Resolución TP10:

Ejercicio 6 - a

Dado el campo vectorial F y la superficie S, calcular el flujo correspondiente con la normal que apunta hacia arriba.

$$F(x,y,z) = (x^2y, xz, y^2z)$$

S: es el triangulo determinado por la intersección del plano de ecuación $x + y + z = 1$ y los planos coordenados.

Resolviendo:

$$I = \iint_S F \cdot dS = \iint_{R_\Phi} F(\Phi) \cdot (\Phi_u X \Phi_v) dudv$$

Considerando que se trata solo del triangulo podemos nombrar a la superficie con la siguiente descripción:

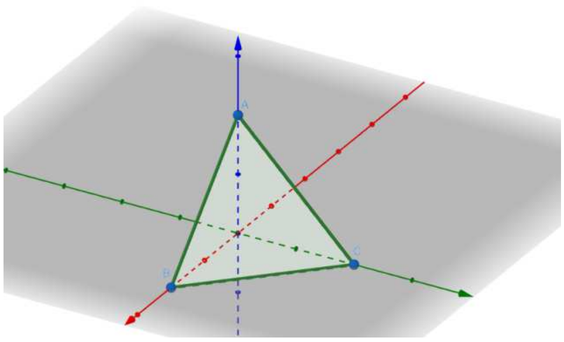
$$S: \begin{cases} x + y + z = 1 \\ y \geq 0 \\ x \geq 0 \\ z \geq 0 \end{cases}$$

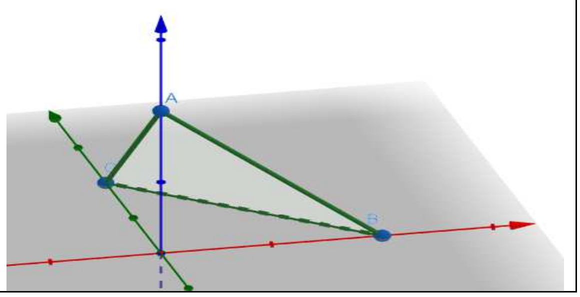
Dibujamos calculando las trazas y los vertices:

$$S \xrightarrow{x=0} \begin{cases} y + z = 1 \\ y \geq 0 \\ z \geq 0 \end{cases} \xrightarrow{y=0} \begin{cases} z = 1 \\ A = (0,0,1) \end{cases}$$

$$S \xrightarrow{y=0} \begin{cases} x + z = 1 \\ x \geq 0 \\ z \geq 0 \end{cases} \xrightarrow{z=0} \begin{cases} x = 1 \\ B = (1,0,0) \end{cases}$$

$$S \xrightarrow{z=0} \begin{cases} x + y = 1 \\ x \geq 0 \\ y \geq 0 \end{cases} \xrightarrow{x=0} \begin{cases} y = 1 \\ C = (0,1,0) \end{cases}$$





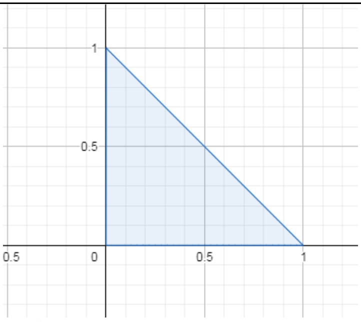
$$S:\begin{cases}\Phi(x,y)=\big(x,y,1-(x+y)\big)\\ R_\Phi:\begin{cases}y\geq 0\\ x\geq 0\\ x+y\leq 1\end{cases}\end{cases}$$

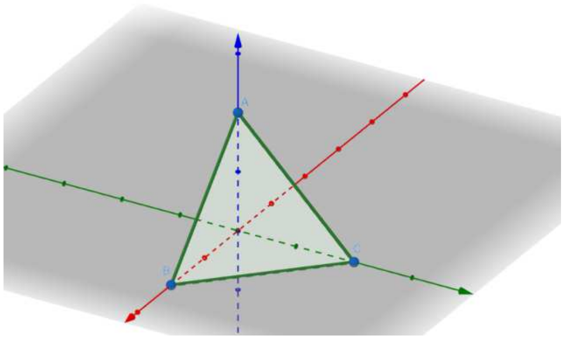
Tipo I

$$S:\begin{cases}\Phi(x,y)=\big(x,y,1-(x+y)\big)\\ R_\Phi:\begin{cases}0\leq y\leq 1-x\\ 0\leq x\leq 1\end{cases}\end{cases}$$

Tipo II

$$S:\begin{cases}\Phi(x,y)=\big(x,y,1-(x+y)\big)\\ R_\Phi:\begin{cases}0\leq x\leq 1-y\\ 0\leq y\leq 1\end{cases}\end{cases}$$





$$\Phi_x = (1,0,-1)$$

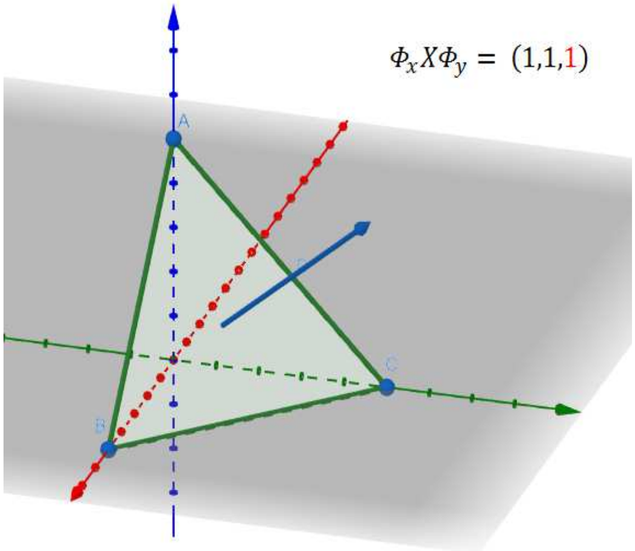
$$\Phi_y = (0,1,-1)$$

$$\Phi_x X \Phi_y = \begin{bmatrix} i & j & k \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} = \left(\begin{vmatrix} 0 & -1 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} 1 & -1 \\ 0 & -1 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\right)$$

$$\Phi_x X \Phi_y = \Big(\big(0-(-1)\big), -(-1-0), (1-0)\Big)$$

$$\Phi_x X \Phi_y = (1,1,1)$$

" calcular el flujo correspondiente con la <u>normal que apunta hacia arriba</u> "



$$\begin{cases} \Phi(x,y) = (x,y,1-x-y) \\ F(x,y,z) = (x^2y, xz, y^2z) \end{cases} \rightarrow F(\Phi)$$

$$F(\Phi) = (x^2y, x(1-x-y), y^2(1-x-y))$$

$$F(\Phi) = (x^2y, x-x^2-xy, y^2-xy^2-y^3)$$

$$F(\Phi)\cdot(\Phi_x X\Phi_y) = (x^2y, x-x^2-xy, y^2-xy^2-y^3)(1,1,1)$$

$$F(\Phi)\cdot(\Phi_x X\Phi_y) = x^2y + x - x^2 - xy + y^2 - xy^2 - y^3$$

$$I = \iint_S F\cdot dS = \iint_{R_\Phi} F(\Phi)\cdot(\Phi_x X\Phi_y)dudv$$

$$I = \iint_{R_\Phi} (x^2y + x - x^2 - xy + y^2 - xy^2 - y^3)dR = \frac{3}{40}$$

$$recomendado\ verificar\ por\ Tipo\ I\ y\ Tipo\ II, ver\ C/A$$

Resolución Tipo I

$$I = \int_0^1 \int_0^{1-x} (x^2y + x - x^2 - xy + y^2 - xy^2 - y^3)\, dydx$$

$$I = \int_0^1 \left[\frac{x^2y^2}{2} + xy - x^2y - \frac{xy^2}{2} + \frac{y^3}{3} - \frac{xy^3}{3} - \frac{y^4}{4}\right]_0^{1-x} dx$$

$$I = \int_0^1 \frac{x^2(1-x)^2}{2} + x(1-x) - x^2(1-x) - \frac{x(1-x)^2}{2} + \frac{(1-x)^3}{3} - \frac{x(1-x)^3}{3} - \frac{(1-x)^4}{4} dx$$

$$I = \int_0^1 \frac{7}{12}x^4 - \frac{5}{6}x^3 + \frac{1}{6}x + \frac{1}{12} dx$$

$$I = \left[\frac{7}{12}\left(\frac{1}{5}x^5\right) - \frac{5}{6}\left(\frac{1}{4}x^4\right) + \frac{1}{6}\left(\frac{1}{2}x^2\right) + \frac{x}{12}\right]_0^1$$

$$I = \left[\frac{7}{60}x^5 - \frac{5}{24}x^4 + \frac{1}{12}x^2 + \frac{x}{12}\right]_0^1$$

$$I = \left(\frac{7}{60} - \frac{5}{24} + \frac{1}{12} + \frac{1}{12}\right) = \frac{3}{40}$$

Resolución Tipo II

$$I = \int_0^1 \int_0^{1-y} (x^2y + x - x^2 - xy + y^2 - xy^2 - y^3)\, dxdy$$

$$I = \int_0^1 \left[\frac{x^3y}{3} + \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^2y}{2} + xy^2 - \frac{x^2y^2}{2} - xy^3\right]_0^{1-y} dy$$

$$I = \int_0^1 \left(\frac{(1-y)^3y}{3} + \frac{(1-y)^2}{2} - \frac{(1-y)^3}{3} - \frac{(1-y)^2y}{2} + (1-y)y^2 - \frac{(1-y)^2y^2}{2} - (1-y)y^3\right) dy$$

$$I = \int_0^1 \frac{y^4}{6} - \frac{y^3}{6} - \frac{y}{6} + \frac{1}{6} dy$$

$$I = \left[\frac{y^5}{30} - \frac{y^4}{24} - \frac{y^2}{12} + \frac{y}{6}\right]_0^1$$

$$I = \frac{1}{30} - \frac{1}{24} - \frac{1}{12} + \frac{1}{6} = \frac{3}{40}$$