Resolución TP7:

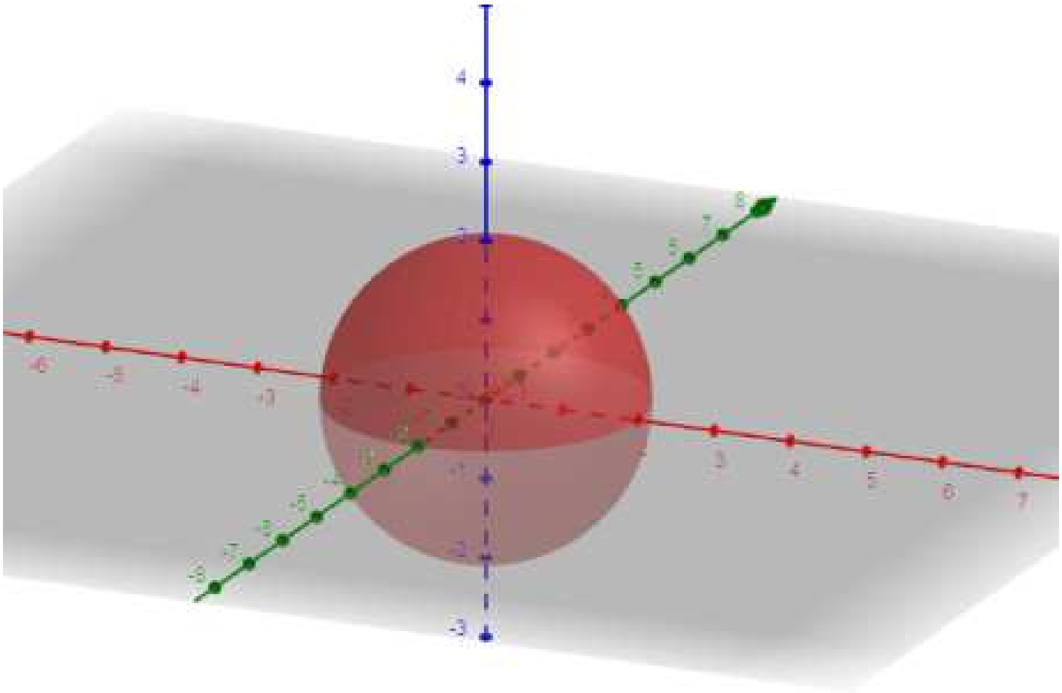
Usando integrales Triples calcular el volumen de las siguientes regiones.

$a)\ V: \{(x, y, z) \epsilon R^3 /\ x^2 + y^2 + z^2 \leq 4\}$

Resolucion:

Si f(x,y,z)=1 esta definida en una region V, entonces el volumen se puede resolver como:

$$V = \iiint_V 1 dxdydz$$



Ejercicio a)

$$x^2 + y^2 + z^2 \leq 4$$

$$z^2 \leq 4 - (x^2 + y^2)$$

$$-\sqrt{4 - (x^2 + y^2)} \leq z \leq \sqrt{4 - (x^2 + y^2)}$$

Buscamos limites de x y z:

$$z_t = \sqrt{4 - (x^2 + y^2)}$$

$$z_p = -\sqrt{4 - (x^2 + y^2)}$$

Por sumas y restas $(1)z_t + (1)z_p$:

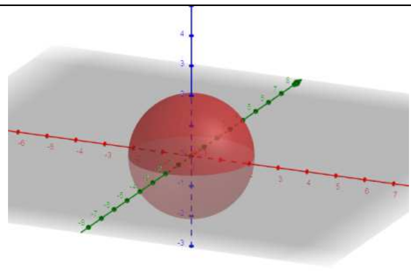
$$2z = 0 \rightarrow z = 0$$

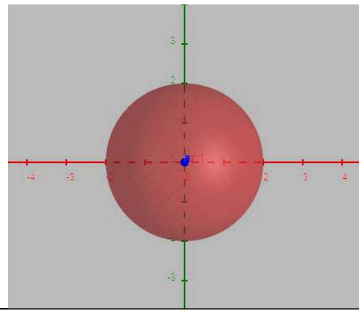
Entonces:

$$x^2 + y^2 + z^2 \leq 4 \rightarrow x^2 + y^2 \leq 4$$

Final mente

$$V:\begin{cases}-\sqrt{4-(x^2+y^2)}\ \leq z \leq \sqrt{4-(x^2+y^2)}\\ x^2+y^2 \leq 4\end{cases}$$





$$V_a = \iiint_V 1dxdydz$$

$$V_a = \iint_{x^2+y^2\leq 4} \int_{-\sqrt{4-(x^2+y^2)}}^{\sqrt{4-(x^2+y^2)}} 1\, dz\, dxdy$$

$$V_a = \iint_{x^2+y^2\leq 4} 2\sqrt{4-(x^2+y^2)}\, dxdy$$

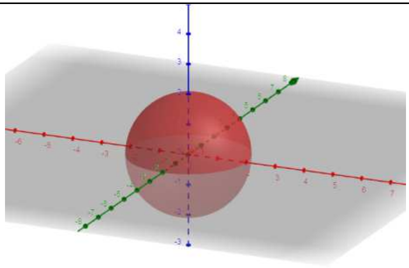
$$V_a \overset{\substack{TRANS\\POLARES}}{\cong} \int_0^2 \int_0^{2\pi} 2\sqrt{4-r^2}\, r\, d\theta dr$$

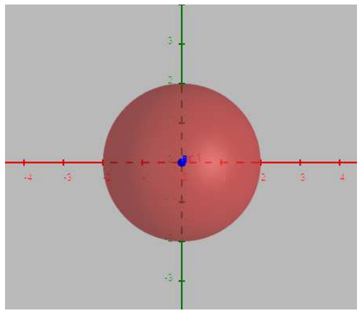
Con coordenadas cilindricas

$$V:\begin{cases}-\sqrt{4-(x^2+y^2)}\ \leq z \leq \sqrt{4-(x^2+y^2)} \\ x^2+y^2 \leq 4\end{cases}$$

$$V:\begin{cases} x = rcos\theta \\ y = rsen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases}-\sqrt{4-r^2}\ \leq z \leq \sqrt{4-r^2} \\ r^2 \leq 4\end{cases}\end{cases}$$

$$V:\begin{cases} x = rcos\theta \\ y = rsen\theta \\ z = z \\ |J| = r \\ V' = \begin{cases}-\sqrt{4-r^2}\ \leq z \leq \sqrt{4-r^2} \\ 0 \leq\ r \leq 2 \\ 0 \leq \theta < 2\pi\end{cases}\end{cases}$$





$$V_a = \iiint\limits_V 1dxdydz$$

$$V_a \overset{\substack{Trans \\ Cilindricas}}{\cong} \iiint\limits_{V'} rd\theta drdz$$

$$V_a = \int_0^2 \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} rdz\ d\theta dr$$

$$V_a = \int_0^2 \int_0^{2\pi} 2\sqrt{4-r^2}\, r\ d\theta dr$$

$$V_a = 2\pi \int_0^2 2\sqrt{4-r^2} r\ dr$$

sustitucion $4-r^2 = t \rightarrow\ -2rdr = dt \rightarrow 2rdr = -dt$

$$V_a = -2\pi \int_{r=0}^{r=2} \sqrt{t}\ dt$$

$$V_a = -2\pi \left[\frac{2}{3}\sqrt{t^3}\right]_{r=0}^{r=2}$$

$$V_a = -2\pi \left[\frac{2}{3}\sqrt{(4-r^2)^3}\right]_{r=0}^{r=2}$$

$$V_a = -\frac{4}{3}\pi\left[\sqrt{(4-r^2)^3}\right]_{r=0}^{r=2}$$

$$V_a = -\frac{4}{3}\pi\left[\sqrt{(4-2^2)^3} - \sqrt{(4-0^2)^3}\right]$$

$$V_a = -\frac{4}{3}\pi[0 - 4\sqrt{4}]$$

$$V_a = -\frac{4}{3}\pi[-8]$$

$$V_a = \frac{32}{3}\pi$$

Verificacion: