{\rightleftarrows} \begin{cases} y = 1 \\ C = (0,1,0) \end{cases}$
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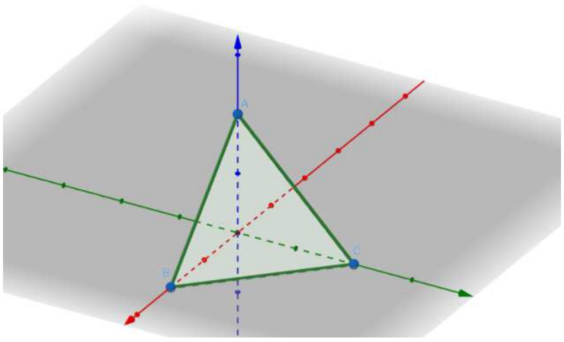
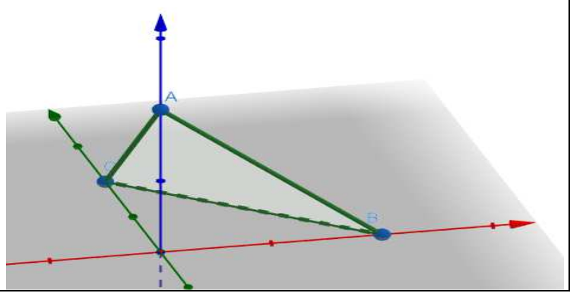
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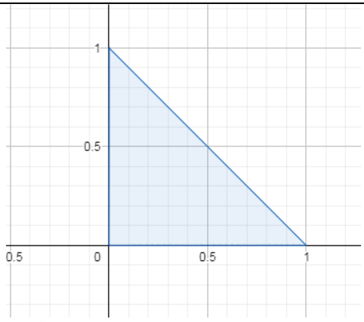
$$S_1:\begin{cases}\Phi_1(x,y)=\big(x,y,1-(x+y)\big)\\ R_{\Phi_1}:\begin{cases}y\geq 0\\ x\geq 0\\ x+y\leq 1\end{cases}\end{cases}$$

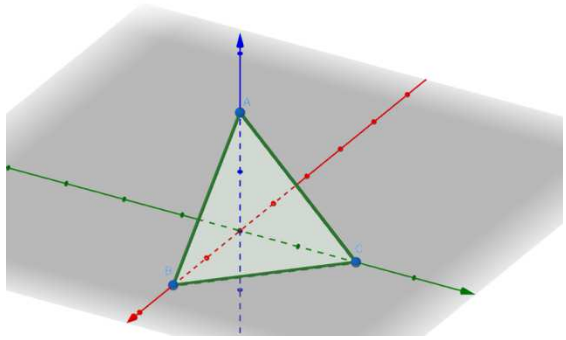
Tipo I

$$S_1:\begin{cases}\Phi_1(x,y)=\big(x,y,1-(x+y)\big)\\ R_{\Phi_1}:\begin{cases}0\leq y\leq 1-x\\ \\ 0\leq x\leq 1\end{cases}\end{cases}$$

Tipo II

$$S_1:\begin{cases}\Phi_1(x,y)=\big(x,y,1-(x+y)\big)\\ R_{\Phi_1}:\begin{cases}0\leq x\leq 1-y\\ \\ 0\leq y\leq 1\end{cases}\end{cases}$$





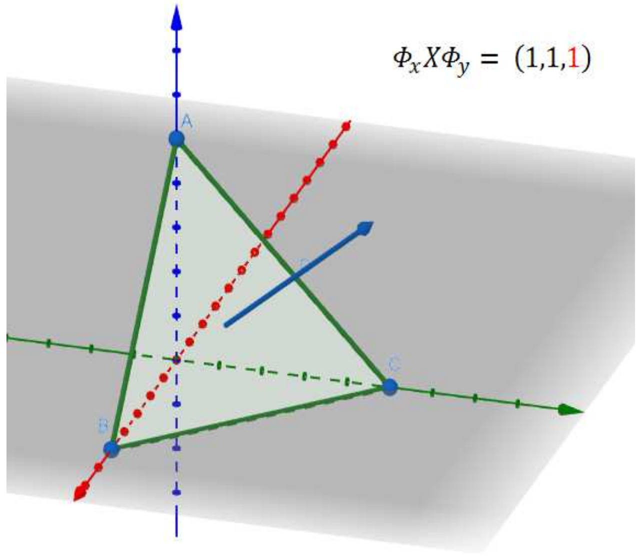
$$\Phi_{1x}=(1,0,-1)$$

$$\Phi_{1y}=(0,1,-1)$$

$$\Phi_{1x}X\Phi_{1y}=\begin{bmatrix}i & j & k\\ 1 & 0 & -1\\ 0 & 1 & -1\end{bmatrix}=\left(\begin{vmatrix}0 & -1\\ 1 & -1\end{vmatrix},-\begin{vmatrix}1 & -1\\ 0 & -1\end{vmatrix},\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}\right)$$

$$\Phi_{1x}X\Phi_{1y}=\Big(\big(0-(-1)\big),-(-1-0),(1-0)\Big)$$

$$\Phi_{1x}X\Phi_{1y}=\ (1,1,1)$$

" calcular el flujo saliente"	

$$\begin{cases} \Phi_1(x,y) = (x, y, 1-x-y) \\ F(x,y,z) = (x^2y, xz, y^2z) \end{cases} \rightarrow F(\Phi_1)$$

$$F(\Phi_1) = (x^2y, x(1-x-y), y^2(1-x-y))$$

$$F(\Phi_1) = (x^2y, x - x^2 - xy, y^2 - xy^2 - y^3)$$

$$F(\Phi_1)\cdot(\Phi_{1x} X \Phi_{1y}) = (x^2y, x - x^2 - xy, y^2 - xy^2 - y^3)(1,1,1)$$

$$F(\Phi_1)\cdot(\Phi_{1x} X \Phi_{1y}) = x^2y + x - x^2 - xy + y^2 - xy^2 - y^3$$

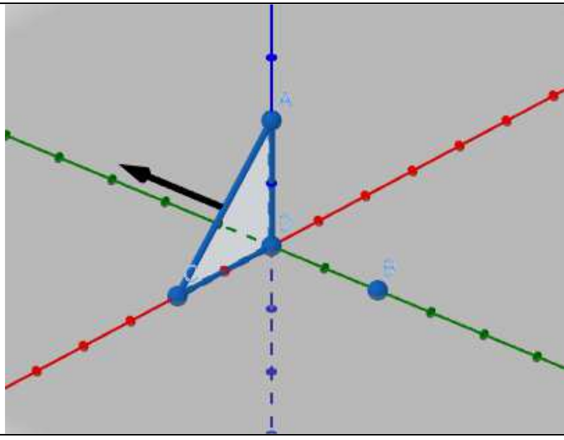
$$I_1 = \iint_{S_1} F \cdot dS_1 = \iint_{R_{\Phi_1}} F(\Phi_1)\cdot(\Phi_{1x} X \Phi_{1y})dudv$$

$$I_1 = \iint_{R_{\Phi_1}} (x^2y + x - x^2 - xy + y^2 - xy^2 - y^3)dR_1 = \frac{3}{40}$$

$$recomendado\ verificar\ por\ Tipo\ I\ y\ Tipo\ II, ver\ C/A$$

$$S_2:\begin{cases}\Phi_2(x,z)=(x,0,z)\\ R_{\Phi_2}:\begin{cases}z\geq 0\\ x\geq 0\\ x+z\leq 1\end{cases}\end{cases}$$

$$S_2:\begin{cases}\Phi_2(x,z)=(x,0,z)\\ R_{\Phi_2}:\begin{cases}0\leq x\leq\ 1\\ \\ 0\leq z\leq 1-x\end{cases}\end{cases}$$



$$\Phi_{2x}=(1,0,0)$$

$$\Phi_{2z}=(0,0,1)$$

$$\Phi_{2x}X\Phi_{2z}=\begin{bmatrix} i & j & k\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}=\left(\begin{vmatrix}0 & 0\\ 0 & 1\end{vmatrix},-\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix},\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix}\right)$$

$$\Phi_{2x}X\Phi_{2z}=\ (0,-(1),0)$$

$$\Phi_{2x}X\Phi_{1z}=\ (0,-1,0)$$

$$Es\ la\ normal\ buscada$$

$$F(x,y,z)=(x^2y,xz,y^2z)$$

$$F(\Phi_2)=(x^20,xz,0^2z)$$

$$F(\Phi_2)\cdot(\Phi_{2x}X\Phi_{2z})=(0,xz,0)(0,-1,0)$$

$$F(\Phi_2)\cdot(\Phi_{2x}X\Phi_{2z})=-xz$$

$$I_2=\iint_{S_2}F\cdot dS_2=\iint_{R_{\Phi_2}}F(\Phi_2)\cdot(\Phi_{2x}X\Phi_{2z})dR_2$$

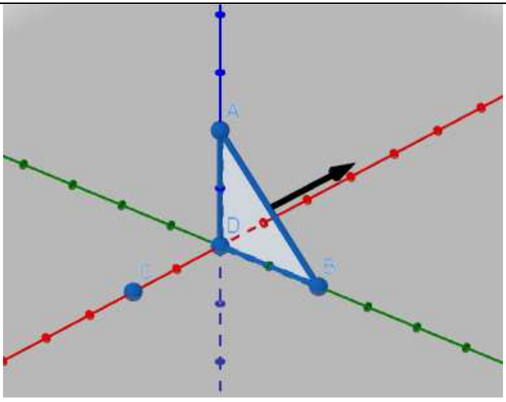
$$I_2=\iint_{R_{\Phi_2}}(-xz)dR_2=-\int_0^1\int_0^{1-x}xz\,dzdx$$

$$I_2=-\int_0^1\left[\frac{xz^2}{2}\right]_0^{1-x}dx=-\int_0^1\frac{x(1-x)^2}{2}dx=-\int_0^1\frac{x}{2}-x^2+\frac{x^3}{2}dx$$

$$I_2=-\left[\frac{x^2}{4}-\frac{x^3}{3}+\frac{x^4}{8}\right]_0^1=-\frac{1}{24}$$

$$S_3 : \begin{cases} \Phi_3(y,z) = (0,y,z) \\ R_{\Phi_3} : \begin{cases} z \geq 0 \\ y \geq 0 \\ y + z \leq 1 \end{cases} \end{cases}$$

$$S_3 : \begin{cases} \Phi_3(x,z) = (0,y,z) \\ R_{\Phi_3} : \begin{cases} 0 \leq y \leq 1 \\ \\ 0 \leq z \leq 1 - y \end{cases} \end{cases}$$



$$\Phi_{3y} = (0,1,0)$$

$$\Phi_{3z} = (0,0,1)$$

$$\Phi_{3y} X \Phi_{3z} = \begin{bmatrix} i & j & k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \left(\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}, - \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \right)$$

$$\Phi_{3y} X \Phi_{3z} = (1,0,0)$$

$$-\Phi_{3y} X \Phi_{3z} = (-1,0,0)$$

$$Es\ la\ normal\ buscada$$

$$F(x,y,z) = (x^2 y, xz, y^2 z)$$

$$F(\Phi_3) = (0^2 y, 0z, y^2 z)$$

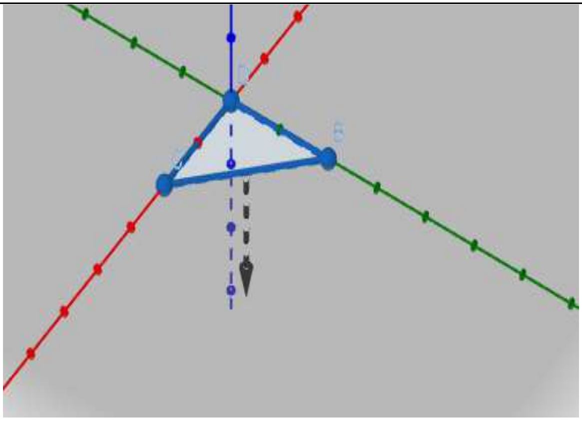
$$F(\Phi_3) \cdot (\Phi_{3y} X \Phi_{3z}) = (0,0,y^2 z)(-1,0,0)$$

$$F(\Phi_3) \cdot (\Phi_{3y} X \Phi_{3z}) = 0$$

$$I_3 = \iint_{S_3} F \cdot dS_2 = \iint_{R_{\Phi_3}} F(\Phi_3) \cdot (\Phi_{3y} X \Phi_{3z}) dR_3 = 0$$

$$S_4:\begin{cases}\Phi_4(x,y)=(x,y,0)\\ R_{\Phi_4}:\begin{cases}x\geq 0\\ y\geq 0\\ y+x\leq 1\end{cases}\end{cases}$$

$$S_4:\begin{cases}\Phi_4(x,y)=(x,y,0)\\ R_{\Phi_4}:\begin{cases}0\leq x\leq\ 1\\ \\ 0\leq y\leq 1-x\end{cases}\end{cases}$$



$$\Phi_{4x}=(1,0,0)$$

$$\Phi_{4y}=(0,1,0)$$

$$\Phi_{4x}X\Phi_{4y}=\begin{bmatrix} i & j & k\\ 1 & 0 & 0\\ 0 & 1 & 0\end{bmatrix}=\left(\begin{vmatrix}0 & 0\\ 1 & 0\end{vmatrix},-\begin{vmatrix}1 & 0\\ 0 & 0\end{vmatrix},\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}\right)$$

$$\Phi_{4x}X\Phi_{4y}=\ (0,0,1)$$

$$-\Phi_{4x}X\Phi_{4y}=\ (0,0,-1)$$

$$Es\ la\ normal\ buscada$$

$$F(x,y,z)=(x^2y,xz,y^2z)$$

$$F(\Phi_4)=(x^2y,x0,y^20)$$

$$F(\Phi_4)\cdot(\Phi_{4x}X\Phi_{4y})=(x^2y,0,0)(0,0,-1)$$

$$F(\Phi_4)\cdot(\Phi_{4x}X\Phi_{4y})=0$$

$$I_4=\iint\limits_{S_4}F\cdot dS_4=\iint\limits_{R_{\Phi_4}}F(\Phi_4)\cdot(\Phi_{4x}X\Phi_{4y})dR_4=0$$

$$I = \iint_S F \cdot dS = \sum_i \iint_{R_{\Phi_i}} F(\Phi_i) \cdot (\Phi_{iu} X \Phi_{iv}) du dv$$

$$I = I_1 + I_2 + I_3 + I_4$$

$$I = \frac{3}{40} - \frac{1}{24} + 0 + 0$$

$$I = \frac{1}{30}$$