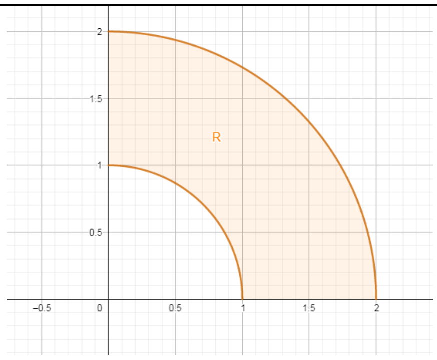
Resolución TP7:

Ejercicio 4 - c

Graficar la región de integración R y resolver la integral I.

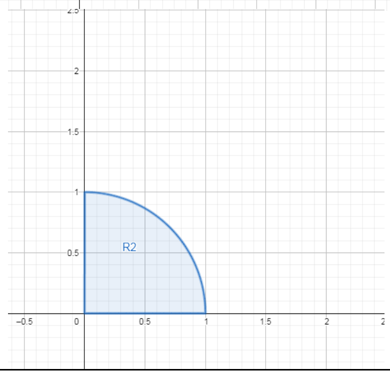
$$R: \{(x,y) \epsilon R^2 / \ 1 \ \leq x^2 + y^2 \leq 4 \ \wedge x \geq 0 \ \wedge y \geq 0\}$$

$$I = \iint\limits_R x + y dxdy$$



Resolviendo con Método de Resta de Recintos.

Como podemos ver en el siguiente grafico, Si tomamos el recinto debajo de cada curva y restamos sus integrales correspondientemente podremos llegar al mismo resultado. Asimismo ambas integrales se pueden resolver con tipo1 o tipo2.



$$R1: \begin{cases} 0 < x < 2 \\ 0 < y < \sqrt{4 - x^2} \end{cases}$$

$$R2: \begin{cases} 0 < x < 1 \\ 0 < y < \sqrt{1 - x^2} \end{cases}$$

$$I = \iint\limits_R (x+y)dxdy = \iint\limits_{R1} (x+y)dydx - \iint\limits_{R2} (x+y)dydx$$

$$I_1 = \iint_{R1} (x+y)dydx = \int_{x=0}^{x=2} \int_{y=0}^{y=\sqrt{4-x^2}} (x+y)dy\,dx$$

$$I_1 = \int_{x=0}^{x=2} \left[xy + \frac{y^2}{2}\right]_{y=0}^{y=\sqrt{4-x^2}} dx$$

$$I_1 = \int_{x=0}^{x=2} \left[\left(x\sqrt{4-x^2} + \frac{(\sqrt{4-x^2})^2}{2}\right) - 0\right] dx$$

$$I_1 = \int_{x=0}^{x=2} x\sqrt{4-x^2} + \frac{(\sqrt{4-x^2})^2}{2} \; dx$$

$$I_1 = \int_{x=0}^{x=2} x\sqrt{4-x^2}\,dx + \int_{x=0}^{x=2} 2 - \frac{x^2}{2}\,dx$$

Ver calculo auxiliar

$$I_1 = -\frac{8}{3}\left[\cos^3\left(arcsen\left(\frac{x}{2}\right)\right)\right]_{x=0}^{x=2} + \left[2x - \frac{x^3}{6}\right]_{x=0}^{x=2}$$

$$I_1 = -\frac{8}{3}\left[\cos^3\big(arcsen(1)\big) - \cos^3\big(arcsen(0)\big)\right] + \left[\left(4 - \frac{8}{6}\right) - 0\right]$$

$$I_1 = -\frac{8}{3}[0-1] + \left[\left(4 - \frac{8}{6}\right)\right] = \frac{16}{3}$$

$$I_2 = \iint_{R2} (x+y)dydx = \int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} (x+y)dy\,dx$$

$$I_2 = \int_{x=0}^{x=1} \left[xy + \frac{y^2}{2}\right]_{y=0}^{y=\sqrt{1-x^2}} dx$$

$$I_2 = \int_{x=0}^{x=1} \left[\left(x\sqrt{1-x^2} + \frac{(\sqrt{1-x^2})^2}{2}\right) - 0\right] dx$$

$$I_2 = \int_{x=0}^{x=1} x\sqrt{1-x^2} + \frac{1}{2} - \frac{x^2}{2}\ dx$$

$$I_2 = \int_{x=0}^{x=1} x\sqrt{1-x^2}\,dx + \int_{x=0}^{x=1} \frac{1}{2} - \frac{x^2}{2}\ dx$$

Ver calculo auxiliar

$$I_2 = -\frac{1}{3}\left[\cos^3(arcsen(x))\ \right]_{x=0}^{x=1} + \left[\frac{1}{2}x - \frac{x^3}{6}\right]_{x=0}^{x=1}$$

$$I_2 = -\frac{1}{3}\left[\cos^3(arcsen(1)) \ - \cos^3(arcsen(0))\ \right] + \left[\left(\frac{1}{2} - \frac{1}{6}\right) - 0\right]$$

$$I_2 = -\frac{1}{3}[0-1] + \left[\left(\frac{1}{2} - \frac{1}{6}\right) - 0\right] = \frac{2}{3}$$

Finalmente:

$$I = \iint_R (x+y)dxdy = I_1 - I_2 = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

$$I = \frac{14}{3}$$

C/A

$$\int \; x\sqrt{a^2 - x^2}\, dx$$

$sustitucion\ en\ x\sqrt{a^2 - x^2}$

$$x = asen\ t$$

$$dx = acos\ tdt$$

$$a^2 - x^2 = a^2 - a^2sen^2\ t = a^2 \cos^2 t$$

$$\sqrt{a^2 - x^2} = acost$$

$$x\sqrt{a^2 - x^2}dx = asentacostacostdt = a^3sentcos^2tdt$$

$$\int \; x\sqrt{a^2 - x^2}\, dx = \int \; a^3sentcos^2tdt$$

$sustitucion\ en\ sentcos^2tdt$

$$u = cos\ t$$

$$du = -sen\ tdt$$

$$a^3 \int \; sentcos^2tdt \; = -a^3 \int u^2du = \; -\frac{a^3}{3}\left[\cos^3\left(arcsen\left(\frac{x}{a}\right)\right)\right]$$