Ejercicio 9.b: Calcular la integral de línea del siguiente campo vectorial. Siendo:

$\mathbb{C} = \{(x, y) \in R^2 \,/\, x^2 + y^2 = a^2\}$ recorrida en sentido positivo.

$$\oint \frac{(x+y).dx \; - \; (x-y).dy}{x^2 + y^2}$$

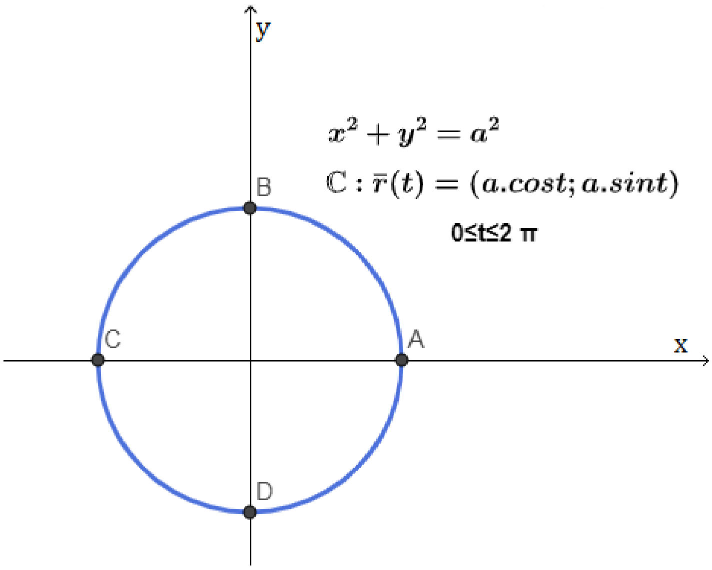
Sabiendo que:

$\bar{F}(x, y) = \big(P(x, y)\,;\; Q(x, y)\big)$

$\overline{ds} = (dx, dy)$

Entonces:

Sí recorremos la curva en sentido positivo, esto es siguiendo la secuencia de puntos $\boldsymbol{A} \to \boldsymbol{B} \to \boldsymbol{C} \to \boldsymbol{D}$, tenemos:



$$\oint_{\mathbb{C}^+} \bar{F} \bullet \overline{ds} \; = \oint_{\mathbb{C}^+} \big(P(x, y)\,;\; Q(x, y)\big) \bullet (dx\,;\; dy) = \oint_{\mathbb{C}^+} \bar{F}[\bar{r}(t)] \bullet \dot{r}(t).dt$$

$$\oint_{\mathbb{C}^+} \bar{F} \bullet \overline{ds} \; = \oint_{\mathbb{C}^+} P(x, y).dx \; + \; Q(x, y).dy = \oint_{\mathbb{C}^+} \bar{F}[\bar{r}(t)] \bullet \dot{r}(t).dt \quad \to$$

$$\oint_{\mathbb{C}^+} \boldsymbol{\bar{F} \bullet \overline{ds}} \; = \oint_{\mathbb{C}^+} \boldsymbol{\bar{F}[\bar{r}(t)] \bullet \dot{r}(t).dt}$$

Ahora bien:

$$\oint \frac{(x+y).dx-(x-y).dy}{x^2+y^2} = \oint \left(\frac{x+y}{x^2+y^2}\right).dx + \left(\frac{-x+y}{x^2+y^2}\right).dx$$

Entonces:

$$\bar{F}(x,y) = \left(\frac{x+y}{x^2+y^2}\ ;\frac{-x+y}{x^2+y^2}\right)$$

$$\mathbb{C}: \bar{r}(t) = (a.cos\,t\ ;\ a.sin\,t) \quad \rightarrow \quad 0 \le t \le 2.\pi$$

$$\boldsymbol{\dot{r}(t) = (-a.sin\,t\ ;\ a.cos\,t)}$$

$$\bar{F}[\bar{r}(t)] = \left(\frac{a.cos\,t + a.sin\,t}{a^2.cos^2(t)+a^2.sen^2(t)}\ ;\ \frac{-a.cos\,t + a.sin\,t}{a^2.cos^2(t)+a^2.sen^2(t)}\right)$$

$$\bar{F}[\bar{r}(t)] = \left(\frac{\cancel{a}.(cos\,t + sin\,t)}{a^{\cancel{2}}.\left(cos^2(t)+.sen^2(t)\right)}\ ;\ \frac{\cancel{a}.(-cos\,t + sin\,t)}{a^{\cancel{2}}.\left(cos^2(t)+sen^2(t)\right)}\right)$$

$$\boldsymbol{\bar{F}[\bar{r}(t)] = \left(\frac{cos\,t + sin\,t}{a}\ ;\ \frac{-cos\,t + sin\,t}{a}\right)}$$

Con lo cual, reemplazando en:

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = \oint_{\mathbb{C}^+} \bar{F}[\bar{r}(t)]\bullet\dot{r}(t).dt \quad \rightarrow$$

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = \int_0^{2\pi} \left(\frac{cos\,t + sin\,t}{a}\ ;\ \frac{-cos\,t + sin\,t}{a}\right)\bullet(-a.sin\,t\ ;\ a.cos\,t).dt \quad \rightarrow$$

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = \int_0^{2\pi} (-sin(t).cos(t) - sin^2(t) - cos^2(t) + sin(t).cos(t)).dt$$

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = \int_0^{2\pi} \left(-sin^2(t) - cos^2(t)\right).dt \quad \rightarrow$$

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = -\int_0^{2\pi} dt \quad \rightarrow$$

$$\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = -2.\pi \quad \rightarrow$$

$$\boldsymbol{\oint_{\mathbb{C}^-} \bar{F}\bullet\overline{ds} = -\oint_{\mathbb{C}^+} \bar{F}\bullet\overline{ds} = 2.\pi}$$