Resolución TP10:

Ejercicio 4 - f

Calcular el área la superficie del volumen definido por la intersección del cilindro $x^2 + y^2 = 1$ con del cilindro $x^2 + z^2 = 1$

Resolviendo:

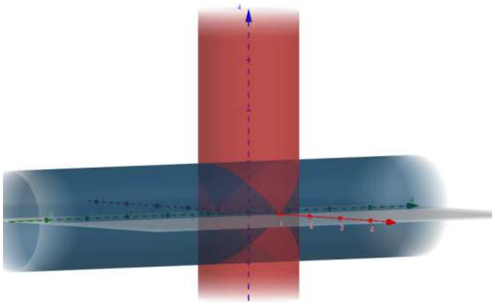
Considerando que se trata de un volumen se puede nombrar la siguiente descripción:

$$V: \begin{cases} x^2 + y^2 \leq 1 \\ \cap \\ x^2 + z^2 \leq 1 \end{cases}$$

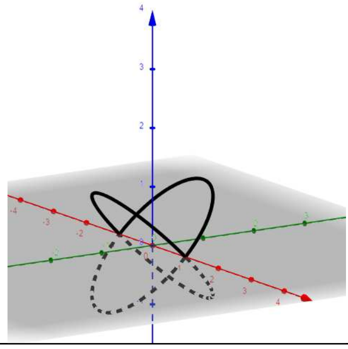
Aplicando sumas y restas podremos diferenciar los limites de los cilindros

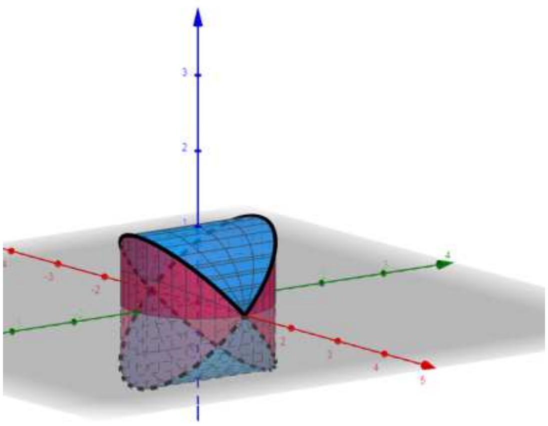
$$\begin{cases} x^2 + y^2 = 1 \ (a) \\ x^2 + z^2 = 1 \ (b) \end{cases} \rightarrow (a) - (b) \rightarrow y^2 - z^2 = 0 \rightarrow |y| = |z|$$

$$\begin{cases} x^2 + y^2 = 1 \\ x^2 + z^2 = 1 \end{cases}$$

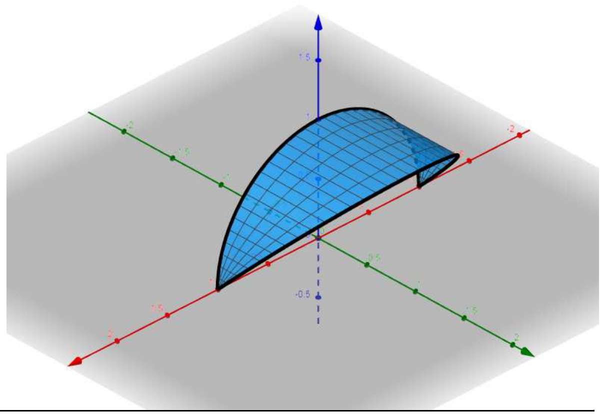


$$|y| = |z|$$





$$A(S) = 8A(S_i)$$



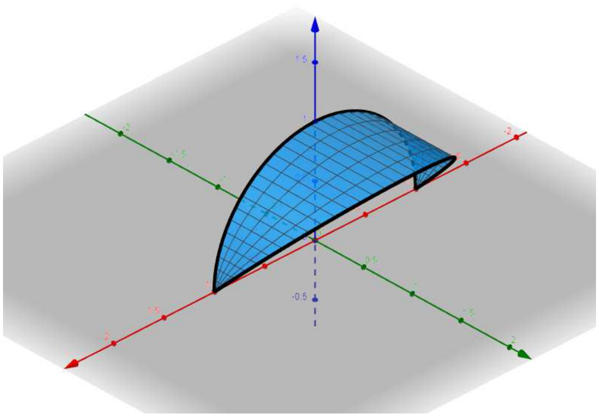
Considerando que se trata de un volumen se puede nombrar la siguiente descripción:

$$V:\begin{cases} x^2+y^2\leq 1 \\ \cap \\ x^2+z^2\leq 1 \end{cases}$$

Conformando su superficie en 2 partes:

$S_1:\begin{cases} x^2+y^2=1 \\ \cap \\ x^2+z^2\leq 1 \end{cases}$ Grafico rojo	$S_2:\begin{cases} x^2+y^2\leq 1 \\ \cap \\ x^2+z^2=1 \end{cases}$ Grafico azul

Buscaremos nuestra respuesta en una porción del Grafico azul ubicada en $y\geq 0 \ \wedge z\geq 0$



$$\begin{cases} x^2+y^2\leq 1 \\ \cap \\ x^2+z^2=1 \end{cases}\rightarrow\begin{cases} x^2=1-z^2 \\ \cap \\ x^2+y^2\leq 1 \end{cases}\rightarrow 1-z^2+y^2\leq 1\rightarrow\ y^2\leq z^2$$

$$\begin{cases} y^2\leq z^2 \\ y\geq 0 \ \ \wedge z\geq 0 \end{cases}\rightarrow\begin{cases} |y|\leq |z| \\ y\geq 0 \ \ \wedge z\geq 0 \end{cases}\rightarrow\begin{cases} y\leq z \\ y\geq 0 \end{cases}\rightarrow 0\leq y\leq z$$

Finalmente parametrizamos

$$\begin{cases} x^2+z^2=1 \\ 0\leq y\leq z \end{cases}\rightarrow\begin{cases} \Phi(\alpha,y)=\left(cos(\alpha),y,sen(\alpha)\right) \\ Dom\Phi=\begin{cases} 0\leq\ \alpha\leq\pi \\ 0\leq y\leq sen(\alpha) \end{cases} \end{cases}$$

$$\Phi(\alpha, y) = \left(cos(\alpha), y, sen(\alpha)\right)$$

$$\Phi_\alpha(\alpha, y) = (-sen(\alpha), 0, \cos(\alpha))$$

$$\Phi_y(\alpha, y) = (0,1,0)$$

$$\left|\Phi_\alpha X \Phi_y\right| = \begin{bmatrix} i & j & k \\ -\operatorname{sen}(\alpha) & 0 & \cos(\alpha) \\ 0 & 1 & 0 \end{bmatrix}$$

$$\left|\Phi_\alpha X \Phi_y\right| = \left(\begin{bmatrix} 0 & \cos(\alpha) \\ 1 & 0 \end{bmatrix}, -\begin{bmatrix} -\operatorname{sen}(\alpha) & \cos(\alpha) \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -\operatorname{sen}(\alpha) & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$\left|\Phi_\alpha X \Phi_y\right| = (-\cos(\alpha), 0, -\operatorname{sen}(\alpha))$$

$$\left|\left|\Phi_\alpha X \Phi_y\right|\right| = \sqrt{(-\cos(\alpha))^2 + 0^2 + (-\operatorname{sen}(\alpha))^2} = 1$$

$$Area(S) = 8Area(S_i) = 8\int_0^\pi \int_0^{sen(\alpha)} dy\, d\alpha$$

$$Area(S) = 8\int_0^\pi sen(\alpha) d\alpha$$

$$Area(S) = 8\left[-\cos(\alpha) \quad \right]_0^\pi$$

$$Area(S) = 8[(-cos(\pi)) - (-\cos(0))]$$

$$Area(S) = 8[(-(-1)) - (-1)]$$

$$Area(S) = 8$$