Resolución TP6:

Ejercicio 17 - iv

Hallar los puntos extremos para $f(x,y) = 5 + x^2 + y^2$ dado la siguiente restricción $x^2 + y^2 - 2x - 2y + 1 = 0$, y clasificar como máximo o mínimo.

Para empezar:

- El dominio de ambas funciones es todo $\mathbb{R}^2$ por lo que no tenemos restricción alguna para los puntos que hallaremos

Primeras Derivadas:

$$f_x = 2x$$
$$f_y = 2y$$
$$g_x = 2x - 2$$
$$g_y = 2y - 2$$

Sistema de ecuaciones:

$$\begin{cases} g(x,y) = 0 \\ \nabla f(x,y) = \ell \nabla g(x,y) \end{cases} \rightarrow \begin{cases} x^2 + y^2 - 2x - 2y + 1 = 0 \\ 2x = \ell(2x - 2) \\ 2y = \ell(2y - 2) \end{cases}$$

$$\rightarrow \begin{cases} x^2 + y^2 - 2x - 2y + 1 = 0 \\ 2x = 2x\ell - 2\ell \\ 2y = 2y\ell - 2\ell \end{cases}$$

$$\rightarrow \begin{cases} x^2 + y^2 - 2x - 2y + 1 = 0 \\ 2x - 2x\ell = -2\ell \\ 2y - 2y\ell = -2\ell \end{cases}$$

$$\rightarrow \begin{cases} x^2 + y^2 - 2x - 2y + 1 = 0 \\ 2x(1 - \ell) = -2\ell \\ 2y(1 - \ell) = -2\ell \end{cases}$$

$$\ell = 1 \rightarrow Absurdo\ 0 = -2$$

$$\ell \neq 1 \rightarrow \begin{cases} x^2 + y^2 - 2x - 2y + 1 = 0 \\ x = -\dfrac{\ell}{(1 - \ell)} \\ y = -\dfrac{\ell}{(1 - \ell)} \end{cases}$$

Sustitución en $g(x,y) = 0$

$$x^2 + y^2 - 2x - 2y + 1 = 0$$

$$\left(-\frac{\ell}{(1-\ \ell)}\right)^2+\left(-\frac{\ell}{(1-\ \ell)}\right)^2-2\left(-\frac{\ell}{(1-\ \ell)}\right)-2\left(-\frac{\ell}{(1-\ \ell)}\right)+1=0$$

$$2\left((-1)^2\frac{\ell^2}{(1-\ \ell)^2}\right)\quad -4\left(-\frac{\ell}{(1-\ \ell)}\right)+1=0$$

$$2\frac{\ell^2}{(1-\ \ell)^2}+4\frac{\ell}{(1-\ \ell)}\ +1=0$$

$$\frac{2\ell^2+4\ell(1-\ell)+1(1-\ell)^2}{(1-\ \ell)^2}=0$$

$$2\ell^2+4\ell(1-\ell)+1(1-\ell)^2=0$$

$$2\ell^2+4\ell-4\ell^2+1-2\ell+\ell^2\ =0$$

$$-\ell^2+2\ell+1\quad=0$$

$$\ell=\frac{-2\pm\sqrt{4-4(-1)1}}{2(-1)}$$

$$\ell=\frac{-2\pm2\sqrt{2}}{-2}$$

$$\ell=\frac{-2+2\sqrt{2}}{-2}\text{ o }\ell=\frac{-2-2\sqrt{2}}{-2}$$

$$\ell=1-\sqrt{2}\text{ o }\ell=1+\sqrt{2}$$

Entonces

$$\begin{cases}\ell=1-\sqrt{2}\text{ o }\ell=1+\sqrt{2}\\ x=-\dfrac{\ell}{(1-\ \ell)}\\ y\ =-\dfrac{\ell}{(1-\ \ell)}\end{cases}$$

$$\begin{cases}\ell=1-\sqrt{2}\\ x=-\dfrac{\ell}{(1-\ \ell)}\\ y\ =-\dfrac{\ell}{(1-\ \ell)}\end{cases}\rightarrow\begin{cases}\ell=1-\sqrt{2}\\ x=-\dfrac{1-\sqrt{2}}{(1-\ 1+\sqrt{2})}\\ y\ =-\dfrac{1-\sqrt{2}}{(1-\ 1+\sqrt{2})}\end{cases}\rightarrow\begin{cases}\ell=1-\sqrt{2}\\ x=-\dfrac{1-\sqrt{2}}{\sqrt{2}}\\ y\ =-\dfrac{1-\sqrt{2}}{\sqrt{2}}\end{cases}\rightarrow PC_1$$

$$PC_1=\left(-\frac{1-\sqrt{2}}{\sqrt{2}},-\frac{1-\sqrt{2}}{\sqrt{2}}\right)$$

$$\begin{cases} \ell = 1+\sqrt{2} \\ x = -\dfrac{\ell}{(1-\ \ell)} \\ y\ = -\dfrac{\ell}{(1-\ \ell)} \end{cases} \rightarrow \begin{cases} \ell = 1+\sqrt{2} \\ x = -\dfrac{1+\sqrt{2}}{\left(1-\ 1-\sqrt{2}\right)} \\ y\ = -\dfrac{1-\sqrt{2}}{\left(1-\ 1-\sqrt{2}\right)} \end{cases} \rightarrow \begin{cases} \ell = 1+\sqrt{2} \\ x = -\dfrac{1+\sqrt{2}}{-\sqrt{2}} \\ y\ = -\dfrac{1+\sqrt{2}}{-\sqrt{2}} \end{cases} \rightarrow PC_2$$

$$PC_2 = \left(\frac{1+\sqrt{2}}{\sqrt{2}}, \frac{1+\sqrt{2}}{\sqrt{2}}\right)$$

Clasificación:

Método 1: Ya sabemos que ambos puntos cumplen la condición, debemos compáralos entre sí para saber si son máximo o mínimo.

Se evalúan en $f(x,y) = 5 + x^2 + y^2$

- $f(Pc_1) = f\left(-\frac{1-\sqrt{2}}{\sqrt{2}}, -\frac{1-\sqrt{2}}{\sqrt{2}}\right) = 5 + \left(1-\sqrt{2}\right)^2$
- $f(Pc_2) = f\left(\frac{1+\sqrt{2}}{\sqrt{2}}, \frac{1+\sqrt{2}}{\sqrt{2}}\right) = 5 + \left(1+\sqrt{2}\right)^2$

Pc_2 es máximo condicionados de $g(x,y) = 0$

Pc_1 es mínimo condicionados de $g(x,y) = 0$