Resolución TP4:

Ejercicio 24-a

Determinar si existen puntos de la ecuación: $x^2 + y^2 + z^2 = 4$ para los que $N = (2,2,2)$ es normal a la superficie.

Herramientas:

- Si $F(x, y, z)$ es Diferenciable $\nabla F(P)$es perpendicular a la superficie de nivel a la que pertenece P
- Si N y $\nabla F(P)$ son normales entonces se da $\nabla F(P) = kN$

Resolviendo:

$$\nabla F(x, y, z) = (2x, 2y, 2z)$$
$$(2\mathrm{x}, 2\mathrm{y}, 2\mathrm{z}) = k(2,2,2)$$

$$\begin{cases} x^2 + y^2 + z^2 = 4 \\ 2x = 2k \\ 2y = 2k \\ 2z = 2k \end{cases}$$

$$\begin{cases} x^2 + y^2 + z^2 = 4 \\ x = k \\ y = k \\ z = k \end{cases}$$

$$k^2 + k^2 + k^2 = 4$$
$$3k^2 = 4$$
$$k_1 = \frac{2}{3}\sqrt{3}$$
$$k_2 = -\frac{2}{3}\sqrt{3}$$

Los puntos resultantes son:

$$P1 = (\frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3})$$

Y

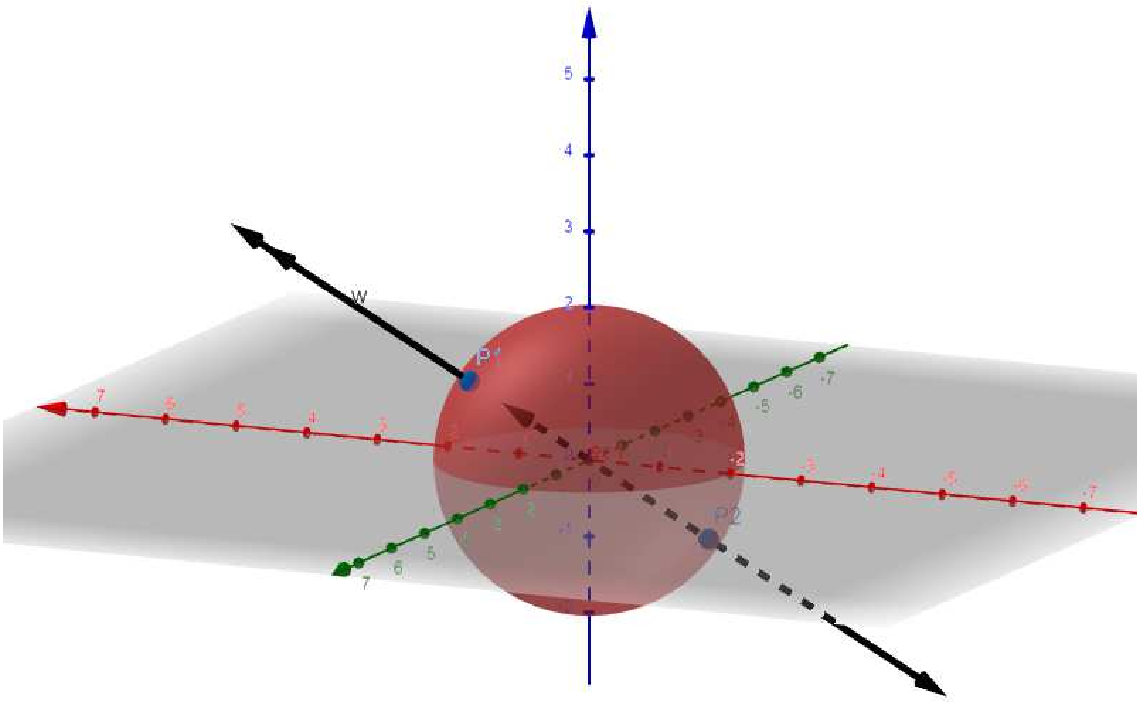
$$P2 = (-\frac{2}{3}\sqrt{3}, -\frac{2}{3}\sqrt{3}, -\frac{2}{3}\sqrt{3})$$

Corolario: A cada punto calcularle su Recta normal y Plano tangente.

Calcular la ecuación del plano tangente y la recta normal para la ecuación: $x^2 + y^2 + z^2 = 4$ en $P1 = (\frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3}, \frac{2}{3}\sqrt{3})$

Calcular la ecuación del plano tangente y la recta normal para la ecuación: $x^2 + y^2 + z^2 = 4$ en $P2 = (-\frac{2}{3}\sqrt{3}, -\frac{2}{3}\sqrt{3}, -\frac{2}{3}\sqrt{3})$

$$\nabla F(x, y, z) = (2x, 2y, 2z)$$
$$\nabla F(P1) = \left(\frac{4}{3}\sqrt{3}, \frac{4}{3}\sqrt{3}, \frac{4}{3}\sqrt{3}\right) \simeq (2{,}3\ ;\ 2{,}3\ ;\ 2{,}3)$$
$$\nabla F(P2) = \left(-\frac{4}{3}\sqrt{3}, -\frac{4}{3}\sqrt{3}, -\frac{4}{3}\sqrt{3}\right) \simeq (-2{,}3\ ;\ -2{,}3\ ;\ -2{,}3)$$



Con esta informacion se pueden calcular sus planos tangentes y rectas normales

$$N(x, y, z) = NP_1$$
$$\nabla F(P_1)(x, y, z) = \nabla F(P_1)P_1$$

$$RnP_1 = P_1 + tN$$
$$RnP_1 = P_1 + t\nabla F(P_1)$$

$$N(x,y,z) = NP_1 \rightarrow (2,2,2)(x,y,z) = (2,2,2)\left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right)$$

$$N(x,y,z) = NP_1 \rightarrow 2x + 2y + 2z = 4\sqrt{3}$$

$$N(x,y,z) = NP_1 \rightarrow x + y + z = 2\sqrt{3}$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow \left(\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3}\right)(x,y,z) = \left(\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3}\right)\left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right)$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow \frac{4}{3}\sqrt{3}x + \frac{4}{3}\sqrt{3}y + \frac{4}{3}\sqrt{3}z = 3\left(\frac{4}{3}\sqrt{3}\right)\left(\frac{2}{3}\sqrt{3}\right)$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow \frac{4}{3}\sqrt{3}x + \frac{4}{3}\sqrt{3}y + \frac{4}{3}\sqrt{3}z = \left(\frac{8}{3}3\right)$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow \frac{4}{3}\sqrt{3}x + \frac{4}{3}\sqrt{3}y + \frac{4}{3}\sqrt{3}z = 8$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow x + y + z = \frac{8}{\frac{4}{3}\sqrt{3}}$$

$$\nabla F(P_1)(x,y,z) = \nabla F(P_1)P_1 \rightarrow x + y + z = \frac{2*3}{\sqrt{3}} * \frac{\sqrt{3}}{\sqrt{3}} = 2\sqrt{3}$$

$$RnP_1 = P_1 + tN = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + t(2,2,2) = \left(\frac{2}{3}\sqrt{3} + 2t,\frac{2}{3}\sqrt{3} + 2t,\frac{2}{3}\sqrt{3} + 2t\right)$$

$$RnP_1 = P_1 + t\nabla F(P_1) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + t\left(\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3}\right)$$

$$RnP_1 = P_1 + t\nabla F(P_1) = \left(\frac{2}{3}\sqrt{3} + \frac{4}{3}\sqrt{3}t,\frac{2}{3}\sqrt{3} + \frac{4}{3}\sqrt{3}t,\frac{2}{3}\sqrt{3} + \frac{4}{3}\sqrt{3}t\right)$$

$$RnP_1 = P_1 + t\nabla F(P_1) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + t\left(\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3},\frac{4}{3}\sqrt{3}\right)$$

$$RnP_1 = P_1 + t\nabla F(P_1) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + \left(\frac{2}{3}\sqrt{3}\right)t(2,2,2)$$

$$RnP_1(t) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + \left(\frac{2}{3}\sqrt{3}\right)t(2,2,2)$$

$$RnP_1(k) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + k(2,2,2)$$

$$RnP_1 = P_1 + t\nabla F(P_1) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + \left(\frac{4}{3}\sqrt{3}\right)t(1,1,1)$$

$$RnP_1(a) = \left(\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3},\frac{2}{3}\sqrt{3}\right) + a(1,1,1)$$