Forecasting Nuclear Proliferation

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Contents

1 Abstract

This article models the time between the first test of a nuclear weapon by one nation and the next over the 74 years of history since the first such test by the US. We use those results to forecast nuclear proliferation over the next 74 years. The maximum likelihoood estimate of the time between "first tests" 74 / 8 = 9.3 years (using the standard formula for censored estimation of a constant exponential distribution). However, a plot of the times between "first tests" of the 9 nuclear powers as of 2020-02-06 suggests a nonhomogeneous renewal process that suggests a decrease over time in log(Poisson mean) of the number of "first tests" each year by new nuclear-weapon states. This might be modeled using `glm(..., family=poisson)`. Unfortunately, the linear trend is not statistically signficant. We therefore use Bayesian Model Averaging (BMA), considering two related BMA mixtures. This average of the Monte Carlo simulations for the constant-linear mixture forecasts an addition 5.4 new nuclear-weapon states by 2093, using a forecasting period to match the available history. The average *nonlinear* mixture forecasts an addition 7.3 new nuclear-weapon states by 2093. Eighty percent percent prediction limits run from 0 to 14 new nuclear-weapon states, within the typical life span of babies born today. The risk of a nuclear war leading to the extinction of civilization is increasing. Nuclear proliferation will likely continue until it becomes effectively impossible for anyone to make more nuclear weapons for a very

long time. This could come from a nuclear war or a massive and unprecidented strengthening of international law that provides effective judicial recourse for grievances of the poor, weak and disfranchised.

2 Introduction

A plot of times between "first tests" by the world's nuclear-weapon states as of 2020-02-06 suggests that the process of nuclear proliferation has slowed; see Figure 1.

```
library(Ecdat)
data(nuclearWeaponStates)
ymax <- max(
  nuclearWeaponStates$yearsSinceLastFirstTest,
            na.rm=TRUE)
ylim0 <- c(0, ymax)

NPTdate = as.Date('1970-03-05')
plotNucStates <- function(type.='n', xlim., ylim.,
    line_mtext=3:2, cex.=1, mtext.=TRUE, log.='',
    ...){
##
## Write a function to create this desired plot
## that is general enough to be customized
## to make other similar but different plots
## later.
##
## Obviously, during the process of writing
## this vignette, it requires revising this
## function later as the needs become clearer.
##
## The advantage of doing it this way, it that
## it makes the code easier to read, because
## it's clearer it clearer what is the same and
## what is different between similar plots.
##
#   Start with an internal function
#   to add the 2-letter country codes.
  addCountries <- function(line_mtext=3:2, cex.=1,
                           mtext.=TRUE){
#  Add the country codes ("ctry") to a plot
#  showing the time between "first tests"
#  of nuclear-weapon states
# ... to save copying code
#  and hopefully make the logic clearer
    xlab. <- paste(c(
        'Note: The US is not on this plot,',
        'because it had no predecessors.'),
              collapse='\n')
    if(mtext.){
        mtext(xlab., 1, line_mtext[1], cex=cex.)
        mtext('years from the\nprevious "first test"',
          2, line_mtext[2], cex=cex.)
    }
    with(nuclearWeaponStates,
        text(firstTest, yearsSinceLastFirstTest,
```

```r
          ctry, xpd=TRUE, cex=cex.))
  }
#  xlim and ylim?
  if(missing(xlim.))xlim. <- range(
    nuclearWeaponStates$firstTest)
  if(missing(ylim.))ylim. <- range(
    nuclearWeaponStates$yearsSinceLastFirstTest[-1])
# If very wide log scale on y,
# make the margins wider and move the label out:
  if((log.=='y') && (diff(log(ylim.))>5)){
    op <- par(mar=c(5, 6, 4, 2)+0.1)
    on.exit(par(op))
    line_mtext[2] <- 4
  }
#
  plot(yearsSinceLastFirstTest~firstTest,
       nuclearWeaponStates, type=type.,
       xlab='', ylab='', las=1,
       xlim=xlim., ylim=ylim., log=log., bty='n',
       ...)
  abline(v=NPTdate, lty='dashed', col='grey')
  ymid <- {
    if(log.=='y') sqrt(ylim.[1]*ylim.[2]) else
      mean(ylim.)
  }
  text(NPTdate-.017*diff(xlim.),
       ymid, 'NPT', col='grey', srt=90)
  addCountries(line_mtext=line_mtext, cex.=cex.,
               mtext.=mtext.)
}
plotNucStates(type.='h', ylim.=ylim0)
```

It stretches credibility to suggest that nuclear proliferation has stopped. There were only 5 nuclear-weapon states when the Treaty on the Non-Proliferation of Nuclear Weapons (NPT, Non-proliferation treaty) entered into force in 1970.[1] When US President George W. Bush decried an "Axis of evil" in his State of the Union message, 2002-01-29,[2] there were 8. As this is written 2020-02-06, there are 9. Toon et al. (2007) noted that in 2003 another 32 had sufficient fissile material to make nuclear weapons if they wished.

Moreover, those 32 do *NOT* include either Turkey nor Saudi Arabia. On 2019-09-04, Turkish President Erdogan said it was unacceptable for nuclear-armed states to forbid Turkey from acquiring its own nuclear weapons.[3]

Similarly, in 2006 *Forbes* reported that Saudi Arabia has "a secret underground city and dozens of underground silos for" Pakistani nuclear weapons and missiles.[4] In 2018 the *Middle East Monitor* reported that "Israel 'is selling nuclear information' to Saudi Arabia."[5] This is particularly disturbing, because of the substantial evidence that Saudi Arabia may have been and may still be the primary recruiter and funder of Islamic terrorism.[6]

This analysis suggests that the number of nuclear-weapon states will likely continue to grow until some

[1] United Nations Office for Disarmament Affairs (1970). See also ("Treaty on the Non-Proliferation of Nuclear Weapons" n.d.).
[2] Bush (2002); see also ("Axis of Evil" n.d.).
[3] Toksabay (2019); O'Connor (2019).
[4] Forbes (2006); see also ("Nuclear Program of Saudi Arabia" n.d.).
[5] Middle East Monitor (2018); see also ("Nuclear Program of Saudi Arabia" n.d.).
[6] Benjamin (2016); see also ("Winning the War on Terror" n.d.).

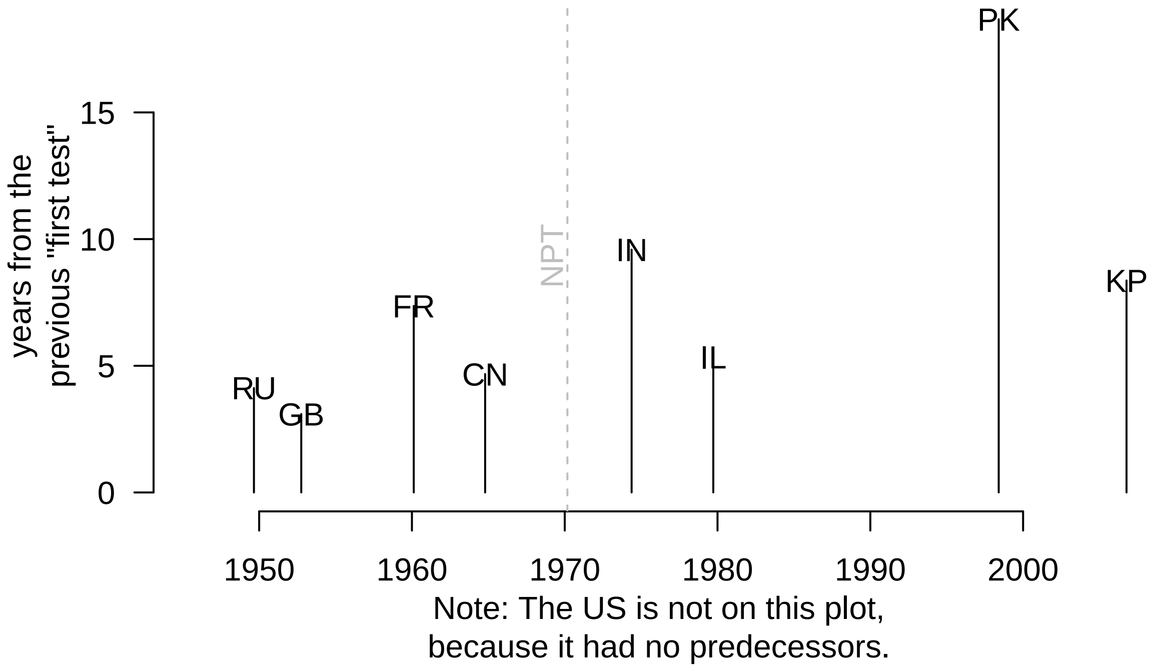

Figure 1: Time between new nuclear-weapon states

dramatic break with the past makes further nuclear proliferation either effectively impossible or sufficiently undesirable. One way this could happen is as a result of a nuclear war that effectively destroys the ability of anyone to make more nuclear weapons for a very long time.

This vignette first reviews the data and history on this issue. We then consider modeling these data as a series of annual Poisson observations of the number of states conducting a first test of a nuclear weapon each year (1 in each of 8 years since 1945; 0 in the others).

The simplest such model that considers the possible inhomogeniety visible in Figure 1 is Poisson regression assuming that log(Poisson mean) is linear in the time since the first test of a nuclear weapon by the US on 1945-07-16. We estimate this using `glm(..., family=poisson)`. This model is plausible to the extent that this trend might represent a growing international awareness of the threat represented by nuclear weapons including a hypothesized increasing reluctance of existing nuclear-weapon states to share their technology. The current process of ratifying the Treaty on the Prohibition of Nuclear Weapons supports the hypothesis of such a trend, while the lack of universal support for it and the trend visible in Figure 1 clearly indicate that nuclear proliferation is still likely to continue. We use this model to extend the 74 years of history of nuclear proliferation available as this is being written 2020-02-06 into predicting another 74 years into the future.

3 How did the existing nuclear-weapon states develop this capability?

There are, of course, multiple issues in nuclear proliferation: a new nuclear-weapon state requires at least four distinct things to produce a nuclear weapon: motivation, money, knowledge, and material.

Disclaimer: Complete answers to each of these questions for every nuclear-weapon state can never be known with certainty. The literature found by the present authors is summarized in the accompanying table with citations to the literature in the following discussion but should not be considered any more authoritative than the sources cited, some of which may not be adequate to support all the details and the generalizations in the accompanying table.

However, this analysis should be sufficient to support the general conclusions of this article.

```
howHist <- t(as.matrix(data.frame(
  US = c('Nazi threat', 'self',
    paste('own scientists + immigrants,',
      'esp. fr. Germany and Italy, plus',
      'collaboration w the UK & Canada'), 'Congo + self'),
  USSR = c(paste('Hiroshima & Nagasaki bombs +',
      'western invasions during WW II,',
      'after WW I, and before'), 'self',
    paste('own scientists + espionage',
      'in the US & captured Germans'), 'self'),
  UK = c('USSR', 'self', 'Manhattan Project',
    'Canada'),
  FR = c('USSR + Suez Crisis', rep('self', 3)),
  China = c(paste('1st Taiwan Strait Crisis 1954-1955,',
        'Korean Conflict, etc.'),
    'self', 'USSR', 'self'),
  India = c(paste('loss of territory:',
      'China-Himalayan border-1962'), 'self',
    'students in UK, US',
    'Canadian nuc reactor'),
  Israel = c('hostile neighbors', 'self',
    'self + France', 'France, Argentina'),
  Pakistan=c('Loss of E. Pakistan in 1971',
    'Saudis + self', 'US, maybe China?', 'self?'),
  N.Korea = c('threats fr. US', "self?",
    'US via Pakistan?', 'self?'),
  stringsAsFactors=FALSE)))
colnames(howHist) <- c('Motivation',
          'Money', "Knowledge", "Material")

library(knitr)
library(kableExtra)

emTot <- 41
em. <- c(nation=2.2, motive=10, money=2,
         knowledge=10, material=3.4)
em <- paste0(emTot*em./sum(em.), 'em')

kable(howHist) %>%
  column_spec(1, width=em[1]) %>%
  column_spec(2, width=em[2]) %>%
  column_spec(3, width=em[3]) %>%
  column_spec(4, width=em[4]) %>%
  column_spec(5, width=em[5])
```

	Motivation	Money	Knowledge	Material
US	Nazi threat	self	own scientists + immigrants, esp. fr. Germany and Italy, plus collaboration w the UK & Canada	Congo + self
USSR	Hiroshima & Nagasaki bombs + western invasions during WW II, after WW I, and before	self	own scientists + espionage in the US & captured Germans	self
UK	USSR	self	Manhattan Project	Canada
FR	USSR + Suez Crisis	self	self	self
China	1st Taiwan Strait Crisis 1954–1955, Korean Conflict, etc.	self	USSR	self
India	loss of territory: China-Himalayan border-1962	self	students in UK, US	Canadian nuc reactor
Israel	hostile neighbors	self	self + France	France, Argentina
Pakistan	Loss of E. Pakistan in 1971	Saudis + self	US, maybe China?	self?
N.Korea	threats fr. US	self?	US via Pakistan?	self?

To help us understand the differences in sizes of the different nuclear-weapon states, Figure 2 plots the populations and Gross Domestic Products (GDP) of the current nuclear-weapon states.[7] The following provides analysis with references behind the summary in Figure 2.

```
#op <- par(mar=c(5,5,4,2)+.1)
plot(GDP_B/1000 ~ popM, nuclearWeaponStates, type='n',
     log='xy', las=1, xlab='', ylab='')
title(ylab='nominal GDP (USD trillions)')
title(xlab='population (millions)', line = 1.7)
Xlab <- paste0(
  'CN = China, GB, FR (overplotted) = UK & France',
  ' IL = Israel,\nIN = India, KP = North Korea',
  'PK = Pakistan, RU = Russia')
nNucStates <- nrow(nuclearWeaponStates)
title(xlab=Xlab, line=3.5, cex.lab=0.85)

#lines(GDP_B/1000 ~ popM, nuclearWeaponStates,
#       type='c', lty='dotted', col='grey')
#with(nuclearWeaponStates, arrows(head(popM, -1),
#  head(GDP_B/1000, -1), tail(popM, -1),
#  tail(GDP_B/1000, -1), col='grey'))
i0 <- c(1:2, 4:(nNucStates-1))
with(nuclearWeaponStates, arrows(
  popM[i0], GDP_B[i0]/1000,
  popM[i0+1], GDP_B[i0+1]/1000, col='grey',
  lty='dotted'))
cols <- c(US='blue', RU='red', GB='red',
   FR='blue', CN='red', IN='orange',
   IL='blue', PK='green', KP='red')
with(nuclearWeaponStates,
     text(popM, GDP_B/1000, ctry,
          col=cols) )
leg <- with(nuclearWeaponStates,
    paste(ctry, '=', nation))
```

[7] Data for different years, 2017-2020, depending on what was available from Wikipedia on 2020-02-05.

```r
with(nuclearWeaponStates,
  legend('bottomright', legend=leg,
         bty='n', text.col=cols))
```

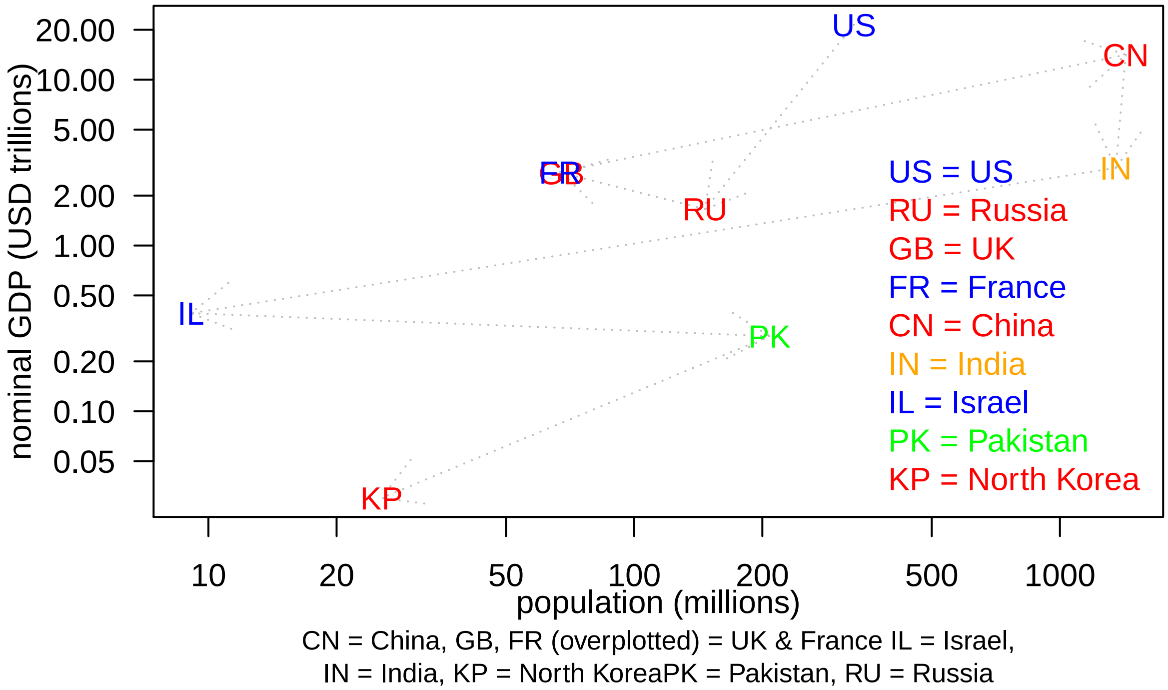

Figure 2: Gross Domestic Product and Populaton of Nuclear-Weapon States

```r
#par(op)
```

3.1 Motivation

Virtually any country that feels threatened would like to have some counterweight against aggression by a potential enemy. The US funded the Manhattan project believing that Nazi Germany likely had a similar project.

Soviet leaders might have felt a need to defend themselves from nuclear coercion after having been invaded by Nazi Germany only a few years earlier, and having defeated foreign invasions from the West and the East after world War I trying to put the Tsar back in power.[8]

The United Kingdom and France felt nuclear threats from the Soviet Union.[9]

France's concern about the Soviets increased after the US refused to support them during the 1956 Suez Crisis: If the US would not support a British-French-Israeli invasion of Egypt, the US might not defend France against a possible Soviet invasion.[10]

China reportedly decided to initiate its nuclear weapons program during the First Taiwan Strait Crisis of

[8] Fogelsong (2014). That doesn't count numerous other invasions that are a sordid part of Russian history, which educated Russians throughout history would likely remember, even if their invadors may not.

[9] The UK and France would have had many reasons to fear the intentions of the USSR during the early period of the Cold War: The first test of a nuclear weapon by the USSR came just over three months after the end of the 1948-49 Berlin Blockade. Other aspects of Soviet repression in countries they occupied in Eastern Europe contributed to the failed Hungarian Revolution of 1956.

[10] Fromkin (2006). See also ("France and Weapons of Mass Destruction" n.d.)

1954-55,[11] following nuclear threats from the US regarding Korea[12]

India lost territory to China in the 1962 Sino-Indian War, which reportedly convinced India to abandon a policy of avoiding nuclear weapons.[13]

Pakistan's nuclear weapons program began in 1972 in response to the loss of East Pakistan (now Bangledesh) in the 1971 Bangladesh Liberation War.[14]

Israel has faced potentially hostile neighbors since its declaration of independence in 1948.[15]

A commonly accepted date for North Korea's first test of a nuclear weapon is 2006-10-09,[16] not quite 5 years after having been declared part of an "Axis of evil"[17] by US President George W. Bush, and watching repeated threats by the US against Iran and an invasion of Iraq, the other two countries denounced as part of Bush's "Axis of evil".

All this suggests that it will be difficult to reduce the threat of nuclear proliferation and nuclear war without somehow changing the nature of international relations so weaker countries have less to fear from the demands of stronger countries.

3.2 Money

It's no accident that most of the world's nuclear-weapon states are large countries with substantial populations and economies. That's not true of Israel with only roughly 9 million people nor North Korea with roughly 26 million people in 2018. France and the UK have only about 67 and 68 million people, but they are also among the world leaders in the size of their economies.

Pakistan is a relatively poor country; it has reportedly received financial assistance from Saudi Arabia for its nuclear program.[18]

Another reason for a possible decline in the rate of nuclear proliferation apparent in Figure 1 is the fact that among nuclear-weapon states, those with higher GDPs tended to acquire this capability earlier, as is evident in Figure 2.

3.3 Knowledge

In 1976, John Aristotle Phillips, an "underachieving" undergraduate at Princeton University, "designed a nuclear weapon using publicly available books and papers."[19] Nuclear weapons experts disagreed on whether the design would have worked. Whether Phillips' design would have worked or not, it should be clear that the continuing progress in human understanding of nuclear physics inevitably makes it easier for people interested in making such weapons to acquire the knowledge of how to do so.

Before that, the nuclear age arguably began with the 1896 discovery of radioactivity by the French scientist Henri Becquerel and further developed in France by Pierre and Marie Curie, in England by Ernest Rutherford

[11] ("China and Weapons of Mass Destruction" n.d.); see also ("First Taiwan Strait Crisis" n.d.), Halperin (1966), and ("Daniel Ellsberg" n.d.).

[12] Pierson (2017). See also ("Daniel Ellsberg" n.d.).

[13] Riedel (2012). See also ("India and Weapons of Mass Destruction" n.d.). India and China have continued to have conflicts, e.g., in 1967, and the Doklam standoff in 2017.

[14] ("Pakistan and Weapons of Mass Destruction" n.d.). India-Pakistan relations have been marked by frequent conflict since the two nations were born with the dissolution of the British Raj in 1947. This history might help people understand the need that Pakistani leaders may have felt and still feel for nuclear parity with India, beyond the loss of half their population and 15 percent of their land area in the 1971 Bangladesh Liberation War.

[15] ("Arab-Israeli Conflict" n.d.). These have continued, including the Gaza border protests that have continued at least into 2020. One might therefore reasonably understand why Israel might feel a need for nuclear weapons and why others might believe that the 1979-09-22 Vela incident was an Israeli nuclear test.

[16] Medalia (2013); ("Comprehensive Nuclear-Test-Ban Treaty: Background and Current Developments" 2016). See also ("2006 North Korean Nuclear Test" n.d.).

[17] Bush (2002); see also ("Axis of Evil" n.d.).

[18] Riedel (2008).

[19] Chronicle (1976). See also ("John Aristotle Phillips" n.d.).

plus others, especially in France, England and Germany.[20] In 1933 after Adolf Hitler came to power in Germany, Leo Szilard moved from Germany to England. The next year he patented the idea of a nuclear fission reactor. After World War II began, the famous Manhattan Project became a joint British-American project, which produced the very first test of a nuclear weapon.[21]

After Soviet premier Joseph Stalin learned of the atomic bombings of Hiroshima and Nagasaki, the USSR (now Russia) increased the funding for their nuclear-weapons program. That program was helped by intelligence gathering about the German nuclear weapon project and the American Manhattan Project.[22]

The UK's nuclear-weapons program was built in part on their wartime participation in the Manhattan Project, as noted above. France was among the leaders in nuclear research until World War II. They still had people with the expertise needed after the Suez Crisis convinced them they needed to build nuclear bombs, as noted above.[23]

China got some help from the Soviet Union during the initial phases of their nuclear program.[24]

The first country to get nuclear weapons after the Non-Proliferation Treaty was India, whose Atomic Energy Commission was founded in 1948, chaired by Homi J. Bhabha; he had published important research in nuclear physics while a graduate student in England in the 1930s, working with some of the leading nuclear physicists of that day.[25]

Meanwhile, Israel's nuclear weapons program initially included sending students abroad to study under leading physicist like Enrico Fermi at the University of Chicago. It also included extensive collaboration with the French nuclear-weapons program.[26]

Pakistan got secret help from the US in the 1980s in violation of US law to secure Pakistani cooperation with US support for anti-Soviet resistance in Afghanistan.[27] Robert Gallucci, a special adviser on WMDs to President Clinton, said that the nuclear programs of Iran, Libya and North Korea got US technology from Pakistan, and would not have gotten off the ground without US support for the Pakistani nuclear program.[28]

Western sources have claimed that China also helped Pakistan's nuclear-weapons program, but Chna has denied those claims.[29]

And now the US is helping Saudi Arabia obtain nuclear power, in spite of (a) the evidence that the Saudi government including members of the Saudi royal family were involved in preparations for the suicide mass murders of September 11, 2001, at least as early as 1999,[30] and (b) their on-going support for Al Qaeda in Yemen, reported as recently as 2019.[31]

[20] ("Nuclear Physics" n.d.)

[21] ("History of Nuclear Weapons" n.d.)

[22] OSTI (n.d.). See also ("Soviet Atomic Bomb Project" n.d.).

[23] See also ("History of Nuclear Weapons" n.d.)

[24] ("China and Weapons of Mass Destruction" n.d.).

[25] ("Homi J. Bhabha" n.d.); see also ("Timeline of Nuclear Weapons Development" n.d.).

[26] ("Nuclear Weapons and Israel" n.d.).

[27] Burr (2012), Burr (2013). There have also been reports that China helped Pakistan obtain nuclear weapons. However, China has vigorously denied those charges, many if not all of which may not be credible, having originated with the US government. See ("Pakistan and Weapons of Mass Destruction" n.d.).

[28] Levy and Scott-Clark (2007) say that Gallucci was a special adviser on WDMs to US Presidents Clinton and G. W. Bush. The Wikipedia article on him says he was US Assistant Secretary of State for Political-Military Affairs from July 13, 1992 to October 11, 1994 under Presidents George H. W. Bush and Bill Clinton but not G. W. Bush. Later, per Gallucci (2001), "In March 1998, the Department of State announced his appointment as Special Envoy to deal with the threat posed by the proliferation of ballistic missiles and weapons of mass destruction. He held this position until January 2001." G. W. Bush became US President 2001-01-20. Thus, if Gallucci served under G. W. Bush, it was only for a few days. Similar remarks about the US helping Pakistan's nuclear program were made by Richard Barlow, a CIA analyst who reported these questionable activities to a committee of the US House as noted by Levy and Scott-Clark (2007). Barlow was reportedly severely punished for honestly answering questions in a classified briefing to an oversight committee of the US House. Barlow said that US assistance to Pakistan's nuclear weapons program was in exchange for Pakistan's help in supplying rebels in Afghanistan fighting Soviet occupation. This was during the Iran-Contra affair, which exposed actions of officials of the Reagan administration to pursue foreign policy objectives in Central America in blatant violation of law passed by Congress and signed by the President.

[29] ("Pakistan and Weapons of Mass Destruction" n.d.).

[30] Graham et al. (2003). See also ("The 28 Pages" n.d.).

[31] See, for example, Bazzi (2019) and ("Saudi Arabian-Led Intervention in Yemen" n.d.), more generally.

3.4 Material

Reportedly the most difficult part of making nuclear weapons today is obtaining sufficient fissile material. Toon et al. (2007) said, "Thirteen countries operate plutonium and/or uranium enrichment facilities, including Iran", but Iran did not have sufficient fissile material in 2003 to make a nuclear weapon. Another 20 were estimated to have had sufficient stockpiles of fissile material acquired elsewhere to make nuclear weapons. They concluded that 32 (being 13 minus 1 plus 20) additional countries have sufficient fissile material to make nuclear weapons if they want.[32]

Toon et al. (2007) also said, "In 1992 the International Atomic Energy Agency safeguarded less than 1% of the world's HEU [Highly Enriched Uranium] and only about 35% of the world inventory of Pu [Plutonium] . . . . Today [in 2007] a similarly small fraction is safeguarded."

HUE is obtained by separating ^{235}U, which is only 0.72 percent of naturally occurring uranium.[33] Weapons-grade uranium has at least 85 percent ^{235}U.[34] Thus, at least $0.85/0.0072 = 118$ kg of naturally occurring uranium are required to obtain 1 kg that's weapons-grade, and Toon et al. (2007) estimated that 25 kg of HEU would be used on average for each ^{235}U-based nuclear weapon. Plutonium, by contrast, is a byproduct of energy production in standard ^{238}U nuclear reactors.

Much of the uranium for the very first test of a nuclear weapon by the US came from the Congo,[35] but domestic sources provided most of the uranium for later US nuclear-weapons production.[36] The Soviet Union (USSR, now Russia) also seems to have had adequate domestic sources for its nuclear-weapons program, especially including Kazakhstan, which was part of the USSR until 1990; Kazakhstan has historically been the third largest source of uranium worldwide after Canada and the US.[37] The UK presumably got most of its uranium from Canada.

The French nuclear-weapons program seems to have been built primarily on plutonium.[38] This required them to first build standard ^{238}U nuclear reactors to make the plutonium. Then they didn't need nearly as much uranium to sustain their program.

China has reportedly had sufficient domestic reserves of uranium to support its own needs,[39], even exporting some to the USSR in the 1950s in exchange for other assistance with their nuclear defense program.[40]

India's nuclear weapons program seems to have been entirely (or almost entirely) based on plutonium.[41]

Israel seems not to have had sufficient uranium deposits to meet its own needs. Instead, they purchased some from France until France ended their nuclear-weapons collaboration with Israel in the 1960s. Israel also purchased uranium from Argentina.[42] To minimize the amount of uranium needed, nearly all Israeli nuclear weapons seem to be plutonium bombs.[43]

It's not clear where Pakistan got most of its uranium: Its reserves in 2015 were estimated at zero, and its historical production to that point was relatively low.[44] By comparison with the first seven nuclear-weapon states, it's not clear where Pakistan might have gotten enough uranium to produce 83 plutonium bombs and

[32] pp. 1975, 1977. The 32 countries they identified included 12 of the 13 that "operate plutonium and/or uranium enrichement facilities", excepting Iran as noted. The other 20 countries acquired stockpiles elsewhere. In addition to the 32 with sufficient fissile material to make a nuclear weapon, Egypt, Iraq and the former Yugoslavia were listed as having abandoned a nuclear-weapons program.

[33] ("Weapons of Mass Destruction (WMD): Uranium Isotopes" n.d.).

[34] ("Enriched Uranium" n.d.), section on "Highly enriched uranium (HEU)".

[35] ("Manhattan Project" n.d.).

[36] ("List of Countries by Uranium Reserves" n.d.).

[37] ("List of Countries by Uranium Reserves" n.d.).

[38] ("France and Weapons of Mass Destruction" n.d.). See also Table 2 in Toon et al. (2007), which claims that in 2003, France had enough fissile material for roughly 24,000 plutonium bombs and 1,350 ^{235}U bombs.

[39] ("List of Countries by Uranium Reserves" n.d.).

[40] ("China and Weapons of Mass Destruction" n.d.).

[41] ("India and Weapons of Mass Destruction" n.d.); see also Toon et al. (2007) and ("List of Countries by Uranium Reserves" n.d.).

[42] ("Nuclear Weapons and Israel" n.d.).

[43] Toon et al. (2007).

[44] ("List of Countries by Uranium Reserves" n.d.).

44 uranium bombs, as estimated by Toon et al. (2007)[45] As previously noted, the US helped the Pakistani nuclear-weapons program in the 1980s and accused China of providing similar assistance, a charge that China has repeatedly and vigorously denied. Both France and China have provided civlian nuclear reactors, which could help produce plutonium but not ^{235}U.[46]

According to the Federation of American Scientists, "North Korea maintains uranium mines with an estimated four million tons of exploitable high-quality uranium ore ... that ... contains approximately 0.8% extractable uranium.[47] If that's accurate, processing all that would produce 4,000,000 times 0.008 = 32,000 tons of pure natural uranium, which should be enough to produce the weapons they apparently have today.

3.5 Conclusions regarding motivation, money, knowledge and materials

1. There seems to be no shortage of motivations for other countries to acquire nuclear weapons. The leaders of the Soviet Union had personal memories of being invaded not only by Germany during World War II but also by the US and others after World War I. The UK had reason to fear the Soviets in their occupation of Eastern Europe. The French decided after Suez they couldn't trust the US to defend them. China had been forced to yield to nuclear threats before starting their nuclear program, as did India, Pakistan and North Korea. Israel has fought multiple wars since their independence in 1948.
2. The knowledge and materials required to make such weapons in a relatively short order are also fairly widely available, even without the documented willingness of current nuclear powers to secretly help other countries acquire such weapons in some cases.[48]
3. Unless there is some fundamental change in the structure of international relations, it seems unwise to assume that there will not be more nuclear-weapon states in the future, with the time to the next "first test" of a nuclear weapon following a probability distribution consistent with the previous times between "first tests" of nuclear weapons by the current nuclear-weapon states.

4 Distribution of the time between Poisson "first tests"

Possibly the simplest model for something like the time between "first tests" in an application like this is to assume they come from one exponential distribution with 8 observed times between the 9 current nuclear-weapon states plus one censored observation of the time between the most recent one and a presumed next one. This simple theory tells us that the maximum likelihood estimate of the mean time between such "first tests" is the total time from the US "Trinity" test to the present, 74.6 years, divided by the number of new nuclear-weapon states 8, not counting the first, which had no predicessors. Conclusion: Mean time between "first tests" = 9.3 years.[49]

However, Figure 1 suggests that the time between "first tests" of succeeding nuclear-weapon states is increasing. The decreasing hazard suggested by this figure requires mathematics that are not as easy as the censored data estimation as just described.

To understand the current data better, we redo Figure 1 with a log scale on the y axis in Figure 3.

```
plotNucStates(log.='y')
```

```
# optionally write to a file
if(plot2file){
```

[45] Table 2, p. 1976.

[46] ("Pakistan and Weapons of Mass Destruction" n.d.).

[47] ("DPRK: Nuclear Weapons Program" n.d.); see also ("North Korea and Weapons of Mass Destruction" n.d.).

[48] In addition to the 32 currently non-nuclear-weapon states with "sufficient fissile material to make nuclear weapons if they wished", per Toon et al. (2007), the inspector general of the US Department of Energy concluded in 2009 (in its most recent public accounting) that enough highly enriched uranium was missing from US inventories to make at least five nuclear bombs comparable to those that destroyed substantial portions of Hiroshima and Nagasaki in 1945. Substantially more weapons-grade materials may be missing in other countries, especially Russia (Malone and Smith (2018)).

[49] For precursors to the current study that involve censored estimation of time to a nuclear war, see ("Time to the Extinction of Civilization" n.d.) and ("Time to Nuclear Armageddon" n.d.).

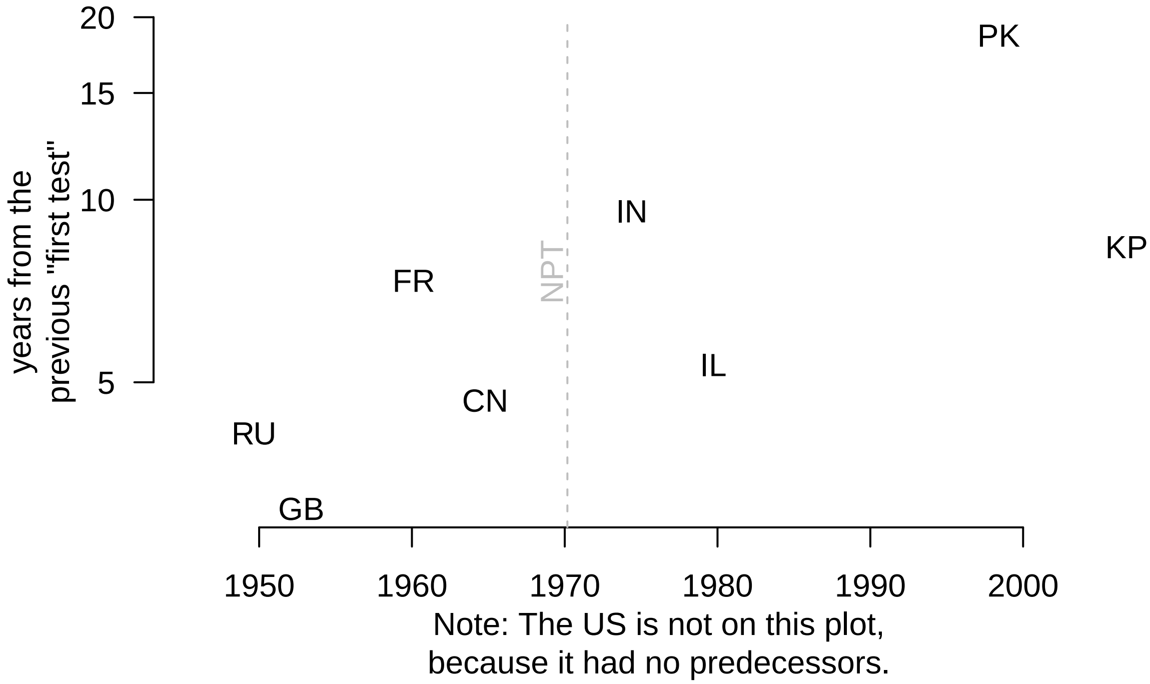

Figure 3: Semilog plot of time between new nuclear-weapon states

```
  svg('nucProlif.svg', width=9, height=7)
#  png('nucProlif.png', width=1300, height=1000)
  cex2 <- 1.6
  par(cex=cex2, mar=c(5, 4, 2, 2)+.1)
  plotNucStates(log.='y', cex.=cex2)
  dev.off()
}
```

Figures 1 and 3 seem consistent with the following:

- If the mean time between "first tests" is increasing over time, as suggested by Figures 1 and 3, then the distribution cannot be exponential, becaause that requires a constant hazard rate. [For the exponential distribution, $h(t) = (-d/dt \log S(t)) = \lambda$, writing the exponential survival function as $S(t) = \exp(-\lambda t)$.]

- Even though nuclear proliferation has been slowing since 1950, it could *accelerate* in the future if more states began to perceive greater threats from other nations.

- Fortunately we can simplify this modeling problem by useing the famous duality between exponential time between events and a Poisson distribution for numbers of events in specific intervals of time. By modeling Poisson counts of "first tests" each year, we can use techniques for Poisson regression for models suggested by Figure 3. The simplest such model might consider log(Poisson mean numbers of "first tests" each year) to be linear in the time since the first test of a nuclear weapon (code-named "Trinity".[50] However, the image in Figure 3 suggests the line may not be straight. Easily tested alternatives to linearity could be second, third and fourth powers of the "timeSinceTrinity."[51]

[50] Rhodes (1986). See also ("Trinity (Nuclear Test)" n.d.).

[51] One might also consider a model with the log(Poisson mean) behaving like a "Wiener process" (also called a "Brownian motion"). This stochastic formulation would mean that the variance of the increments in log(hazard) between "first tests" is proportional to the elapsed time. See ("Wolfram, "Wiener Process"" n.d.) and ("Wikipedia, "Wiener Process"" n.d.). The `bssm` package provides a reasonable framework for modeling this. Its `ng_bsm` function supports modeling a normal random walk in log(Poisson mean) of the number of "first tests" each year. In this article, we model the trend as deterministic and leave

We could use Poisson regression modeling this as a series of the number of events each year, month, week, or day. For present purposes, we will use a series of annual observation. Changing to monthly, weekly or daily observations might give us slightly better answers while possibly increasing the compute time more than it's worth.

5 Parameter estimation

For modeling and parameter estimation, we use `glm(firstTests ~ timeSinceTrinity, poisson)` with:

- `firstTests` = the number of "first tests" of a nuclear-weapon by a new nuclear-weapon state each year, and

- `timeSinceTrinity` = number of years since 1945-07-16, when the first nuclear weapon was tested, code-named "Trinity".

We use the `lubridate` package for dates. The first thing we want is the current year. We get that starting with `today`:

```
library(lubridate)
(Today <- today())
```

```
## [1] "2020-02-13"
```

From this we get the year:

```
(currentYear <- year(Today))
```

```
## [1] 2020
```

We include an observation for the current year only if it's more than 6 months since January 1 and since the last "first test".

```
if((month(Today)<7) ||
   (difftime(Today,
          tail(nuclearWeaponStates$firstTest, 1),
          units = 'days')<(366/2)))
      currentYear <- (year(Today)-1)
```

Start after the year of the first test of a nuclear weapon.

```
firstTstYr <- year(nuclearWeaponStates$firstTest)
(firstYear <- firstTstYr[1])
```

```
## [1] 1945
```

We use this to create a vector of the number of `firstTests` by year and put this a `tibble` with `Year`.

```
(nYrs <- currentYear - firstYear)
```

```
## [1] 74
```

```
firstTests <- ts(rep(0, nYrs), firstYear+1)
firstTstYrSinceFirst <- firstTstYr - firstYear
firstTests[firstTstYrSinceFirst] <- 1

library(tibble)
(FirstTsts <- tibble(Year=time(firstTests),
      nFirstTests=firstTests))
```

consideration of a Gaussian random walk and similar stochastic formulations for future work.

```
## # A tibble: 74 x 2
##     Year nFirstTests
##    <dbl>       <dbl>
##  1  1946           0
##  2  1947           0
##  3  1948           0
##  4  1949           1
##  5  1950           0
##  6  1951           0
##  7  1952           1
##  8  1953           0
##  9  1954           0
## 10  1955           0
## # ... with 64 more rows
```

We add `ctry` to this `tibble` for future reference.

```
Ctry <- rep('', nYrs)
Ctry[firstTstYrSinceFirst] <-
        nuclearWeaponStates$ctry[-1]
FirstTests <- cbind(FirstTsts, ctry=Ctry)
```

We add `timeSinceTrinity`, which we will use in modeling.

```
FirstTests$timeSinceTrinity <- 1:nYrs
```

We then fit a model with log(Poisson mean number of first tests each year) linear in `timeSinceTrinity`.

```
summary(fitProlif1 <- glm(
  firstTests ~ timeSinceTrinity,
  poisson, FirstTests))
```

```
##
## Call:
## glm(formula = firstTests ~ timeSinceTrinity, family = poisson,
##     data = FirstTests)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.6610  -0.5157  -0.4114  -0.3281   1.9584
##
## Coefficients:
##                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)      -1.49869    0.60462  -2.479   0.0132 *
## timeSinceTrinity -0.02232    0.01767  -1.263   0.2066
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 35.594  on 73  degrees of freedom
## Residual deviance: 33.891  on 72  degrees of freedom
## AIC: 53.891
##
## Number of Fisher Scoring iterations: 6
```

This says that the time trend visible in Figures 1 and 3 is not statistically significant.

George Box famously said that, "All models are wrong, but some are useful."[52]

Burnham and Anderson (1998) and others claim that better predictions can generally be obtained using Bayesian Model Averaging.[53] In this case, we have two models: `log(Poisson mean)` being constant or linear in `timeSinceTrinity`. The `bic.glm` function in the `BMA` package can estimate these two models and compute posterior probabilities.

```
library(BMA)
```

```
## Loading required package: survival

## Loading required package: leaps

## Loading required package: robustbase

##
## Attaching package: 'robustbase'

## The following object is masked from 'package:survival':
##
##     heart

## Loading required package: inline

## Loading required package: rrcov

## Scalable Robust Estimators with High Breakdown Point (version 1.5-2)
```

```
fitProlif <- bic.glm(
  FirstTests['timeSinceTrinity'],
  FirstTests$nFirstTests,
  "poisson")
summary(fitProlif)
```

```
##
## Call:
## bic.glm.data.frame(x = FirstTests["timeSinceTrinity"], y = FirstTests$nFirstTests,     glm.family =
##
##
##   2  models were selected
##  Best  2  models (cumulative posterior probability =  1 ):
##
##                    p!=0   EV         SD       model 1     model 2
## Intercept          100    -2.069208  0.51495    -2.22462    -1.49869
## timeSinceTrinity   21.4   -0.004779  0.01228       .        -0.02232
##
## nVar                                               0           1
## BIC                                           -278.60277  -276.00191
## post prob                                        0.786       0.214
```

It is standard in the BMA literature to assume a priori an approximate uniform distribution over all models considered with a penalty for estimating each additional parameter to correct for the tendancy of the models to overfit the data. With these standard assumptions, this comparison of these two models estimates a 21 percent posterior probability for the model linear in `timeSinceTrinity`, leaving 79 percent probability for the model with a constant Poisson mean. Figure 4 adds these lines to Figure 3.[54]

[52] Box and Draper (1987); ("All Models Are Wrong" n.d.).

[53] See also Raftery (1995) and Claeskens and Hjort (2008).

[54] For Figure 3, we use the standard duality between the Poisson and exponential distributions, ignoring the fact that changes over time in the Poisson rate imply that the hazard rate is not constant, as required for the exponential distribution.

```r
plotNucStates(log.='y')

predProlif <- with(fitProlif,
    outer(rep(1, nYrs+1), mle[, 1]) +
    outer(0:nYrs, mle[, 2]))
lgnd <- paste0(c('constant', 'linear'),
    ' (', 100*round(fitProlif$postprob, 2), '%)')
firstTest_nYrs <- as.Date(paste0(
  trunc(nuclearWeaponStates$firstTestYr[1])+0:nYrs,
  '-07-01') )
matlines(firstTest_nYrs, exp(-predProlif),
         lty=c('dashed', 'dotted'),
         col=c('red', 'blue'))
legend('topleft', lty=c('dashed', 'dotted'),
      col=c('red', 'blue'), lgnd,
      bty='n')
```

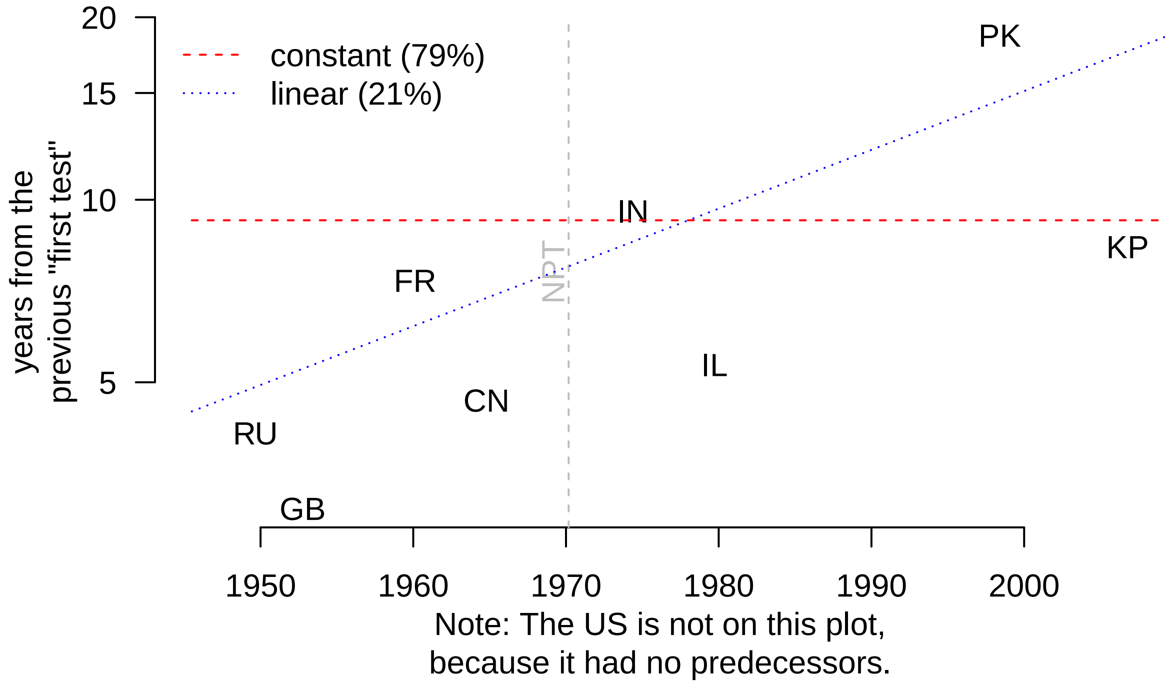

Figure 4: BMA fit to time between new nuclear-weapon states

The lines in this figure seem higher than the mean of the points and a linear trend through the points. This bias might be explained by the difference between ordinary least squares and `glm` used in this case.

It's well known that extrapolation is problematic. Bayesian Model Averaging offers on average more plausible predictions than using a single model. Before proceeding, let's consider a similar BMA fit with quadratic, cubic, and quartic terms.

```r
FirstTests$time2 <- (1:nYrs)^2
FirstTests$time3 <- (1:nYrs)^3
FirstTests$time4 <- (1:nYrs)^4
FirstTests$time5 <- (1:nYrs)^5

fitProlif4 <- bic.glm(
```

```
  FirstTests[4:7], FirstTests$nFirstTests, "poisson")
fitProlif4$postprob
```

```
## [1] 0.4858642 0.1323559 0.1321233 0.1265517 0.1231049
```

```
fitProlif4$mle
```

```
##      (Intercept) timeSinceTrinity         time2         time3         time4
## [1,]   -2.224624        0.0000000  0.0000000000  0.000000e+00  0.000000e+00
## [2,]   -1.498694       -0.0223236  0.0000000000  0.000000e+00  0.000000e+00
## [3,]   -1.748115        0.0000000 -0.0003227533  0.000000e+00  0.000000e+00
## [4,]   -1.854253        0.0000000  0.0000000000 -4.789539e-06  0.000000e+00
## [5,]   -1.912813        0.0000000  0.0000000000  0.000000e+00 -7.158141e-08
```

```
fitProlif5 <- try(bic.glm(
  FirstTests[4:8], FirstTests$nFirstTests, "poisson"))
```

```
## Error in solve.default(cov[-1, -1]) :
##   system is computationally singular: reciprocal condition number = 3.45488e-18
```

When quadratic, cubic and quartic terms are considered, the `BMA:::bic.glm` algorithm keeps only the highest order term, and their regression coefficients are all negative. This means that for each model in the Poisson mixture, the minimum of the mean time to the next "first test" occurs when `timeSinceTrinity` is zero. When a fifth order term is included, one of the models the algorithm tries to fit is computationally singular. Both these resuts make some sense, as there are only 8 years with one "first test"; all the others have zero "first tests", and no year had more than one.

We add the extra lines of `fitProlif4` to Figure 4 to get Figure 5.

```
plotNucStates(log.='y')

predProlif4 <- matrix(NA, nYrs+1, 5)
predProlif4[, 1] <- fitProlif4$mle[1,1]
for(pwr in 1:4){
  predProlif4[, pwr+1] <- with(fitProlif4,
      mle[pwr+1, 1] + ((0:nYrs)^pwr)*
      fitProlif4$mle[pwr+1, pwr+1])
}
lgnd4 <- paste0(c('constant', 'linear', 'quadratic',
        'cubic', 'quartic'),
    ' (', 100*round(fitProlif4$postprob, 4), '%)')
matlines(firstTest_nYrs, exp(-predProlif4),
         lty=1:5, col=1:5)
legend('topleft', lty=1:5, col=1:5, lgnd4,
     bty='n', cex=0.95)
```

Comparing predictions between `fitProlif` and `fitProlif4` might help us understand better the limits of what we can learn from the available data.

Next, we compute central 60 and 80 percent confidence limits plus 80 percent prediction, and (0.8, 0.8) tolerance limits for future nuclear proliferation based on `fitProlif` and `fitProlif4`.[55]

[55] "Confidence intervals" bound the predicted mean number of nuclear-weapon states for each future year considered. Central 80 percent "prediction intervals" are limits that include the central 80 percent of distribution of the number of nuclear-weapon states. They add the uncertainty in the modeled Poisson process to the uncertainty of estimating the mean of that process for each future year considered. We will also compute (0.8, 0.8) "tolerance intervals"; $(p, 1-\alpha)$ tolerance intervals have a probability of $(1-\alpha)$ of containing a proportion of at least p of all future observations.

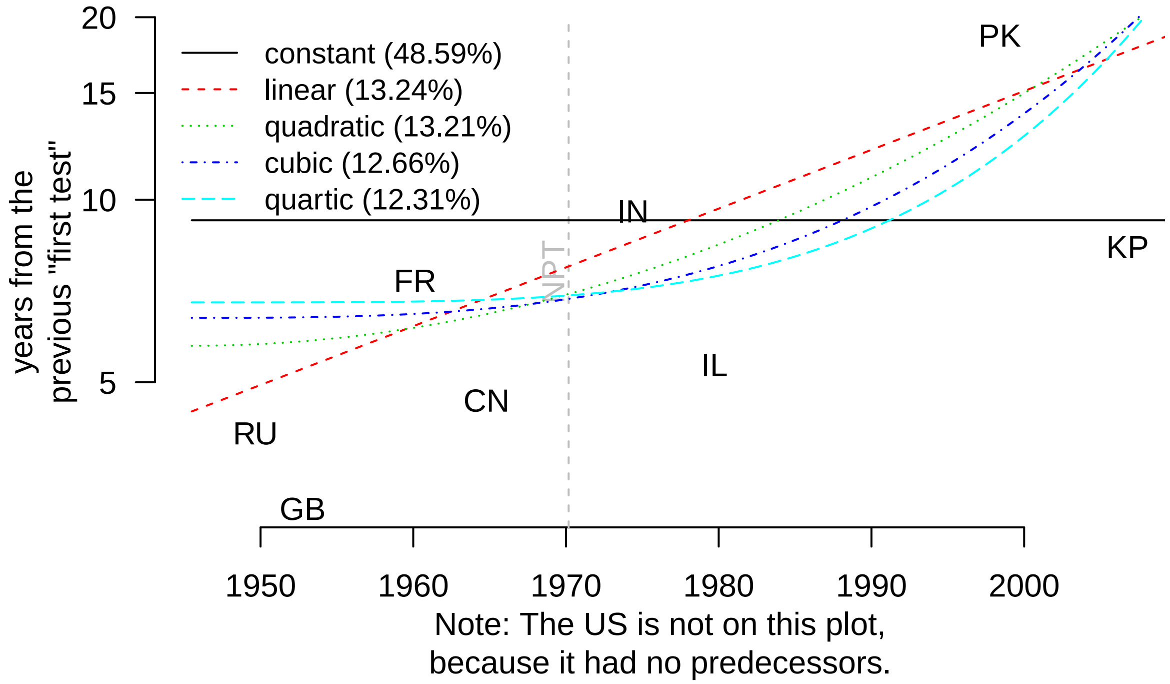

Figure 5: BMA quartic fit to time between new nuclear-weapon states

6 Confidence limits

We start by computing `nSims` simulated Poisson mean numbers of "first tests" by new nuclear-weapon states for each of the `nYrs` years used in `fitProlif` and `fitProlif4` and another `nYrs` years beyond. These simulations will later be used to compute confidence limits for the model fit and prediction and tolerance limits for the actual number of nuclear-weapon states.

```
nSims <- 5000
timeSncT <- 1:(2*nYrs)
pastfut <- tibble(Year=firstYear+timeSncT,
                  timeSinceTrinity=timeSncT,
                  time2=timeSncT^2, time3=timeSncT^3,
                  time4=timeSncT^4)
library(Ecfun)
simMeans <- simulate(fitProlif, nSims, seed=3,
    newdata=pastfut[2], type='response')
dim(simMeans)
```

```
## [1]  148 5000
```

```
# earlier simulations showed a curve
# for the mean of simMeans4 that
# that was quite extreme.
# Check it by set.seed(1) here
# and 2, 3, ... later
simMeans4 <- simulate(fitProlif4, nSims, seed=1,
    newdata=pastfut[2:5], type='response')
dim(simMeans4)
```

```
## [1]  148 5000
```

We invert these simulated Poisson means to get simulated exponential times, then summarize them in a format compatible with `yearsSinceLastFirstTest` in `nuclearWeaponStates`.

```
sumSims <- function(x, Year=pastfut$Year){
##
## return data.frame of Year with
## mean and (.1, .5, .9) quantiles of x
##
  Yr <- as.Date(paste0(Year, '-07-01'))
  xMean <- apply(x, 1, mean)
  xCI <- apply(x, 1, quantile,
          probs=c(.1, .2, .5, .8, .9))
# fix names
  rownames(xCI) <- c(
    'L10', 'L20', 'median', 'U20', 'U10')
  xSum <- data.frame(Year=Yr,
        mean=xMean, data.frame(t(xCI)))
  xSum
}
sumExpMeans <- sumSims(1/simMeans)
sumExpMeans4 <- sumSims(1/simMeans4)
```

These numbers are added to Figure 3 to produce Figures 6 and 7.

```
plotNucStatesPred <- function(x, ...){
##
## plotNucStates with future predictions
## summarized in x
##
  xlim. <- range(x$Year)
  ylim. <- range(nuclearWeaponStates$yearsSinceLastFirstTest,
      head(x[-1], 1), tail(x[-1], 1), na.rm=TRUE)

  plotNucStates(xlim.=xlim., ylim.=ylim., log.='y', ...)
  with(x, lines(Year, mean))
  with(x, lines(Year, median, lty='dashed', col='blue'))
  with(x, lines(Year, U10, lty='dotted', col='red'))
  with(x, lines(Year, L10, lty='dotted', col='red'))
  with(x, lines(Year, U20, lty='dotted', col='red'))
  with(x, lines(Year, L20, lty='dotted', col='red'))
  legend('topleft', c('60, 80% confidence limits for the mean',
          'mean', 'median'),
      col=c('red', 'black', 'blue'),
      lty=c('dotted', 'solid', 'dashed'), bty='n')
  abline(h=200, lty='dotted', col='grey')
}
plotNucStatesPred(sumExpMeans)
```

The fairly flat shape of the median and lower 10 percent lines in Figure 6 seem consistent with a model that is 79 percent constant and 21 percent linear on the log scale, reported with `summary(fitProlif)` above. The substantial curvature of the solid line forecast looks hopeful, with a mean of simulated means being almost 200 years between successive "first tests" by new nuclear-weapon states by the end of the forecasted period, 2093.

The fact that the mean of the simulations exceeds the upper confidence limit for 2093 seems odd but can be explained by noting that this is a mixture with a smaller component being positively skewed with a mean

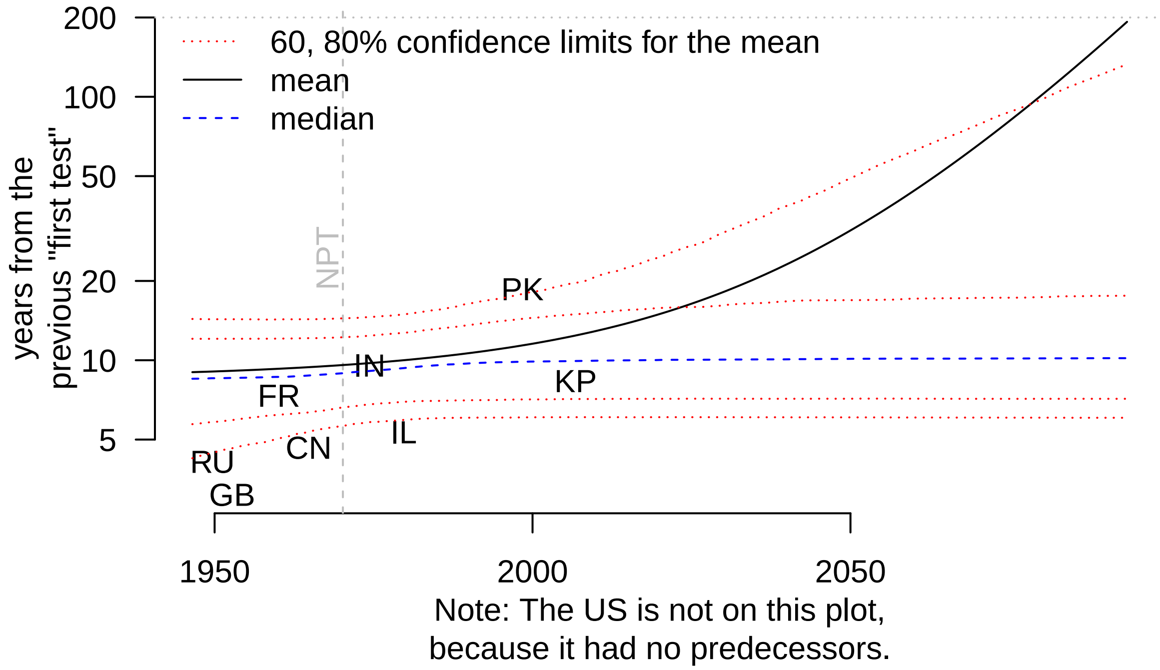

Figure 6: Estimated mean time between "first tests," past and future

substantially larger than the more common component. To understand this consider a toy example

$$X = PX_1 + (1 - P)X_2$$

with the random variable P being either 0 or 1 with $\mathrm{E}P = p$, X_i being lognormal with mean and standard deviation of the logarithms being μ_i and σ_i, respectively, and with P and X_i, $i = 1, 2$, being independent of one another. Consider the further simplification with $\mu_1 = 0 = \sigma_1$. Then the $(1 - p/2)$ upper limit for X is the median of the second component, $\exp(\mu_2)$, while $\mathrm{E}X = (1 - p)\exp(\mu_2 + \sigma_2^2/2)$. Then the mean is greater than this upper limit whenever $(1 - p)\exp(\sigma_2^2/2) > 1$, which will be true if $\sigma_2 > \sqrt{-2\log(1 - p)}$. The case considered here is more complicated than this toy example, but the same principle applies.

Note further that the distribution for each year in Figure 6 is a mixture of lognormal distributions, which means that their reciprocals, the mean numbers of "first tests" each year, will also be a mixture of lognormals with the same standard deviations on the log scale. This standard deviation is larger the farther we extrapolate into the future.

```
plotNucStatesPred(sumExpMeans4)
```

The increase over time in the *mean* time between "first tests" in Figures 6 and 7 suggests a desirable decrease in the rate of nuclear proliferation.

However, we are more concerned with the shorter times between "first tests", and they seem all too probable, as we shall see when we simulate and `cumsum` them. To do that, we append these simulated predictions to a plot of the evolution of the number of nuclear-weapon states through the historical period.[56]

[56] In these simulations, we assume a zero probability of a nuclear power giving up their nuclear weapons, even though South Africa reportedly discontinued their nuclear weapons program in 1989, prior to its first universal elections in 1994. We could potentially add South Africa to `nuclearWeaponStates` with the same date as Israel, then model the distribution of the time to when a nuclear-weapon state gives up its nuclear weapons using an exponential distribution. For that, we have one observed time and eight such times that are censored. Standard theory in that case says that the maximum likelihood estimate of the mean time to relinquishing nuclear weapons assuming an exponential distribution is the sum of all the times, censored or observed, divided by the number of times observed, not including the censored times in the denominator. For purposes of illustration, we will assume that South Africa dismanteled its nuclear weapons 1989-12-31, though a report of an inspection by the International

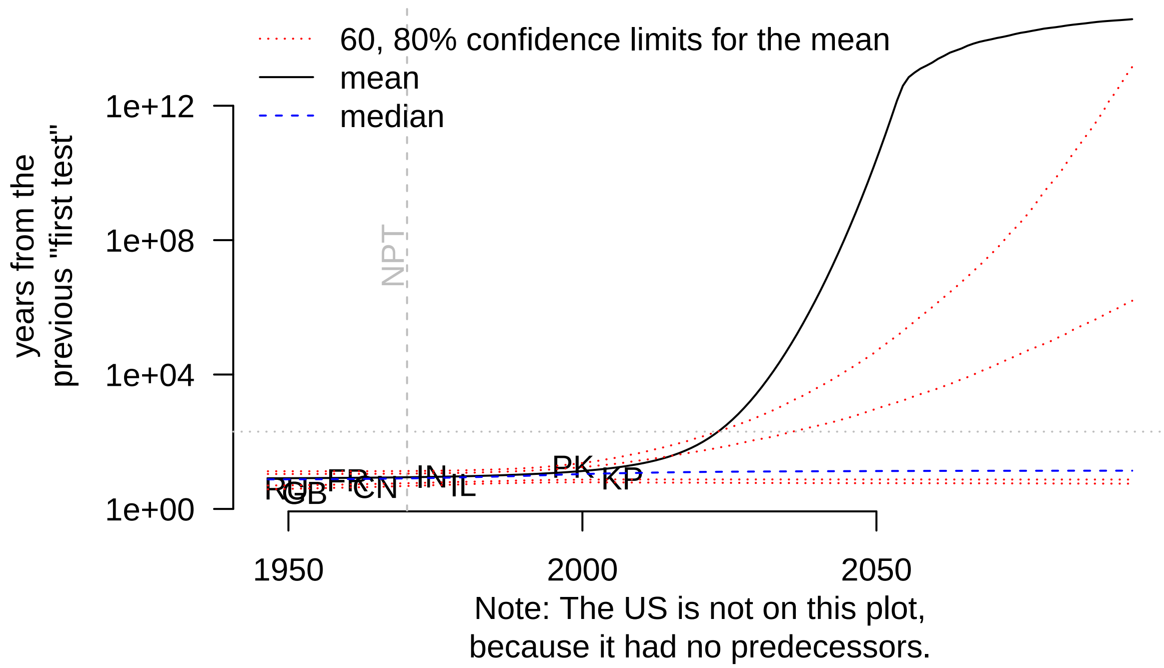

Figure 7: Estimated mean time between "first tests" considering up to a quartic model

```
str(cumMeans <- apply(simMeans[-(1:nYrs), ],
                              2, cumsum))
```

```
##  num [1:74, 1:5000] 0.114 0.228 0.342 0.455 0.569 ...
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:74] "75" "76" "77" "78" ...
##   ..$ : chr [1:5000] "sim_1" "sim_2" "sim_3" "sim_4" ...
```

```
quantile(cumMeans[nYrs,])
```

```
##           0%          25%          50%          75%         100%
##   0.05537032   5.05256923   7.29710712   9.67619756 390.35503361
```

```
str(cumCI <- sumSims(
    nNucStates+rbind(0, cumMeans),
    pastfut$Year[-(1:(nYrs-1))]))
```

```
## 'data.frame':    75 obs. of  7 variables:
##  $ Year  : Date, format: "2019-07-01" "2020-07-01" ...
##  $ mean  : num  9 9.1 9.21 9.31 9.42 ...
##  $ L10   : num  9 9.04 9.08 9.12 9.16 ...
##  $ L20   : num  9 9.06 9.13 9.19 9.25 ...
##  $ median: num  9 9.1 9.2 9.3 9.4 ...
##  $ U20   : num  9 9.14 9.28 9.42 9.56 ...
##  $ U10   : num  9 9.16 9.33 9.49 9.66 ...
```

These numbers are plotted in Figure 8.

Atomic Energy Agency dated 1994-08-19 said they had dismantled six nuclear weapons and were still working to dismantle one more. Based on this, the estimated mean lifetime of a nuclear-weapon state can be estimated at 491 years. We could potentially add this to the current modeling effort, but it would not likely change the answers enough to justify the additional effort.

```r
plotNNucStates <- function(xfuture,
        xpast=nuclearWeaponStates,
        lwd.=c(1,1,1,2,2), ...){
##
## plot stairsteps for xpast and lines for xfuture
## with either 5 or 7 columns in xfuture
##
  xlim. <- c(xpast$firstTest[1], tail(xfuture$Year, 1))
  nColsFut <- length(xfuture)
  ylim. <- c(0, tail(xfuture[[nColsFut]], 1))

  plot(xlim., ylim., type='n', xlab='', ylab='', las=1,
     bty='n')
##
## 1.  plot xpast
##
  fT_date <- c(xpast$firstTest[1],
      xpast$firstTest, Today)
  lines(fT_date,
      c(0:nNucStates, nNucStates), type='s')
  abline(v=NPTdate, lty='dashed', col='grey')
# abline(h=20)
  xlim20. <- c(xlim.[1]-7*365, xlim.[1])
  lines(xlim20., rep(20, 2), lty='dashed', col='grey',
        xpd=NA)
  xlim20 <- c(as.Date('1980-01-01'), xlim.[2])
  lines(xlim20, rep(20, 2), lty='dashed', col='grey')
#
  ymid <- mean(ylim.)
  text(NPTdate-.017*diff(xlim.),
       ymid, 'NPT', col='grey', srt=90)
##
## 2.  plot xfuture
##
  with(xfuture, lines(Year, mean),
       lwd=lwd.[1])
  with(xfuture, lines(Year, median, lty='dashed',
              col='blue', lwd=lwd.[2]))
  with(xfuture, lines(Year, U10, lty='dotted',
              col='red', lwd=lwd.[3]))
  with(xfuture, lines(Year, L10, lty='dotted',
              col='red', lwd=lwd.[3]))
  with(xfuture, lines(Year, U20, lty='dotted',
              col='red', lwd=lwd.[3]))
  with(xfuture, lines(Year, L20, lty='dotted',
              col='red', lwd=lwd.[3]))
  ncols <- 3
  leg <- c('60, 80% confidence limits\nfor the mean',
          'mean', 'median')
  col. <- c('red', 'black', 'blue')
#
  if('predU10' %in% names(xfuture)){
    leg <- c(leg, '80% prediction limits')
```

```
    col. <- c(col., 'green')
    with(xfuture, lines(Year, predU10, lty='dashed',
            col='green', lwd=lwd.[4]))
    with(xfuture, lines(Year, predL10, lty='dashed',
            col='green', lwd=lwd.[4]))
    ncols <- 4
  }
  if('tolU10' %in% names(xfuture)){
    leg <- c(leg, '(0.8, 0.8) tolerance limits')
    col. <- c(col., 'purple')
    with(xfuture, lines(Year, tolU10, lty='dashed',
            col='purple', lwd=lwd.[5]))
    with(xfuture, lines(Year, tolL10, lty='dashed',
            col='purple', lwd=lwd.[5]))
    ncols <- ncols+1
  }
##
## 3.  legend
##
  lty. <- c('dotted', 'solid', rep('dashed', 3))
#  lwd. <- c(rep(par('lwd'), 3),
#            rep(par('lwd')*2, 2) )
  legend('topleft', leg[1:ncols], col=col.[1:ncols],
         lty=lty.[1:ncols], lwd=lwd., bty='n')
}
plotNNucStates(cumCI)
```

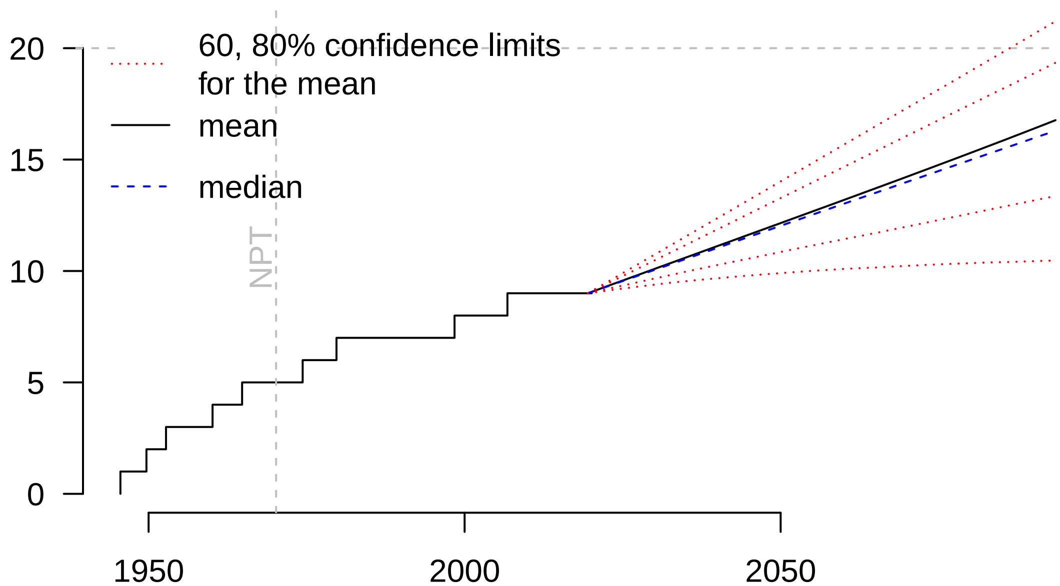

Figure 8: Number of nuclear-weapon states, past and predicted mean

```
# optionally write to a file
if(plot2file){
#  svg('nucProlifPred.svg', width=9, height=7)
  png('nucProlifPred.png', width=2600, height=1500)
  cex2 <- 8
```

```
  par(cex=cex2, mar=c(2,2, 0, 2)+.1, lwd=2)
  plotNNucStates(cumCI, lwd.=rep(8,5))
  dev.off()
}
```

The slopes of the mean and median lines are steeper than the recent history, but the statistical evidence does not support the naive interpretation of a slowing in nuclear proliferation that one might get from considering only the most recent data.

We repeat this analysis with the quartic BMA mixture in Figure 9.

```
str(cumMeans4 <- apply(simMeans4[-(1:nYrs), ],
                              2, cumsum))
```

```
##  num [1:74, 1:5000] 0.0571 0.1142 0.1713 0.2284 0.2855 ...
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:74] "75" "76" "77" "78" ...
##   ..$ : chr [1:5000] "sim_1" "sim_2" "sim_3" "sim_4" ...
```

```
quantile(cumMeans4[nYrs,])
```

```
##           0%          25%          50%          75%         100%
## 1.428091e-04 4.382988e-01 5.485718e+00 8.966752e+00 1.987436e+23
```

```
str(cumCI4 <- sumSims(
    nNucStates+rbind(0, cumMeans4),
    pastfut$Year[-(1:(nYrs-1))]))
```

```
## 'data.frame':    75 obs. of  7 variables:
##  $ Year  : Date, format: "2019-07-01" "2020-07-01" ...
##  $ mean  : num  9 9.09 9.19 9.28 9.38 ...
##  $ L10   : num  9 9.01 9.01 9.02 9.02 ...
##  $ L20   : num  9 9.02 9.04 9.05 9.07 ...
##  $ median: num  9 9.08 9.16 9.24 9.31 ...
##  $ U20   : num  9 9.13 9.26 9.4 9.53 ...
##  $ U10   : num  9 9.17 9.33 9.5 9.66 ...
```

```
plotNNucStates(cumCI4)
```

Comparing Figures 8 and 9 shows that the higher order terms in the quartic BMA mixture widens the confidence limits, making the 10th percentile essentially flat with almost no additional nuclear proliferation, while the mean quickly escapes the upper limit. That sharply rising mean suggests that less than 10 percent of the simulations predict nuclear arms races that involve many nation states and many more non-state armed groups. These outcomes are not likely, but the probabilities of such outcomes seem too large to be dismissed without further consideration, especially when gambling with the future of civilization.

We replicate the simulations summarized in `cumCI4` to see how stable the numbers are for the final year in Figure 9; see Figure 10.

```
cumCI2 <- rbind(tail(cumCI, 1), tail(cumCI4, 1))
rownames(cumCI2) <- c('BMA2', 'BMA4')

for(i in 2:10){
  simMeans4b <- simulate(fitProlif4, nSims, seed=i,
      newdata=pastfut[2:5], type='response')
  cumMeans4b <- apply(simMeans4b[-(1:nYrs), ],
                              2, cumsum)
  cumCI4b <- sumSims(
```

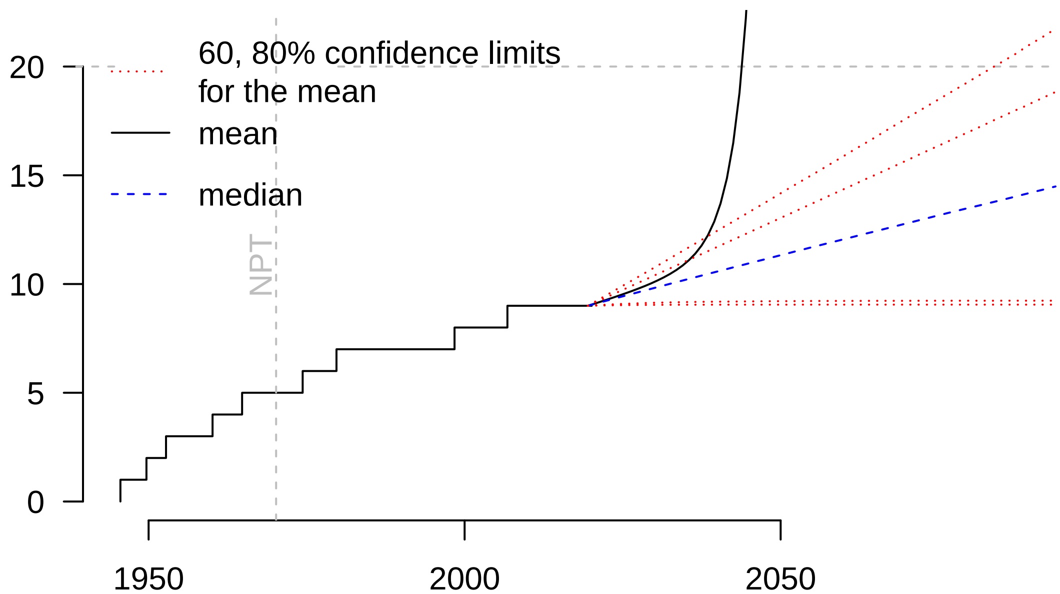

Figure 9: Number of nuclear-weapon states, past and predicted mean, BMA quartic fit

```
    nNucStates+rbind(0, cumMeans4b),
    pastfut$Year[-(1:(nYrs-1))])
  cumCI2 <- rbind(cumCI2, tail(cumCI4b, 1))
  rownames(cumCI2)[i+1] <- paste0(
    'BMA4', letters[i])
}

qqCI2 <- as.data.frame(
  qqnorm(cumCI2$mean[-1], datax=TRUE,
       log='x', ylab='simulated mean'))
with(qqCI2[1, , drop=FALSE],
     points(x, y, pch=15))
with(qqCI2[1, , drop=FALSE],
     text(x, y, adj=c(0, 0.5),
          '  <- used in other figures'))
```

```
cumCI2
```

```
##              Year         mean       L10       L20   median      U20      U10
## BMA2  2093-07-01 1.676818e+01 10.468776 13.366253 16.29711 19.34506 21.20451
## BMA4  2093-07-01 4.132400e+19  9.052477  9.234085 14.48572 18.83051 21.72690
## BMA4b 2093-07-01 1.156695e+26  9.059790  9.252236 14.48593 18.93504 22.03171
## BMA4c 2093-07-01 2.256389e+22  9.061431  9.264471 14.35693 18.84294 21.96172
## BMA4d 2093-07-01 2.258986e+20  9.056517  9.246345 14.54828 18.90989 21.59459
## BMA4e 2093-07-01 1.292328e+21  9.055521  9.250400 14.56862 18.98892 22.07166
## BMA4f 2093-07-01 1.969363e+15  9.057691  9.232413 14.38031 18.79668 21.49412
## BMA4g 2093-07-01 3.838722e+19  9.054800  9.257839 14.43597 18.64658 21.71724
## BMA4h 2093-07-01 5.593417e+22  9.059264  9.245404 14.43115 18.91518 22.17685
## BMA4i 2093-07-01 3.670922e+22  9.069038  9.262718 14.60146 18.92376 21.89457
## BMA4j 2093-07-01 1.407930e+20  9.057179  9.241011 14.40076 18.78418 21.80093
```

These replications establish that the simulated `mean` number of nuclear-weapon states in the last simulated year, 2093, in Figure 9 is slightly conservative relative to the simulated replicates and is definitely *not* a

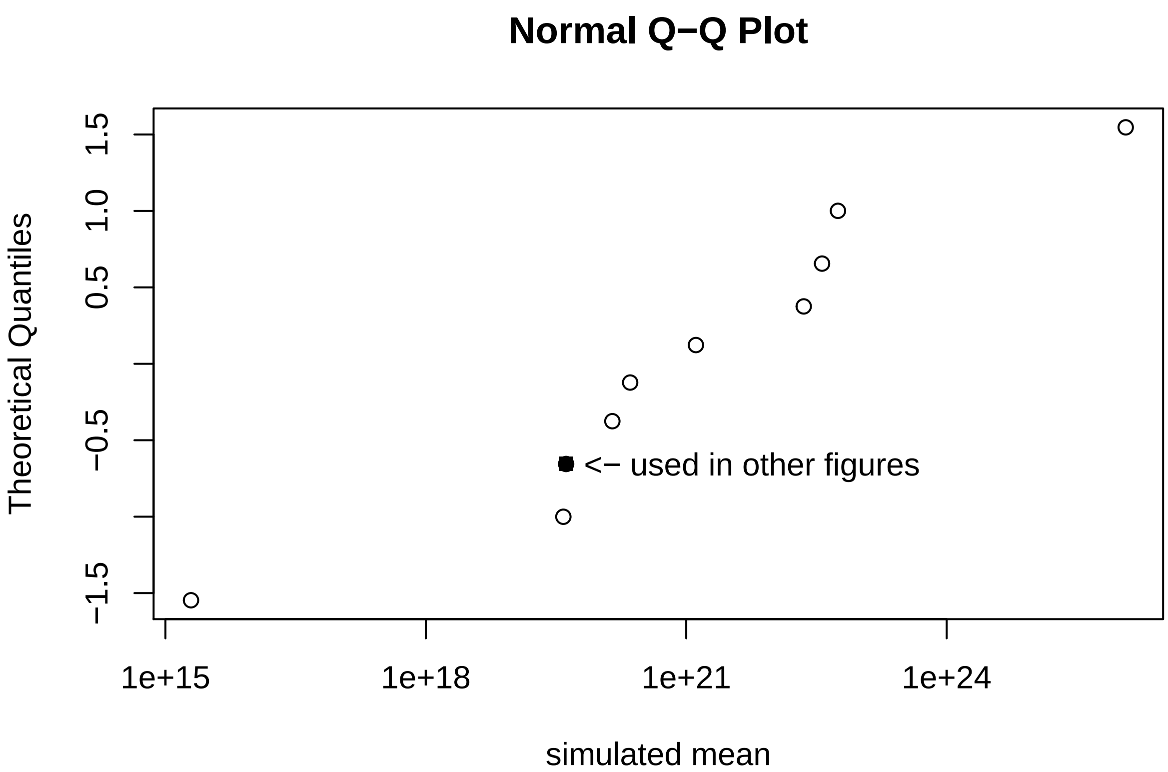

Figure 10: Lognormal probability plot of mean numbers of nuclear-weapon states per the quartic mixture model in the last simulated year

complete outlier.

Beyond that, comparing Figures 8 and 9 establishes that the median and lower limit are lower for the quartic BMA mixture while the upper limit is slightly higher, and the mean for the quartic BMA mixture is too large to be credible.

Even ignoring the simulations of uncontrolled nuclear arms races, the median lines in Figures 8 and 9 predict between 16.3 and 14.5 at the end of the current simulated period, 2093, which is within the anticipated lifespan of a substantial portion of babies born today. Those *median* numbers are a little less than double the number of nuclear-weapon states today.

We extend this analysis by adding prediction intervals to these plots.

7 Prediction limits

The simplest bounds on the future are prediction intervals, which combine the statistical uncertainty in the estimates of mean numbers of nuclear-weapon states with the random variability in the outcomes. For this we use `rpois(., simMeanNucStByYr)`.

```
set.seed(9)
rpois. <- function(n, lambda){
##
## Some of the means are so large that
## rpois sometimes returns NAs.
## Avoid this by outputting numerics
## rather than integers in those cases.
## NOTE:  This was discussed on
```

```r
## r-devel@r-project.org 2020-01-19 and 20
## with the tentative conclusion that
## a change such as documented here
## might be implemented in a future version
## of R.  If that happens, this
## function will no longer be needed.
##
      n2 <- max(n, length(lambda))
      n. <- rep_len(n, n2)
      lam <- rep_len(lambda, n2)
#  If Poisson mean = 0.9*.Machine$integer.max,
#  an observation would have to be over
#  4600 standard deviations above the mean
#  to generate an error.
      big <- (lam>0.9*.Machine$integer.max)
      out <- rep(NA, n2)
      out[big] <- round(rnorm(sum(big),
                  lam[big], sqrt(lam[big])))
      out[!big] <- rpois(sum(!big), lam[!big])
      out
}
cumsumPred <- function(x, ...){
##
## cumsum of rpois predictions based on x
##
#
  simPred <- data.frame(lapply(
      x[-(1:nYrs),], rpois., n=nYrs))
  cumPred <- data.frame(lapply(
      simPred, cumsum))
  cumPred
}
cumPred <- cumsumPred(simMeans)

cumsumC.PI <- function(cumsumPred, cumsumCI, ...){
  cumPI <- sumSims(
      nNucStates+rbind(0, cumsumPred),
      pastfut$Year[-(1:(nYrs-1))])
  prd. <- which(names(cumPI) %in%
            c('L10', 'L20', 'U20', 'U10'))
  names(cumPI)[prd.] <- paste0('pred',
            c('L10', 'L20', 'U20', 'U10'))
# checks
  dYr <- difftime(cumsumCI$Year, cumPI$Year, 'days')
  if(any(as.numeric(dYr)>0))
      stop('Years do not match')
  rd.mean <- ((cumsumCI$mean-cumPI$mean) /
      (cumsumCI$mean+cumPI$mean) )
  if(any(rd.mean>0.01))
    stop('means do not match')
# cbind
  cumC.PI <- cbind(cumsumCI, cumPI[prd.])
  cumC.PI
```

```
}
cumC.PI <- cumsumC.PI(cumPred, cumCI)
```

We add this to the image in Figure 8 to create Figure 11.

```
plotNNucStates(xfuture=cumC.PI)
```

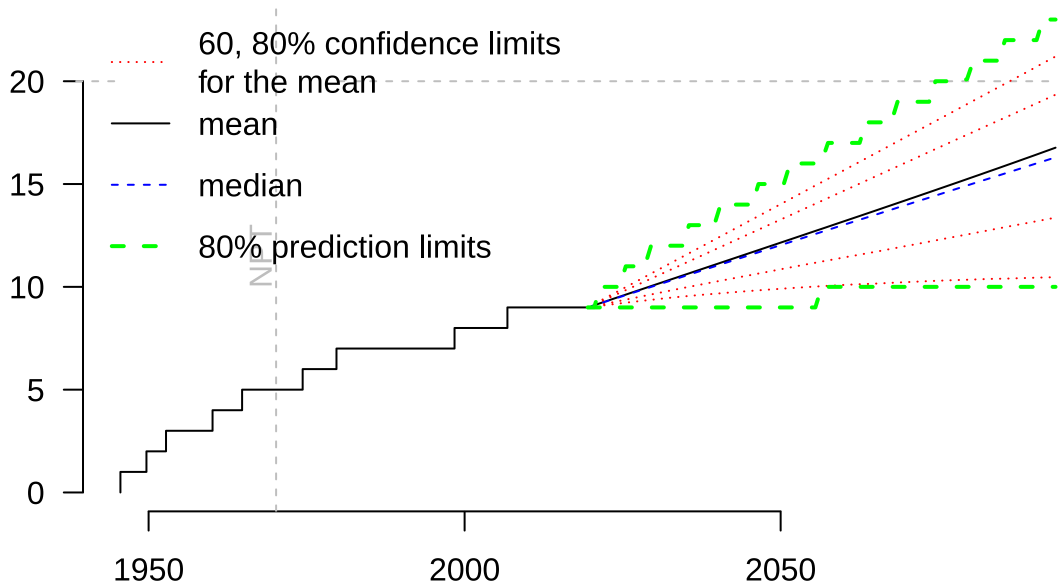

Figure 11: Number of nuclear-weapon states, past and predicted

We do the same for the quartic BMA model in Figure 9 to produce Figure 12. In both Figures 11 and 12 the most likely scenarios, especially the median line and the space more generally between the 80 percent confidence limits, predict a continuation of nuclear proliferation. It's difficult to imagine how that could continue without also substantively increasing the risk of nuclear war and therefore also of the extinction of civilization.

```
cumPred4 <- cumsumPred(simMeans4)
cumC.PI4 <- cumsumC.PI(cumPred4, cumCI4)
plotNNucStates(xfuture=cumC.PI4)
```

We also plot the probability distribution of further nuclear proliferation by summarizing `cumPred` in Figure 13.

```
plotProbs <- function(x, label_year=2050, ...){
# adj.=matrix(c(.53, -.4, .55, 1.15), 2, byrow=TRUE), ...){
##
## Probability distribution of the next 1:5
## new nuclear-weapon states
##
## from x = cumPred, a data.frame
##
  maxNewNucSt <- 5
  probs <- function(x, n=maxNewNucSt){
    p <- colMeans(outer(x, 0:(n-1), '>'))
    p
  }
  probProlif. <- apply(as.matrix(x), 1, probs)
```

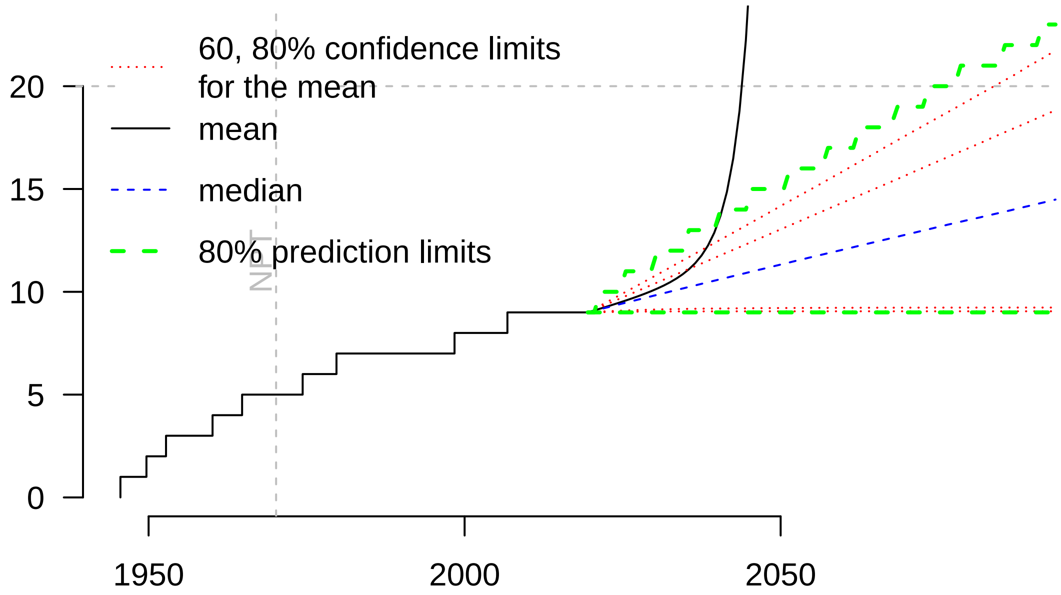

Figure 12: Number of nuclear-weapon states, past and predicted per quartic BMA model

```
  probProlif <- ts(t(probProlif.),
                   currentYear+1)
  colnames(probProlif) <- 1:maxNewNucSt
  ylims <- probProlif.[c(1, maxNewNucSt), c(1, nYrs)]
  midLine <- round(mean(ylims[c(1, 4)]), 2)
  matplot(time(probProlif), probProlif,
        type='l', xlab='', las=1, ylab='',
        ...)
  abline(h=midLine, lty='dotted', ...)
#
  midTime <- rep(NA, maxNewNucSt)
  for(i in 1:maxNewNucSt){
    it <- min(which(probProlif[,i]>=midLine))
#    cat(time(probProlif)[it], '')
    midTime[i] <- time(probProlif)[it]
    text(midTime[i], midLine,
       paste0(midTime[i], '\n', i))
  }
  text(currentYear+0.95*nYrs, midLine,
       paste0('p =\n', midLine) )
# lable_year
  abline(v=label_year, lty='dotted')
  iyr <- which(time(probProlif) == label_year)
  label_p <- probProlif[iyr, ]
  lines(time(probProlif)[c(1, iyr)],
        rep(label_p[1], 2), lty='dotted', ...)
  text(time(probProlif)[1+iyr/4], label_p[1],
    paste0('p =\n', round(label_p[1], 2), ...) )
#
  list(midLine=midLine, midTime=midTime,
       probProlif=probProlif,
```

```
       label_year=c(label_year=label_year,
                    label_prob=label_p))
}
probProlif <- plotProbs(cumPred)
```

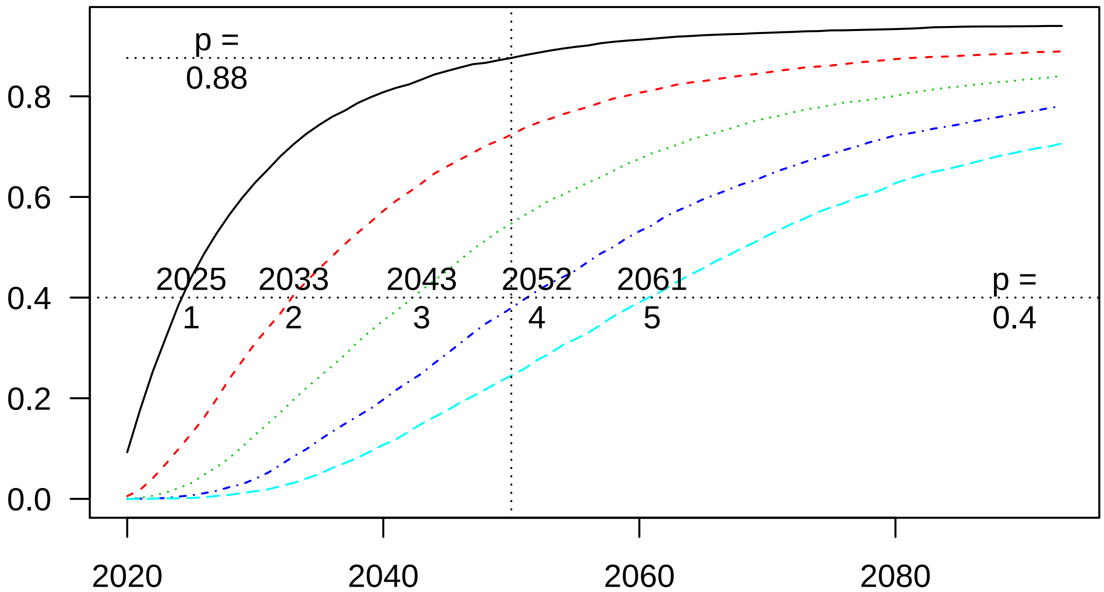

Figure 13: Probability distribution of the time to the next 1:5 new nuclear-weapon states

Figure 13 says there is a 94 percent chance of another nuclear-weapon state by 2093 with a 40 percent chance of at least 1 by 2025.

We replicate Figure 13 for the quartic BMA model in Figure 14.

```
probProlif4 <- plotProbs(cumPred4)
```

The quartic BMA model summarized in Figure 14 is more optimistic than the constant-linear BMA model summarized in Figure 13: The estimated probability of one more nuclear-weapon state by 2050 drops from 0.88 for the constant-linear mixture to 0.69 for the quartic BMA model. However, the conclusions from even this more optimistic model remain the same:

The current structure of international relations threatens the extinction of civilization.

To better quantify the uncertainty in modeling, we next construct tolerance intervals for the time to the next new nuclear-weapon state.

8 Tolerance limits

We want to add tolerance limits to Figures 8 and 9 in addition to the prediction limits. To do this, we add `rpois` simulations to the 80 percent confidence limits in those figures, rather than `rpois` simulations all the indivudal simulations summarized to construct Figures 8 and 9. The results for the constant-linear mixture appear in Figure 15.

```
cumsumTol <- function(x=cumCI, ...){
##
## cumsum of rpois predictions based on x
```

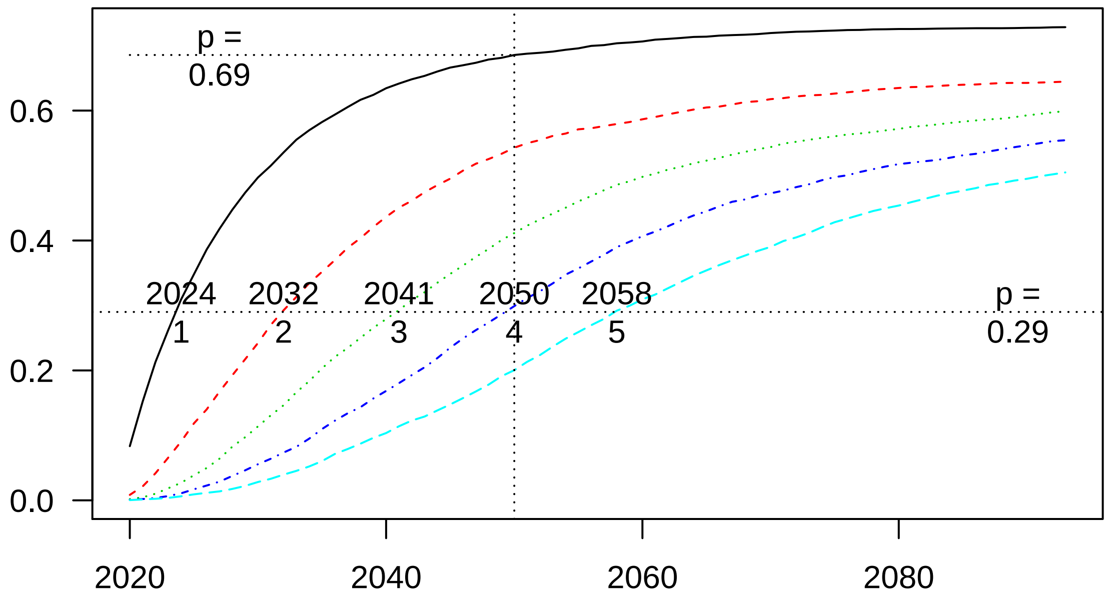

Figure 14: Probability distribution of the time to the next 1:5 new nuclear-weapon states using the quartic BMA model

```
##
#
  simTolU. <- rpois.(nSims*nYrs, diff(x$U10))
  simTolU <- matrix(rpois.(nSims*nYrs,
                  diff(x$U10)), nYrs)
  simTolL <- matrix(rpois.(nSims*nYrs,
                  diff(x$L10)), nYrs)
#
  cumTolU <- apply(simTolU, 2, cumsum)
  cumTolL <- apply(simTolL, 2, cumsum)
#
  cumTolU10 <- apply(cumTolU, 1, quantile, probs=0.9)
  cumTolL10 <- apply(cumTolL, 1, quantile, probs=0.1)
#
  x$tolL10 <- (nNucStates + c(0, cumTolL10))
  x$tolU10 <- (nNucStates + c(0, cumTolU10))
#
  as.data.frame(x)
}
cumTol <- cumsumTol(cumC.PI)
```

```
plotNNucStates(cumTol)
```

The upper limit line in Figure 15 is higher than that in Figure 8. It gives us a bit more humility regarding the value of current knowledge. However, the difference is not enough to substantively alter our conclusions, namely that nuclear proliferation is likely and should not be ignored.

Do we get the same considering the quartic BMA model? See Figure 16.

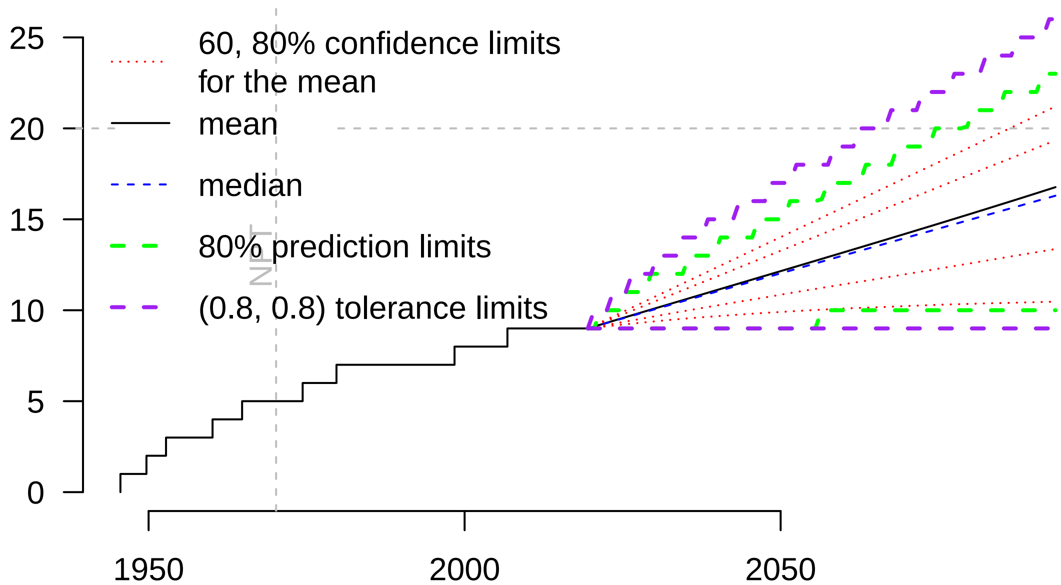

Figure 15: Number of nuclear-weapon states, past and predicted

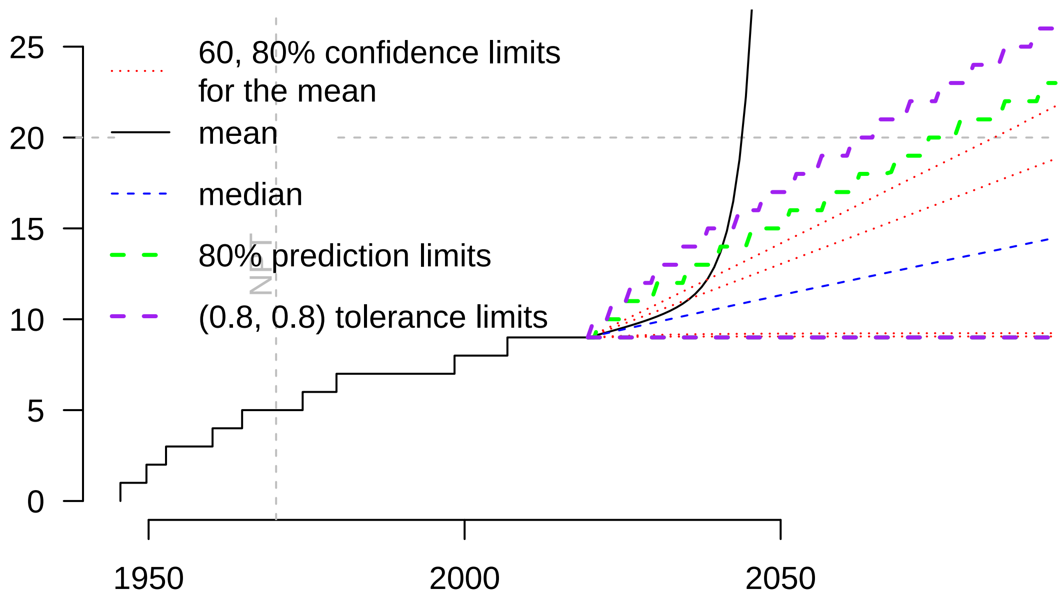

Figure 16: Number of nuclear-weapon states, past and predicted per quartic BMA model

```r
cumTol4 <- cumsumTol(cumC.PI4)
plotNNucStates(cumTol4)
```

Indeed, the conclusion from Figure 16 is the same as before: Nuclear proliferation is likely to continue until something makes it impossible for anyone to make more nuclear weapons for a very long time. This might happen either as a result of (a) a nuclear war leading to the destruction of civilization or (b) a major and unprecidented political movement that strengthens international law to the point that the poor, weak, and disfranchised have effective judicial redress for grievances.

9 Discussion

A growing number of leading figures have said that as long as the world maintains large nuclear arsenals, it is only a matter of time before there is a nuclear war. Concerns like this have been expressed by two former US Secretaries of Defense (Robert McNamara[57] and William Perry), two former US Secretaries of State (Henry Kissinger and George Schultz), former US Senator Sam Nunn,[58] and others with, for example, the Nuclear Threat Initiative. Daniel Ellsberg has said that a nuclear war will most likely generate a nuclear winter that lasts several years during which 98 percent of humanity will starve to death if they do not die of something else sooner.[59]

Banerjee and Duflo, two of the three who won the 2019 Nobel Memorial Prize in Economics, have noted that neither economic nor political stability are assured for any country, including the United States, China and India. In particular, they predict that economic growth will almost certainly slow substantailly in the latter two, leaving many poor people in desparate economic straits.[60] Various journalists and academic researchers remind us that President Trump has demonized various different groups and promoted an increase in ethnic violence. He refused in 2016 and has continued to refuse to say if he would accept the results of an election if he lost in 2020.[61] Internal problems in the US, China, India or any other nuclear-weapon state could push political leaders to pursue increasingly risky foreign adventures, like Argentina did in 1982,[62] possibly leading to a war that could produce nuclear Armageddon.

The present work only increases the urgency of limiting the threat of nuclear war and nuclear proliferation in particular.

In the 20 years following the first test of a nuclear weapon on 1945-07-16 by the US, four more nations acquired such weapons. In the 50 years since the Non-Proliferation Treaty took effect in 1970, another four acquired them.[63] Our analysis of the available data considering only the dates of these first tests suggests that nuclear proliferation may have been slowing throughout this period. However, that apparent trend was not statistically significant in the model we fit.

Bayesian Model Averages (BMA) is known to generally produce better predictions than single model fits. Accordingly, we've estimated confidence, prediction, and tolerance limits for the number of new nuclear-weapon states 74 years into the future based on two BMA models with mixtures of either a constant with a linear model or a constant with terms up to quartic in the time since the very first test of a nuclear weapon.

We can expect that some non-nuclear nations and terrorist groups would eagerly pursue nuclear weapons if such seemed feasible unless some unprecidented change in international law provided them with effective judicial recourse to perceived threats.

Moreover, these weapons will become more available with the passage of time unless (a) a nuclear war destroys everyone's ability to make more such weapons for a long time, or (b) an international movement has far more

[57] McNamara and Blight (2003)
[58] Shultz, Perry, and Nunn (2019)
[59] Ellsberg, Goodman, and González (2017)
[60] Banerjee and Duflo (2019)
[61] Klaas:2019
[62] ("Falklands War" n.d.)
[63] This uses a commonly accepted list of existing nuclear-weapon states and when they each first tested a nuclear weapon.

success than similar previous efforts in giving the poor, weak and disfranchised effective nonviolent means for pursuing a redress of grievances.

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