Ejercicio 23-TP4

$f(x,y) = \frac{x^2-y^2}{x^2+y^2}$ Tomando P=(1;1)

Como $f(x,y)$ $es\ diferenciable\ en\ (1;1)\ se\ podrá\ usar\ \ f_v'(1;1) = \nabla f(1;1) \cdot v$ (1)

$$f_x' = \frac{2x(x^2+y^2)-(x^2-y^2)2x}{(x^2+y^2)^2}$$

$$f_x' = \frac{2x^3+2xy^2-2x^3+2xy^2}{(x^2+y^2)^2}$$

$$f_x' = \frac{4xy^2}{(x^2+y^2)^2} \quad \rightarrow \quad f_x'(1;1) = 1$$

$$f_y' = \frac{-2y(x^2+y^2)-(x^2-y^2)2y}{(x^2+y^2)^2}$$

$$f_y' = \frac{-2yx^2-2y^3-2yx^2+2y^3}{(x^2+y^2)^2}$$

$$f_y' = \frac{-4yx^2}{(x^2+y^2)^2} \quad \rightarrow \quad f_y'(1;1) = -1$$

$\rightarrow$ $\nabla f(1;1) = (1;-1)$

$v = (a;b)$

De (1) $f_v'(1;1) = (1;-1).(a;b) = 0$

a-b = 0 $\rightarrow$ b=a

$$v = (a;a)$$

$$\|v\|^2 = a^2 + a^2 = 1$$

$$a^2 = \frac{1}{2}$$

$$a_1 = \frac{1}{\sqrt{2}} \qquad \wedge \qquad a_2 = \frac{-1}{\sqrt{2}}$$

$$v_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \qquad \wedge \qquad v_2 = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$