TP6-Ej. 12 h

> Hallar los puntos críticos de la función dada y determinar si es máximo, mínimo o punto de ensilladura:
>
> $$f(x,y) = 2x - 3y + \frac{1}{2} ln(x^2 + y^2) + 5\, arctg\left(\frac{y}{x}\right)$$

$$D_f = \{(x,y)\epsilon\mathbb{R}^2/x^2 + y^2 \neq 0 \wedge x \neq 0\}$$

$$D_f = \{(x,y)\epsilon\mathbb{R}^2/\ x \neq 0\}$$

$$f'_x = 2 + \frac{1}{2}\frac{2x}{(x^2+y^2)} + 5\frac{1}{1+\frac{y^2}{x^2}}\left(-\frac{y}{x^2}\right)$$

$$f'_x = 2 + \frac{x}{x^2+y^2} - \frac{5y}{x^2+y^2}$$

$$f'_x = 2 + \frac{x-5y}{x^2+y^2}$$

$$f'_x = 2 + \frac{x-5y}{x^2+y^2} = 0$$

$$\frac{x-5y}{x^2+y^2} = -2$$

$$-\frac{x}{2} + \frac{5}{2}y = x^2 + y^2 \qquad (1)$$

$$f'_y = -3 + \frac{1}{2}\frac{2y}{(x^2+y^2)} + 5\frac{1}{\left(1+\frac{y^2}{x^2}\right)}\frac{1}{x}$$

$$f'_y = -3 + \frac{y}{x^2+y^2} + \frac{5x}{x^2+y^2}$$

$$f'_y = -3 + \frac{5x+y}{x^2+y^2} = 0$$

$$\frac{5}{3}x + \frac{1}{3}y = x^2 + y^2 \qquad (2)$$

De (1) ^ (2) $-\frac{x}{2}+\frac{5}{2}y=\frac{5}{3}x+\frac{y}{3}$

$$-3x+15y=10x+2y$$

$$13y=13x$$

$$\boxed{x=y}$$

Reemplazando en (1):

$$-\frac{x}{2}+\frac{5x}{2}=x^2+x^2$$

$x = x^2 \rightarrow \ x^2 - x = 0 \rightarrow x(x-1) = 0$

x = 1	^	y = 1

→ Hay un solo punto crítico en (1,1)

$$f'_x=2+\frac{x-5y}{x^2+y^2}$$

$$f''_{xx}=\frac{x^2+y^2-(x-5y)2x}{(x^2+y^2)^2}$$

$$f''_{xx}=\frac{y^2-x^2+10xy}{(x^2+y^2)^2}$$

$$f''_{xy}=\frac{-5(x^2+y^2)-(x-5y)2y}{(x^2+y^2)^2}$$

$$f''_{xy}=\frac{5y^2-5x^2-2xy}{(x^2+y^2)^2}$$

$$f'_y=-3+\frac{5x+y}{x^2+y^2}$$

$$f''_{yy}=\frac{x^2+y^2-(5x+y)2y}{(x^2+y^2)^2}$$

$$f''_{yy} = \frac{x^2 - y^2 - 10xy}{(x^2+y^2)^2}$$

$f''_{xx}(1,1) = \frac{5}{2}$ $\qquad$ $f''_{yy}(1,1) = -\frac{5}{2}$ $\qquad$ $f''_{xy}(1,1) = -\frac{1}{2}$

$$\rightarrow H_f(1,1) = \begin{vmatrix} \frac{5}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{5}{2} \end{vmatrix} = -\frac{13}{2} < 0$$

$\rightarrow en\ (1,1)\ hay\ un\ punto\ de\ ensilladura$