Supplementary material: Pair equations

Endemic disease, awareness, and local behavioural response

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\begin{aligned}
\frac{d}{dt}S_- &= -\hat{\beta}\left[S_-I_-\right]^d - \sigma_I\hat{\beta}\left[S_-I_+\right]^d \\
&\quad +\delta R_- \\
&\quad +\lambda S_+ \\
&\quad -\hat{\alpha}\left[S_-S_+\right]^a - \hat{\alpha}\left[S_-I_+\right]^a - \hat{\alpha}\left[S_-R_+\right]^a \\
\frac{d}{dt}I_- &= +\hat{\beta}\left[S_-I_-\right]^d + \sigma_I\hat{\beta}\left[S_-I_+\right]^d \\
&\quad -\gamma I_- \\
&\quad +\lambda I_+ \\
&\quad -\omega I_- \\
&\quad -\hat{\alpha}\left[I_-S_+\right]^a - \hat{\alpha}\left[I_-I_+\right]^a - \hat{\alpha}\left[I_-R_+\right]^a \\
\frac{d}{dt}R_- &= +\gamma I_- \\
&\quad -\delta R_- \\
&\quad +\lambda R_+ \\
&\quad -\hat{\alpha}\left[R_-S_+\right]^a - \hat{\alpha}\left[R_-I_+\right]^a - \hat{\alpha}\left[R_-R_+\right]^a \\
\frac{d}{dt}S_+ &= -\sigma_S\sigma_I\hat{\beta}\left[S_+I_+\right]^d - \sigma_S\hat{\beta}\left[I_-S_+\right]^d \\
&\quad +\phi\delta R_+ \\
&\quad -\lambda S_+ \\
&\quad +\hat{\alpha}\left[S_-S_+\right]^a + \hat{\alpha}\left[S_-I_+\right]^a + \hat{\alpha}\left[S_-R_+\right]^a
\end{aligned}
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$$
\begin{aligned}
\frac{d}{dt} I_+ &= +\sigma_S \sigma_I \hat{\beta} \left[S_+ I_+ \right]^d + \sigma_S \hat{\beta} \left[I_- S_+ \right]^d \\
&\quad -\epsilon \gamma I_+ \\
&\quad -\lambda I_+ \\
&\quad +\omega I_- \\
&\quad +\hat{\alpha} \left[I_- S_+ \right]^a + \hat{\alpha} \left[I_- I_+ \right]^a + \hat{\alpha} \left[I_- R_+ \right]^a \\
\frac{d}{dt} R_+ &= +\epsilon \gamma I_+ \\
&\quad -\phi \delta R_+ \\
&\quad -\lambda R_+ \\
&\quad +\hat{\alpha} \left[R_- S_+ \right]^a + \hat{\alpha} \left[R_- I_+ \right]^a + \hat{\alpha} \left[R_- R_+ \right]^a \\
\frac{d}{dt} \left[S_- S_- \right]^d &= 2 \Big(-\hat{\beta} \left[S_- S_- I_- \right]^{dd} - \sigma_I \hat{\beta} \left[S_- S_- I_+ \right]^{dd} \\
&\quad +\delta \left[S_- R_- \right]^d \\
&\quad +\lambda \left[S_- S_+ \right]^d \\
&\quad -\hat{\alpha} \left[S_- S_- S_+ \right]^{da} - \hat{\alpha} \left[S_- S_- I_+ \right]^{da} - \hat{\alpha} \left[S_- S_- R_+ \right]^{da} \\
&\quad \Big) \\
\frac{d}{dt} \left[S_- S_- \right]^a &= 2 \Big(-\hat{\beta} \left[S_- S_- I_- \right]^{ad} - \sigma_I \hat{\beta} \left[S_- S_- I_+ \right]^{ad} \\
&\quad +\delta \left[S_- R_- \right]^a \\
&\quad +\lambda \left[S_- S_+ \right]^a \\
&\quad -\hat{\alpha} \left[S_- S_- S_+ \right]^{aa} - \hat{\alpha} \left[S_- S_- I_+ \right]^{aa} - \hat{\alpha} \left[S_- S_- R_+ \right]^{aa} \\
&\quad \Big) \\
\frac{d}{dt} \left[S_- I_- \right]^d &= -\hat{\beta} \left[S_- I_- \right]^d + \hat{\beta} \left[S_- S_- I_- \right]^{dd} - \hat{\beta} \left[I_- S_- I_- \right]^{dd} \\
&\quad +\sigma_I \hat{\beta} \left[S_- S_- I_+ \right]^{dd} - \sigma_I \hat{\beta} \left[I_+ S_- I_- \right]^{dd} \\
&\quad -\gamma \left[S_- I_- \right]^d \\
&\quad +\delta \left[I_- R_- \right]^d \\
&\quad +\lambda \left[I_- S_+ \right]^d + \lambda \left[S_- I_+ \right]^d \\
&\quad -\omega \left[S_- I_- \right]^d \\
&\quad -\hat{\alpha} \left[S_+ S_- I_- \right]^{ad} - \hat{\alpha} \left[I_+ S_- I_- \right]^{ad} - \hat{\alpha} \left[R_+ S_- I_- \right]^{ad} \\
&\quad -\hat{\alpha} \left[S_- I_- S_+ \right]^{da} - \hat{\alpha} \left[S_- I_- I_+ \right]^{da} - \hat{\alpha} \left[S_- I_- R_+ \right]^{da}
\end{aligned}
$$

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\begin{aligned}
\frac{d}{dt}\left[S\_I\_\right]^{a} &= +\hat{\beta}\left[S\_S\_I\_\right]^{ad} - \hat{\beta}\left[I\_S\_I\_\right]^{da} + \sigma_I\hat{\beta}\left[S\_S\_I_+\right]^{ad} - \sigma_I\hat{\beta}\left[I_+S\_I\_\right]^{da} \\
&\quad -\gamma\left[S\_I\_\right]^{a} \\
&\quad +\delta\left[I\_R\_\right]^{a} \\
&\quad +\lambda\left[I\_S_+\right]^{a} + \lambda\left[S\_I_+\right]^{a} \\
&\quad -\omega\left[S\_I\_\right]^{a} \\
&\quad -\hat{\alpha}\left[S_+S\_I\_\right]^{aa} - \hat{\alpha}\left[I_+S\_I\_\right]^{aa} - \hat{\alpha}\left[R_+S\_I\_\right]^{aa} \\
&\quad -\hat{\alpha}\left[S\_I\_S_+\right]^{aa} - \hat{\alpha}\left[S\_I\_I_+\right]^{aa} - \hat{\alpha}\left[S\_I\_R_+\right]^{aa} \\
&\quad -q_{d|a}\hat{\beta}\left[S\_I\_\right]^{a} \\
\frac{d}{dt}\left[S\_R\_\right]^{d} &= -\hat{\beta}\left[I\_S\_R\_\right]^{dd} - \sigma_I\hat{\beta}\left[I_+S\_R\_\right]^{dd} \\
&\quad +\gamma\left[S\_I\_\right]^{d} \\
&\quad -\delta\left[S\_R\_\right]^{d} + \delta\left[R\_R\_\right]^{d} \\
&\quad +\lambda\left[R\_S_+\right]^{d} + \lambda\left[S\_R_+\right]^{d} \\
&\quad -\hat{\alpha}\left[S_+S\_R\_\right]^{ad} - \hat{\alpha}\left[I_+S\_R\_\right]^{ad} - \hat{\alpha}\left[R_+S\_R\_\right]^{ad} \\
&\quad -\hat{\alpha}\left[S\_R\_S_+\right]^{da} - \hat{\alpha}\left[S\_R\_I_+\right]^{da} - \hat{\alpha}\left[S\_R\_R_+\right]^{da} \\
\frac{d}{dt}\left[S\_R\_\right]^{a} &= -\hat{\beta}\left[I\_S\_R\_\right]^{da} - \sigma_I\hat{\beta}\left[I_+S\_R\_\right]^{da} \\
&\quad +\gamma\left[S\_I\_\right]^{a} \\
&\quad -\delta\left[S\_R\_\right]^{a} + \delta\left[R\_R\_\right]^{a} \\
&\quad +\lambda\left[R\_S_+\right]^{a} + \lambda\left[S\_R_+\right]^{a} \\
&\quad -\hat{\alpha}\left[S_+S\_R\_\right]^{aa} - \hat{\alpha}\left[I_+S\_R\_\right]^{aa} - \hat{\alpha}\left[R_+S\_R\_\right]^{aa} \\
&\quad -\hat{\alpha}\left[S\_R\_S_+\right]^{aa} - \hat{\alpha}\left[S\_R\_I_+\right]^{aa} - \hat{\alpha}\left[S\_R\_R_+\right]^{aa} \\
\frac{d}{dt}\left[S\_S_+\right]^{d} &= -\hat{\beta}\left[I\_S\_S_+\right]^{dd} - \sigma_S\sigma_I\hat{\beta}\left[S\_S_+I_+\right]^{dd} - \sigma_I\hat{\beta}\left[I_+S\_S_+\right]^{dd} - \sigma_S\hat{\beta}\left[S\_S_+I\_\right]^{dd} \\
&\quad +\delta\left[R\_S_+\right]^{d} + \phi\delta\left[S\_R_+\right]^{d} \\
&\quad -\lambda\left[S\_S_+\right]^{d} + \lambda\left[S_+S_+\right]^{d} \\
&\quad +\hat{\alpha}\left[S\_S\_S_+\right]^{da} - \hat{\alpha}\left[S_+S\_S_+\right]^{ad} + \hat{\alpha}\left[S\_S\_I_+\right]^{da} \\
&\quad -\hat{\alpha}\left[I_+S\_S_+\right]^{ad} + \hat{\alpha}\left[S\_S\_R_+\right]^{da} - \hat{\alpha}\left[R_+S\_S_+\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[S\_S_+\right]^{d}
\end{aligned}
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\begin{aligned}
\frac{d}{dt}\left[S_{-}S_{+}\right]^{a} &= -\hat{\beta}\left[I_{-}S_{-}S_{+}\right]^{da} - \sigma_S\sigma_I\hat{\beta}\left[S_{-}S_{+}I_{+}\right]^{ad} - \sigma_I\hat{\beta}\left[I_{+}S_{-}S_{+}\right]^{da} - \sigma_S\hat{\beta}\left[S_{-}S_{+}I_{-}\right]^{ad} \\
&\quad +\delta\left[R_{-}S_{+}\right]^{a} + \phi\delta\left[S_{-}R_{+}\right]^{a} \\
&\quad -\lambda\left[S_{-}S_{+}\right]^{a} + \lambda\left[S_{+}S_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{-}S_{+}\right]^{a} + \hat{\alpha}\left[S_{-}S_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[S_{+}S_{-}S_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}S_{-}I_{+}\right]^{aa} \\
&\quad -\hat{\alpha}\left[I_{+}S_{-}S_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}S_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}S_{-}S_{+}\right]^{aa} \\
\frac{d}{dt}\left[S_{-}I_{+}\right]^{d} &= -\hat{\beta}\left[I_{-}S_{-}I_{+}\right]^{dd} + \sigma_S\sigma_I\hat{\beta}\left[S_{-}S_{+}I_{+}\right]^{dd} - \sigma_I\hat{\beta}\left[S_{-}I_{+}\right]^{d} \\
&\quad -\sigma_I\hat{\beta}\left[I_{+}S_{-}I_{+}\right]^{dd} + \sigma_S\hat{\beta}\left[S_{-}S_{+}I_{-}\right]^{dd} \\
&\quad -\epsilon\gamma\left[S_{-}I_{+}\right]^{d} \\
&\quad +\delta\left[R_{-}I_{+}\right]^{d} \\
&\quad +\lambda\left[S_{+}I_{+}\right]^{d} - \lambda\left[S_{-}I_{+}\right]^{d} \\
&\quad +\omega\left[S_{-}I_{-}\right]^{d} \\
&\quad -\hat{\alpha}\left[S_{+}S_{-}I_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}S_{-}I_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}S_{-}I_{+}\right]^{ad} \\
&\quad +\hat{\alpha}\left[S_{-}I_{-}S_{+}\right]^{da} + \hat{\alpha}\left[S_{-}I_{-}I_{+}\right]^{da} + \hat{\alpha}\left[S_{-}I_{-}R_{+}\right]^{da} \\
&\quad -q_{a|d}\hat{\alpha}\left[S_{-}I_{+}\right]^{d} \\
\frac{d}{dt}\left[S_{-}I_{+}\right]^{a} &= -\hat{\beta}\left[I_{-}S_{-}I_{+}\right]^{da} + \sigma_S\sigma_I\hat{\beta}\left[S_{-}S_{+}I_{+}\right]^{ad} - \sigma_I\hat{\beta}\left[I_{+}S_{-}I_{+}\right]^{da} + \sigma_S\hat{\beta}\left[S_{-}S_{+}I_{-}\right]^{ad} \\
&\quad -\epsilon\gamma\left[S_{-}I_{+}\right]^{a} \\
&\quad +\delta\left[R_{-}I_{+}\right]^{a} \\
&\quad +\lambda\left[S_{+}I_{+}\right]^{a} - \lambda\left[S_{-}I_{+}\right]^{a} \\
&\quad +\omega\left[S_{-}I_{-}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}S_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[S_{-}I_{+}\right]^{a} - \hat{\alpha}\left[I_{+}S_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}S_{-}I_{+}\right]^{aa} \\
&\quad +\hat{\alpha}\left[S_{-}I_{-}S_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}I_{-}I_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}I_{-}R_{+}\right]^{aa} \\
&\quad -q_{d|a}\sigma_I\hat{\beta}\left[S_{-}I_{+}\right]^{a}
\end{aligned}
$$

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\begin{aligned}
\frac{d}{dt}\left[S_{-}R_{+}\right]^{d} &= -\hat{\beta}\left[I_{-}S_{-}R_{+}\right]^{dd} - \sigma_{I}\hat{\beta}\left[I_{+}S_{-}R_{+}\right]^{dd} \\
&\quad +\epsilon\gamma\left[S_{-}I_{+}\right]^{d} \\
&\quad +\delta\left[R_{-}R_{+}\right]^{d} - \phi\delta\left[S_{-}R_{+}\right]^{d} \\
&\quad +\lambda\left[S_{+}R_{+}\right]^{d} - \lambda\left[S_{-}R_{+}\right]^{d} \\
&\quad -\hat{\alpha}\left[S_{+}S_{-}R_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}S_{-}R_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}S_{-}R_{+}\right]^{ad} \\
&\quad +\hat{\alpha}\left[S_{-}R_{-}S_{+}\right]^{da} + \hat{\alpha}\left[S_{-}R_{-}I_{+}\right]^{da} + \hat{\alpha}\left[S_{-}R_{-}R_{+}\right]^{da} \\
&\quad -q_{a|d}\hat{\alpha}\left[S_{-}R_{+}\right]^{d} \\
\frac{d}{dt}\left[S_{-}R_{+}\right]^{a} &= -\hat{\beta}\left[I_{-}S_{-}R_{+}\right]^{da} - \sigma_{I}\hat{\beta}\left[I_{+}S_{-}R_{+}\right]^{da} \\
&\quad +\epsilon\gamma\left[S_{-}I_{+}\right]^{a} \\
&\quad +\delta\left[R_{-}R_{+}\right]^{a} - \phi\delta\left[S_{-}R_{+}\right]^{a} \\
&\quad +\lambda\left[S_{+}R_{+}\right]^{a} - \lambda\left[S_{-}R_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}S_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[I_{+}S_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[S_{-}R_{+}\right]^{a} - \hat{\alpha}\left[R_{+}S_{-}R_{+}\right]^{aa} \\
&\quad +\hat{\alpha}\left[S_{-}R_{-}S_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}R_{-}I_{+}\right]^{aa} + \hat{\alpha}\left[S_{-}R_{-}R_{+}\right]^{aa} \\
\frac{d}{dt}\left[I_{-}I_{-}\right]^{d} &= 2\Big(+\hat{\beta}\left[S_{-}I_{-}\right]^{d} + \hat{\beta}\left[I_{-}S_{-}I_{-}\right]^{dd} + \sigma_{I}\hat{\beta}\left[I_{+}S_{-}I_{-}\right]^{dd} \\
&\quad -\gamma\left[I_{-}I_{-}\right]^{d} \\
&\quad +\lambda\left[I_{-}I_{+}\right]^{d} \\
&\quad -\omega\left[I_{-}I_{-}\right]^{d} \\
&\quad -\hat{\alpha}\left[I_{-}I_{-}S_{+}\right]^{da} - \hat{\alpha}\left[I_{-}I_{-}I_{+}\right]^{da} - \hat{\alpha}\left[I_{-}I_{-}R_{+}\right]^{da} \\
&\quad \Big) \\
\frac{d}{dt}\left[I_{-}I_{-}\right]^{a} &= 2\Big(+\hat{\beta}\left[I_{-}S_{-}I_{-}\right]^{da} + \sigma_{I}\hat{\beta}\left[I_{+}S_{-}I_{-}\right]^{da} \\
&\quad -\gamma\left[I_{-}I_{-}\right]^{a} \\
&\quad +\lambda\left[I_{-}I_{+}\right]^{a} \\
&\quad -\omega\left[I_{-}I_{-}\right]^{a} \\
&\quad -\hat{\alpha}\left[I_{-}I_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}I_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}I_{-}R_{+}\right]^{aa} \\
&\quad +q_{d|a}\hat{\beta}\left[S_{-}I_{-}\right]^{a}\Big)
\end{aligned}
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\begin{aligned}
\frac{d}{dt}\left[I_{-}R_{-}\right]^{d} &= +\hat{\beta}\left[I_{-}S_{-}R_{-}\right]^{dd} + \sigma_I\hat{\beta}\left[I_{+}S_{-}R_{-}\right]^{dd} \\
&\quad +\gamma\left[I_{-}I_{-}\right]^{d} - \gamma\left[I_{-}R_{-}\right]^{d} \\
&\quad -\delta\left[I_{-}R_{-}\right]^{d} \\
&\quad +\lambda\left[R_{-}I_{+}\right]^{d} + \lambda\left[I_{-}R_{+}\right]^{d} \\
&\quad -\omega\left[I_{-}R_{-}\right]^{d} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}R_{-}\right]^{ad} - \hat{\alpha}\left[I_{+}I_{-}R_{-}\right]^{ad} - \hat{\alpha}\left[R_{+}I_{-}R_{-}\right]^{ad} \\
&\quad -\hat{\alpha}\left[I_{-}R_{-}S_{+}\right]^{da} - \hat{\alpha}\left[I_{-}R_{-}I_{+}\right]^{da} - \hat{\alpha}\left[I_{-}R_{-}R_{+}\right]^{da} \\
\frac{d}{dt}\left[I_{-}R_{-}\right]^{a} &= +\hat{\beta}\left[I_{-}S_{-}R_{-}\right]^{da} + \sigma_I\hat{\beta}\left[I_{+}S_{-}R_{-}\right]^{da} \\
&\quad +\gamma\left[I_{-}I_{-}\right]^{a} - \gamma\left[I_{-}R_{-}\right]^{a} \\
&\quad -\delta\left[I_{-}R_{-}\right]^{a} \\
&\quad +\lambda\left[R_{-}I_{+}\right]^{a} + \lambda\left[I_{-}R_{+}\right]^{a} \\
&\quad -\omega\left[I_{-}R_{-}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}R_{-}\right]^{aa} - \hat{\alpha}\left[I_{+}I_{-}R_{-}\right]^{aa} - \hat{\alpha}\left[R_{+}I_{-}R_{-}\right]^{aa} \\
&\quad -\hat{\alpha}\left[I_{-}R_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}R_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}R_{-}R_{+}\right]^{aa} \\
\frac{d}{dt}\left[I_{-}S_{+}\right]^{d} &= +\hat{\beta}\left[I_{-}S_{-}S_{+}\right]^{dd} - \sigma_S\sigma_I\hat{\beta}\left[I_{-}S_{+}I_{+}\right]^{dd} + \sigma_I\hat{\beta}\left[I_{+}S_{-}S_{+}\right]^{dd} \\
&\quad -\sigma_S\hat{\beta}\left[I_{-}S_{+}\right]^{d} - \sigma_S\hat{\beta}\left[I_{-}S_{+}I_{-}\right]^{dd} \\
&\quad -\gamma\left[I_{-}S_{+}\right]^{d} \\
&\quad +\phi\delta\left[I_{-}R_{+}\right]^{d} \\
&\quad -\lambda\left[I_{-}S_{+}\right]^{d} + \lambda\left[S_{+}I_{+}\right]^{d} \\
&\quad -\omega\left[I_{-}S_{+}\right]^{d} \\
&\quad +\hat{\alpha}\left[S_{+}S_{-}I_{-}\right]^{ad} + \hat{\alpha}\left[I_{+}S_{-}I_{-}\right]^{ad} + \hat{\alpha}\left[R_{+}S_{-}I_{-}\right]^{ad} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}S_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}I_{-}S_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}I_{-}S_{+}\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[I_{-}S_{+}\right]^{d}
\end{aligned}
$$

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\begin{aligned}
\frac{d}{dt}\left[I_{-}S_{+}\right]^{a} &= +\hat{\beta}\left[I_{-}S_{-}S_{+}\right]^{da} - \sigma_S\sigma_I\hat{\beta}\left[I_{-}S_{+}I_{+}\right]^{ad} + \sigma_I\hat{\beta}\left[I_{+}S_{-}S_{+}\right]^{da} - \sigma_S\hat{\beta}\left[I_{-}S_{+}I_{-}\right]^{ad} \\
&\quad -\gamma\left[I_{-}S_{+}\right]^{a} \\
&\quad +\phi\delta\left[I_{-}R_{+}\right]^{a} \\
&\quad -\lambda\left[I_{-}S_{+}\right]^{a} + \lambda\left[S_{+}I_{+}\right]^{a} \\
&\quad -\omega\left[I_{-}S_{+}\right]^{a} \\
&\quad +\hat{\alpha}\left[S_{+}S_{-}I_{-}\right]^{aa} + \hat{\alpha}\left[I_{+}S_{-}I_{-}\right]^{aa} + \hat{\alpha}\left[R_{+}S_{-}I_{-}\right]^{aa} - \hat{\alpha}\left[I_{-}S_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[I_{+}I_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}I_{-}S_{+}\right]^{aa} \\
&\quad -q_{d|a}\sigma_S\hat{\beta}\left[I_{-}S_{+}\right]^{a} \\
\frac{d}{dt}\left[I_{-}I_{+}\right]^{d} &= +\hat{\beta}\left[I_{-}S_{-}I_{+}\right]^{dd} + \sigma_S\sigma_I\hat{\beta}\left[I_{-}S_{+}I_{+}\right]^{dd} + \sigma_I\hat{\beta}\left[S_{-}I_{+}\right]^{d} \\
&\quad +\sigma_I\hat{\beta}\left[I_{+}S_{-}I_{+}\right]^{dd} + \sigma_S\hat{\beta}\left[I_{-}S_{+}\right]^{d} + \sigma_S\hat{\beta}\left[I_{-}S_{+}I_{-}\right]^{dd} \\
&\quad -\gamma\left[I_{-}I_{+}\right]^{d} - \epsilon\gamma\left[I_{-}I_{+}\right]^{d} \\
&\quad -\lambda\left[I_{-}I_{+}\right]^{d} + \lambda\left[I_{+}I_{+}\right]^{d} \\
&\quad +\omega\left[I_{-}I_{-}\right]^{d} - \omega\left[I_{-}I_{+}\right]^{d} \\
&\quad +\hat{\alpha}\left[I_{-}I_{-}S_{+}\right]^{da} - \hat{\alpha}\left[S_{+}I_{-}I_{+}\right]^{ad} + \hat{\alpha}\left[I_{-}I_{-}I_{+}\right]^{da} \\
&\quad -\hat{\alpha}\left[I_{+}I_{-}I_{+}\right]^{ad} + \hat{\alpha}\left[I_{-}I_{-}R_{+}\right]^{da} - \hat{\alpha}\left[R_{+}I_{-}I_{+}\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[I_{-}I_{+}\right]^{d} \\
\frac{d}{dt}\left[I_{-}I_{+}\right]^{a} &= +\hat{\beta}\left[I_{-}S_{-}I_{+}\right]^{da} + \sigma_S\sigma_I\hat{\beta}\left[I_{-}S_{+}I_{+}\right]^{ad} + \sigma_I\hat{\beta}\left[I_{+}S_{-}I_{+}\right]^{da} + \sigma_S\hat{\beta}\left[I_{-}S_{+}I_{-}\right]^{ad} \\
&\quad -\gamma\left[I_{-}I_{+}\right]^{a} - \epsilon\gamma\left[I_{-}I_{+}\right]^{a} \\
&\quad -\lambda\left[I_{-}I_{+}\right]^{a} + \lambda\left[I_{+}I_{+}\right]^{a} \\
&\quad +\omega\left[I_{-}I_{-}\right]^{a} - \omega\left[I_{-}I_{+}\right]^{a} \\
&\quad +\hat{\alpha}\left[I_{-}I_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[S_{+}I_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}I_{+}\right]^{a} + \hat{\alpha}\left[I_{-}I_{-}I_{+}\right]^{aa} \\
&\quad -\hat{\alpha}\left[I_{+}I_{-}I_{+}\right]^{aa} + \hat{\alpha}\left[I_{-}I_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}I_{-}I_{+}\right]^{aa} \\
&\quad +q_{d|a}\sigma_I\hat{\beta}\left[S_{-}I_{+}\right]^{a} + q_{d|a}\sigma_S\hat{\beta}\left[I_{-}S_{+}\right]^{a}
\end{aligned}
$$

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\begin{aligned}
\frac{d}{dt}\left[I_{-}R_{+}\right]^{d} &= +\hat{\beta}\left[I_{-}S_{-}R_{+}\right]^{dd} + \sigma_{I}\hat{\beta}\left[I_{+}S_{-}R_{+}\right]^{dd} \\
&\quad -\gamma\left[I_{-}R_{+}\right]^{d} + \epsilon\gamma\left[I_{-}I_{+}\right]^{d} \\
&\quad -\phi\delta\left[I_{-}R_{+}\right]^{d} \\
&\quad +\lambda\left[I_{+}R_{+}\right]^{d} - \lambda\left[I_{-}R_{+}\right]^{d} \\
&\quad -\omega\left[I_{-}R_{+}\right]^{d} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}R_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}I_{-}R_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}I_{-}R_{+}\right]^{ad} \\
&\quad +\hat{\alpha}\left[I_{-}R_{-}S_{+}\right]^{da} + \hat{\alpha}\left[I_{-}R_{-}I_{+}\right]^{da} + \hat{\alpha}\left[I_{-}R_{-}R_{+}\right]^{da} \\
&\quad -q_{a|d}\hat{\alpha}\left[I_{-}R_{+}\right]^{d} \\
\frac{d}{dt}\left[I_{-}R_{+}\right]^{a} &= +\hat{\beta}\left[I_{-}S_{-}R_{+}\right]^{da} + \sigma_{I}\hat{\beta}\left[I_{+}S_{-}R_{+}\right]^{da} \\
&\quad -\gamma\left[I_{-}R_{+}\right]^{a} + \epsilon\gamma\left[I_{-}I_{+}\right]^{a} \\
&\quad -\phi\delta\left[I_{-}R_{+}\right]^{a} \\
&\quad +\lambda\left[I_{+}R_{+}\right]^{a} - \lambda\left[I_{-}R_{+}\right]^{a} \\
&\quad -\omega\left[I_{-}R_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}I_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[I_{+}I_{-}R_{+}\right]^{aa} - \hat{\alpha}\left[I_{-}R_{+}\right]^{a} - \hat{\alpha}\left[R_{+}I_{-}R_{+}\right]^{aa} \\
&\quad +\hat{\alpha}\left[I_{-}R_{-}S_{+}\right]^{aa} + \hat{\alpha}\left[I_{-}R_{-}I_{+}\right]^{aa} + \hat{\alpha}\left[I_{-}R_{-}R_{+}\right]^{aa} \\
\frac{d}{dt}\left[R_{-}R_{-}\right]^{d} &= 2\,(+\gamma\left[I_{-}R_{-}\right]^{d} \\
&\quad -\delta\left[R_{-}R_{-}\right]^{d} \\
&\quad +\lambda\left[R_{-}R_{+}\right]^{d} \\
&\quad -\hat{\alpha}\left[R_{-}R_{-}S_{+}\right]^{da} - \hat{\alpha}\left[R_{-}R_{-}I_{+}\right]^{da} - \hat{\alpha}\left[R_{-}R_{-}R_{+}\right]^{da} \\
&\quad) \\
\frac{d}{dt}\left[R_{-}R_{-}\right]^{a} &= 2\,(+\gamma\left[I_{-}R_{-}\right]^{a} \\
&\quad -\delta\left[R_{-}R_{-}\right]^{a} \\
&\quad +\lambda\left[R_{-}R_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[R_{-}R_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[R_{-}R_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[R_{-}R_{-}R_{+}\right]^{aa} \\
&\quad)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[R_{-}S_{+}\right]^{d} &= -\sigma_S\sigma_I\hat{\beta}\left[R_{-}S_{+}I_{+}\right]^{dd} - \sigma_S\hat{\beta}\left[R_{-}S_{+}I_{-}\right]^{dd} \\
&\quad +\gamma\left[I_{-}S_{+}\right]^{d} \\
&\quad -\delta\left[R_{-}S_{+}\right]^{d} + \phi\delta\left[R_{-}R_{+}\right]^{d} \\
&\quad -\lambda\left[R_{-}S_{+}\right]^{d} + \lambda\left[S_{+}R_{+}\right]^{d} \\
&\quad +\hat{\alpha}\left[S_{+}S_{-}R_{-}\right]^{ad} + \hat{\alpha}\left[I_{+}S_{-}R_{-}\right]^{ad} + \hat{\alpha}\left[R_{+}S_{-}R_{-}\right]^{ad} \\
&\quad -\hat{\alpha}\left[S_{+}R_{-}S_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}R_{-}S_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}R_{-}S_{+}\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[R_{-}S_{+}\right]^{d} \\
\frac{d}{dt}\left[R_{-}S_{+}\right]^{a} &= -\sigma_S\sigma_I\hat{\beta}\left[R_{-}S_{+}I_{+}\right]^{ad} - \sigma_S\hat{\beta}\left[R_{-}S_{+}I_{-}\right]^{ad} \\
&\quad +\gamma\left[I_{-}S_{+}\right]^{a} \\
&\quad -\delta\left[R_{-}S_{+}\right]^{a} + \phi\delta\left[R_{-}R_{+}\right]^{a} \\
&\quad -\lambda\left[R_{-}S_{+}\right]^{a} + \lambda\left[S_{+}R_{+}\right]^{a} \\
&\quad +\hat{\alpha}\left[S_{+}S_{-}R_{-}\right]^{aa} + \hat{\alpha}\left[I_{+}S_{-}R_{-}\right]^{aa} + \hat{\alpha}\left[R_{+}S_{-}R_{-}\right]^{aa} - \hat{\alpha}\left[R_{-}S_{+}\right]^{a} \\
&\quad -\hat{\alpha}\left[S_{+}R_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[I_{+}R_{-}S_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}R_{-}S_{+}\right]^{aa} \\
\frac{d}{dt}\left[R_{-}I_{+}\right]^{d} &= +\sigma_S\sigma_I\hat{\beta}\left[R_{-}S_{+}I_{+}\right]^{dd} + \sigma_S\hat{\beta}\left[R_{-}S_{+}I_{-}\right]^{dd} \\
&\quad +\gamma\left[I_{-}I_{+}\right]^{d} - \epsilon\gamma\left[R_{-}I_{+}\right]^{d} \\
&\quad -\delta\left[R_{-}I_{+}\right]^{d} \\
&\quad -\lambda\left[R_{-}I_{+}\right]^{d} + \lambda\left[I_{+}R_{+}\right]^{d} \\
&\quad +\omega\left[I_{-}R_{-}\right]^{d} \\
&\quad +\hat{\alpha}\left[S_{+}I_{-}R_{-}\right]^{ad} + \hat{\alpha}\left[I_{+}I_{-}R_{-}\right]^{ad} + \hat{\alpha}\left[R_{+}I_{-}R_{-}\right]^{ad} \\
&\quad -\hat{\alpha}\left[S_{+}R_{-}I_{+}\right]^{ad} - \hat{\alpha}\left[I_{+}R_{-}I_{+}\right]^{ad} - \hat{\alpha}\left[R_{+}R_{-}I_{+}\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[R_{-}I_{+}\right]^{d} \\
\frac{d}{dt}\left[R_{-}I_{+}\right]^{a} &= +\sigma_S\sigma_I\hat{\beta}\left[R_{-}S_{+}I_{+}\right]^{ad} + \sigma_S\hat{\beta}\left[R_{-}S_{+}I_{-}\right]^{ad} \\
&\quad +\gamma\left[I_{-}I_{+}\right]^{a} - \epsilon\gamma\left[R_{-}I_{+}\right]^{a} \\
&\quad -\delta\left[R_{-}I_{+}\right]^{a} \\
&\quad -\lambda\left[R_{-}I_{+}\right]^{a} + \lambda\left[I_{+}R_{+}\right]^{a} \\
&\quad +\omega\left[I_{-}R_{-}\right]^{a} \\
&\quad +\hat{\alpha}\left[S_{+}I_{-}R_{-}\right]^{aa} + \hat{\alpha}\left[I_{+}I_{-}R_{-}\right]^{aa} + \hat{\alpha}\left[R_{+}I_{-}R_{-}\right]^{aa} - \hat{\alpha}\left[S_{+}R_{-}I_{+}\right]^{aa} \\
&\quad -\hat{\alpha}\left[R_{-}I_{+}\right]^{a} - \hat{\alpha}\left[I_{+}R_{-}I_{+}\right]^{aa} - \hat{\alpha}\left[R_{+}R_{-}I_{+}\right]^{aa}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[R_{-}R_{+}\right]^{d} &= +\gamma\left[I_{-}R_{+}\right]^{d}+\epsilon\gamma\left[R_{-}I_{+}\right]^{d} \\
&\quad -\delta\left[R_{-}R_{+}\right]^{d}-\phi\delta\left[R_{-}R_{+}\right]^{d} \\
&\quad -\lambda\left[R_{-}R_{+}\right]^{d}+\lambda\left[R_{+}R_{+}\right]^{d} \\
&\quad +\hat{\alpha}\left[R_{-}R_{-}S_{+}\right]^{da}-\hat{\alpha}\left[S_{+}R_{-}R_{+}\right]^{ad}+\hat{\alpha}\left[R_{-}R_{-}I_{+}\right]^{da} \\
&\quad -\hat{\alpha}\left[I_{+}R_{-}R_{+}\right]^{ad}+\hat{\alpha}\left[R_{-}R_{-}R_{+}\right]^{da}-\hat{\alpha}\left[R_{+}R_{-}R_{+}\right]^{ad} \\
&\quad -q_{a|d}\hat{\alpha}\left[R_{-}R_{+}\right]^{d} \\
\frac{d}{dt}\left[R_{-}R_{+}\right]^{a} &= +\gamma\left[I_{-}R_{+}\right]^{a}+\epsilon\gamma\left[R_{-}I_{+}\right]^{a} \\
&\quad -\delta\left[R_{-}R_{+}\right]^{a}-\phi\delta\left[R_{-}R_{+}\right]^{a} \\
&\quad -\lambda\left[R_{-}R_{+}\right]^{a}+\lambda\left[R_{+}R_{+}\right]^{a} \\
&\quad +\hat{\alpha}\left[R_{-}R_{-}S_{+}\right]^{aa}-\hat{\alpha}\left[S_{+}R_{-}R_{+}\right]^{aa}+\hat{\alpha}\left[R_{-}R_{-}I_{+}\right]^{aa}-\hat{\alpha}\left[I_{+}R_{-}R_{+}\right]^{aa} \\
&\quad -\hat{\alpha}\left[R_{-}R_{+}\right]^{a}+\hat{\alpha}\left[R_{-}R_{-}R_{+}\right]^{aa}-\hat{\alpha}\left[R_{+}R_{-}R_{+}\right]^{aa} \\
\frac{d}{dt}\left[S_{+}S_{+}\right]^{d} &= 2\Big(-\sigma_{S}\sigma_{I}\hat{\beta}\left[S_{+}S_{+}I_{+}\right]^{dd}-\sigma_{S}\hat{\beta}\left[S_{+}S_{+}I_{-}\right]^{dd} \\
&\quad +\phi\delta\left[S_{+}R_{+}\right]^{d} \\
&\quad -\lambda\left[S_{+}S_{+}\right]^{d} \\
&\quad +\hat{\alpha}\left[S_{+}S_{-}S_{+}\right]^{ad}+\hat{\alpha}\left[I_{+}S_{-}S_{+}\right]^{ad}+\hat{\alpha}\left[R_{+}S_{-}S_{+}\right]^{ad} \\
&\quad +q_{a|d}\hat{\alpha}\left[S_{-}S_{+}\right]^{d}\Big) \\
\frac{d}{dt}\left[S_{+}S_{+}\right]^{a} &= 2\Big(-\sigma_{S}\sigma_{I}\hat{\beta}\left[S_{+}S_{+}I_{+}\right]^{ad}-\sigma_{S}\hat{\beta}\left[S_{+}S_{+}I_{-}\right]^{ad} \\
&\quad +\phi\delta\left[S_{+}R_{+}\right]^{a} \\
&\quad -\lambda\left[S_{+}S_{+}\right]^{a} \\
&\quad +\hat{\alpha}\left[S_{-}S_{+}\right]^{a}+\hat{\alpha}\left[S_{+}S_{-}S_{+}\right]^{aa}+\hat{\alpha}\left[I_{+}S_{-}S_{+}\right]^{aa}+\hat{\alpha}\left[R_{+}S_{-}S_{+}\right]^{aa} \\
&\quad)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[S_+I_+\right]^d &= -\sigma_S\sigma_I\hat{\beta}\left[S_+I_+\right]^d + \sigma_S\sigma_I\hat{\beta}\left[S_+S_+I_+\right]^{dd} - \sigma_S\sigma_I\hat{\beta}\left[I_+S_+I_+\right]^{dd} \\
&\quad +\sigma_S\hat{\beta}\left[S_+S_+I_-\right]^{dd} - \sigma_S\hat{\beta}\left[I_-S_+I_+\right]^{dd} \\
&\quad -\epsilon\gamma\left[S_+I_+\right]^d \\
&\quad +\phi\delta\left[I_+R_+\right]^d \\
&\quad -\lambda\left[S_+I_+\right]^d - \lambda\left[S_+I_+\right]^d \\
&\quad +\omega\left[I_-S_+\right]^d \\
&\quad +\hat{\alpha}\left[S_+S_-I_+\right]^{ad} + \hat{\alpha}\left[I_+S_-I_+\right]^{ad} + \hat{\alpha}\left[R_+S_-I_+\right]^{ad} \\
&\quad +\hat{\alpha}\left[S_+I_-S_+\right]^{ad} + \hat{\alpha}\left[I_+I_-S_+\right]^{ad} + \hat{\alpha}\left[R_+I_-S_+\right]^{ad} \\
&\quad \\
&\quad +q_{a|d}\hat{\alpha}\left[S_-I_+\right]^d + q_{a|d}\hat{\alpha}\left[I_-S_+\right]^d \\
\frac{d}{dt}\left[S_+I_+\right]^a &= +\sigma_S\sigma_I\hat{\beta}\left[S_+S_+I_+\right]^{ad} - \sigma_S\sigma_I\hat{\beta}\left[I_+S_+I_+\right]^{da} + \sigma_S\hat{\beta}\left[S_+S_+I_-\right]^{ad} - \sigma_S\hat{\beta}\left[I_-S_+I_+\right]^{da} \\
&\quad -\epsilon\gamma\left[S_+I_+\right]^a \\
&\quad +\phi\delta\left[I_+R_+\right]^a \\
&\quad -\lambda\left[S_+I_+\right]^a - \lambda\left[S_+I_+\right]^a \\
&\quad +\omega\left[I_-S_+\right]^a \\
&\quad +\hat{\alpha}\left[S_+S_-I_+\right]^{aa} + \hat{\alpha}\left[S_-I_+\right]^a + \hat{\alpha}\left[I_+S_-I_+\right]^{aa} + \hat{\alpha}\left[R_+S_-I_+\right]^{aa} \\
&\quad +\hat{\alpha}\left[I_-S_+\right]^a + \hat{\alpha}\left[S_+I_-S_+\right]^{aa} + \hat{\alpha}\left[I_+I_-S_+\right]^{aa} + \hat{\alpha}\left[R_+I_-S_+\right]^{aa} \\
&\quad -q_{d|a}\sigma_S\sigma_I\hat{\beta}\left[S_+I_+\right]^a \\
\frac{d}{dt}\left[S_+R_+\right]^d &= -\sigma_S\sigma_I\hat{\beta}\left[I_+S_+R_+\right]^{dd} - \sigma_S\hat{\beta}\left[I_-S_+R_+\right]^{dd} \\
&\quad +\epsilon\gamma\left[S_+I_+\right]^d \\
&\quad -\phi\delta\left[S_+R_+\right]^d + \phi\delta\left[R_+R_+\right]^d \\
&\quad -\lambda\left[S_+R_+\right]^d - \lambda\left[S_+R_+\right]^d \\
&\quad +\hat{\alpha}\left[S_+S_-R_+\right]^{ad} + \hat{\alpha}\left[I_+S_-R_+\right]^{ad} + \hat{\alpha}\left[R_+S_-R_+\right]^{ad} \\
&\quad +\hat{\alpha}\left[S_+R_-S_+\right]^{ad} + \hat{\alpha}\left[I_+R_-S_+\right]^{ad} + \hat{\alpha}\left[R_+R_-S_+\right]^{ad} \\
&\quad +q_{a|d}\hat{\alpha}\left[S_-R_+\right]^d + q_{a|d}\hat{\alpha}\left[R_-S_+\right]^d
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[S_+R_+\right]^a &= -\sigma_S\sigma_I\hat{\beta}\left[I_+S_+R_+\right]^{da} - \sigma_S\hat{\beta}\left[I_-S_+R_+\right]^{da} \\
&\quad +\epsilon\gamma\left[S_+I_+\right]^a \\
&\quad -\phi\delta\left[S_+R_+\right]^a + \phi\delta\left[R_+R_+\right]^a \\
&\quad -\lambda\left[S_+R_+\right]^a - \lambda\left[S_+R_+\right]^a \\
&\quad +\hat{\alpha}\left[S_+S_-R_+\right]^{aa} + \hat{\alpha}\left[I_+S_-R_+\right]^{aa} + \hat{\alpha}\left[S_-R_+\right]^a + \hat{\alpha}\left[R_+S_-R_+\right]^{aa} \\
&\quad +\hat{\alpha}\left[R_-S_+\right]^a + \hat{\alpha}\left[S_+R_-S_+\right]^{aa} + \hat{\alpha}\left[I_+R_-S_+\right]^{aa} + \hat{\alpha}\left[R_+R_-S_+\right]^{aa} \\
\frac{d}{dt}\left[I_+I_+\right]^d &= 2\Big(+\sigma_S\sigma_I\hat{\beta}\left[S_+I_+\right]^d + \sigma_S\sigma_I\hat{\beta}\left[I_+S_+I_+\right]^{dd} + \sigma_S\hat{\beta}\left[I_-S_+I_+\right]^{dd} \\
&\quad -\epsilon\gamma\left[I_+I_+\right]^d \\
&\quad -\lambda\left[I_+I_+\right]^d \\
&\quad +\omega\left[I_-I_+\right]^d \\
&\quad +\hat{\alpha}\left[S_+I_-I_+\right]^{ad} + \hat{\alpha}\left[I_+I_-I_+\right]^{ad} + \hat{\alpha}\left[R_+I_-I_+\right]^{ad} \\
&\quad +q_{a|d}\hat{\alpha}\left[I_-I_+\right]^d\Big) \\
\frac{d}{dt}\left[I_+I_+\right]^a &= 2\Big(+\sigma_S\sigma_I\hat{\beta}\left[I_+S_+I_+\right]^{da} + \sigma_S\hat{\beta}\left[I_-S_+I_+\right]^{da} \\
&\quad -\epsilon\gamma\left[I_+I_+\right]^a \\
&\quad -\lambda\left[I_+I_+\right]^a \\
&\quad +\omega\left[I_-I_+\right]^a \\
&\quad +\hat{\alpha}\left[S_+I_-I_+\right]^{aa} + \hat{\alpha}\left[I_-I_+\right]^a + \hat{\alpha}\left[I_+I_-I_+\right]^{aa} + \hat{\alpha}\left[R_+I_-I_+\right]^{aa} \\
&\quad +q_{d|a}\sigma_S\sigma_I\hat{\beta}\left[S_+I_+\right]^a\Big) \\
\frac{d}{dt}\left[I_+R_+\right]^d &= +\sigma_S\sigma_I\hat{\beta}\left[I_+S_+R_+\right]^{dd} + \sigma_S\hat{\beta}\left[I_-S_+R_+\right]^{dd} \\
&\quad +\epsilon\gamma\left[I_+I_+\right]^d - \epsilon\gamma\left[I_+R_+\right]^d \\
&\quad -\phi\delta\left[I_+R_+\right]^d \\
&\quad -\lambda\left[I_+R_+\right]^d - \lambda\left[I_+R_+\right]^d \\
&\quad +\omega\left[I_-R_+\right]^d \\
&\quad +\hat{\alpha}\left[S_+I_-R_+\right]^{ad} + \hat{\alpha}\left[I_+I_-R_+\right]^{ad} + \hat{\alpha}\left[R_+I_-R_+\right]^{ad} \\
&\quad +\hat{\alpha}\left[S_+R_-I_+\right]^{ad} + \hat{\alpha}\left[I_+R_-I_+\right]^{ad} + \hat{\alpha}\left[R_+R_-I_+\right]^{ad} \\
&\quad +q_{a|d}\hat{\alpha}\left[I_-R_+\right]^d + q_{a|d}\hat{\alpha}\left[R_-I_+\right]^d
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[I_+R_+\right]^a &= +\sigma_S\sigma_I\hat{\beta}\left[I_+S_+R_+\right]^{da} + \sigma_S\hat{\beta}\left[I_-S_+R_+\right]^{da} \\
&\quad +\epsilon\gamma\left[I_+I_+\right]^a - \epsilon\gamma\left[I_+R_+\right]^a \\
&\quad -\phi\delta\left[I_+R_+\right]^a \\
&\quad -\lambda\left[I_+R_+\right]^a - \lambda\left[I_+R_+\right]^a \\
&\quad +\omega\left[I_-R_+\right]^a \\
&\quad +\hat{\alpha}\left[S_+I_-R_+\right]^{aa} + \hat{\alpha}\left[I_+I_-R_+\right]^{aa} + \hat{\alpha}\left[I_-R_+\right]^a + \hat{\alpha}\left[R_+I_-R_+\right]^{aa} \\
&\quad +\hat{\alpha}\left[S_+R_-I_+\right]^{aa} + \hat{\alpha}\left[R_-I_+\right]^a + \hat{\alpha}\left[I_+R_-I_+\right]^{aa} + \hat{\alpha}\left[R_+R_-I_+\right]^{aa} \\
\frac{d}{dt}\left[R_+R_+\right]^d &= 2\left(+\epsilon\gamma\left[I_+R_+\right]^d\right. \\
&\quad -\phi\delta\left[R_+R_+\right]^d \\
&\quad -\lambda\left[R_+R_+\right]^d \\
&\quad +\hat{\alpha}\left[S_+R_-R_+\right]^{ad} + \hat{\alpha}\left[I_+R_-R_+\right]^{ad} + \hat{\alpha}\left[R_+R_-R_+\right]^{ad} \\
&\quad \left.+q_{a|d}\hat{\alpha}\left[R_-R_+\right]^d\right) \\
\frac{d}{dt}\left[R_+R_+\right]^a &= 2\left(+\epsilon\gamma\left[I_+R_+\right]^a\right. \\
&\quad -\phi\delta\left[R_+R_+\right]^a \\
&\quad -\lambda\left[R_+R_+\right]^a \\
&\quad +\hat{\alpha}\left[S_+R_-R_+\right]^{aa} + \hat{\alpha}\left[I_+R_-R_+\right]^{aa} + \hat{\alpha}\left[R_-R_+\right]^a + \hat{\alpha}\left[R_+R_-R_+\right]^{aa} \\
&\quad \left.\right)
\end{aligned}
$$