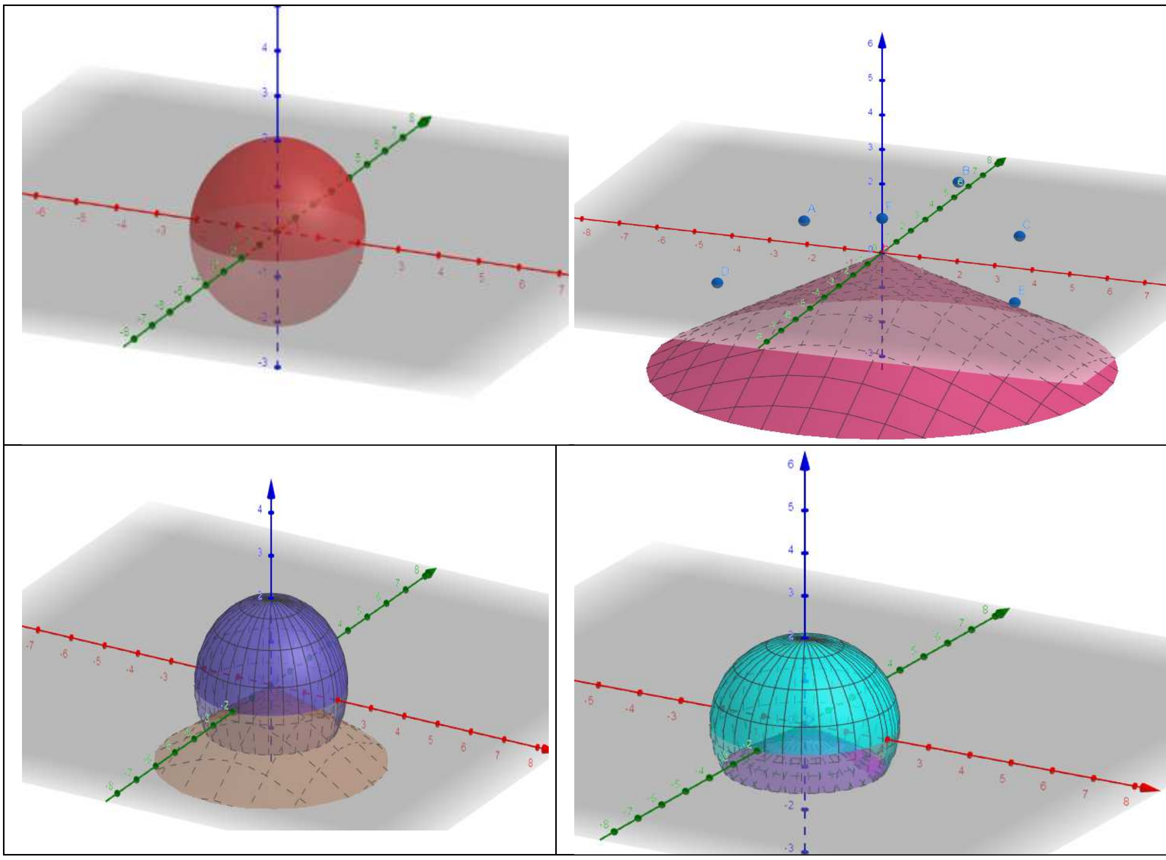
Resolución TP7:

Resolver I usando V

$$V: \{(x,y,z)\epsilon R^3 /\ x^2+y^2+z^2 \leq 4\ \wedge z \geq -\frac{\sqrt{3}}{3}\sqrt{x^2+y^2}\}$$

$$I = \iiint_V \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}}dxdydz$$



Con coordenadas Esfericas

$$V: \begin{cases} x^2+y^2+z^2 \leq 4 \\ z \geq -\frac{\sqrt{3}}{3}\sqrt{x^2+y^2} \end{cases}$$

$$V: \begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} ? \leq r \leq ? \\ ? \leq \varphi \leq ? \\ ? \leq \theta \leq ? \end{cases} \end{cases}$$

$$I = \iiint_V \frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}}dxdydz$$

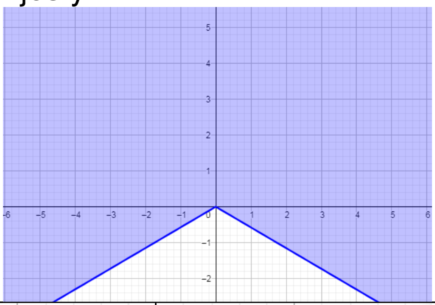
$$I = \iiint_{V'} \frac{e^{x(r,\theta,\varphi)^2+y(r,\theta,\varphi)^2+z(r,\theta,\varphi)^2}}{\sqrt{x(r,\theta,\varphi)^2+y(r,\theta,\varphi)^2+z(r,\theta,\varphi)^2}}|J(r,\theta,\varphi)|drd\theta d\varphi$$

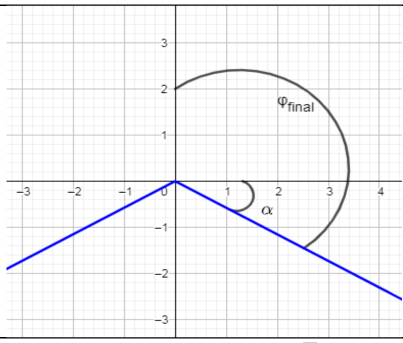
$$I = \iiint_{V'} \frac{e^{r^2}}{\sqrt{r^2}}r^2 sen(\varphi)drd\theta d\varphi$$

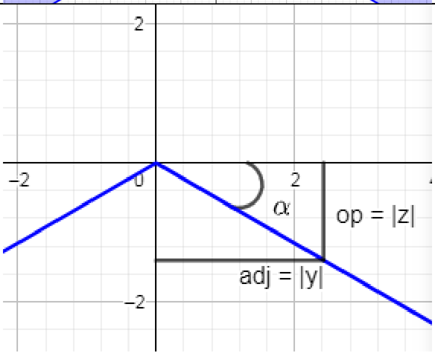
$$I = \iiint_{V'} e^{r^2} rsen(\varphi)drd\theta d\varphi$$

si $x = 0 \rightarrow z \geq -\frac{\sqrt{3}}{3}\sqrt{0^2 + y^2} \rightarrow z \geq -\frac{\sqrt{3}}{3}|y|$

Ejes yz







$$\varphi_{final} = \frac{\pi}{2} + \alpha$$

$$tg(\alpha) = \frac{|z|}{|y|} = \frac{\frac{\sqrt{3}}{3}|-|y||}{|y|} = \frac{\sqrt{3}}{3}$$

$$\alpha = arctg\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\varphi_{final} = \frac{2}{3}\pi$$

si $z = 0 \rightarrow x^2 + y^2 + 0^2 \leq 4 \rightarrow x^2 + y^2 \leq 4 \rightarrow r \leq 2 \; 0 \leq \theta \leq 2\pi$

Con coordenadas Esfericas

$$V: \begin{cases} x^2 + y^2 + z^2 \leq 4 \\ z \geq -\frac{\sqrt{3}}{3}\sqrt{x^2 + y^2} \end{cases}$$

$$V: \begin{cases} x = rcos\theta sen(\varphi) \\ y = rsen\theta sen(\varphi) \\ z = rcos(\varphi) \\ |J| = r^2 sen(\varphi) \\ V' = \begin{cases} 0 \leq r \leq 2 \\ 0 \leq \varphi \leq \frac{2}{3}\pi \\ 0 \leq \theta \leq 2\pi \end{cases} \end{cases}$$

$$I = \iiint_{V'} e^{r^2} rsen(\varphi) dr d\theta d\varphi$$

$$I = \int_0^{\frac{2}{3}\pi} \int_0^{2\pi} \int_0^{2} e^{r^2} rsen(\varphi) dr d\theta d\varphi$$

$$I = \int_0^{\frac{2}{3}\pi} \int_0^{2\pi} \int_0^{2} e^{r^2} r^2 sen(\varphi) dr d\theta d\varphi = \int_0^{\frac{2}{3}\pi} sen(\varphi) d\varphi \int_0^{2\pi} d\theta \int_0^{2} e^{r^2} r dr$$

$$I = [-\cos(\varphi)]_0^{\frac{2}{3}\pi} [\theta]_0^{2\pi} \left[\frac{e^{r^2}}{2}\right]_0^2$$

$$I = \left[\left(-(-\frac{1}{2})\right) - (-1)\right] [2\pi - 0] \left[\frac{e^4}{2} - \frac{e^0}{2}\right]$$

$$I = \left[\frac{1}{2} + 1\right][2\pi]\left[\frac{e^4}{2} - \frac{1}{2}\right]$$

$$I = 3\pi\left[\frac{e^4}{2} - \frac{1}{2}\right]$$

C/A

$$\int e^{r^2} r dr \overset{\substack{r^2=t \\ 2rdr=dt \\ rdr=\frac{dt}{2}}}{\cong} \int \frac{e^t dt}{2} = \frac{e^t}{2} \overset{r^2=t}{\cong} \frac{e^{r^2}}{2}$$