Supplementary material

The spread of awareness and its impact on epidemic outbreaks

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In the following, we will present and analyze the system of ordinary differential equations describing the mean-field dynamics, and carry out the calculations leading to the basic reproductive number in the individual-based model.

Mean-field analysis

Subscript $_i$ denotes information at the i-th generation, i.e. diminished i times by a factor ρ. Subscript $_0$ denotes the 0-th generation, information which has not been transmitted or lost quality yet. Disregarding the generation of information for the moment by setting $\omega = 0$, the full set of mean-field equations is

$$\frac{dS_i}{dt} = -(1-\rho^i)\beta\frac{S_i}{N}I \qquad - \alpha\frac{S_i}{N}\left(\sum_{j=0}^{i-2}N_j\right) + \alpha\frac{N_{i-1}}{N}\left(\sum_{j=i+1}^{\infty}S_j\right) - \lambda S_i + \lambda S_{i-1}, \tag{1}$$

$$\frac{dI_i}{dt} = +(1-\rho^i)\beta\frac{S_i}{N}I - \gamma I_i - \alpha\frac{I_i}{N}\left(\sum_{j=0}^{i-2}N_j\right) + \alpha\frac{N_{i-1}}{N}\left(\sum_{j=i+1}^{\infty}I_j\right) - \lambda I_i + \lambda I_{i-1}, \tag{2}$$

$$\frac{dR_i}{dt} = +\gamma I_i \qquad - \alpha\frac{R_i}{N}\left(\sum_{j=0}^{i-2}N_j\right) + \alpha\frac{N_{i-1}}{N}\left(\sum_{j=i+1}^{\infty}R_j\right) - \lambda R_i + \lambda R_{i-1}, \tag{3}$$

where $I = \sum_i I_i$ and $N_i = S_i + I_i + R_i$, and $S_{-1} = I_{-1} = R_{-1} = N_{-1} = 0$. The dynamical equation for I is

$$\frac{dI}{dt} = \sum\frac{dI_i}{dt} = +\left(\sum_{i=0}^{\infty}(1-\rho^i)\beta\frac{S_i}{N}\right)I - \gamma I \tag{4}$$

and the initial rate of increase of infected starting with a small number of infected in an otherwise completely susceptible and uninformed population is $\exp(\beta-\gamma)t$, such that initially number of infected will always increase if $\beta/\gamma > 1$.

Disease dynamics

Rephrasing the system in terms of the dynamical variables $S = \sum_i S_i$, $I = \sum_i I_i$ and $R = \sum_i R_i$ reduces the system to SIR dynamics:

$$\frac{dS}{dt} = -\beta'(\rho,t)\frac{S}{N}I, \tag{5}$$

$$\frac{dI}{dt} = \beta'(\rho,t)\frac{S}{N}I - \gamma I, \tag{6}$$

$$\frac{dR}{dt} = \gamma I, \tag{7}$$

in which $\beta'(\rho,t) = \beta(1-g(\rho,t))$, and $g(\rho,t)$ is the probability generating function of the distribution of information among susceptibles at time t, $g(\rho,t) = \sum(S_i(t)/S(t))\rho^i$, $i = 0, 1, 2, \ldots$.

Information dynamics

Rephrasing the system in terms of the N_i yields the information dynamics (if $\omega = 0$):

$$\begin{aligned}\frac{dN_i}{dt} &= -\alpha\frac{N_i}{N}\left(\sum_{j=0}^{i-1} N_j\right) + \alpha\frac{N_{i-1}}{N}\left(N - \sum_{j=0}^{i-1} N_j\right) - \lambda N_i + \lambda N_{i-1} \\ &= -\alpha\frac{N_i}{N} N_{<i} + \alpha\frac{N_{i-1}}{N}(N - N_{<i}) - \lambda(N_i - N_{i-1}),\end{aligned} \tag{8}$$

where $N_{<i} = \sum_0^{i-1} N_j$ is the sum over all more informed parts of the population.
In equilibrium, we have

$$N_i = \frac{1 + \frac{\alpha}{\lambda}\frac{N - N_{<i}}{N}}{1 + \frac{\alpha}{\lambda}\frac{N_{<i}}{N}} N_{i-1}. \tag{9}$$

for all $i > 0$. Therefore, the condition for the maximum of the distribution of information is

$$N_{<i} = \frac{1}{2}N, \tag{10}$$

such that, in equilibrium, the maximum of the distribution is always its median.
The total amount of information in the population, $Q = \sum_{i=0}^{\infty} \rho^i N_i$ changes in time as

$$\begin{aligned}\frac{dQ}{dt} &= -\frac{\alpha}{N}\sum_{i=0}^{\infty}\sum_{j=0}^{i-1} \rho^i N_i N_j + \frac{\alpha}{N}\sum_{i=0}^{\infty} \rho^i N_{i-1} N - \frac{\alpha}{N}\sum_{i=0}^{\infty}\sum_{j=0}^{i-1} \rho^i N_{i-1} N_j - \lambda\sum_{i=0}^{\infty} \rho^i (N_i - N_{i-1}) \\ &= -(1+\rho)\frac{\alpha}{N}\sum_{i=1}^{\infty} \rho^i N_i N_{<i} + \alpha\rho Q - \lambda(1-\rho)Q,\end{aligned} \tag{11}$$

and the initial rate of increase of awareness starting with a small number of informed in an otherwise completely uninformed population is $\alpha\rho - \lambda(1-\rho))$. Hence, awareness in the population will increase if

$$\frac{\rho}{(1-\rho)}\frac{\alpha}{\lambda} > 1. \tag{12}$$

Information generation

The inclusion of information generation into the model adds the following terms:

$$\begin{aligned}\frac{dI_0}{dt} &= \ldots + \omega(I - I_0) \\ \frac{dI_{i>0}}{dt} &= \ldots - \omega I_i,\end{aligned} \tag{13}$$

which do not change the disease equations.
With information generation, the dynamical equation for the most informed part of the population N_0 is

$$\frac{dN_0}{dt} = -\lambda N_0 + \omega(I - I_0). \tag{14}$$

Note that, however, $S_0 = 0$ at all times as susceptibles will always have their her information passed on to them by someone.

Final sizes

Examples of the possible reduction in the final size of the outbreak are given in Supporting Figures 1-3, which show the fraction of the population having been infected by the end of an outbreak as calculated from the ODE system in terms of the information transmission rate α for different values of the decay constant ρ and the rate of information generation ω.

Individual-based analysis

In our individual-based model, every individual has a number of contacts with other individuals, distributed with mean $\overline{k}$ and variance $\mathrm{Var}(k)$. Each of these contacts have independent probability of disease transmission, in case one of the two individuals at its ends is infected and the other susceptible. Given the average transmission probability T and the distribution of the number of contacts each individual possesses, the average number of secondary infections caused by an infected individual is called the *basic reproductive number* $\hat{R}_0$. In a conventional SIR model on a network, it is given by [1; 2; 3; 4]

$$\hat{R}_0 = TD_k = \frac{\hat{\beta}}{\hat{\beta}+\gamma}D_k \tag{15}$$

where $T = \hat{\beta}/(\hat{\beta}+\gamma)$ is the per-contact probability of infection [5] (or the probability that infection happens before recovery) and

$$D_k = \left(\overline{k} - 1 + \frac{\mathrm{Var}(k)}{\overline{k}}\right) \tag{16}$$

takes into account the variation in the number of contacts each individual has.

In our model, variation in the state of awareness of susceptibles changes the transmission probabilities. If the susceptible contact at the end of a connection for potential disease transmission is in awareness state i, the transmission probability for a given contact between a susceptible and an infected is given by

$$T_i = \frac{\hat{\beta}(1-\rho^i)}{\hat{\beta}(1-\rho^i)+\gamma}, \tag{17}$$

and the modified basic reproductive number by

$$\hat{R}_0' = \left(\sum_{i=1}^{\infty} p_i T_i\right) D_k \tag{18}$$

where p_i is the probability of the susceptible to possess information having gone through i hands at the time of potential infection, and T_i is the probability of infection over that contact given that i.

If we restrict ourselves to one-step of information transmission, only the transmission probabilities p_0 and p_1 are non-zero. In fact, p_0 is the probability that infection or recovery happens before information is generated plus the probability that, if information is generated before infection or recovery happens, one of the two still occurs before information is transmitted. Therefore, we have

$$p_0 = \frac{\hat{\beta}+\gamma}{\hat{\beta}+\gamma+\omega} + \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\beta}+\gamma}{\hat{\beta}+\gamma+\hat{\alpha}} \tag{19}$$

$$p_1 = \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\alpha}}{\hat{\beta}+\gamma+\hat{\alpha}}. \tag{20}$$

To assess the maximum impact the information process can have on the spreading disease, let us assume the networks for the spread of awareness and the disease to be the same. The modified basic reproductive number, $\hat{R}_0' = (p_0T_0 + p_1T_1)D_k$ is then given by

$$\begin{aligned}
\hat{R}_0' &= \left(\left(\frac{\hat{\beta}+\gamma}{\hat{\beta}+\gamma+\omega} + \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\beta}+\gamma}{\hat{\beta}+\gamma+\hat{\alpha}}\right)\frac{\hat{\beta}}{\hat{\beta}+\gamma} + \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\alpha}}{\hat{\beta}+\gamma+\hat{\alpha}}\frac{\hat{\beta}(1-\rho)}{\hat{\beta}(1-\rho)+\gamma}\right)D_k \\
&= \left(\frac{\hat{\beta}}{\hat{\beta}+\gamma+\omega} + \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\beta}}{\hat{\beta}+\gamma+\hat{\alpha}} + \frac{\omega}{\hat{\beta}+\gamma+\omega}\frac{\hat{\alpha}}{\hat{\beta}+\gamma+\hat{\alpha}}\frac{\hat{\beta}(1-\rho)}{\hat{\beta}(1-\rho)+\gamma}\right)D_k \\
&= \left(\frac{\hat{\beta}}{\hat{\beta}+\gamma+\omega}\left(1 + \frac{\omega}{\hat{\beta}+\gamma+\hat{\alpha}}\left(1 + \frac{(1-\rho)\hat{\alpha}}{(1-\rho)\hat{\beta}+\gamma}\right)\right)\right)D_k \\
&= \left(\frac{\hat{\beta}+\gamma}{\hat{\beta}+\gamma+\omega}\left(1 + \frac{\omega}{\hat{\beta}+\gamma+\hat{\alpha}}\left(1 + \frac{(1-\rho)\hat{\alpha}}{(1-\rho)\hat{\beta}+\gamma}\right)\right)\right)\hat{R}_0
\end{aligned} \tag{21}$$

The mean-field limit is taken by setting $\overline{k} \to \infty$, $\hat{\alpha} \to 0$, $\hat{\beta} \to 0$, $\overline{k}\hat{\alpha} \to \alpha$, and $\overline{k}\hat{\beta} \to \beta$. In that case, we get $\hat{R}_0' \to R_0 = \beta/\gamma$.

Solving $\hat{R}_0' = 1$ for $\hat{R}_0$ yields

$$\begin{aligned} \hat{R}_0^{\text{crit}} &= \frac{(\hat{\beta}+\gamma+\omega)(\hat{\beta}+\gamma+\hat{\alpha})((1-\rho)\hat{\beta}+\gamma)}{(\hat{\beta}+\gamma)\left[(\hat{\beta}+\gamma+\omega+\hat{\alpha})((1-\rho)\hat{\beta}+\gamma)+\omega\hat{\alpha}(1-\rho)\right]} \\ &= 1+\frac{\omega\alpha(1-\rho)\gamma}{(\hat{\beta}+\gamma)\left[(\hat{\beta}+\gamma+\omega+\hat{\alpha})((1-\rho)\hat{\beta}+\gamma)+\omega\hat{\alpha}(1-\rho)\right]} \end{aligned} \tag{22}$$

The following limits can be applied to Eq. 21 before solving for $\hat{R}_0$:

$$\lim_{\hat{\beta}\to\infty} \hat{R}_0^{\text{crit}} = 1, \tag{23}$$

and, using $\hat{\beta} = \gamma\hat{R}_0^{\text{crit}}/(D_k - \hat{R}_0^{\text{crit}})$ (Eq. 15)

$$\lim_{\substack{\omega\to\infty\\ \hat{\alpha}\to\infty}} \hat{R}_0^{\text{crit}} = \frac{1}{1-\rho\left(1-D_k^{-1}\right)}, \tag{24}$$

$$\lim_{\substack{\omega\to\infty\\ \rho\to1}} \hat{R}_0^{\text{crit}} = \frac{\gamma+\hat{\alpha}}{\gamma+\hat{\alpha}D_k^{-1}}, \tag{25}$$

$$\lim_{\substack{\hat{\alpha}\to\infty\\ \rho\to1}} \hat{R}_0^{\text{crit}} = \frac{\gamma+\omega}{\gamma+\omega D_k^{-1}}. \tag{26}$$

References

[1] Anderson RM, Medley GF, May RM, Johnson AM (1986) A preliminary study of the transmission dynamics of the human immunodeficiency virus (HIV), the causative agent of AIDS. *IMA J Math Appl Med Biol* 3:229-263. 136-142.

[2] Andersson H (1997) Epidemics in a Population with Social Structures. *Math Biosci* 140:79-84.

[3] Diekmann O, Heesterbeek JAP (2000) Mathematical Epidemiology of Infectious Diseases. (Wiley, Chichester).

[4] Meyers LA (2007) Contact Network Epidemiology: Bond Percolation Applied to Infectious Disease Prediction and Control *Bull Amer Math Soc* 44:63-86.

[5] Keeling MJ, Grenfell BT (2000) Individual-based Perspectives on R_0. *J Theor Biol* 203:51-61.

Supporting Videos

Video 1

Simulated outbreak of just a disease spreading without associated awareness on a triangular lattice. Red color indicates nodes which have been infected, with brighter red indicating those that are still infectious.

Video 2

Simulated outbreak of a disease with associated awareness spreading on a triangular lattice. Red color indicates nodes which have been infected, with brighter red indicating those that are still infectious. Darker shades of gray indicate higher levels of awareness.

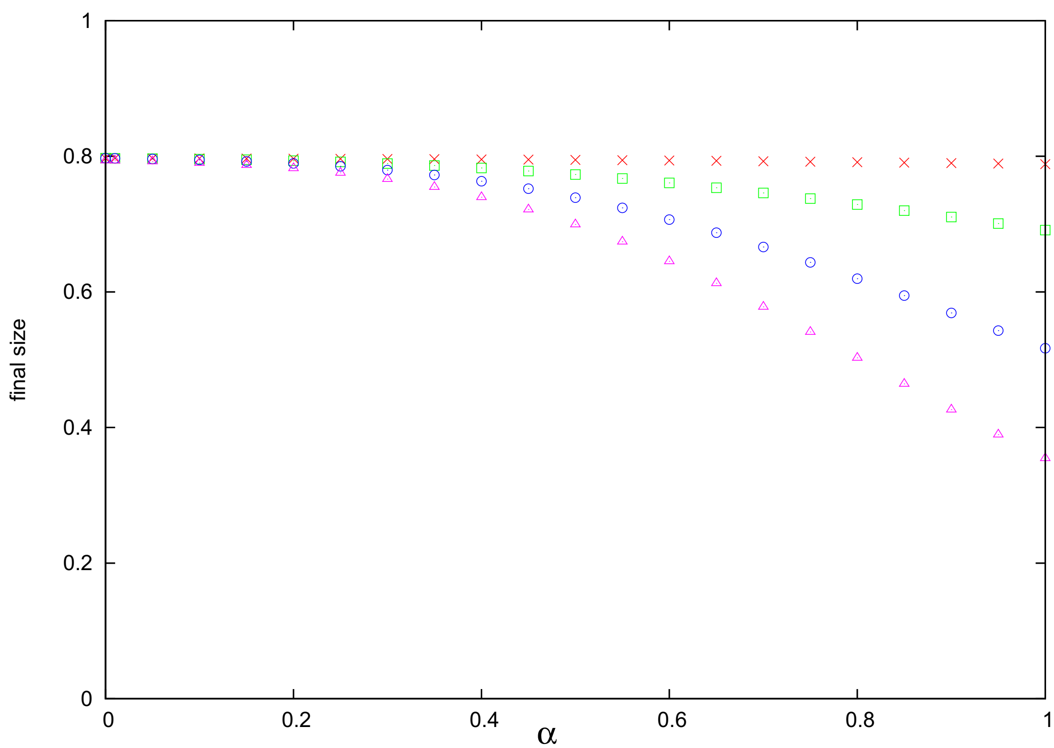


Fig. S1. Fraction of the population having been infected by the end of an outbreak as calculated from the ODE system in terms of the information transmission rate α. Different values of the decay constant ρ are indicated as crosses (ρ = 0.5), squares (ρ = 0.8), circles (ρ = 0.9), and triangles (ρ = 0.95); the rate of information generation ω is 0.01; the other parameters are β = 1, γ = 0.5, and λ = 0.5.

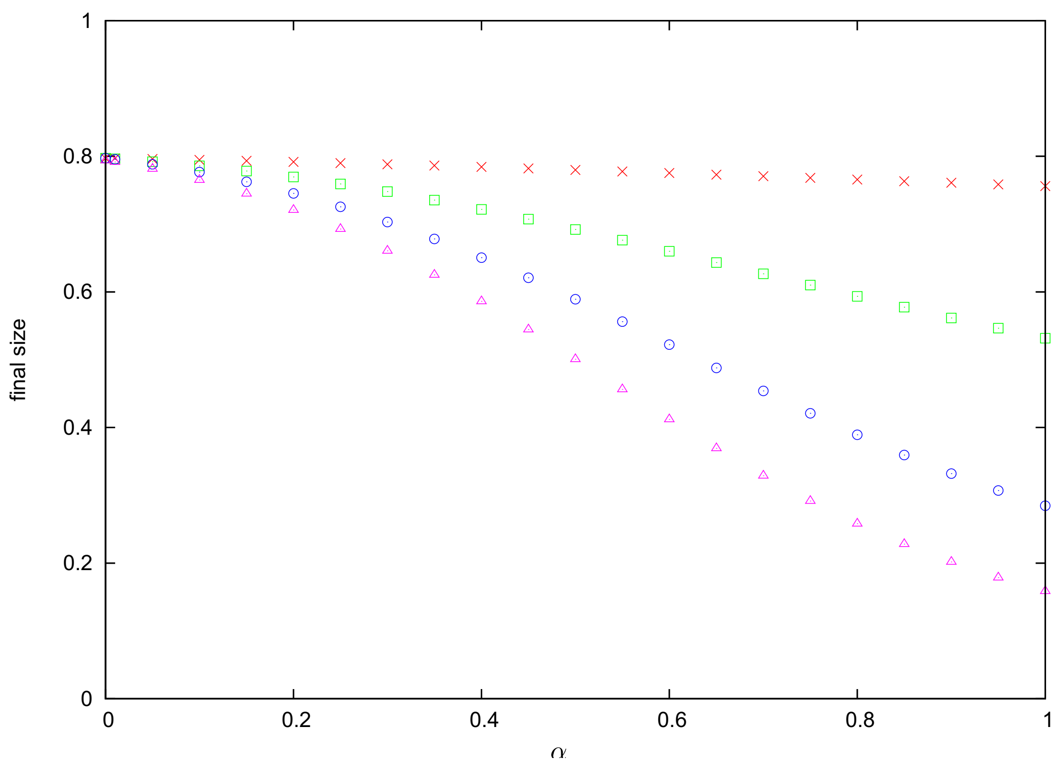


Fig. S2. Fraction of the population having been infected by the end of an outbreak as calculated from the ODE system in terms of the information transmission rate α. Different values of the decay constant ρ are indicated as crosses (ρ = 0.5), squares (ρ = 0.8), circles (ρ = 0.9), and triangles (ρ = 0.95); the rate of information generation ω is 0.1; the other parameters are β = 1, γ = 0.5, and λ = 0.5.

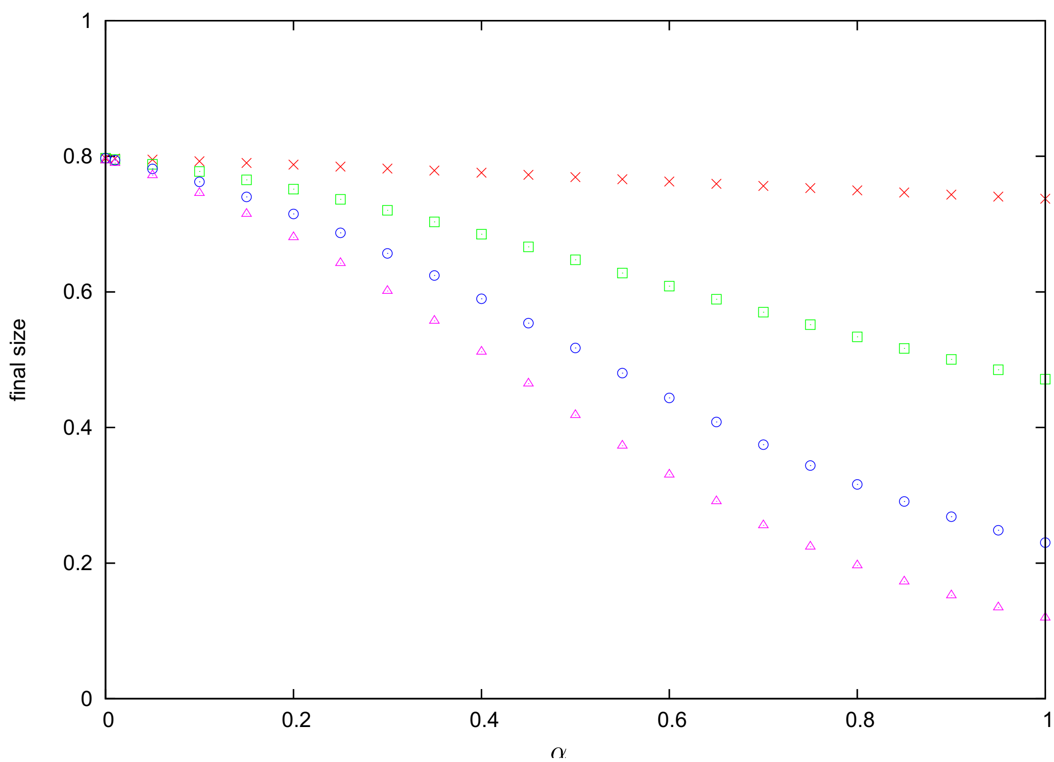
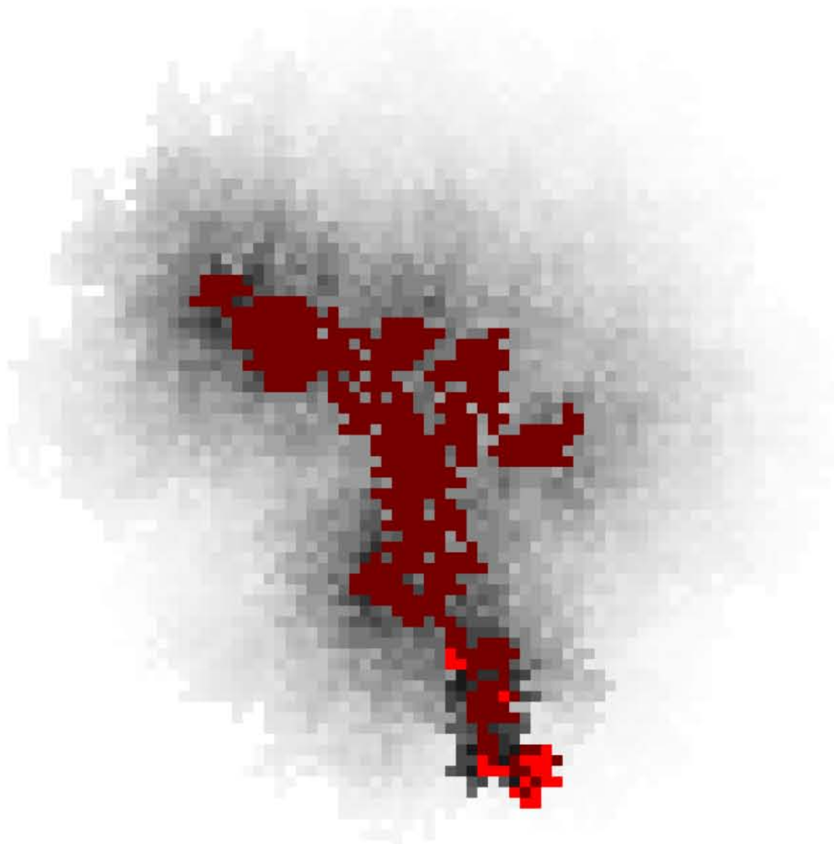


Fig. S3. Fraction of the population having been infected by the end of an outbreak as calculated from the ODE system in terms of the information transmission rate α. Different values of the decay constant ρ are indicated as crosses (ρ = 0.5), squares (ρ = 0.8), circles (ρ = 0.9), and triangles (ρ = 0.95); the rate of information generation ω is 0.2; the other parameters are β = 1, γ = 0.5, and λ = 0.5.

Movie S1. Simulated outbreak of only a disease spreading on a triangular lattice. Red color indicates nodes that have been infected, with brighter red indicating those that are still infectious.

Movie S1 (MOV)



Movie S2. Simulated outbreak of a disease with associated awareness spreading on a triangular lattice. Red color indicates nodes that have been infected, with brighter red indicating those that are still infectious. Darker shades of gray indicate higher levels of awareness.

Movie S2 (MOV)