Resolución TP7:

Ejercicio adicional

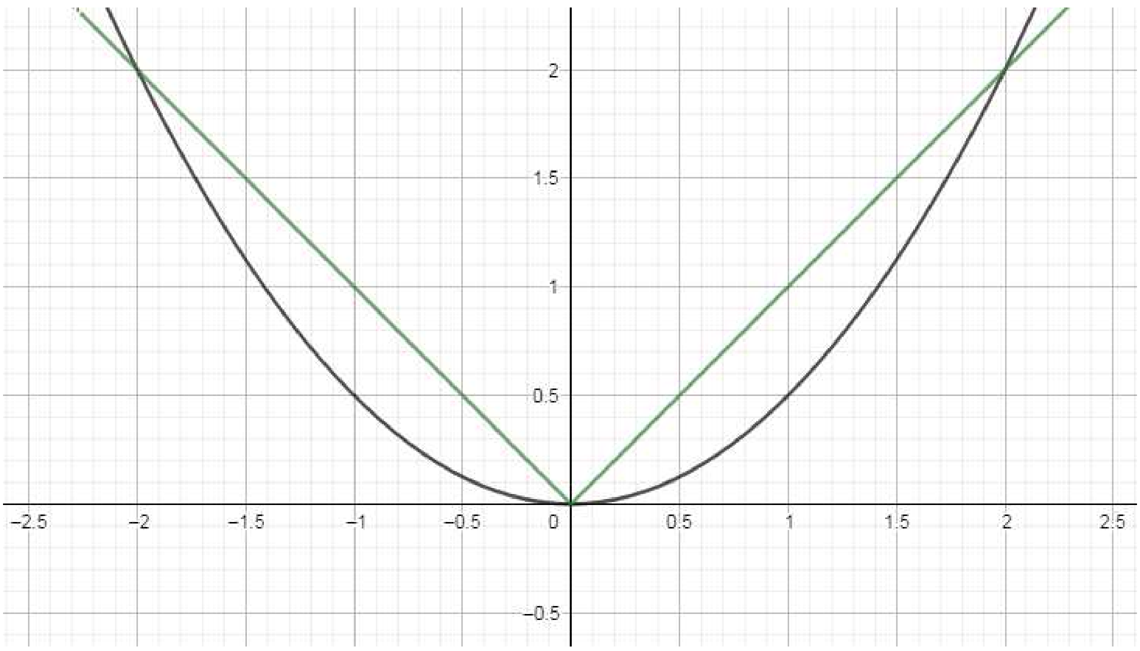
Resolver la integral triple I con él recinto V.

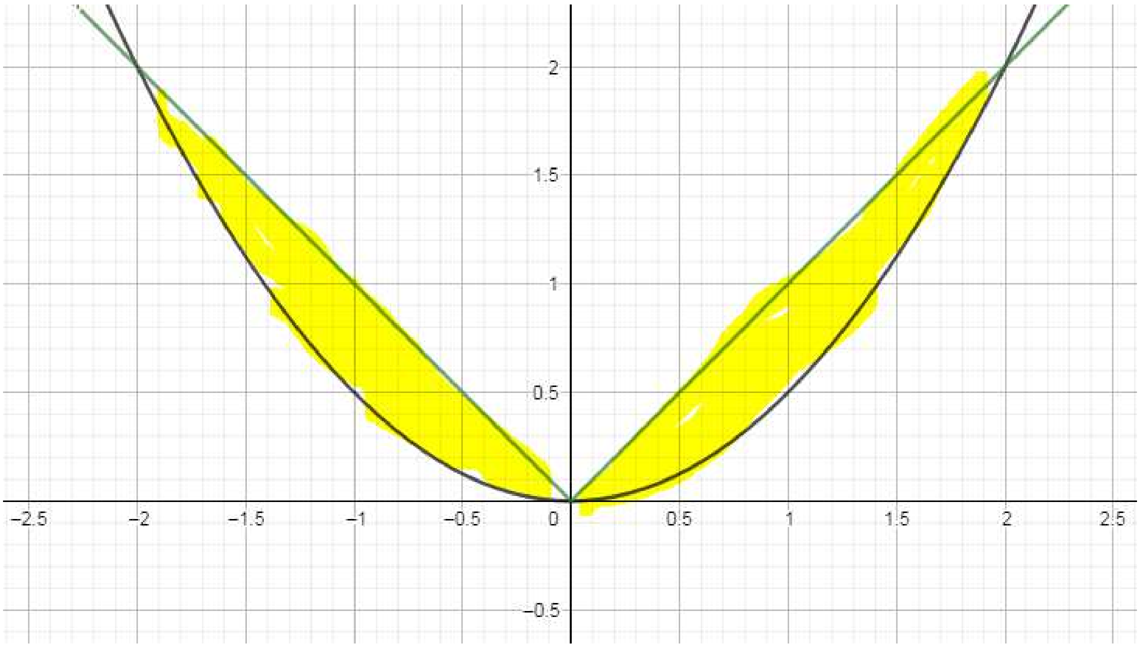
$$V:\{(x,y,z)\epsilon R^3:\ figura\ entre\ \ z=\sqrt{x^2+y^2}\ \wedge\ 2z=x^2+y^2\ \}$$

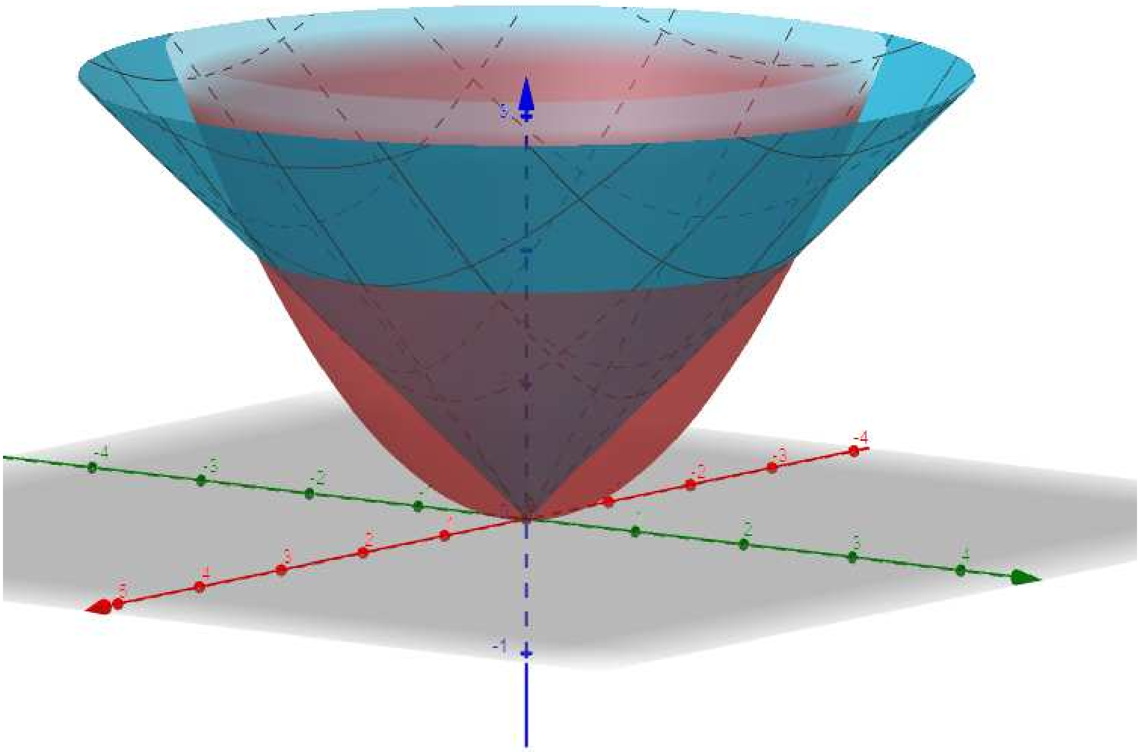
$$I=\ \ \iint T(x,y)-P(x,y)dxdy$$

$$x=0\rightarrow z=\sqrt{0^2+y^2}\ \wedge\ z=\frac{y^2}{2}$$

$$x=0\rightarrow z=|y|\wedge\ z=\frac{y^2}{2}$$







$$V: \{(x, y, z) \epsilon R^3: \ z \leq \sqrt{x^2 + y^2} \wedge z \geq \frac{x^2}{2} + \frac{y^2}{2}\}$$

$$V: \{(x, y, z) \epsilon R^3: \frac{x^2}{2} + \frac{y^2}{2} \leq z \leq \sqrt{x^2 + y^2} \ \}$$

$$\frac{x^2}{2}+\frac{y^2}{2}=\sqrt{x^2+y^2}$$

$$\underbrace{\frac{x^2+y^2}{2}=z \qquad \overbrace{z=\sqrt{x^2+y^2}}^{z^2=x^2+y^2}}_{\frac{z^2}{2}=z}$$

$$\frac{z^2}{2}=z\rightarrow z=0\ o\ z=2\rightarrow x^2+y^2\leq 4$$

$$V:\{(x,y,z)\epsilon R^3:\frac{x^2}{2}+\frac{y^2}{2}\leq z\leq\sqrt{x^2+y^2}\ \wedge\ x^2+y^2\leq 4\}$$

$$\iint\limits_{x^2+y^2\leq 4}\sqrt{x^2+y^2}-\frac{x^2+y^2}{2}dxdy$$

Transformacion de Coordenas polares

$$\int_0^2\int_0^{2\pi}\left(r-\frac{r^2}{2}\right)\overbrace{r}^{|J|}d\alpha dr$$

$$\int_0^2\int_0^{2\pi}\left(r^2-\frac{r^3}{2}\right)d\alpha dr$$

$$2\pi\int_0^2\left(r^2-\frac{r^3}{2}\right)dr$$

$$2\pi\left[\frac{r^3}{3}-\frac{r^4}{8}\right]_0^2=\frac{4}{3}\pi$$