21. d) $A = \begin{pmatrix} a & b & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ¿a, b? es diagonalizable

1°) $|A - \lambda I| = 0$ $\begin{vmatrix} a-\lambda & b & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$ $(a-\lambda).[(-1-\lambda)(1-\lambda)] = 0$

Autovalores : $a, -1, 1$

Si $a \neq -1$ y $a \neq 1$ $(\forall b)$, A es diagonalizable porque

$m(a)\ a = m(g)\ a = 1$

$m(a)\ 1 = m(g)\ 1 = 1$ $\exists$ autobase

$m(a) -1 = m(g) -1 = 1$

Si $a = -1$

$A = \begin{pmatrix} -1 & b & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$; $|A - \lambda I| = \begin{vmatrix} -1-\lambda & b & 0 \\ 0 & -1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$ $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$\underline{\lambda_1 = \lambda_2 = -1}$ $\begin{pmatrix} 0 & b & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{pmatrix} \to \begin{cases} by = 0 \longrightarrow b.y = 0 \to \begin{cases} b = 0 \\ y = 0 \end{cases} \\ 2z = 0 \to z = 0 \end{cases}$

$b = 0$, $y \neq 0 \to \begin{pmatrix} \alpha \\ \beta \\ 0 \end{pmatrix} \to E_{-1} = gen\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$; $\dim E_{-1} = 2$

$m(a) -1 = 2$

$m(g) -1 =$

* $y = 0$ $(b \neq 0) \to \begin{pmatrix} \alpha \\ 0 \\ 0 \end{pmatrix} \longrightarrow E_{-1} = gen\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\}$; $\dim E_1 = 1$

CONCLUSIÓN : Si $a = -1$ $\begin{cases} b = 0 \text{ ; A es diagonalizable} \\ b \neq 0 \text{ ; A no es "} \end{cases}$

$a = 1$; $\underline{\lambda_1 = \lambda_2 = 1}$

$\begin{pmatrix} 0 & b & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & b & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \to \begin{cases} by = 0 \longrightarrow b.0 = 0 \ (\forall b \in \mathbb{R}) \\ 1.y = 0 \to y = 0 \end{cases}$

$\begin{pmatrix} x \\ 0 \\ z \end{pmatrix} \to E_1 = gen\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$; $\dim E_1 = 2$

$m(a)\ 1 = m(g)\ 1 = 2$

CONCLUSIÓN : Si $a = 1$; para todo b , la matriz A es diagonalizable