Time to next new nuclear-weapon state

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Contents

1 Abstract

This article models the time between the "first test" of a nuclear weapon by one nation and the next. It uses this model to obtain confidence bounds on the number of nuclear-weapon states decades into the future. A plot of the times between "first tests" of the 9 nuclear powers as of 2020-01-10 suggests a nonhomogeneous renewal process that suggests a linear decrease over time in log(Poisson mean) of the number of "first tests" each year by new nuclear-weapon states. This can be modeled using `glm(..., family=poisson)`. We fit this model and use it to predict the probabilities of further nuclear proliferation. In brief, we estimate a probability of at least 61 percent that there will be at least one more nuclear-weapon state by 2050 and a 10 percent chance of at least 5 more by that time, unless there is a massive and unprecedented strengthening of international law that provides effective judicial recourse for grievances of the poor, weak and disfranchised.

2 Introduction

A plot of times between "first tests" by the world's nuclear-weapon states as of 2020-01-10 suggests that the process of nuclear proliferation has slowed over time; see Figure 1.

```r
library(Ecdat)
```

```
## Loading required package: Ecfun

##
## Attaching package: 'Ecfun'

## The following object is masked from 'package:base':
##
##     sign

##
## Attaching package: 'Ecdat'

## The following object is masked from 'package:datasets':
##
##     Orange
```

```r
data(nuclearWeaponStates)
ymax <- max(
  nuclearWeaponStates$yearsSinceLastFirstTest,
          na.rm=TRUE)
ylim0 <- c(0, ymax)

NPTdate = as.Date('1970-03-05')
plotNucStates <- function(type.='n', xlim., ylim.,
    line_mtext=3:2, cex.=1, mtext.=TRUE, log.='',
    ...){
##
## Write a function to create this desired plot
## that is general enough to be customized
## to make other similar but different plots
## later.
##
## Obviously, during the process of writing
## this vignette, it requires revising this
## function later as the needs become clearer.
##
## The advantage of doing it this way, it that
## it makes the code easier to read, because
## it's clearer it clearer what is the same and
## what is different between similar plots.
##
#    Start with an internal function
#    to add the 2-letter country codes.
  addCountries <- function(line_mtext=3:2, cex.=1,
                           mtext.=TRUE){
#  Add the country codes ("ctry") to a plot
#  showing the time between "first tests"
#  of nuclear-weapon states
# ... to save copying code
#  and hopefully make the logic clearer
```

```
    xlab. <- paste(c(
        'Note: The US is not on this plot,',
        'because it had no predecessors.'),
                collapse='\n')
    if(mtext.){
        mtext(xlab., 1, line_mtext[1], cex=cex.)
        mtext('years from the\nprevious "first test"',
          2, line_mtext[2], cex=cex.)
    }
    with(nuclearWeaponStates,
        text(firstTest, yearsSinceLastFirstTest,
          ctry, xpd=TRUE, cex=cex.))
  }
# xlim and ylim?
  if(missing(xlim.))xlim. <- range(
    nuclearWeaponStates$firstTest)
  if(missing(ylim.))ylim. <- range(
    nuclearWeaponStates$yearsSinceLastFirstTest[-1])
#
  plot(yearsSinceLastFirstTest~firstTest,
      nuclearWeaponStates, type=type.,
      xlab='', ylab='', las=1,
      xlim=xlim., ylim=ylim., log=log., bty='n',
      ...)
  abline(v=NPTdate, lty='dashed', col='grey')
  ymid <- {
    if(log.=='y') sqrt(ylim.[1]*ylim.[2]) else
      mean(ylim.)
  }
  text(NPTdate-.017*diff(xlim.),
      ymid, 'NPT', col='grey', srt=90)
  addCountries(line_mtext=line_mtext, cex.=cex.,
              mtext.=mtext.)
}
plotNucStates(type.='h', ylim.=ylim0)
```

However, it stretches credibility to suggest that nuclear proliferation has stopped. There were only 5 nuclear-weapon states when the Treaty on the Non-Proliferation of Nuclear Weapons (NPT, Non-proliferation treaty) entered into force in 1970.[1] When US President George W. Bush decried an "Axis of evil" in his State of the Union message, 2002-01-29,[2] there were 8. As this is written 2020-01-10, there are 9. Toon et al. (2007) noted that in 2003 another 32 had sufficient fissile material to make nuclear weapons if they wished.

Moreover, those 32 do *NOT* include either Turkey nor Saudi Arabia. On 2019-09-04, Turkish President Erdogan said it was unacceptable for nuclear-armed states to forbid Turkey from acquiring its own nuclear weapons.[3]

In 2006 *Forbes* reported that Saudi Arabia has "a secret underground city and dozens of underground silos for" Pakistani nuclear weapons and missiles.[4] In 2018 the *Middle East Monitor* reported that "Israel 'is selling nuclear information' to Saudi Arabia."[5] This is particularly disturbing, because of the substantial evidence

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[1]United Nations Office for Disarmament Affairs (1970). See also ("Treaty on the Non-Proliferation of Nuclear Weapons" n.d.).

[2]Bush (2002); see also ("Axis of Evil" n.d.).

[3]Toksabay (2019); O'Connor (2019).

[4]Forbes (2006); see also ("Nuclear Program of Saudi Arabia" n.d.).

[5]Middle East Monitor (2018); see also ("Nuclear Program of Saudi Arabia" n.d.).
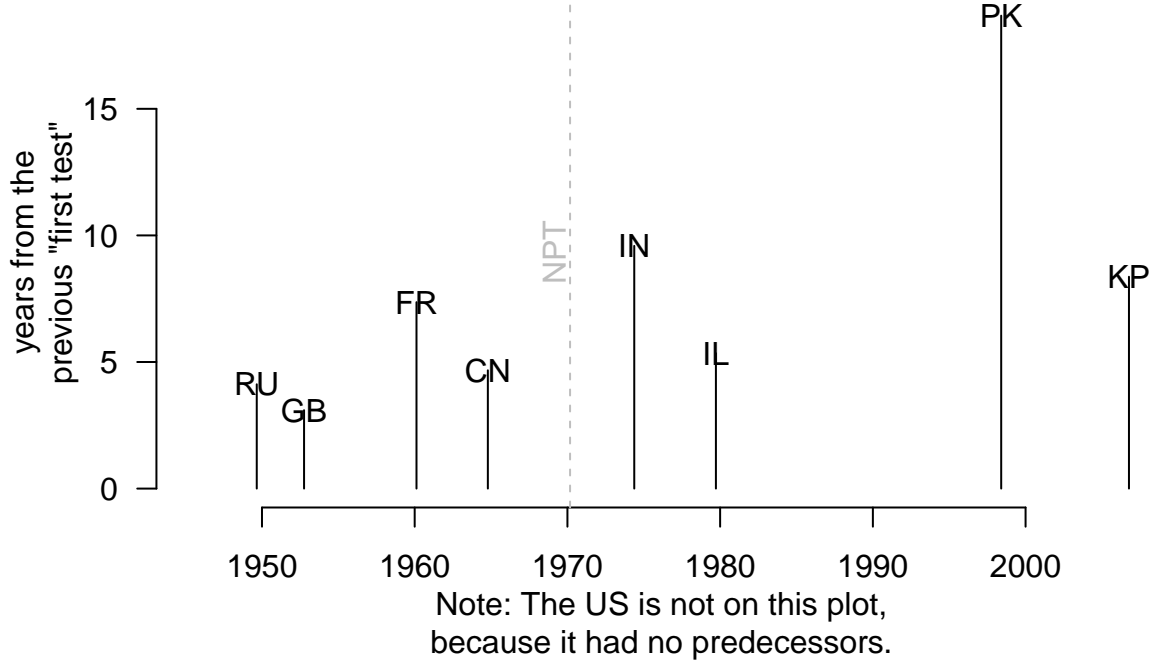
Figure 1: Time between new nuclear-weapon states

that Saudi Arabia may have been and may still be the primary recruiter and funder of Islamic terrorism.[6]

This analysis suggests that the number of nuclear-weapon states will likely continue to grow until either (i) a nuclear war destroys the ability of anyone to make more nuclear weapons, or (ii) the fundamental structure of international relations changes to the point that any nation that perceives an external threat can confidently rely on international law for protection, without a military that might threaten other countries.

This vignette first reviews the data and history on this issue including brief discussions of what might have motivated different nations to pursue nuclear weapons. We then consider modeling these data as a series of annual Poisson observations of the number of states conducting a first test of a nuclear weapon each year (1 in each of 8 years since 1945; 0 in the others).

The simplest such model that considers the possible inhomogeniety visible in Figure 1 is Poisson regression assuming that log(Poisson mean) is linear in the time since the first test of a nuclear weapon by the US on 1945-07-16. We estimate this using `glm(..., family=poisson)`. This model is plausible to the extent that this trend might represent a growing international awareness of the threat represented by nuclear weapons including a hypothesized increasing reluctance of existing nuclear-weapon states to share their technology. The current process of ratifying the Treaty on the Prohibition of Nuclear Weapons supports the hypothesis of such a trend, while the lack of universal support for it clearly indicates that nuclear proliferation is still likely to continue. We use this model to extend the 74 years of history of nuclear proliferation available as this is being written 2020-01-10 into predicting another 74 years into the future.

3   How did the existing nuclear-weapon states develop this capability?

There are, of course, multiple issues in nuclear proliferation: a new nuclear-weapon state requires at least four distinct things to produce a nuclear weapon: motivation, money, knowledge, and material.

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[6]Benjamin (2016); see also ("Winning the War on Terror" n.d.).

3.1 Motivation

Virtually any country that feels threatened would like to have some counterweight against aggression by a potential enemy. The US funded the Manhattan project believing that Nazi Germany likely had a similar project.

Ellsberg (2017) noted that every US president since Truman (president: 1945-1953) considered the use of nuclear weapons. With the possible exception of Ford, all threatened to use them, sometimes publicly, sometimes in secret. Countries threatened include the Soviet Union (now Russia), the People's Republic of China, Iraq, Iran, Libya, India, and North Korea.[7]

One might therefore understand the motivation of North Korea to accelerate their nuclear program in the early 2000s after hearing themselves along with Iran and Iraq described as the "Axis of evil" by US President George W. Bush[8] and after then seeing Iran repeatedly threatened and Iraq invaded with estimated deaths ranging from over 100,000 to over a million[9] out of a population in 2016 of roughly 37 million. Indeed, any reasonable person might understand the eagerness of North Korean leaders for something that could protect them from similar threats.

Similar logic might explain why Soviet leaders might have felt a need to defend themselves from nuclear coercion after having been invaded by the US and over a dozen other countries trying to put the Tsar back in power after World War I.[10] The United Kingdom and France likely felt nuclear threats from the Soviet Union.[11] China faced nuclear threats from the US regarding Korea[12] and the Taiwan Strait.[13] India faced major threats from China,[14] Pakistan from India,[15] and Israel from its neighbors.[16]

More generally the history of US belicosity includes invading Canada in 1812[17] and numerous other foreign interventions including invading Afghanistan in 2001, and Iraq in 2003, plus continued threats against Iran, North Korea, and more recently Venezuela. This history might help people understand how leaders in many countries may be concerned about their own security if they fail to do what the US demands of them. The extra-judicial execution of Osama bin Laden and four others in his household by SEAL Team 6 on 2011-05-02 has reportedly increased the risks that a Pakistani nuclear weapon might be stolen by Islamic terrorists intent on retaliating against the US for its interventions in Pakistan and neighboring countries.[18]

The September 11th attacks
might have been a mushroom cloud.

All this suggests that it will be difficult to reduce the threat of nuclear proliferation and nuclear war without

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[7]Ellsberg (2017). See also ("Daniel Ellsberg" n.d.).

[8]Bush (2002); see also ("Axis of Evil" n.d.).

[9]For a survey of casualty estimates by many different groups, see ("Casualties of the Iraq War" n.d.). The highest serious counts appear to have been Opinion Research Business (2018).

[10]Fogelsong (2014). Many more references can be found in ("Allied Intervention in the Russian Civil War" n.d.).

[11]The UK and France would have had many reasons to fear the intentions of the USSR during the early period of the Cold War: The first test of a nuclear weapon by the USSR came just over three months after the end of the 1948-49 Berlin Blockade. Other aspects of Soviet repression in countries they occupied in Eastern Europe contributed to the failed Hungarian Revolution of 1956.

[12]Pierson (2017). See also ("Daniel Ellsberg" n.d.).

[13]Halperin (1966). See also ("Daniel Ellsberg" n.d.).

[14]India had military conflicts with China in 1962 and 1967 before India's first nuclear weapon's test 1974-05-18. India and China have continued to have conflicts, including, e.g., the Doklam standoff in 2017.

[15]India-Pakistan relations have been marked by frequent conflict since the two nations were born with the dissolution of the British Raj in 1947. This history might help people understand the need that Pakistani leaders may have felt and still feel for nuclear parity with India.

[16]Israel faced many threats from its Arab neighbors before it became an independent nation in 1949. These have continued, including the Gaza border protests that have continued at least to mid 2019. One might therefore reasonably understand why Israel might feel a need for nuclear weapons and why others might believe that the 1979-09-22 Vela incident was an Israeli nuclear test.

[17]Berton (1980) claims that the modern Canadian national identity was forged in the War of 1812. Casual conversations with Canadians in the 2000s and 2010s have confirmed that this is one issue that still separates Canadians from the US.

[18]Cohen (2008); see also Borger (2010).

somehow changing the nature of international relations so weaker countries have less to fear from the demands of nuclear-weapon states like the US.

3.2 Money

It's no accident that most of the world's nuclear-weapon states are large countries with substantial populations and economies. That's not true of Israel with only roughly 9 million people nor North Korea with roughly 26 million people in 2018. France and the UK have only about 67 and 68 million people, but they are also among the world leaders in the size of their economies.

Pakistan is a relatively poor country that that got secret help from the US in the 1980s in violation of US law to secure Pakistani cooperation with US support for anti-Soviet resistance in Afghanistan.[19] Robert Gallucci, a special adviser on WMDs to President Clinton, said that the nuclear programs of Iran, Libya and North Korea got US technology from Pakistan, and would not have gotten off the ground without US support for the Pakistani nuclear program.[20]

And now the US is helping Saudi Arabia obtain nuclear power, in spite of (a) the evidence that the Saudi government including members of the Saudi royal family were involved in preparations for the suicide mass murders of September 11, 2001, at least as early as 1999,[21] and (b) their on-going support for Al Qaeda in Yemen, reported as recently as 2019.[22]

3.3 Knowledge

In 1976, John Aristotle Phillips, an "underachieving" undergraduate at Princeton University, "designed a nuclear weapon using publicly available books and papers."[23] Nuclear weapons experts disagreed on whether the design would have worked. Whether Phillips' design would have worked or not, it should be clear that the continuing progress in human understanding of atomic physics inevitably make it easier for people interested in making such weapons to acquire the knowledge of how to do so.

That was less true in the 1940s. After Soviet premier Joseph Stalin learned of the atomic bombings of Hiroshima and Nagasaki, the USSR (now Russia) increased the funding for their nuclear program. That nuclear-weapons program was helped by intelligence gathering about the German nuclear weapon project and the American Manhattan Project.[24]

3.4 Material

Reportedly the most difficult part of making nuclear weapons today is obtaining sufficient fissile material. As noted above, Toon et al. (2007) said that 32 currently non-nuclear powers had sufficient fissile material in 2003 to make nuclear weapons if they wished.

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[19]Burr (2012), Burr (2013). There have also been reports that China helped Pakistan obtain nuclear weapons. However, China has vigorously denied those charges, many if not all of which may not be credible, having originated with the US government. See ("Pakistan and Weapons of Mass Destruction" n.d.).

[20]See Levy and Scott-Clark (2007). That article claims that Gallucci was a special adviser on WDMs to both Clinton and G. W. Bush. However, the Wikipedia article and Gallucci (2001) both indicate he left government service in January 2001; G. W. Bush took office 2001-01-20. Similar comments were made by Richard Barlow, a CIA analyst who reported these questionable activities to a committee of the US House as noted by Levy and Scott-Clark (2007). Barlow was reportedly severely punished for honestly answering questions in a classified briefing to an oversight committee of the US House. Barlow said that US assistance to Pakistan's nuclear weapons program was in exchange for Pakistan's help in supplying rebels in Afghanistan fighting Soviet occupation. This was during the Iran-Contra affair, which exposed actions of officials of the Reagan administration to pursue foreign policy objectives in Central America in blatant violation of law passed by Congress and signed by the President.

[21]Graham et al. (2003). See also ("The 28 Pages" n.d.).

[22]See, for example, Bazzi (2019) and ("Saudi Arabian-Led Intervention in Yemen" n.d.), more generally.

[23]Chronicle (1976). See also ("John Aristotle Phillips" n.d.).

[24]OSTI (n.d.). See also ("Soviet Atomic Bomb Project" n.d.).

3.5 Conclusions regarding motivation, money, knowledge and materials

1. There seems to be no shortage of motivations for other countries to acquire nuclear weapons.

2. The knowledge and materials required to make such weapons in a relatively short order are also fairly widely available, even without the documented willingness of current nuclear powers to secretly help other countries acquire such weapons in some cases.[25]

3. Unless there is some fundamental change in the structure of international relations, it seems unwise to assume that there will not be more nuclear-weapon states in the future, with the time to the next "first test" of a nuclear weapon following a probability distribution consistent with the previous times between "first tests" of nuclear weapons by the current nuclear-weapon states.

4 Exponential time between Poisson "first tests"

Possibly the simplest model for something like the time between "first tests" in a situation like this is to assume they come from one exponential distribution with 8 observed times between the 9 current nuclear-weapon states plus one censored observation of the time between the most recent one and a presumed next one.[26]

However, Figure 1 suggests that the time between "first tests" of succeeding nuclear-weapon states is increasing. This makes it difficult to use censored data estimation as just described.

To understand the current data better, we redo Figure 1 with a log scale on the y axis in Figure 2.

```
plotNucStates(log.='y')
```

Figures 1 and 2 seem consistent with the following:

- Time between events might be exponentially distributed but with a hazard rate that varies over time. [Recall that the hazard rate for the exponential distribution is constant: $h(t) = (-d/dt \log S(t)) = \lambda$, writing the exponential survival function as $S(t) = \exp(-\lambda t)$.] A non-constant hazard is consistent with the apparent increase in the time to the next "first test" by a new nuclear-weapon state.

- Even though nuclear proliferation has been slowing since 1950, it could *accelerate* in the future if more states began to perceive greater threats from other nations.

- Log hazard that is either linear in the time since the first test of a nuclear weapon (code-named "Trinity")[27] or behaves like a "Wiener process" (also called a "Brownian motion").[28] In this article, we model the trend as linear and leave consideration of a Gaussian random walk and similar stochastic formulations for future work.[29]

Because of the well-known duality between the exponential distribution and the Poisson distribution, we can also model this as a series of the number of events each year, month, week, or day. For present purposes, we will use a series of annual observation. Changing to monthly, weekly or daily observations might give us slightly better answers while possibly increasing the compute time more than it's worth.

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[25]In addition to the 32 currently non-nuclear-weapon states with "sufficient fissile material to make nuclear weapons if they wished", per Toon et al. (2007), the inspector general of the US Department of Energy concluded in 2009 (in its most recent public accounting) that enough highly enriched uranium was missing from US inventories to make at least five nuclear bombs comparable to those that destroyed substantial portions of Hiroshima and Nagasaki in 1945. Substantially more weapons-grade materials may be missing in other countries, especially Russia (Malone and Smith (2018)).

[26]For precursors to the current study that involve censored estimation of time to a nuclear war, see ("Time to the Extinction of Civilization" n.d.) and ("Time to Nuclear Armageddon" n.d.).

[27]Rhodes (1986). See also ("Trinity (Nuclear Test)" n.d.).

[28]This stochastic formulation would mean that the variance of the increments in log(hazard) between "first tests" is proportional to the elapsed time. See ("Wolfram, "Wiener Process"" n.d.) and ("Wikipedia, "Wiener Process"" n.d.).

[29]The `bssm` package provides a reasonable framework for modeling this. Its `ng_bsm` function supports modeling a normal random walk in log(Poisson mean) of the number of "first tests" each year. This uses the fact that the probability distribution of the time between events in a homogeneous Poisson point process is exponential.
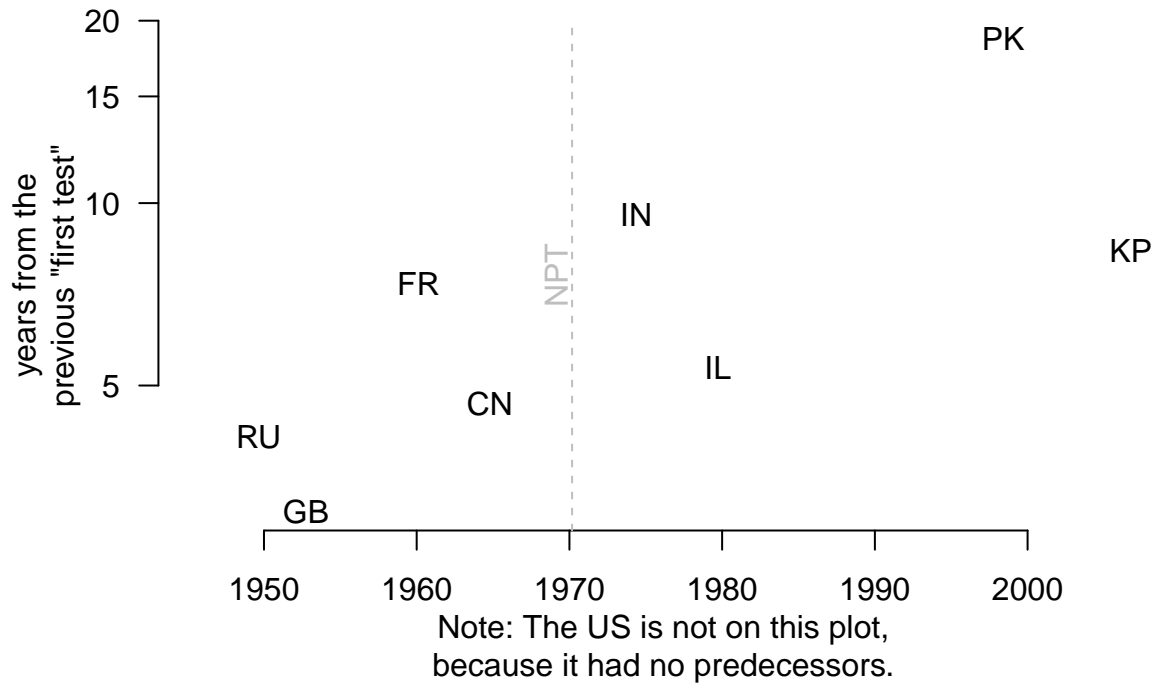
Figure 2: Semilog plot of time between new nuclear-weapon states

5 Parameter estimation

For modeling and parameter estimation, we use `glm(firstTests ~ timeSinceTrinity, poisson)` with:

- `firstTests` = the number of "first tests" of a nuclear-weapon by a new nuclear-weapon state each year, and

- `timeSinceTrinity` = number of years since 1945-07-16, when the first nuclear weapon was tested, code-named "Trinity".

We use the `lubridate` package for dates. The first thing we want is the current year. We get that starting with `today`:

```
library(lubridate)
```

```
##
## Attaching package: 'lubridate'
## The following object is masked from 'package:base':
##
##     date
```

```
(Today <- today())
```

```
## [1] "2020-01-11"
```

From this we get the year:

```
(currentYear <- year(Today))
```

```
## [1] 2020
```

We include an observation for the current year only if it's more than 6 months since January 1 and since the

last "first test".

```
if((month(Today)<7) ||
   (difftime(Today,
         tail(nuclearWeaponStates$firstTest, 1),
         units = 'days')<(366/2)))
      currentYear <- (year(Today)-1)
```

Start after the year of the first test of a nuclear weapon.

```
firstTstYr <- year(nuclearWeaponStates$firstTest)
(firstYear <- firstTstYr[1])
```

```
## [1] 1945
```

We use this to create a vector of the number of `firstTest`s by year and put this a `tibble` with `Year`.

```
(nYrs <- currentYear - firstYear)
```

```
## [1] 74
```

```
firstTests <- ts(rep(0, nYrs), firstYear+1)
firstTstYrSinceFirst <- firstTstYr - firstYear
firstTests[firstTstYrSinceFirst] <- 1

library(tibble)
(FirstTsts <- tibble(Year=time(firstTests),
      nFirstTests=firstTests))
```

```
## # A tibble: 74 x 2
##      Year nFirstTests
##     <dbl>       <dbl>
##  1  1946           0
##  2  1947           0
##  3  1948           0
##  4  1949           1
##  5  1950           0
##  6  1951           0
##  7  1952           1
##  8  1953           0
##  9  1954           0
## 10  1955           0
## # ... with 64 more rows
```

We add `ctry` to this `tibble` for future reference.

```
Ctry <- rep('', nYrs)
Ctry[firstTstYrSinceFirst] <-
      nuclearWeaponStates$ctry[-1]
FirstTests <- cbind(FirstTsts, ctry=Ctry)
```

We add `timeSinceTrinity`, which we use in modeling.

```
FirstTests$timeSinceTrinity <- 1:nYrs
```

We then fit a model with log(Poisson mean number of first tests each year) linear in `timeSinceTrinity`.

```
summary(fitProlif1 <- glm(
  firstTests ~ timeSinceTrinity,
  poisson, FirstTests))
```

```
##
## Call:
## glm(formula = firstTests ~ timeSinceTrinity, family = poisson,
##     data = FirstTests)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -0.6610  -0.5157  -0.4114  -0.3281   1.9584
##
## Coefficients:
##                   Estimate Std. Error z value Pr(>|z|)
## (Intercept)       -1.49869    0.60462  -2.479   0.0132 *
## timeSinceTrinity  -0.02232    0.01767  -1.263   0.2066
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for poisson family taken to be 1)
##
##     Null deviance: 35.594  on 73   degrees of freedom
## Residual deviance: 33.891  on 72   degrees of freedom
## AIC: 53.891
##
## Number of Fisher Scoring iterations: 6
```

This says that the time trend visible in Figures 1 and 2 is not statistically significant.

George Box famously said that, "All models are wrong, but some are useful."[30]

Burnham and Anderson (1998) and others claim that better predictions can generally be obtained using Bayesian Model Averaging.[31] In this case, we have two models: `log(Poisson mean)` being constant or linear in `timeSinceTrinity`. The `bic.glm` function in the `BMA` package can estimate these two models and compute posterior probabilities.

```
library(BMA)
```

```
## Loading required package: survival

## Loading required package: leaps

## Loading required package: robustbase

##
## Attaching package: 'robustbase'

## The following object is masked from 'package:survival':
##
##     heart

## Loading required package: inline

## Loading required package: rrcov

## Scalable Robust Estimators with High Breakdown Point (version 1.4-9)
```

```
fitProlif <- bic.glm(
  FirstTests['timeSinceTrinity'],
  FirstTests$nFirstTests,
```

---

[30]Box and Draper (1987); ("All Models Are Wrong" n.d.).
[31]See also Raftery (1995) and Claeskens and Hjort (2008).

```
  "poisson")
summary(fitProlif)
```

```
##
## Call:
## bic.glm.data.frame(x = FirstTests["timeSinceTrinity"], y = FirstTests$nFirstTests,    glm.family =
##
##
##   2  models were selected
##  Best  2  models (cumulative posterior probability =  1 ):
##
##                   p!=0   EV         SD        model 1     model 2
## Intercept         100   -2.069208  0.51495    -2.22462    -1.49869
## timeSinceTrinity  21.4  -0.004779  0.01228       .        -0.02232
##
## nVar                                              0           1
## BIC                                          -278.60277  -276.00191
## post prob                                       0.786      0.214
```

It is standard in the BMA literature to assume an approximate uniform distribution over all models considered with a penalty for estimating each additional parameter to correct for the tendancy of the models to overfit the data. With these standard assumptions, this comparison of these two models estimates a 21 percent posterior probability for the model linear in `timeSinceTrinity`, leaving 79 percent probability for the model with a constant Poisson mean. Figure 3 adds these lines to Figure 2.

```
plotNucStates(log.='y')

predProlif <- with(fitProlif,
    outer(rep(1, nYrs+1), mle[, 1]) +
    outer(0:nYrs, mle[, 2]))
lgnd <- paste0(c('constant', 'linear'),
    ' (', 100*round(fitProlif$postprob, 2), '%)')
firstTest_nYrs <- as.Date(paste0(
  trunc(nuclearWeaponStates$firstTestYr[1])+0:nYrs,
  '-07-01') )
matlines(firstTest_nYrs, exp(-predProlif),
        lty=c('dashed', 'dotted'),
        col=c('red', 'blue'))
legend('topleft', lty=c('dashed', 'dotted'),
    col=c('red', 'blue'), lgnd,
    bty='n')
```

The lines in this figure seem higher than the mean of the points and a linear trend through the points. This bias might be explained by the difference between ordinary least squares and `glm` used in this case.

We shall compute here central 80 percent confidence and prediction intervals for future nuclear proliferation based on the fitted BMA model. ("Confidence intervals" bound the predicted mean number of nuclear-weapon states for each future year considered. Central 80 percent "prediction intervals" are limits that include the central 80 percent of distribution of the number of nuclear-weapon states. They add the uncertainty in the modeled Poisson process to the uncertainty of estimating the mean of that process for each future year considered.)[32]

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[32]We could also compute $(p, 1 - \alpha)$ "tolerance intervals" that would have a probability of $(1 - \alpha)$ of containing a proportion of at least p of all future observations. That's more complicated than what seems needed for the present application.
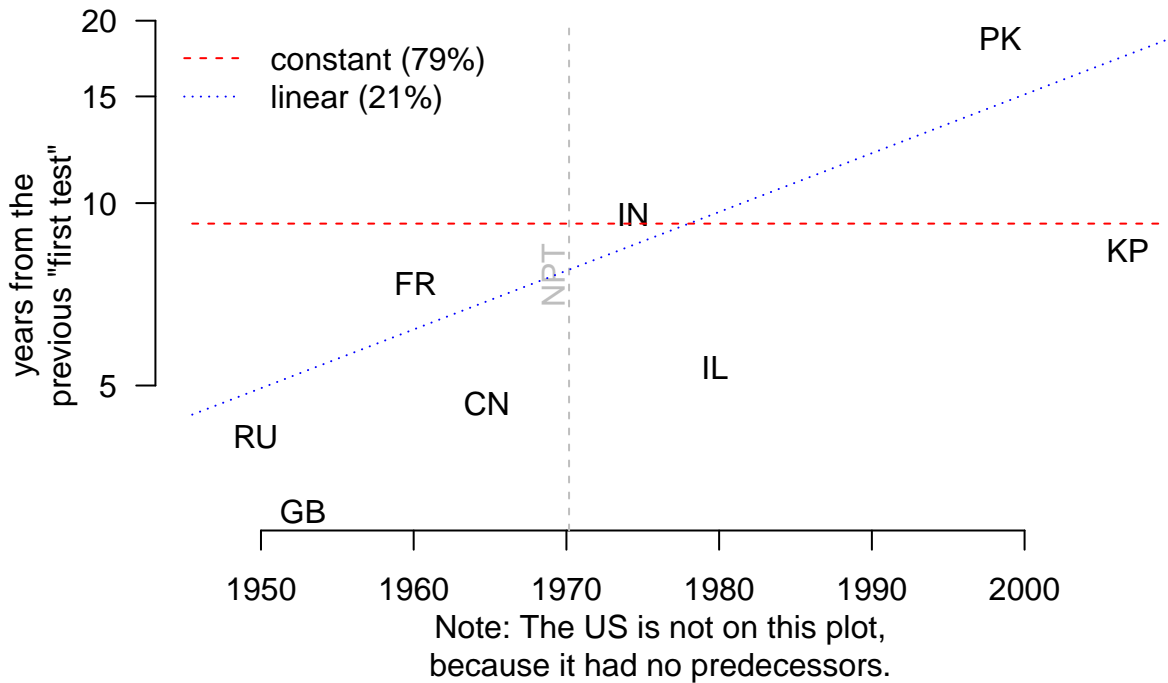
Figure 3: BMA fit to time between new nuclear-weapon states

6 Confidence intervals

We will start by computing `nSims` simulated Poisson mean numbers of "first tests" by new nuclear-weapon states for each of the `nYrs` years used in `fitProlif` and another `nYrs` years beyond. These simulations will later be used to compute confidence intervals for the model fit and predictions.

```
nSims <- 5000
timeSncT <- 1:(2*nYrs)
pastfut <- tibble(Year=firstYear+timeSncT,
                  timeSinceTrinity=timeSncT)
library(Ecfun)
simMeans <- simulate(fitProlif, nSims, seed=3,
    newdata=pastfut['timeSinceTrinity'], type='response')
dim(simMeans)
```

```
## [1]  148 5000
```

We invert these simulated Poisson means to get simulated exponential times, then summarize them in a format compatible with `nuclearWeaponStates`.

```
sumSims <- function(x, Year=pastfut$Year){
##
## return data.frame of Year with
## mean and (.1, .5, .9) quantiles of x
##
  Yr <- as.Date(paste0(Year, '-07-01'))
  xMean <- apply(x, 1, mean)
  xCI <- apply(x, 1, quantile,
                probs=c(.1, .5, .9))
# fix names
```

```
  rownames(xCI) <- c('L10', 'median', 'U10')
  xSum <- data.frame(Year=Yr,
         mean=xMean, data.frame(t(xCI)))
  xSum
}
sumExpMeans <- sumSims(1/simMeans)
```

These numbers are added to Figure 2 to produce Figure 4.

```
plotNucStatesPred <- function(x, ...){
##
## plotNucStates with future predictions
## summarized in x
##
  xlim. <- range(x$Year)
  ylim. <- range(nuclearWeaponStates$yearsSinceLastFirstTest,
      head(x[-1], 1), tail(x[-1], 1), na.rm=TRUE)

  plotNucStates(xlim.=xlim., ylim.=ylim., log.='y', ...)

  with(x, lines(Year, mean))
  with(x, lines(Year, median, lty='dashed', col='blue'))
  with(x, lines(Year, U10, lty='dotted', col='red'))
  with(x, lines(Year, L10, lty='dotted', col='red'))
  legend('topleft', c('80% confidence interval for the mean',
         'mean', 'median'),
      col=c('red', 'black', 'blue'),
      lty=c('dotted', 'solid', 'dashed'), bty='n')
  abline(h=200, lty='dotted', col='grey')
}
plotNucStatesPred(sumExpMeans)
```

The fairly flat shape of the median and lower 10 percent lines seem consistent with a model that is 79 percent constant and 21 percent linear on the log scale, reported with `summary(fitProlif)` above. The substantial curvature of the solid line forecast looks hopeful, with a mean of simulated means being almost 200 years between successive "first tests" by new nuclear-weapon states by the end of the forecasted period, 2093.

However, those are simulated lognormal distributions, which means that its reciprocal, the mean number of "first tests" each year, will also be lognormal with the same standard deviation on the log scale. This standard deviation is larger the farther we extrapolate into the future. That increasing standard deviation explains how the average or mean line in Figure 4 can eventually exceed the upper 10 percent line.

A more interesting and useful view of this is to plot the evolution of the number of nuclear-weapon states through the historical period followed by the forecasts per these simulations.[33]

---

[33]In doing this, we assume a zero probability of a nuclear power giving up their nuclear weapons, even though South Africa reportedly discontinued their nuclear weapons program in 1989, prior to its first universal elections in 1994. We could potentially add South Africa to `nuclearWeaponStates` with the same date as Israel, then model the distribution of the time to when a nuclear-weapon state gives up its nuclear weapons using an exponential distribution. For that, we have one observed time and nine such times that are censored. Standard theory in that case says that the maximum likelihood estimate of the mean time to relinquishing nuclear weapons assuming an exponential distribution is the sum of all the times, censored or observed, divided by the number of observations. For purposes of illustration, we will assume that South Africa dismanteled its nuclear weapons 1989-12-31, though a report of an inspection by the International Atomic Energy Agency dated 1994-08-19 said they had dismantled six nuclear weapons and were still working to dismantle one more. Based on this, the estimated mean lifetime of a nuclear-weapon state can be estimated at 490 years. We could potentially add this to the current modeling effort, but it would not likely change the answers enough to justify the additional effort.
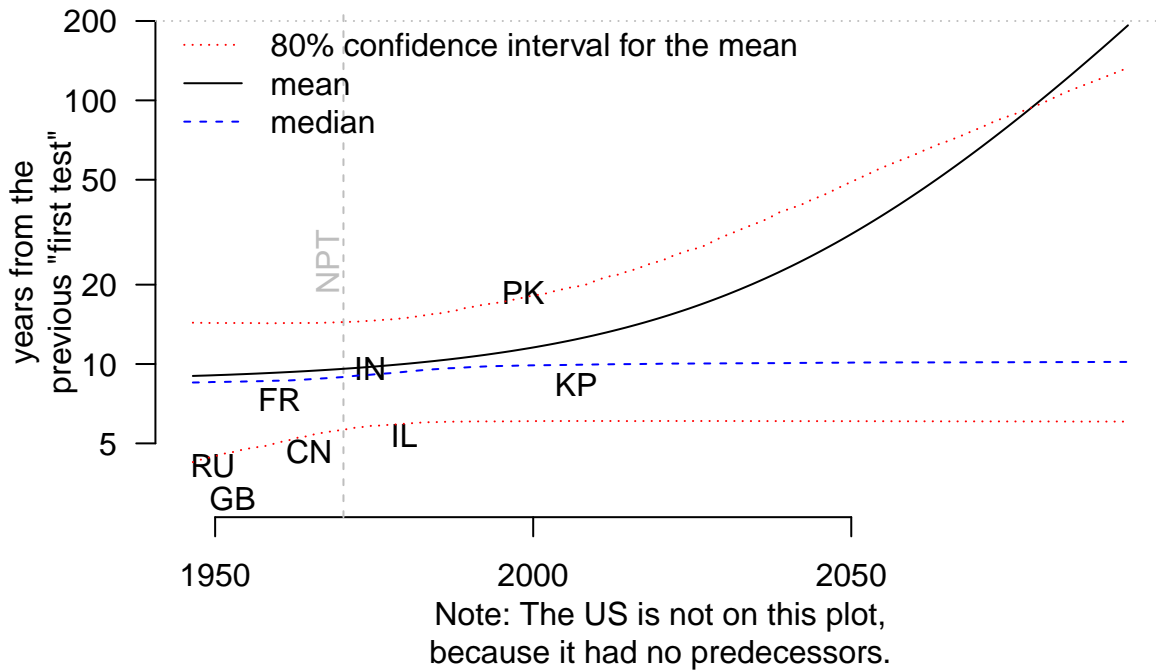
Figure 4: Estimated mean time between "first tests," past and future

```
str(cumMeans <- apply(simMeans[-(1:nYrs), ],
                          2, cumsum))
```

```
##  num [1:74, 1:5000] 0.114 0.228 0.342 0.455 0.569 ...
##  - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:74] "75" "76" "77" "78" ...
##   ..$ : chr [1:5000] "sim_1" "sim_2" "sim_3" "sim_4" ...
```

```
quantile(cumMeans[nYrs,])
```

```
##          0%         25%         50%         75%        100%
##   0.05537032   5.05256923   7.29710712   9.67619756 390.35503361
```

```
(nNucStates <- nrow(nuclearWeaponStates))
```

```
## [1] 9
```

```
str(cumCI <- sumSims(
    nNucStates+rbind(0, cumMeans),
    pastfut$Year[-(1:(nYrs-1))]))
```

```
## 'data.frame':     75 obs. of  5 variables:
##  $ Year  : Date, format: "2019-07-01" "2020-07-01" ...
##  $ mean  : num  9 9.1 9.21 9.31 9.42 ...
##  $ L10   : num  9 9.04 9.08 9.12 9.16 ...
##  $ median: num  9 9.1 9.2 9.3 9.4 ...
##  $ U10   : num  9 9.16 9.33 9.49 9.66 ...
```

These numbers are plotted in Figure 5.

```
plotNNucStates <- function(xfuture,
        xpast=nuclearWeaponStates, ...){
```

```
##
## plot stairsteps for xpast and lines for xfuture
## with either 5 or 7 columns in xfuture
##
  xlim. <- c(xpast$firstTest[1], tail(xfuture$Year, 1))
  nColsFut <- length(xfuture)
  ylim. <- c(0, tail(xfuture[[nColsFut]], 1))

  plot(xlim., ylim., type='n', xlab='', ylab='', las=1,
     bty='n')
##
## 1.  plot xpast
##
  fT_date <- c(xpast$firstTest[1],
      xpast$firstTest, Today)
  lines(fT_date,
      c(0:nNucStates, nNucStates), type='s')
  abline(v=NPTdate, lty='dashed', col='grey')
  ymid <- mean(ylim.)
  text(NPTdate-.017*diff(xlim.),
       ymid, 'NPT', col='grey', srt=90)
##
## 2.  plot xfuture
##
  with(xfuture, lines(Year, mean))
  with(xfuture, lines(Year, median, lty='dashed', col='blue'))
  with(xfuture, lines(Year, U10, lty='dotted', col='red'))
  with(xfuture, lines(Year, L10, lty='dotted', col='red'))
  ncols <- 3
  if(nColsFut>5){
    with(xfuture, lines(Year, predU10, lty='dashed',
              col='green', lwd=2))
    with(xfuture, lines(Year, predL10, lty='dashed',
              col='green', lwd=2))
    ncols <- 4
  }
##
## 3.  legend
##
  leg <- c('80% confidence level for the mean', 'mean', 'median',
           'predictions')
  col. <- c('red', 'black', 'blue', 'green')
  lty. <- c('dotted', 'solid', 'dashed', 'dashed')
  lwd. <- c(rep(1, 3), 2)

  legend('topleft', leg[1:ncols], col=col.[1:ncols],
        lty=lty.[1:ncols], lwd=lwd., bty='n')
}
plotNNucStates(cumCI)
```

The slopes of the mean and median lines are steeper than the recent history, but the statistical evidence does not support the naive interpretation of a slowing in nuclear proliferation that one might get from considering only the most recent data.
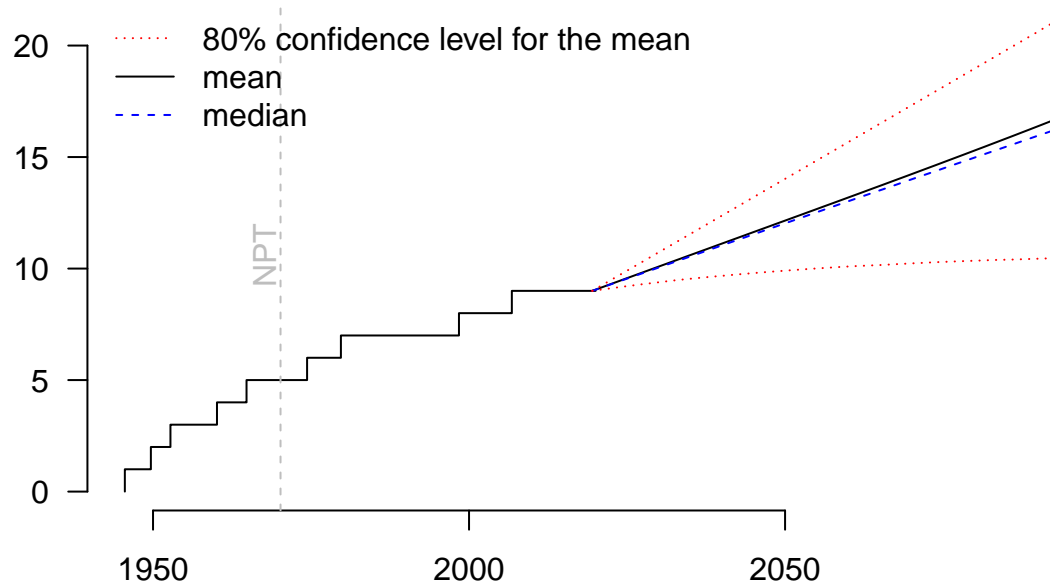
Figure 5: Number of nuclear-weapon states, past and predicted mean

Let's add prediction intervals to this plot.

7 Prediction intervals

The simplest bounds on the future are prediction intervals, which combine the statistical uncertainty in the models with the random variability in the outcomes. For this we use `rpois(., simMeanNucStByYr)`.

```r
set.seed(9)
cumsumPred <- function(x, ...){
##
## cumsum of rpois predictions based on x
##
  simPred <- data.frame(lapply(
      x[-(1:nYrs),], rpois, n=nYrs))
  cumPred <- data.frame(lapply(
      simPred, cumsum))
  cumPred
}
cumPred <- cumsumPred(simMeans)

cumsumC.PI <- function(cumsumPred, cumsumCI, ...){
  cumPI <- sumSims(
      nNucStates+rbind(0, cumsumPred),
      pastfut$Year[-(1:(nYrs-1))])
  names(cumPI)[c(3, 5)] <- paste0('pred',
                            c('L', 'U'), 10)
  cumC.PI <- cbind(cumsumCI, cumPI[c(3, 5)])
  cumC.PI
}
cumC.PI <- cumsumC.PI(cumPred, cumCI)
```

We add this to the image in Figure 5 to create Figure 6.

```
plotNNucStates(xfuture=cumC.PI)
```
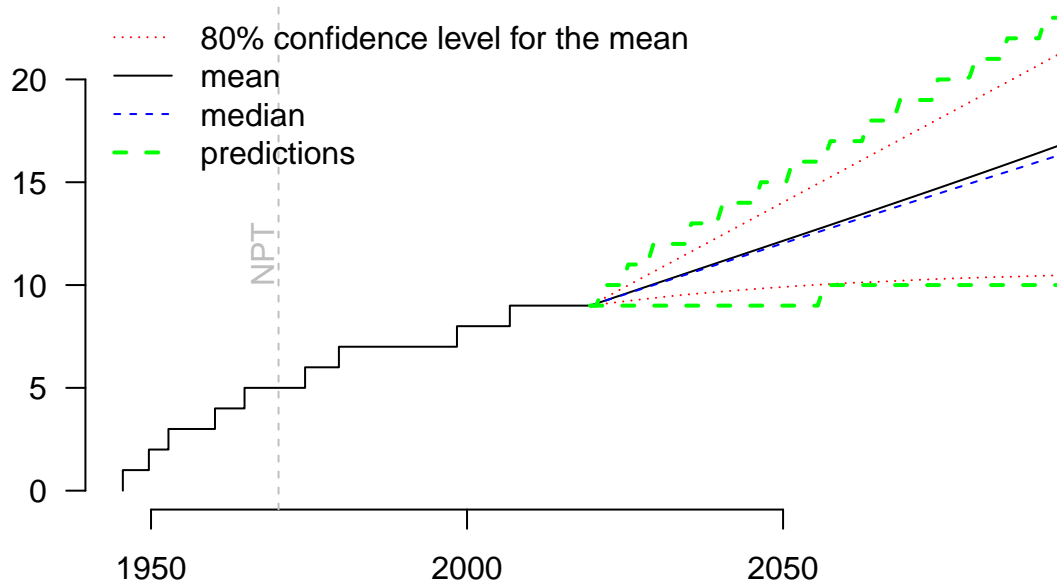

Figure 6: Number of nuclear-weapon states, past and predicted

We also plot the probability distribution of further nuclear proliferation by summarizing `simCumNewNucSt` in Figure 7.

```
plotProbs <- function(x, label_year=2050, ...){
# adj.=matrix(c(.53, -.4, .55, 1.15), 2, byrow=TRUE), ...){
##
## Probability distribution of the next 1:5
## new nuclear-weapon states
##
## from x = cumPred, a data.frame
##
  maxNewNucSt <- 5
  probs <- function(x, n=maxNewNucSt){
    p <- colMeans(outer(x, 0:(n-1), '>'))
    p
  }
  probProlif. <- apply(as.matrix(x), 1, probs)
  probProlif <- ts(t(probProlif.),
                   currentYear+1)
  colnames(probProlif) <- 1:maxNewNucSt
  ylims <- probProlif.[c(1, maxNewNucSt), c(1, nYrs)]
  midLine <- round(mean(ylims[c(1, 4)]), 2)
  matplot(time(probProlif), probProlif,
       type='l', xlab='', las=1, ylab='',
       ...)
  abline(h=midLine, lty='dotted', ...)
#
  midTime <- rep(NA, maxNewNucSt)
  for(i in 1:maxNewNucSt){
    it <- min(which(probProlif[,i]>=midLine))
```

```r
#     cat(time(probProlif)[it], '')
    midTime[i] <- time(probProlif)[it]
    text(midTime[i], midLine,
        paste0(midTime[i], '\n', i))
  }
  text(currentYear+0.95*nYrs, midLine,
        paste0('p =\n', midLine) )
# lable_year
  abline(v=label_year, lty='dotted')
  iyr <- which(time(probProlif) == label_year)
  label_p <- probProlif[iyr, ]
  lines(time(probProlif)[c(1, iyr)],
        rep(label_p[1], 2), lty='dotted', ...)
  text(time(probProlif)[1+iyr/4], label_p[1],
    paste0('p =\n', round(label_p[1], 2), ...) )
#
  list(midLine=midLine, midTime=midTime,
       probProlif=probProlif,
       label_year=c(label_year=label_year,
                    label_prob=label_p))
}
probProlif <- plotProbs(cumPred)
```
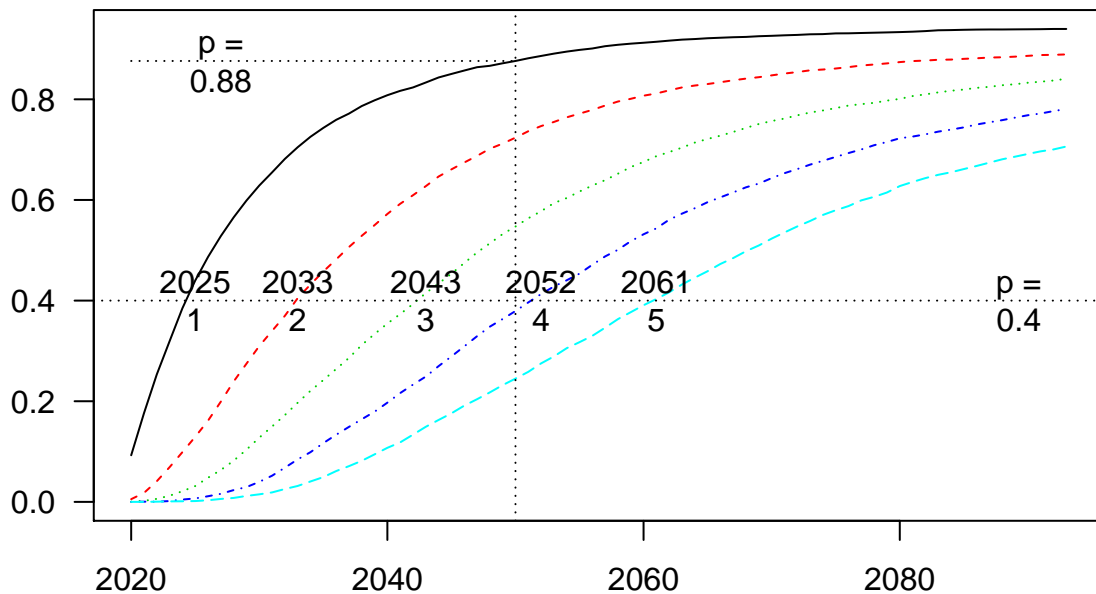

Figure 7: Probability distribution of the time to the next 1:5 new nuclear-weapon states

Figure 7 says there is a 94 percent chance of another nuclear-weapon state by 2093 with a 40 percent chance of at least 1 by 2025.

This image may seem too pessimistic to be credible if we believe that the rate of nuclear proliferation will continue to decline, as suggested by Figures 1 through 6 above. However, that decline was not statistically significant in the `glm` model `fitProlif1` above. This encouraged us to use Bayesian Model Averaging (BMA), which placed more weight on the constant model.

One way to get more optimistic estimates is to pretend that the model `fitProlif1` above has 100 percent posterior probability rather than the mere 21 percent computed with `fitProlif`. We next look at what that

estimates for future nuclear proliferation.

8 Predictions per the linear-only model

We want analogues to Figures 4, 6, and 7, as summarized in the following table.

```
tabOfFigs <- data.frame(Function=
    c('', 'plotNucStatesPred', 'plotNNucStates',
      'plotProbs'),
  BMA=c('Figure (data)', '4 (sumExpMeans)',
        '6 (cumC.PI)', '7 (cumPred)'),
  glm=c('Figure (data)', '8 (sumExpMeans1)',
        '9 (cumC.PI1)', '10 (cumPred1)'))
names(tabOfFigs)[1] <- 'function'
rownames(tabOfFigs) <- c('',
  'Estimated mean time between "first tests"',
  'Number of nuclear-weapon states',
  paste("dist'n of time to",
        'more nuc-weapon states') )
knitr::kable(tabOfFigs)
```

	function	BMA	glm
		Figure (data)	Figure (data)
Estimated mean time between "first tests"	plotNucStatesPred	4 (sumExpMeans)	8 (sumExpMeans1)
Number of nuclear-weapon states	plotNNucStates	6 (cumC.PI)	9 (cumC.PI1)
dist'n of time to more nuc-weapon states	plotProbs	7 (cumPred)	10 (cumPred1)

As before, we use simulations.

```
simMeans1 <- simulate(fitProlif1, nSims, seed=3,
    newdata=pastfut['timeSinceTrinity'], type='response')
```

These numbers are added to Figure 2 to produce Figure 8, comparable to Figure 4, assuming the linear model had 100 percent of the posterior probability, not just 21 percent.

```
sumExpMeans1 <- sumSims(1/simMeans1)


op <- par(mar=c(5,5,4,2))
plotNucStatesPred(sumExpMeans1, line_mtext=c(3, 3))

par(op)
```

This model intuitively seems more consistent with the data than the BMA model. However, to find it plausible, we need to to believe other information we have about nuclear proliferation beyond the estimates of uncertainty documented in the BMA model, `fitProlif`.

We similary modify Figure 6 to produce Figure 9.

```
cumPred1 <- cumsumPred(simMeans1)


str(cumMeans1 <- apply(simMeans1[-(1:nYrs), ],
                        2, cumsum))

## num [1:74, 1:5000] 0.0514 0.1022 0.1525 0.2021 0.2511 ...
## - attr(*, "dimnames")=List of 2
##   ..$ : chr [1:74] "75" "76" "77" "78" ...
```
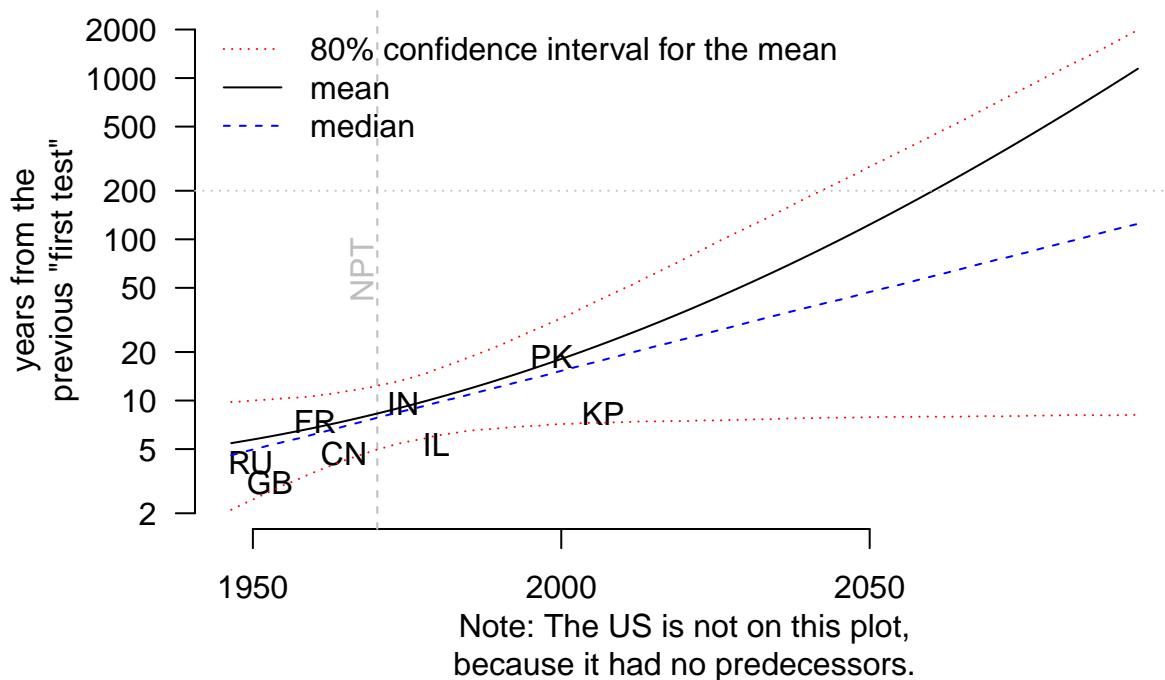
19

Figure 8: Estimated mean time between "first tests" with log-linear model

```
##    ..$ : chr [1:5000] "sim_1" "sim_2" "sim_3" "sim_4" ...
str(cumCI1 <- sumSims(
    nNucStates+rbind(0, cumMeans1),
    pastfut$Year[-(1:(nYrs-1))]))
```

```
## 'data.frame':    75 obs. of  5 variables:
## $ Year  : Date, format: "2019-07-01" "2020-07-01" ...
## $ mean  : num  9 9.06 9.12 9.19 9.25 ...
## $ L10   : num  9 9.01 9.03 9.04 9.05 ...
## $ median: num  9 9.04 9.08 9.12 9.16 ...
## $ U10   : num  9 9.13 9.27 9.4 9.53 ...
```

```
cumC.PI1 <- cumsumC.PI(cumPred1, cumCI1)
plotNNucStates(xfuture=cumC.PI1)
```

We also sumarize the probability distribution of the time to more nuclear-weapon states in Figure 10, similar to Figure 7.

```
probProlif1 <- plotProbs(cumPred1)
```

As expected Figures 8, 9, and 10 provide a more optimistic image than the comparable Figures 4, 6, and 7. The estimated probability of at least one more nuclear-weapon state by the end of the forecast period, 2093, dropped from 94 percent in Figure 7 to 70 percent in Figure 10. Not surprisingly, the probability distributions associated with all aspects of nuclear proliferation are more optimistic with the single `glm` model, `fitProlif1`, than the comparable BMA model, `fitProlif`.

However, to believe the more optimistic analysis, we must use evidence beyond what's in the data.

The safest policy may be to believe that the reality is someplace in between. At any event, it seems unwise to pretend that the future will likely be better than the more optimistic scenario provided by `fitProlif1` and the accompanying Figures 8, 9, and 10 – unless we actually see an international de-nuclearization movement
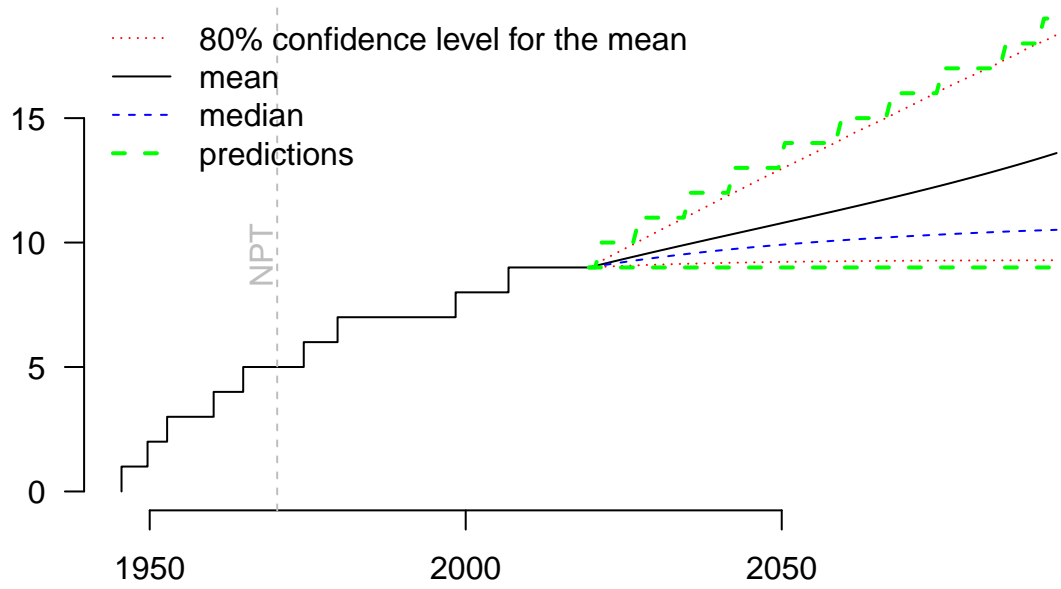
Figure 9: Number of nuclear-weapon states, past and predicted with the glm model
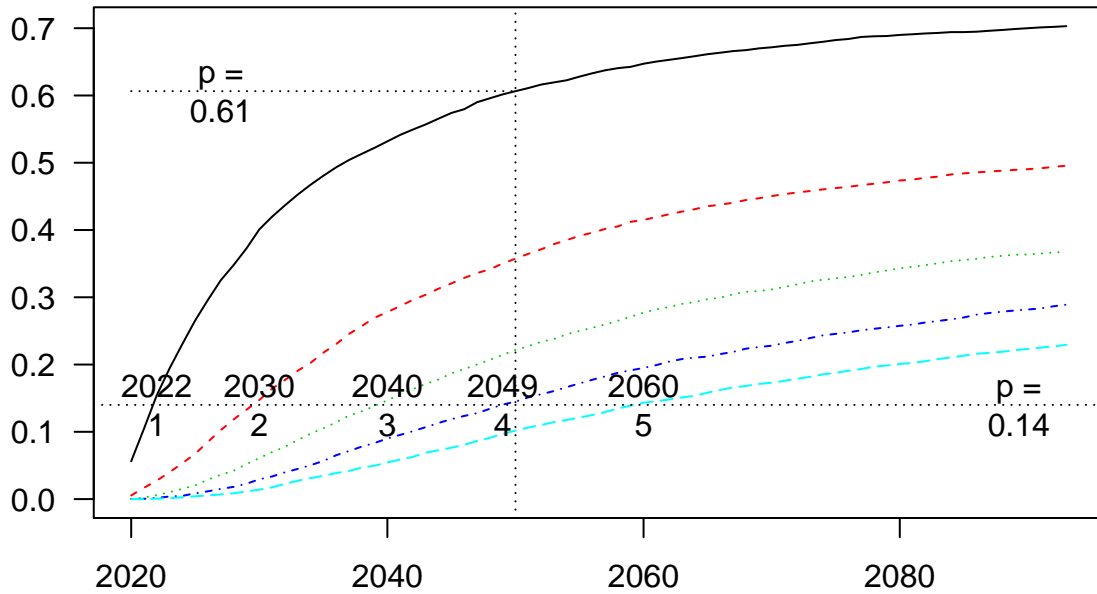

Figure 10: Probability distribution of the time to more nuclear-weapon states per glm model

with momentum far beyond what we currently see with the International Campaign to Abolish Nuclear Weapons (ICAN), the United Nations Treaty on the Prohibition of Nuclear Weapons, and related efforts.

9   Discussion

A growing number of leading figures have said that as long as the world maintains large nuclear arsenals, it is only a matter of time before there is a nuclear war. Concerns like this have been expressed by two former US Secretaries of Defense (Robert McNamara[34] and William Perry), two former US Secretaries of State (Henry Kissinger and George Schultz), former US Senator Sam Nunn,[35] and others with, for example, the Nuclear Threat Initiative. Daniel Ellsberg has said that a nuclear war will most likely generate a nuclear winter that lasts several years during which 98 percent of humanity will starve to death if they do not die of something else sooner.[36] Banerjee and Duflo, two of the three who won the 2019 Nobel Memorial Prize in Economics, have noted that neither economic nor political stability are assured for any country, including the United States, China and India. In particular, they predict that economic growth will almost certainly slow substantailly in the latter two, leaving many poor people in desparate economic straits.[37] Various journalists and academic researchers remind us that President Trump has demonized various different groups and promoted an increase in ethnic violence. He refused in 2016 and has continued to refuse to say if he would accept the results of an election if he lost in 2020.[38] Internal problems in the US, China, India or any other nuclear-weapon state could push political leaders to pursue increasingly risky foreign adventures, like Argentina did in 1982,[39] possibly leading to a war that could produce nuclear Armageddon.

The present work only increases the urgency of limiting the threat of nuclear war and nuclear proliferation in particular. This article has estimated confidence and prediction intervals for the time to the next new nuclear-weapon state. To this end, we built two two models, one being too pessimistic to be credible and the other more optimistic than is justified by the data considered. The optimistic model estimates a 61 percent chance of at least one more nuclear-weapon state by 2050 and a 10 percent chance of 5 or more by that time.

We can expect that some non-nuclear nations and terrorist groups would eagerly pursue nuclear weapons if such seemed feasible unless some unprecidented change in international law provided them with effective judicial recourse to perceived threats.

Moreover, these weapons will become more available with the passage of time unless (a) a nuclear war destroys everyone's ability to make more such weapons for a long time, or (b) an international movement has far more success than similar previous efforts in giving the poor, weak and disfranchised effective nonviolent means for pursuing a redress of grievances.

10   Appendix. Plotting to a file

Plotting the time between "first tests" to a file was surprisingly difficult, because font sizes in the files were smaller than the standard display, and getting larger fonts was surprisingly difficult. Consider the following:

```
if(FALSE){
  png('nuclearProliferation.png', 1440, 1200)
  par(mar=c(15, 12, 4, 2)+.1)
  plot(yearsSinceLastFirstTest~firstTest,
      nuclearWeaponStates, type='h',
      xlab='', ylab='', las=1, cex.axis=4,
```

---

[34]McNamara and Blight (2003)
[35]Shultz, Perry, and Nunn (2019)
[36]Ellsberg, Goodman, and González (2017)
[37]Banerjee and Duflo (2019)
[38]Klaas:2019
[39]("Falklands War" n.d.)

```
    bty='n', axes=FALSE, lwd=2,
    ylim=ylim.)
  xyr <- seq(1950, 2000, 10)
  xt. <- paste0(xyr, '-01-01')
  xticks <- as.Date(xt.)
  axis(1, xticks, labels=FALSE, lwd=2)
  axis(1, xticks, xyr, cex.axis=4, lwd=0, line=2)
#axis(1, seq(1950, 2000, by=10), cex.axis=4, lwd=0, line=1)
  axis(2, cex.axis=4, las=1, lwd=2)
  title('Time between new nuclear nations',
      cex.main=4)
addCountries(line=c(12, 2), cex.=4)
#mtext(paste0(
#'Note: The US is not on this plot,',
#'\nbecause it had no predecessors.'),
#1, 12, cex=4)
#mtext('years from the\nprevious "first test"',
#     2, 5, cex=4)
#  with(nuclearWeaponStates,
#    text(firstTest, yearsSinceLastFirstTest,
#      ctry, xpd=TRUE, cex=4))
  dev.off()
}
```

Plot to an svg file, killing the labels for Wikimedia Commons:

```
if(FALSE){
  svg('nuclearProliferation.svg', 14, 11)
  Label. <- FALSE
  par(mar=c(9, 8, 4, 2)+.1)
  cex. <- 4
  plot(yearsSinceLastFirstTest~firstTest,
      nuclearWeaponStates, type='h',
      xlab='', ylab='', axes=FALSE,
      bty='n', ylim=ylim.)
#  axis(1, cex.axis=cex., line=1)
  axis(1, labels=FALSE)
  axis.Date(1,
    nuclearWeaponStates$firstTest,
    tick=FALSE, cex.axis=cex., line=2)
  axis(2, cex.axis=cex., las=1)
  if(Label.)title(
    'Time between new nuclear nations',
                  cex.main=2)
  if(Label.)addCountries(line=c(6, 4), cex=cex.,
        mtext.=Label.)
#  noUS <- paste0(
#    'Note: The US is not on this plot,',
#    '\nbecause it had no predecessors.')
#  if(Label.)mtext(noUS, 1, 6, cex=cex.)
#  ylab. <- paste(
#    'years from the\nprevious "first test"')
#  if(Label.)mtext(ylab., 2, 4, cex=cex.)
#  with(nuclearWeaponStates,
```

```
#     text(firstTest, yearsSinceLastFirstTest,
#         ctry, xpd=TRUE, cex=cex.))
  dev.off()
}
```

Plot to png, killing the labeling for Wikimedia Commons

```
#ymax <- max(nuclearWeaponStates$yearsSinceLastFirstTest,
#            na.rm=TRUE)
#ylim. <- c(0, ymax)

png. <- FALSE
cex.ax=1
lbl <- TRUE
if(png.){
    png('nuclearProliferation.png', 1440, 1200)
  if(lbl){
#     par(mar=c(15, 15, 7, 2)+.1)
    par(mar=c(15, 15, 2, 2)+.1)
  } else par(mar=c(5, 5, 2, 2)+.1)
    cex.ax=4
}
plotNucStates(type='h', ylim.=ylim0,
    line=3:2, cex.=cex.ax, mtext.=lbl, log.='',
    axes=FALSE)

#plot(yearsSinceLastFirstTest~firstTest,
#     nuclearWeaponStates, type='h',
#     xlab='', ylab='', las=1, cex.axis=cex.ax,
#     bty='n', axes=FALSE, lwd=2, ylim=ylim.)
#if(lbl){
#  title('Figure 1. Time between\nnew nuclear nations',
#     cex.main=cex.ax)
#addCountries(line=c(11, 6), cex=cex.ax)
#  xlab. <- paste(c('Note: The US is not on this plot,',
#                  'because it had no predecessors.'),
#               collapse='\n')
#  mtext(xlab., 1, 11, cex=cex.ax)
#  mtext('years from the\nprevious "first test"',
#        2, 6, cex=cex.ax)
#}
xyr <- seq(1950, 2000, 10)
xt. <- paste0(xyr, '-01-01')
xticks <- as.Date(xt.)
axis(1, xticks, labels=FALSE, lwd=2)
axis(1, xticks, xyr, cex.axis=cex.ax, lwd=0, line=0)
#axis(1, seq(1950, 2000, by=10), cex.axis=4, lwd=0, line=1)
axis(2, cex.axis=cex.ax, las=1, lwd=2)
```

Note: The US is not on this plot,
because it had no predecessors.
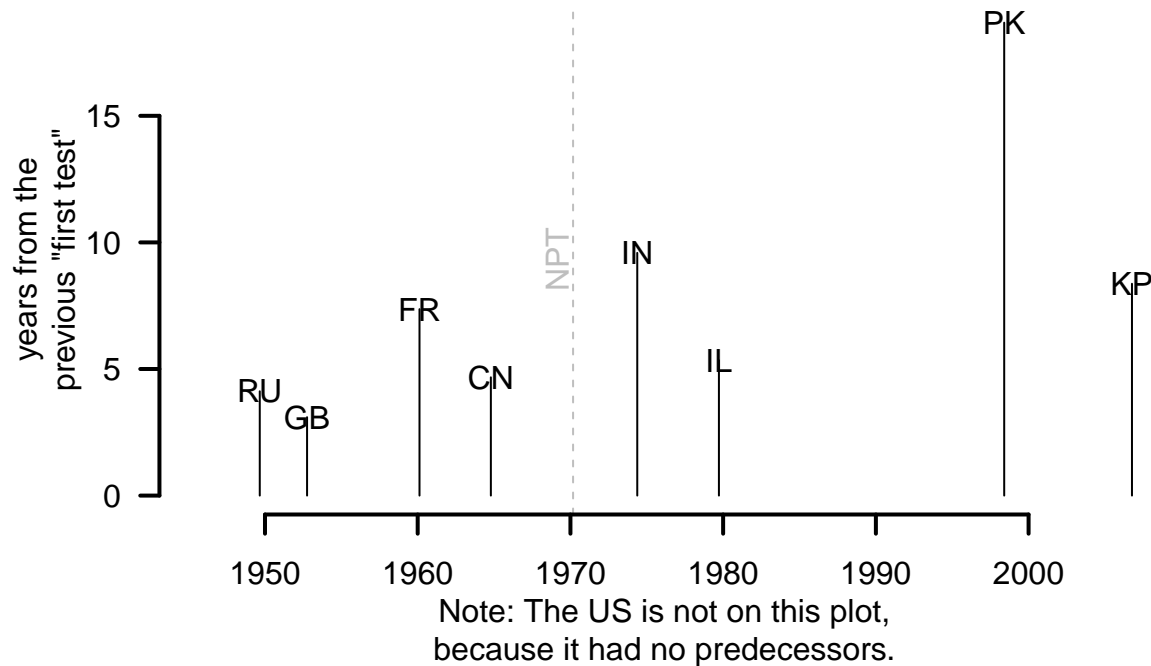
```
#  title('Time between new nuclear nations', cex.main=4)
#xlab. <- paste0('Note: The US is not on this plot,',
#   '\nbecause it had no predecessors.')
# mtext(xlab., 1, 12, cex=4)
#ylab. <- paste0('Note: The US is not on this plot,',
#   '\nbecause it had no predecessors.')
# mtext(ylab., 1, 12, cex=4)
# with(nuclearWeaponStates,
#   text(firstTest, yearsSinceLastFirstTest,
#      ctry, xpd=TRUE, cex=cex.ax))
if(png.) dev.off()
```

11 References

Also, China reportedly wanted a counterweight against the nuclear threat to them posed by India.^[

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