Distributed Synthesis of Surveillance Strategies

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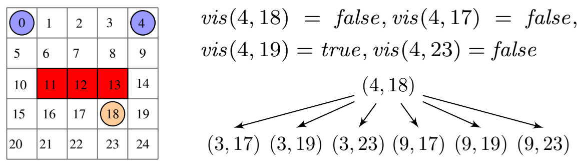

(a) Surveillance arena (b) Transitions from the initial state

Fig. 1: A simple surveillance game on a grid arena. Obstacles are shown in red, the sensors (at locations 0 and 4) and the target (at location 18) are coloured in blue and orange respectively.

***Abstract*—**

I. INTRODUCTION

II. GAMES WITH SURVEILLANCE OBJECTIVES

We begin by defining a formal model for describing multi-agent surveillance strategy synthesis problems, in the form of a two-player game between the mobile sensors and a target, in which the sensors have only partial information about the target's location.

A. Surveillance Game Structures

We define a *multi-agent surveillance game structure* to be a tuple $G = (S, s^{\mathsf{init}}, T, \mathit{Vis})$, with the following components:

- $S = L^a_1 \times L^a_2 \dots L^a_n \times L^t$ is the set of states, with L^a_i the set of locations of sensor i, and L^t the locations of the target;
- $s^{\mathsf{init}} = (l^{\mathsf{init}}_1 \times \dots l^{\mathsf{init}}_n, l^{\mathsf{init}}_t)$ is the initial state;
- $T \subseteq S \times S$ is the transition relation describing the possible joint moves of the sensors and the target; and
- given n *visibility functions* $(\mathit{vis}_1, \dots, \mathit{vis}_n)$ where $\mathit{vis}_i : L^a_i \times L^t \to \mathbb{B}$ is a function that maps a state (l_i, l_t) to true iff *position* l_t *is in the area of sight of* l_i, $\mathit{Vis} = S \to \mathbb{B}$ is a function that maps to true if the set $I = \{i : \mathit{vis}_i(l^a_i, l^t) = \mathit{true}\}$ is non-empty. Simply, Vis outputs true if the target is in view with *at least one* of the sensors.

The transition relation T encodes the one-step move of both the target and the agent, where the target moves first and the agent moves second. For a state (l_a, l_t) we denote $\mathit{succ}_t(l_a, l_t)$ as the set of successor locations of the target:
$\mathit{succ}_t(l_a, l_t) = \{l'_t \in L_t \mid \exists l'_a.\ ((l_a, l_t), (l'_a, l'_t)) \in T\}$.

We extend succ_t to sets of locations of the target by stipulating that the set $\mathit{succ}_t(l_a, L)$ consists of all possible successor locations of the target for states in $\{l_a\} \times L$. Formally, let $\mathit{succ}_t(l_a, L) = \bigcup_{l_t \in L} \mathit{succ}_t(l_a, l_t)$.

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For a state (l_a, l_t) and a successor location of the target l'_t, we denote with $\mathit{succ}_a(l_a, l_t, l'_t)$ the set of successor locations of the agent, given that the target moves to l'_t:
$\mathit{succ}_a(l_a, l_t, l'_t) = \{l'_a \in L_a \mid ((l_a, l_t), (l'_a, l'_t)) \in T\}$.

We assume that, for every $s \in S$, there exists $s' \in S$ such that $(s, s') \in T$, that is, from every state there is at least one move possible (this might be staying in the same state). We also assume that when the target moves to an invisible location, its position does not influence the possible one-step moves of the agent. Formally, we require that if $\mathit{vis}(l_a, l'''_t) = \mathit{vis}(l_a, l''''_t) = \mathit{false}$, then $\mathit{succ}_a(l_a, l'_t, l'''_t) = \mathit{succ}_a(l_a, l''_t, l''''_t)$ for all $l'_t, l''_t, l'''_t, l''''_t \in L_t$. This assumption is natural in the setting when the agent can move in one step only to locations that are in its sight.

Example 1: Figure 1 shows an example of a surveillance game on a grid. The sets of possible locations L_a and L_t for the agent and the target consist of the squares of the grid. The transition relation T encodes the possible one-step moves of both the agent and the target on the grid, and incorporates all desired constraints. For example, moving to an occupied location, or an obstacle, is not allowed. Figure 1b shows the possible transitions from the initial state $(4, 18)$.

The function vis encodes straight-line visibility: a location l_t is visible from a location l_a if there is no obstacle on the straight line between them. Initially the target is not in the area of sight of the agent, but the agent knows the initial position of the target. However, once the target moves to one of the locations reachable in one step, in this case, locations $\{17, 19, 23\}$, this might no longer be the case. More precisely, if the target moves to location 19, then the agent observes its location, but if it moves to one of the others, then the agent no longer knows its exact location. ■

B. Belief-Set Game Structures

In surveillance strategy synthesis we need to state properties of, and reason about, the information which the agent has, i.e. its *belief* about the location of the target. To this end, we can employ a powerset construction which is commonly used to transform a partial-information game into a perfect-information one, by explicitly tracking the knowledge one player has as a set of possible states of the other player.

Given a set B, we denote with $\mathcal{P}(B) = \{B' \mid B' \subseteq B\}$ the powerset (set of all subsets) of B.

For a surveillance game structure $G = (S, s^{\mathsf{init}}, T, \mathit{vis})$ we define the corresponding *belief-set game structure* $G_{\mathsf{belief}} = (S_{\mathsf{belief}}, s^{\mathsf{init}}_{\mathsf{belief}}, T_{\mathsf{belief}})$ with the following components:

- $S_{\mathsf{belief}} = L_a \times \mathcal{P}(L_t)$ is the set of states, with L_a the set of locations of the agent, and $\mathcal{P}(L_t)$ the set of *belief sets* describing information about the location of the target;

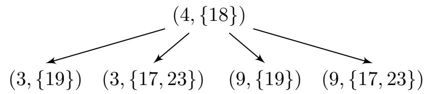

Fig. 2: Transitions from the initial state in the belief-set game from Example 2 where $vis(4, 17) = vis(4, 23) = \mathit{false}$.

- $s^{\mathsf{init}}_{\mathsf{belief}} = (l^{\mathsf{init}}_a, \{l^{\mathsf{init}}_t\})$ is the initial state;
- $T_{\mathsf{belief}} \subseteq S_{\mathsf{belief}} \times S_{\mathsf{belief}}$ is the transition relation where $((l_a, B_t), (l'_a, B'_t)) \in T_{\mathsf{belief}}$ iff $l'_a \in succ_a(l_a, l_t, l'_t)$ for some $l_t \in B_t$ and $l'_t \in B'_t$ and one of these holds:

  (1) $B'_t = \{l'_t\}$, $l'_t \in succ_t(l_a, B_t)$, $vis(l_a, l'_t) = \mathit{true}$;
  (2) $B'_t = \{l'_t \in succ_t(l_a, B_t) \mid vis(l_a, l'_t) = \mathit{false}\}$.

Condition (1) captures the successor locations of the target that can be observed from the agent's current position l_a. Condition (2) corresponds to the belief set consisting of *all possible successor locations of the target not visible from* l_a.

Example 2: Consider the surveillance game structure from Example 1. The initial belief set is $\{18\}$, consisting of the target's initial position. After the first move of the target, there are two possible belief sets: the set $\{19\}$ resulting from the move to a location in the area of sight of the agent, and $\{17, 23\}$ consisting of the two invisible locations reachable in one step from location 18. Figure 2 shows the successor states of the initial state $(4, \{18\})$ in G_{belief}. ■

Based on T_{belief}, we can define the functions $succ_t : S_{\mathsf{belief}} \to \mathcal{P}(\mathcal{P}(L_t))$ and $succ_a : S_{\mathsf{belief}} \times \mathcal{P}(L_t) \to \mathcal{P}(L_a)$ similarly to the corresponding functions defined for G.

A *run* in G_{belief} is an infinite sequence $s_0, s_1, \ldots$ of states in S_{belief}, where $s_0 = s^{\mathsf{init}}_{\mathsf{belief}}$, $(s_i, s_{i+1}) \in T_{\mathsf{belief}}$ for all i.

A *strategy for the target in* G_{belief} is a function $f_t : S^+_{\mathsf{belief}} \to \mathcal{P}(L_t)$ such that $f_t(\pi \cdot s) = B_t$ implies $B_t \in succ_t(s)$ for every $\pi \in S^*_{\mathsf{belief}}$ and $s \in S_{\mathsf{belief}}$. That is, a strategy for the target suggests a move resulting in some belief set reachable from some location in the current belief.

A *strategy for the agent in* G_{belief} is a function $f_a : S^+ \times \mathcal{P}(L_t) \to S$ such that $f_a(\pi \cdot s, B_t) = (l'_a, B'_t)$ implies $B'_t = B_t$ and $l'_a \in succ_a(s, B_t)$ for every $\pi \in S^*_{\mathsf{belief}}$, $s \in S_{\mathsf{belief}}$ and $B_t \in \mathcal{P}(L_t)$. Intuitively, a strategy for the agent suggests a move based on the observed history of the play and the current belief about the target's position.

The outcome of given strategies f_a and f_t for the agent and the target in G_{belief}, denoted $outcome(G_{\mathsf{belief}}, f_a, f_t)$, is a run $s_0, s_1, \ldots$ of G_{belief} such that for every $i \geq 0$, we have $s_{i+1} = f_a(s_0, \ldots, s_i, B^i_t)$, where $B^i_t = f_t(s_0, \ldots, s_i)$.

C. Temporal Surveillance Objectives

Since the states of a belief-set game structure track the information that the agent has, we can state and interpret surveillance objectives over its runs. We now formally define the surveillance properties in which we are interested.

We consider a set of *surveillance predicates* $\mathcal{SP} = \{p_k \mid k \in \mathbb{N}_{>0}\}$, where for $k \in \mathbb{N}_{>0}$ we say that a state (l_a, B_t) in the belief game structure satisfies p_k (denoted $(l_a, B_t) \models p_k$) iff $|\{l_t \in B_t \mid vis(l_a, l_t) = \mathit{false}\}| \leq k$. Intuitively, p_k is satisfied by the states in the belief game structure where the size of the belief set does not exceed the threshold $k \in \mathbb{N}_{>0}$.

We study surveillance objectives expressed by formulas of linear temporal logic (LTL) over surveillance predicates. The LTL surveillance formulas are generated by the grammar $\varphi := p \mid \mathit{true} \mid \mathit{false} \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \bigcirc \varphi \mid \varphi\, \mathcal{U}\, \varphi \mid \varphi\, \mathcal{R}\, \varphi$, where $p \in \mathcal{SP}$ is a surveillance predicate, $\bigcirc$ is the *next* operator, $\mathcal{U}$ is the *until* operator, and $\mathcal{R}$ is the *release* operator. We also define the derived operators *finally*: $\diamondsuit \varphi = \mathit{true}\, \mathcal{U}\, \varphi$ and *globally*: $\square \varphi = \mathit{false}\, \mathcal{R}\, \varphi$.

LTL formulas are interpreted over (infinite) runs. If a run ρ satisfies an LTL formula φ, we write $\rho \models \varphi$. The formal definition of LTL semantics can be found in [**?**]. Here we informally explain the meaning of the formulas we use.

Of special interest will be surveillance formulas of the form $\square p_k$, termed *safety surveillance objective*, and $\square \diamondsuit p_k$, called *liveness surveillance objective*. Intuitively, the safety surveillance formula $\square p_k$ is satisfied if at each point in time the size of the belief set does not exceed k. The liveness surveillance objective $\square \diamondsuit p_k$, on the other hand, requires that infinitely often this size is below or equal to k.

Example 3: We can specify that the agent is required to always know with certainty the location of the target as $\square p_1$. A more relaxed requirement is that the agent's uncertainty never grows above 5 locations, and it infinitely often reduces this uncertainty to at most 2 locations: $\square p_5 \wedge \square \diamondsuit p_2$. ■

D. Incorporating Task Specifications

We can integrate LTL objectives not related to surveillance, i.e., *task specifications*, by considering, in addition to $\mathcal{SP}$, a set $\mathcal{AP}$ of atomic predicates interpreted over states of G. In order to define the semantics of $p \in \mathcal{AP}$ over states of G_{belief}, we restrict ourselves to predicates observable by the agent. Formally, we require that for $p \in \mathcal{AP}$, and states (l_a, l'_t) and (l_a, l''_t) with $vis(l_a, l'_t) = vis(l_a, l''_t) = \mathit{false}$ it holds that $(l_a, l'_t) \models p$ iff $(l_a, l''_t) \models p$. One class of such predicates are those that depend only on the agent's position.

Example 4: Suppose that at_goal is a predicate true exactly when the agent is at some designated goal location. We can then state that the agent visits the goal infinitely often while always maintaining belief uncertainty of at most 10 locations using the LTL formula $\square \diamondsuit\, at\_goal \wedge \square p_{10}$. ■

E. Surveillance Synthesis Problem

A *surveillance game* is a pair (G, φ), where G is a surveillance game structure and φ is a surveillance objective. A *winning strategy for the agent for* (G, φ) is a strategy f_a for the agent in the corresponding belief-set game structure G_{belief} such that for every strategy f_t for the target in G_{belief} it holds that $outcome(G_{\mathsf{belief}}, f_a, f_t) \models \varphi$. Analogously, a *winning strategy for the target for* (G, φ) is a strategy f_t such that, for every strategy f_a for the agent in G_{belief}, it holds that $outcome(G_{\mathsf{belief}}, f_a, f_t) \not\models \varphi$.

Surveillance synthesis problem: Given a surveillance game (G, φ), compute a winning strategy for the agent for (G, φ), or determine that such a strategy does not exist.

It is well-known that two-player perfect-information games with LTL objectives over finite-state game structures are determined, that is exactly one of the players has a winning strategy. This means that the agent does not have a winning strategy for a given surveillance game, if and only if the target has a winning strategy for this game. We refer to winning strategies of the target as *counterexamples*.

III. EXPERIMENTAL EVALUATION

A. Belief subgames

Given a surveillance game (G, φ) where $G = (S, s^{\mathsf{init}}, T, vis)$, we say $G_{\tilde{S}} = (\tilde{S},)$ is a *subgame* of G if

- $\tilde{S} \subseteq S$

B. Decomposition of local surveillance games

Given a surveillance game (G, φ) where $G = (S, s^{\mathsf{init}}, T, vis)$, we aim to decompose the problem into n surveillance games (G_i, φ_i) for $i = 1, \ldots, n$, where $G_i = (S_i, s_i^{\mathsf{init}}, T, vis)$. The decomposition must satisfy the following properties

- If we have winning strategy for the agent in each game f_i such that $outcome(G_{\mathsf{belief}_i}, f_{a_i}, f_{t_i}) \models \varphi_i$, then the composed strategy for all agents $f_a = f_{a_1} \bigoplus f_{a_2} \cdots \bigoplus f_{a_n}$ must be winning for the global property, i.e, $outcome(G_{\mathsf{belief}}, f_a, f_t) \models \varphi$

and $\cup_i S_i = S$.

TODO: **Change definitions of surveillance game to allow for visibility function to be decoupled**

C. Distributed surveillance problem

Given surveillance game (G, φ)