Resolución TP10:

Ejercicio 5 - a

Calcular la integral de línea sobre campo escalar de

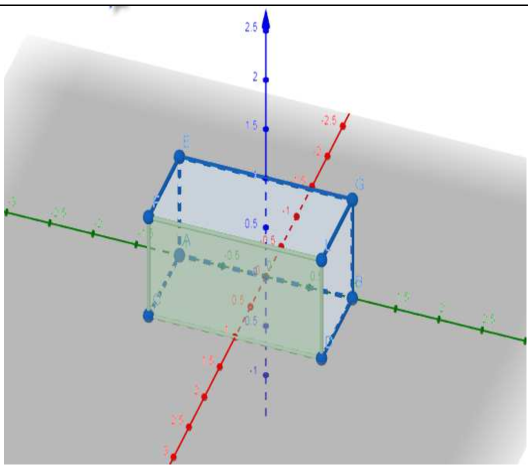
$f(x,y,z) = x + y + z$ con $S: frontera\ de\ [0,1]X[-1,1]X[0,1]$

Resolviendo:

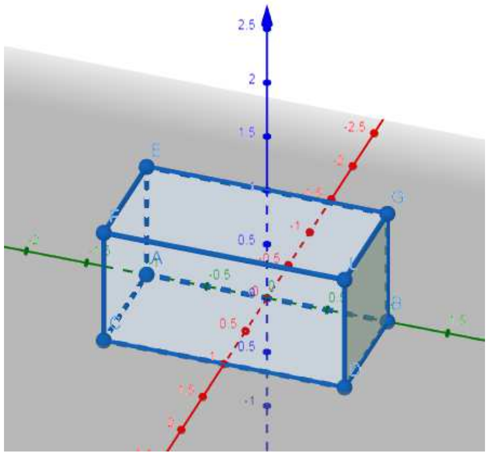
$$\iint_S f dS = \iint_S f(\varphi(u,v))|\varphi_u X \varphi_v| dudv$$

$$\iint_S f dS = \sum \iint_{S_i} f dS_i$$

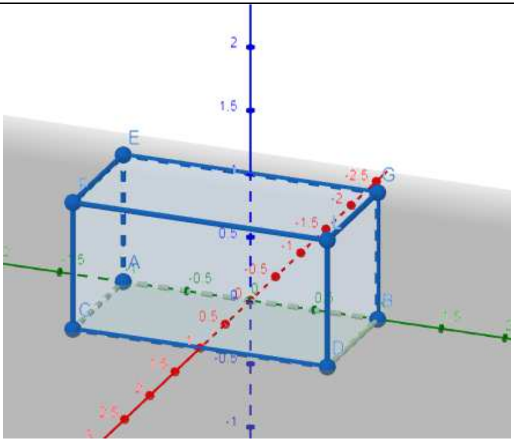
con S_i cada cara del cubo



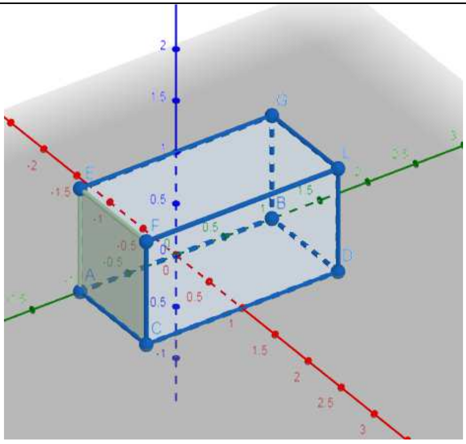
$$\varphi_1 = (1,u,v)$$
$$\varphi_{1u} = (0,1,0)$$
$$\varphi_{1v} = (0,0,1)$$
$$\varphi_{1u} X \varphi_{1v} = (1,0,0)$$
$$|\varphi_{1u} X \varphi_{1v}| = \sqrt{(1^2+0^2+0^2)} = 1$$
$$f(\varphi_1) = 1 + u + v$$
$$R_1: \begin{cases} -1 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{cases}$$



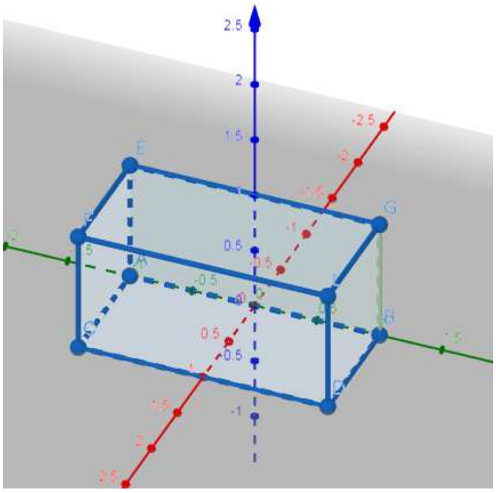
$$\varphi_2 = (u,1,v)$$
$$\varphi_{2u} = (1,0,0)$$
$$\varphi_{2v} = (0,0,1)$$
$$\varphi_{2u} X \varphi_{2v} = (0,1,0)$$
$$|\varphi_{2u} X \varphi_{2v}| = \sqrt{(0^2+1^2+0^2)} = 1$$
$$f(\varphi_2) = u + 1 + v$$
$$R_2: \begin{cases} 0 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{cases}$$



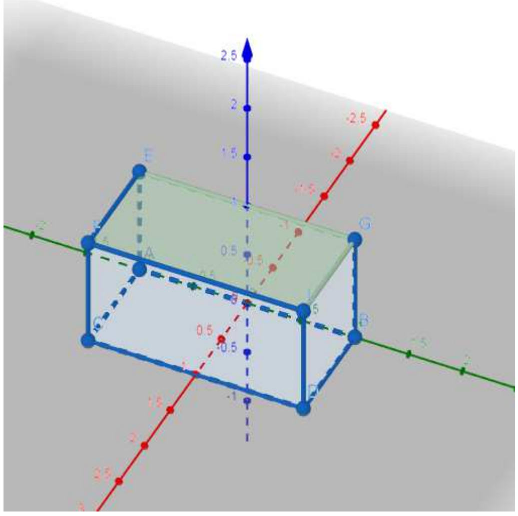
$$\varphi_3 = (u, v, 0)$$
$$\varphi_{3u} = (1,0,0)$$
$$\varphi_{3v} = (0,1,0)$$
$$\varphi_{3u} X \varphi_{3v} = (0,0,1)$$
$$|\varphi_{3u} X \varphi_{3v}| = \sqrt{(0^2 + 0^2 + 1^2)} = 1$$
$$f(\varphi_3) = u + v$$
$$R_3 \colon \begin{cases} 0 \leq u \leq 1 \\ -1 \leq v \leq 1 \end{cases}$$



$$\varphi_4 = (u, -1, v)$$
$$\varphi_{4u} = (1,0,0)$$
$$\varphi_{4v} = (0,0,1)$$
$$\varphi_{4u} X \varphi_{4v} = (0,1,0)$$
$$|\varphi_{4u} X \varphi_{4v}| = \sqrt{(0^2 + 1^2 + 0^2)} = 1$$
$$f(\varphi_4) = u - 1 + v$$
$$R_4 \colon \begin{cases} 0 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{cases}$$



$$\varphi_5 = (0, u, v)$$
$$\varphi_{5u} = (0,1,0)$$
$$\varphi_{5v} = (0,0,1)$$
$$\varphi_{5u} X \varphi_{5v} = (1,0,0)$$
$$|\varphi_{5u} X \varphi_{5v}| = \sqrt{(1^2 + 0^2 + 0^2)} = 1$$
$$f(\varphi_5) = u + v$$
$$R_5 \colon \begin{cases} -1 \leq u \leq 1 \\ 0 \leq v \leq 1 \end{cases}$$



$$\varphi_6 = (u, v, 1)$$
$$\varphi_{6u} = (1,0,0)$$
$$\varphi_{6v} = (0,1,0)$$
$$\varphi_{6u} X \varphi_{6v} = (0,0,1)$$
$$|\varphi_{6u} X \varphi_{6v}| = \sqrt{(0^2 + 0^2 + 1^2)} = 1$$
$$f(\varphi_6) = u + v + 1$$
$$R_6 \colon \begin{cases} 0 \leq u \leq 1 \\ -1 \leq v \leq 1 \end{cases}$$

$$\iint_{S_1} f dS_1 = \int_0^1 \int_{-1}^1 (1 + u + v) du dv = 3$$

$$\iint_{S_2} f dS_2 = \int_0^1 \int_0^1 (u + 1 + v) du dv = 2$$

$$\iint_{S_3} f dS_3 = \int_{-1}^1 \int_0^1 (u + v) du dv = 1$$

$$\iint_{S_4} f dS_4 = \int_0^1 \int_0^1 (u - 1 + v) du dv = 0$$

$$\iint_{S_5} f dS_5 = \int_0^1 \int_{-1}^1 (u + v) du dv = 1$$

$$\iint_{S_6} f dS_6 = \int_{-1}^1 \int_0^1 (u + v + 1) du dv = 1$$

$$\iint_{S} f dS = \sum \iint_{S_i} f dS_i = 8$$