Synthesis of Surveillance Strategies via Belief Abstraction

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***Abstract*— We study the problem of synthesizing a controller for a robot with a *surveillance objective*, that is, the robot is required to maintain knowledge of the location of a moving, possibly adversarial target. We formulate this problem as a one-sided partial-information game in which the winning condition for the agent is specified as a temporal logic formula. The specification formalizes the surveillance requirement given by the user, including additional non-surveillance tasks. In order to synthesize a surveillance strategy that meets the specification, we transform the partial-information game into a perfect-information one, using abstraction to mitigate the exponential blow-up typically incurred by such transformations. This enables the use of off-the-shelf tools for reactive synthesis. We use counterexample-guided refinement to automatically achieve abstraction precision that is sufficient to synthesize a surveillance strategy. We evaluate the proposed method on two case-studies, demonstrating its applicability to large state-spaces and diverse requirements.**

I. INTRODUCTION

Performing surveillance, that is, tracking the location of a target, has many applications. If the target is adversarial, these applications include patrolling and defense, especially in combination with other objectives, such as providing certain services or accomplishing a mission. Techniques for tracking non-adversarial but unpredictable targets have been proposed in settings like surgery to control cameras to keep a patient's organs under observation despite unpredictable motion of occluding obstacles [1]. Mobile robots in airports have also been proposed to carry luggage for clients, requiring the robots to follow the human despite unpredictable motion and possibly sporadically losing sight of the target [2].

When dealing with a possibly adversarial target, a strategy for the surveying agent for achieving its objective can be seen as a strategy in a two-player game between the agent and the target. Since the agent may not always observe, or even know, the exact location of the target, surveillance is, by its very nature, a partial-information problem. It is thus natural to reduce surveillance strategy synthesis to computing a winning strategy for the agent in a two-player partial-information game. Game-based models for related problems have been extensively studied in the literature. Notable examples include pursuit-evasion games [3], patrolling games [4], and graph-searching games [5], where the problem is formulated as enforcing eventual detection, which is, in its essence a search problem – once the target is detected, the game ends. For many applications, this formulation is too restrictive. Often, the goal is not to detect or capture the target, but to maintain certain level of information about its location over an unbounded (or infinite) time duration, or, alternatively, be able to obtain sufficiently precise information over and over again. In other cases, the agent has an additional objective, such as performing certain task, which might prevent him from capturing the target, but allow for satisfying a more relaxed surveillance objective.

In this paper, we study the problem of synthesizing strategies for enforcing *temporal surveillance objectives*, such as the requirement to never let the agent's uncertainty about the target's location exceed a given threshold, or recapturing the target every time it escapes. To this end, we consider surveillance objectives specified in linear temporal logic (LTL), equipped with basic surveillance predicates. This formulation also allows for a seamless combination with other task specifications. Our computational model is that of a two-player game played on a finite graph, whose nodes represent the possible locations of the agent and the target, and whose edges model the possible (deterministic) moves between locations. The agent plays the game with partial information, as it can only observe the target when it is in it's area of sight. The target, on the other hand, always has full information about the agent's location, even when the agent is not in sight. In that way, we consider a model with one-sided partial information, making the computed strategy for the agent robust against a potentially more powerful adversary.

We formulate surveillance strategy synthesis as the problem of computing a winning strategy for the agent in a partial-information game with a surveillance objective. There is a rich theory on partial-information games with LTL objectives [6], [7], and it is well known that even for very simple objectives the synthesis problem is EXPTIME-hard [8], [9]. Moreover, all the standard algorithmic solutions to the problem are based on some form of *belief set construction*, which transforms the imperfect-information game into a perfect-information game and this may be exponentially larger, since the new set of states is the powerset of the original one. Thus, such approaches scale poorly in general, and are not applicable in most practical situations.

We address this problem by using *abstraction*. We introduce an *abstract belief set construction*, which underapproximates the information-tracking abilities of the agent (or, alternatively, overapproximates its belief, i.e., the set of positions it knows the target could be in). We leverage this construction by *reasoning* over the agent's belief in the target location, and this allows us to specify surveillance objectives in LTL over these belief states. Thus, we provide a framework to treat surveillance synthesis as a two-player perfect-information game with an LTL objective, which we then solve using off-the shelf reactive synthesis tools [10].

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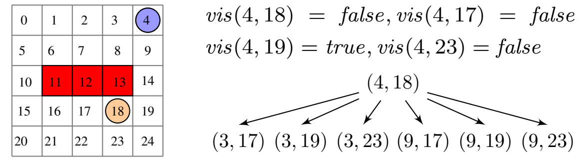
Our construction guarantees that the abstraction is sound, that is, if a surveillance strategy is found in the abstract game, it corresponds to a surveillance strategy for the original game. If, on the other hand, such a strategy is not found, then the method automatically checks if this is due to the coarseness of the abstraction, in which case the abstract belief space is automatically refined. Thus, our method follows the general counterexample guided abstraction refinement (CEGAR) [11] scheme, which has successfully demonstrated its potential in formal verification and reactive synthesis.

Contributions. The main contribution of this paper is the construction of a partial-information game framework that allows the formal specification of surveillance objectives, as well as providing an abstraction method to efficiently synthesize a strategy that will guarantee that the surveillance objectives will be satisfied. Formally, our contributions are as follows:
(1) We propose a *formalization of surveillance objectives* as temporal logic specifications, and frame surveillance strategy synthesis as a partial-information reactive synthesis problem.
(2) We develop an *abstraction method that soundly approximates* surveillance strategy synthesis, thus enabling the application of efficient techniques for reactive synthesis.
(3) We design procedures that *automatically refine a given abstraction* in order to improve its precision when no surveillance strategy exists due to coarseness of the approximation.
(4) We evaluate our approach on different surveillance objectives (e.g, safety, and liveness) combined with task specifications, and discuss the qualitatively different behaviour of the synthesized strategies for the different kinds of specifications.

Related work. While closely related to the surveillance problem we consider, pursuit-evasion games with partial information [3], [12], [13] formulate the problem as eventual detection, and do not consider combinations with other mission specifications. Other work, such as [14] and [15], additionally incorporates map building during pursuit in an unknown environment, but again solely for target detection.

Synthesis from LTL specifications [16], especially from formulae in the efficient GR(1) fragment [17], has been extensively used in robotic planning (e.g. [18], [19]), but surveillance-type objectives, such as the ones we study here, have not been considered so far. Epistemic logic specifications [20] can refer to the knowledge of the agent about the truth-value of logical formulas, but, contrary to our surveillance specifications, are not capable of expressing requirements on the size of the agent's uncertainty.

CEGAR has been developed for verification [11], and later for control [21], of LTL specifications. It has also been extended to infinite-state partial-information games [22], and used for sensor design [23], both in the context of safety specifications. In addition to being focused on safety objectives, the refinement method in [22] is designed to provide the agent with just enough information to achieve safety, and is thus not applicable to surveillance properties whose satisfaction depends on the size of the belief sets.



(a) Surveillance arena (b) Transitions from the initial state

Fig. 1: A simple surveillance game on a grid arena. Obstacles are shown in red, the agent (at location 4) and the target (at location 18) are coloured in blue and orange respectively.

II. GAMES WITH SURVEILLANCE OBJECTIVES

We begin by defining a formal model for describing surveillance strategy synthesis problems, in the form of a two-player game between an agent and a target, in which the agent has only partial information about the target's location.

A. Surveillance Game Structures

We define a *surveillance game structure* to be a tuple $G = (S, s^{\mathsf{init}}, T, vis)$, with the following components:

- $S = L_a \times L_t$ is the set of states, with L_a the set of locations of the agent, and L_t the locations of the target;
- $s^{\mathsf{init}} = (l_a^{\mathsf{init}}, l_t^{\mathsf{init}})$ is the initial state;
- $T \subseteq S \times S$ is the transition relation describing the possible moves of the agent and the target; and
- $vis : S \to \mathbb{B}$ is a function that maps a state (l_a, l_t) to $true$ iff *position* l_t *is in the area of sight of* l_a.

The transition relation T encodes the one-step move of both the target and the agent, where the target moves first and the agent moves second. For a state (l_a, l_t) we denote $succ_t(l_a, l_t)$ as the set of successor locations of the target:
$succ_t(l_a, l_t) = \{l_t' \in L_t \mid \exists l_a'.\ ((l_a, l_t), (l_a', l_t')) \in T\}$.
We extend $succ_t$ to sets of locations of the target by stipulating that the set $succ_t(l_a, L)$ consists of all possible successor locations of the target for states in $\{l_a\} \times L$. Formally, let $succ_t(l_a, L) = \bigcup_{l_t \in L} succ_t(l_a, l_t)$.

For a state (l_a, l_t) and a successor location of the target l_t', we denote with $succ_a(l_a, l_t, l_t')$ the set of successor locations of the agent, given that the target moves to l_t':
$succ_a(l_a, l_t, l_t') = \{l_a' \in L_a \mid ((l_a, l_t), (l_a', l_t')) \in T\}$.
We assume that, for every $s \in S$, there exists $s' \in S$ such that $(s, s') \in T$, that is, from every state there is at least one move possible (this might be staying in the same state). We also assume that when the target moves to an invisible location, its position does not influence the possible one-step moves of the agent. Formally, we require that if $vis(l_a, l_t''') = vis(l_a, l_t'''') = false$, then $succ_a(l_a, l_t', l_t''') = succ_a(l_a, l_t'', l_t'''')$ for all $l_t', l_t'', l_t''', l_t'''' \in L_t$. This assumption is natural in the setting when the agent can move in one step only to locations that are in its sight.

Example 1: Figure 1 shows an example of a surveillance game on a grid. The sets of possible locations L_a and L_t for the agent and the target consist of the squares of the grid. The transition relation T encodes the possible one-step moves of both the agent and the target on the grid, and incorporates all desired constraints. For example, moving to an occupied location, or an obstacle, is not allowed. Figure 1b shows the possible transitions from the initial state $(4, 18)$.

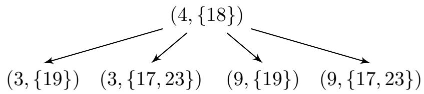


Fig. 2: Transitions from the initial state in the belief-set game from Example 2 where $vis(4, 17) = vis(4, 23) = false$.

The function vis encodes straight-line visibility: a location l_t is visible from a location l_a if there is no obstacle on the straight line between them. Initially the target is not in the area of sight of the agent, but the agent knows the initial position of the target. However, once the target moves to one of the locations reachable in one step, in this case, locations $\{17, 19, 23\}$, this might no longer be the case. More precisely, if the target moves to location 19, then the agent observes its location, but if it moves to one of the others, then the agent no longer knows its exact location. ■

B. Belief-Set Game Structures

In surveillance strategy synthesis we need to state properties of, and reason about, the information which the agent has, i.e. its *belief* about the location of the target. To this end, we can employ a powerset construction which is commonly used to transform a partial-information game into a perfect-information one, by explicitly tracking the knowledge one player has as a set of possible states of the other player.

Given a set B, we denote with $\mathcal{P}(B) = \{B' \mid B' \subseteq B\}$ the powerset (set of all subsets) of B.

For a surveillance game structure $G = (S, s^{\mathsf{init}}, T, vis)$ we define the corresponding *belief-set game structure* $G_{\mathsf{belief}} = (S_{\mathsf{belief}}, s^{\mathsf{init}}_{\mathsf{belief}}, T_{\mathsf{belief}})$ with the following components:

- $S_{\mathsf{belief}} = L_a \times \mathcal{P}(L_t)$ is the set of states, with L_a the set of locations of the agent, and $\mathcal{P}(L_t)$ the set of *belief sets* describing information about the location of the target;
- $s^{\mathsf{init}}_{\mathsf{belief}} = (l^{\mathsf{init}}_a, \{l^{\mathsf{init}}_t\})$ is the initial state;
- $T_{\mathsf{belief}} \subseteq S_{\mathsf{belief}} \times S_{\mathsf{belief}}$ is the transition relation where $((l_a, B_t), (l'_a, B'_t)) \in T_{\mathsf{belief}}$ iff $l'_a \in succ_a(l_a, l_t, l'_t)$ for some $l_t \in B_t$ and $l'_t \in B'_t$ and one of these holds:
  (1) $B'_t = \{l'_t\}$, $l'_t \in succ_t(l_a, B_t)$, $vis(l_a, l'_t) = true$;
  (2) $B'_t = \{l'_t \in succ_t(l_a, B_t) \mid vis(l_a, l'_t) = false\}$.

Condition (1) captures the successor locations of the target that can be observed from the agent's current position l_a. Condition (2) corresponds to the belief set consisting of *all possible successor locations of the target not visible from* l_a.

Example 2: Consider the surveillance game structure from Example 1. The initial belief set is $\{18\}$, consisting of the target's initial position. After the first move of the target, there are two possible belief sets: the set $\{19\}$ resulting from the move to a location in the area of sight of the agent, and $\{17, 23\}$ consisting of the two invisible locations reachable in one step from location 18. Figure 2 shows the successor states of the initial state $(4, \{18\})$ in G_{belief}. ■

Based on T_{belief}, we can define the functions $succ_t : S_{\mathsf{belief}} \to \mathcal{P}(\mathcal{P}(L_t))$ and $succ_a : S_{\mathsf{belief}} \times \mathcal{P}(L_t) \to \mathcal{P}(L_a)$ similarly to the corresponding functions defined for G.

A *run* in G_{belief} is an infinite sequence $s_0, s_1, \ldots$ of states in S_{belief}, where $s_0 = s^{\mathsf{init}}_{\mathsf{belief}}$, $(s_i, s_{i+1}) \in T_{\mathsf{belief}}$ for all i.

A *strategy for the target in* G_{belief} is a function $f_t : S^+_{\mathsf{belief}} \to \mathcal{P}(L_t)$ such that $f_t(\pi \cdot s) = B_t$ implies $B_t \in succ_t(s)$ for every $\pi \in S^*_{\mathsf{belief}}$ and $s \in S_{\mathsf{belief}}$. That is, a strategy for the target suggests a move resulting in some belief set reachable from some location in the current belief.

A *strategy for the agent in* G_{belief} is a function $f_a : S^+ \times \mathcal{P}(L_t) \to S$ such that $f_a(\pi \cdot s, B_t) = (l'_a, B'_t)$ implies $B'_t = B_t$ and $l'_a \in succ_a(s, B_t)$ for every $\pi \in S^*_{\mathsf{belief}}$, $s \in S_{\mathsf{belief}}$ and $B_t \in \mathcal{P}(L_t)$. Intuitively, a strategy for the agent suggests a move based on the observed history of the play and the current belief about the target's position.

The outcome of given strategies f_a and f_t for the agent and the target in G_{belief}, denoted $outcome(G_{\mathsf{belief}}, f_a, f_t)$, is a run $s_0, s_1, \ldots$ of G_{belief} such that for every $i \geq 0$, we have $s_{i+1} = f_a(s_0, \ldots, s_i, B^i_t)$, where $B^i_t = f_t(s_0, \ldots, s_i)$.

C. Temporal Surveillance Objectives

Since the states of a belief-set game structure track the information that the agent has, we can state and interpret surveillance objectives over its runs. We now formally define the surveillance properties in which we are interested.

We consider a set of *surveillance predicates* $\mathcal{SP} = \{p_k \mid k \in \mathbb{N}_{>0}\}$, where for $k \in \mathbb{N}_{>0}$ we say that a state (l_a, B_t) in the belief game structure satisfies p_k (denoted $(l_a, B_t) \models p_k$) iff $|\{l_t \in B_t \mid vis(l_a, l_t) = false\}| \leq k$. Intuitively, p_k is satisfied by the states in the belief game structure where the size of the belief set does not exceed the threshold $k \in \mathbb{N}_{>0}$.

We study surveillance objectives expressed by formulas of linear temporal logic (LTL) over surveillance predicates. The LTL surveillance formulas are generated by the grammar
$\varphi := p \mid true \mid false \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \bigcirc \varphi \mid \varphi \,\mathcal{U}\, \varphi \mid \varphi \,\mathcal{R}\, \varphi$,
where $p \in \mathcal{SP}$ is a surveillance predicate, $\bigcirc$ is the *next* operator, $\mathcal{U}$ is the *until* operator, and $\mathcal{R}$ is the *release* operator. We also define the derived operators *finally*: $\Diamond \varphi = true \,\mathcal{U}\, \varphi$ and *globally*: $\Box \varphi = false \,\mathcal{R}\, \varphi$.

LTL formulas are interpreted over (infinite) runs. If a run ρ satisfies an LTL formula φ, we write $\rho \models \varphi$. The formal definition of LTL semantics can be found in [24]. Here we informally explain the meaning of the formulas we use.

Of special interest will be surveillance formulas of the form $\Box p_k$, termed *safety surveillance objective*, and $\Box\Diamond p_k$, called *liveness surveillance objective*. Intuitively, the safety surveillance formula $\Box p_k$ is satisfied if at each point in time the size of the belief set does not exceed k. The liveness surveillance objective $\Box\Diamond p_k$, on the other hand, requires that infinitely often this size is below or equal to k.

Example 3: We can specify that the agent is required to always know with certainty the location of the target as $\Box p_1$. A more relaxed requirement is that the agent's uncertainty never grows above 5 locations, and it infinitely often reduces this uncertainty to at most 2 locations: $\Box p_5 \wedge \Box\Diamond p_2$. ■

D. Incorporating Task Specifications

We can integrate LTL objectives not related to surveillance, i.e., *task specifications*, by considering, in addition to $\mathcal{SP}$, a set $\mathcal{AP}$ of atomic predicates interpreted over states of G. In order to define the semantics of $p \in \mathcal{AP}$ over states of G_{belief}, we restrict ourselves to predicates observable by the agent. Formally, we require that for $p \in \mathcal{AP}$, and states (l_a, l'_t) and (l_a, l''_t) with $vis(l_a, l'_t) = vis(l_a, l''_t) = false$ it holds that $(l_a, l'_t) \models p$ iff $(l_a, l''_t) \models p$. One class of such predicates are those that depend only on the agent's position.

Example 4: Suppose that at_goal is a predicate true exactly when the agent is at some designated goal location. We can then state that the agent visits the goal infinitely often while always maintaining belief uncertainty of at most 10 locations using the LTL formula $\square\diamondsuit\, at\_goal \wedge \square p_{10}$. ■

E. Surveillance Synthesis Problem

A *surveillance game* is a pair (G, φ), where G is a surveillance game structure and φ is a surveillance objective. A *winning strategy for the agent for* (G, φ) is a strategy f_a for the agent in the corresponding belief-set game structure G_{belief} such that for every strategy f_t for the target in G_{belief} it holds that $outcome(G_{\mathsf{belief}}, f_a, f_t) \models \varphi$. Analogously, a *winning strategy for the target for* (G, φ) is a strategy f_t such that, for every strategy f_a for the agent in G_{belief}, it holds that $outcome(G_{\mathsf{belief}}, f_a, f_t) \not\models \varphi$.

Surveillance synthesis problem: Given a surveillance game (G, φ), compute a winning strategy for the agent for (G, φ), or determine that such a strategy does not exist.

It is well-known that two-player perfect-information games with LTL objectives over finite-state game structures are determined, that is exactly one of the players has a winning strategy. This means that the agent does not have a winning strategy for a given surveillance game, if and only if the target has a winning strategy for this game. We refer to winning strategies of the target as *counterexamples*.

III. BELIEF SET ABSTRACTION

We used the belief-set game structure in order to state the surveillance objective of the agent. While in principle it is possible to solve the surveillance strategy synthesis problem on this game, this is in most cases computationally infeasible, since the size of this game is exponential in the size of the original game. To circumvent this construction when possible, we propose an abstraction-based method, that given a surveillance game structure and a partition of the set of the target's locations, yields an approximation that is sound with respect to surveillance objectives for the agent.

An *abstraction partition* is a family $\mathcal{Q} = \{Q_i\}_{i=1}^n$ of subsets of L_t, $Q_i \subseteq L_t$ such that the following hold:

- $\bigcup_{i=1}^n Q_i = L_t$ and $Q_i \cap Q_j = \emptyset$ for all $i \neq j$;
- For each $p \in \mathcal{AP}$, $Q \in \mathcal{Q}$ and $l_a \in L_a$, it holds that $(l_a, l'_t) \models p$ iff $(l_a, l''_t) \models p$ for every $l'_t, l''_t \in Q$.

Intuitively, these conditions mean that $\mathcal{Q}$ partitions the set of locations of the target, and the concrete locations in each of the sets in $\mathcal{Q}$ agree on the value of the propositions in $\mathcal{AP}$.

If $\mathcal{Q}' = \{Q'_i\}_{i=1}^m$ is a family of subsets of L_t such that $\bigcup_{i=1}^m Q'_i = L_t$ and for each $Q'_i \in \mathcal{Q}'$ there exists $Q_j \in \mathcal{Q}$ such that $Q'_i \subseteq Q_j$, then $\mathcal{Q}'$ is also an abstraction partition, and we say that $\mathcal{Q}'$ *refines* $\mathcal{Q}$, denoted $\mathcal{Q}' \preceq \mathcal{Q}$.

For $\mathcal{Q} = \{Q_i\}_{i=1}^n$, we define a function $\alpha_{\mathcal{Q}} : L_t \to \mathcal{Q}$ by $\alpha(l_t) = Q$ for the unique $Q \in \mathcal{Q}$ with $l_t \in Q$. We denote also

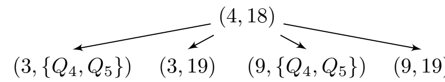


Fig. 3: Transitions from the initial state in the abstract game from Example 5 where $\alpha_{\mathcal{Q}}(17) = Q_4$ and $\alpha_{\mathcal{Q}}(23) = Q_5$.

with $\alpha_{\mathcal{Q}} : \mathcal{P}(L_t) \to \mathcal{P}(\mathcal{Q})$ the *abstraction function* defined by $\alpha_{\mathcal{Q}}(L) = \{\alpha_{\mathcal{Q}}(l) \mid l \in L\}$. We define a *concretization function* $\gamma : \mathcal{P}(\mathcal{Q}) \cup L_t \to \mathcal{P}(L_t)$ such that $\gamma(A) = \{l_t\}$ if $A = l_t \in L_t$, and $\gamma(A) = \bigcup_{Q \in A} Q$ if $A \in \mathcal{P}(\mathcal{Q})$.

Given a surveillance game structure $G = (S, s^{\mathsf{init}}, T, vis)$ and an abstraction partition $\mathcal{Q} = \{Q_i\}_{i=1}^n$ of the set L_t, we define the *abstraction of* G *w.r.t.* $\mathcal{Q}$ to be the game structure $G_{\mathsf{abstract}} = \alpha_{\mathcal{Q}}(G) = (S_{\mathsf{abstract}}, s^{\mathsf{init}}_{\mathsf{abstract}}, T_{\mathsf{abstract}})$, where

- $S_{\mathsf{abstract}} = (L_a \times \mathcal{P}(\mathcal{Q})) \cup (L_a \times L_t)$ is the set of *abstract states*, consisting of states approximating the belief sets in the game structure G_{belief}, as well as the states S;
- $s^{\mathsf{init}}_{\mathsf{abstract}} = (l^{\mathsf{init}}_a, l^{\mathsf{init}}_t)$ is the *initial abstract state*;
- $T_{\mathsf{abstract}} \subseteq S_{\mathsf{abstract}} \times S_{\mathsf{abstract}}$ is the transition relation such that $((l_a, A_t), (l'_a, A'_t)) \in T_{\mathsf{abstract}}$ if and only if one of the following two conditions is satisfied:
  (1) $A'_t = l'_t$, $l'_t \in succ_t(\gamma(A_t))$ and $vis(l_a, l'_t) = true$, and $l'_a \in succ_a(l_a, l_t, l'_t)$ for some $l_t \in \gamma(A_t)$.
  (2) $A'_t = \alpha_{\mathcal{Q}}(\{l'_t \in succ_t(\gamma(A_t)) | vis(l_a, l'_t) = false\})$, and $l'_a \in succ_a(l_a, l_t, l'_t)$ for some $l_t \in \gamma(A_t)$ and some $l'_t \in succ_t(\gamma(A_t))$ with $vis(l_a, l'_t) = false$.

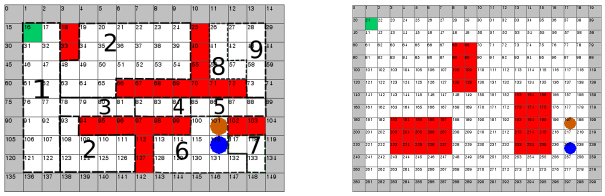
As for the belief-set game structure, the first condition captures the successor locations of the target, which can be observed from the agent's current location l_a. Condition (2) corresponds to the *abstract belief set* whose concretization consists of all possible successors of all positions in $\gamma(A_t)$, which are not visible from l_a. Since the belief abstraction overapproximates the agent's belief, that is, $\gamma(\alpha_{\mathcal{Q}}(B)) \supseteq B$, the next-state abstract belief $\gamma(A'_t)$ may include positions in L_t that are not successors of positions in $\gamma(A_t)$.

Example 5: Consider again the surveillance game from Example 1, and the abstraction partition $\mathcal{Q} = \{Q_1, \ldots, Q_5\}$, where the set Q_i corresponds to the i-th row of the grid. For location 17 of the target we have $\alpha_{\mathcal{Q}}(17) = Q_4$, and for 23 we have $\alpha_{\mathcal{Q}}(23) = Q_5$. Thus, the belief set $B = \{17, 23\}$ is covered by the abstract belief set $\alpha_Q(B) = \{Q_4, Q_5\}$. Figure 3 shows the successors of the initial state $(4, 18)$ of the abstract belief-set game structure. The concretization of the abstract belief set $\{Q_4, Q_5\}$ is the set $\{15, 16, 17, 18, 19, 20, 21, 22, 23, 24\}$ of target locations. ■

An abstract state (l_a, A_t) *satisfies a surveillance predicate* p_k, denoted $(l_a, A_t) \models p_k$, iff $|\{l_t \in \gamma(A_t) \mid vis(l_a, l_t) = false\}| \leq k$. Simply, the number of states in the concretized belief set has to be less than or equal to k. Similarly, for a predicate $p \in \mathcal{AP}$, we define $(l_a, A_t) \models p$ iff for every $l_t \in \gamma(A_t)$ it holds that $(l_a, l_t) \models p$. With these definitions, we can interpret surveillance objectives over runs of G_{abstract}.

Strategies (and wining strategies) for the agent and the target in an abstract belief-set game $(\alpha_{\mathcal{Q}}(G), \varphi)$ are defined analogously to strategies (and winning strategies) in G_{belief}.

In the construction of the abstract game structure, we overapproximate the belief-set of the agent at each step.



(a) Gridworld with an automatically computed partition of size 9.

(b) Gridworld of size 15x20 representing an outdoor environment.

Fig. 4: Gridworlds for the case studies reported in Section IV.

Since we consider surveillance predicates that impose upper bounds on the size of the belief, such an abstraction gives more power to the target (and, dually less power to the agent). This construction guarantees that the abstraction is *sound*, meaning that an abstract strategy for the agent that achieves a surveillance objective corresponds to a winning strategy in the concrete game. This is stated in the following theorem.

Theorem 1: Let G be a surveillance game structure, $\mathcal{Q} = \{Q_i\}_{i=1}^n$ be an abstraction partition, and $G_{\mathsf{abstract}} = \alpha_{\mathcal{Q}}(G)$. For every surveillance objective φ, if there exists a winning strategy for the agent in the abstract belief-set game $(\alpha_{\mathcal{Q}}(G), \varphi)$, then there exists a winning strategy for the agent in the concrete surveillance game (G, φ).

a) Choosing an abstraction partition::

IV. EXPERIMENTAL EVALUATION

We report on the application of our method for surveillance synthesis to two case studies. We have implemented the abstraction-refinement loop in `Python`, using the `slugs` reactive synthesis tool [10]. The experiments were performed on an Intel i5-5300U 2.30 GHz CPU with 8 GB of RAM.

A. Liveness surveillance specification + task specification

Figure 4a shows a gridworld divided into 'rooms'. The surveillance objective requires the agent to infinitely often know precisely the location of the target (either see it, or have a belief consisting of one cell). Additionally, it has to perform the task of patrolling (visiting infinitely often) the green 'goal' cell. Formally, the specification is $\square\diamondsuit p_1 \wedge \square\diamondsuit goal$. The agent can move between 1 and 3 cells at a time.

Starting with an abstract game with 104 states generated by a partition with four elements, our refinement algorithm terminates after 5 iterations (with total running time of 821 s). The resulting partition $\mathcal{Q} = \{Q_1, ..., Q_9\}$ has 9 elements shown as the numbered regions in Figure 4a. Thus, the final refined abstract game has 616 abstract states (2^9 abstract belief states). In contrast, the belief-set game structure would have in the order of 2^{100} states, which is a state-space size that state-of-the-art synthesis tools cannot handle.

A video simulation can be found at `http://goo.gl/YkFuxr`. Note the behaviour of the agent, visiting the goal and then searching for the target. This contrasts with the behaviour under safety surveillance which we look at next.

B. Safety surveillance specification + task specification

Figure 4b depicts a gridworld of an 'outdoor' environment where the red blocks model buildings. In this setting, we enforce the safety surveillance objective $\square p_{30}$ (the belief size should never exceed 30) in addition to infinitely often reaching the green cell. The formal specification is $\square\diamondsuit p_{30} \wedge \square\diamondsuit goal$. Additionally, the target itself is trying to reach the goal cell infinitely often as well, which is known to the agent.

We used an abstraction generated by a partition of size 6, which was sufficiently precise to compute a surveillance strategy in 210 s. This demonstrates that even for larger grids, a coarse abstraction can be sufficient. Again, note that the precise belief-set game would have in the order of 2^{200} states.

We simulated the environment and the synthesized surveillance strategy for the agent in ROS. A video of the simulation can be found at `http://goo.gl/LyC1gQ`. Note the qualitative difference in behaviour compared to the previous example. There, in the case of liveness surveillance, the agent had more leeway to completely lose the target in order to reach its goal location, even though the requirement of reducing the size of the belief to 1 is quite strict. Here, on the other hand, the safety surveillance objective, even with a large threshold of 30, forces the agent to follow the target more closely, in order to prevent its belief from getting too large. The synthesis algorithm thus provides the ability to obtain qualitatively different behaviour as necessary for specific applications by combining different objectives.

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