Resolución TP4:

Ejercicio 5 - a

Utilizando definicion, calcular para $f(x,y) = x + 2xy - 3y^2$ su derivada direccional en $P = (1,2) y\ \vec{v} = \left(\frac{3}{5}, \frac{4}{5}\right)$:

Para empezar:

- Al tratarse de una función de 2 variables $f(x,y)$ posee dos derivadas posibles, una en x y otra en y
  - $f_x(x,y)$
  - $f_y(x,y)$
- $Dom(f) = \mathbb{R}^2$
- Se debe utilizar un vector normalizado $\vec{v} = (a,b)$, es decir $\sqrt{a^2+b^2} = 1$
- La formula por definicion de derivacion direccional:
  - $f_{\vec{v}}(P) = \lim_{t \to 0} \left(\frac{f(x_0+at, y_0+bt) - f(x_0,y_0)}{t}\right)$

Resolvemos:

Primero se verifica que el vector este normalizado:

$$\vec{v} = \left(\frac{3}{5}, \frac{4}{5}\right) \to \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = 1$$

Se puede entonces usar el vector tal como probiene del enunciado.

$$f_{\vec{v}}(1,2) = \lim_{t \to 0} \left(\frac{f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) - f(1,2)}{t}\right)$$

C/A

$$f(x,y) = x + 2xy - 3y^2$$

$$f(1,2) = 1 + 2 \cdot 1 \cdot 2 - 3 \cdot 2^2 = 1 + 4 - 12 = -7$$

$$f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) = 1 + \frac{3}{5}t + 2\left(1+\frac{3}{5}t\right)\left(2+\frac{4}{5}t\right) - 3\left(2+\frac{4}{5}t\right)^2$$

$$f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) = 1 + \frac{3}{5}t + 2\left(2+\frac{4}{5}t+\frac{6}{5}t+\frac{12}{25}t^2\right) - 3\left(4+\frac{16}{5}t+\frac{16}{25}t^2\right)$$

$$f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) = 1 + \frac{3}{5}t + \left(4+\frac{20}{5}t+\frac{24}{25}t^2\right) - \left(12+\frac{48}{5}t+\frac{48}{25}t^2\right)$$

$$f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) = -7 - \frac{25}{5}t - \frac{24}{25}t^2$$

$$f\left(1+\frac{3}{5}t, 2+\frac{4}{5}t\right) = -7 - 5t - \frac{24}{25}t^2$$

$$f_{\vec{v}}(1{,}2) = \lim_{t \to 0}\left(\frac{\left(-7 - 5t - \frac{24}{25}t^2\right) - (-7)}{\mathrm{t}}\right)$$
$$f_{\vec{v}}(1{,}2) = \lim_{t \to 0}\left(\frac{-5t - \frac{24}{25}t^2}{\mathrm{t}}\right)$$
$$f_{\vec{v}}(1{,}2) = \lim_{t \to 0}\left(-5 - \frac{24}{25}t\right)$$
$$f_{\vec{v}}(1{,}2) = -5 - \frac{24}{25} \cdot 0$$

$$f_{\vec{v}}(1{,}2) = -5$$
$$f_{\overrightarrow{\left(\frac{3}{5},\frac{4}{5}\right)}}(1{,}2) = -5$$

$$\vec{w} = (0{,}5) \to |\vec{w}| = \sqrt{0^2 + 5^2} = 5$$
$$\overline{w} = \left(\frac{0}{5},\frac{5}{5}\right) = (0{,}1)$$

- $f_{\vec{w}}(P) = \lim_{t \to 0}\left(\frac{f(x_0+0t,y_0+1t)-f(x_0,y_0)}{\mathrm{t}}\right)$
- $f_{\vec{w}}(P) = \lim_{t \to 0}\left(\frac{f(x_0,y_0+t)-f(x_0,y_0)}{\mathrm{t}}\right)$

$$f_{\vec{w}}(P) = \lim_{\Delta y \to 0}\left(\frac{f(x_0, y_0 + \Delta y) - f(x_0, y_0)}{\Delta y}\right)$$
$$f_{\vec{w}}(1{,}2) = f_y(1{,}2)$$
$$f(x,y) = x + 2xy - 3y^2$$
$$f_y(x,y) = 0 + 2x - 3 * 2y^{2-1} = 2x - 6y$$
$$f_{\vec{w}}(1{,}2) = f_y(1{,}2) = 2 * 1 - 6 * 2 = -10$$