Surveillance Exploration

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We consider the problem of patrolling a known environment while maintaining surveillance of a target.

1 Surveillance

2 Patrol

Let $\mathcal{X}$ be the set of all possible environment configurations. Each $\Omega \in \mathcal{X}$ is an open set representing the free space and Ω^C is a closed set consisting of a finite number of connected components. Let $\mathcal{O} = \{x_i\}_{i=0}^k$ be the sequence of vantage points. For each vantage point, the operator $\mathcal{P}_{x_i}\Omega$ is a projection of Ω along x_i. Then $\mathcal{P}_{x_i}\Omega$ is a set of range measurements defined on the unit sphere. The back projection $\mathcal{Q}$ maps the range measurements to the visibility set $\mathcal{V}_{x_i}\Omega := \mathcal{Q}(\mathcal{P}_{x_i})\Omega$; that is, points in this set are visible from x_i. As more range measurements are acquired, the environment can be approximated by the *cumulatively visible set* Ω_k:

$$\Omega_k = \bigcup_{i=0}^{k} \mathcal{V}_{x_i}\Omega \ .$$

By construction, Ω_k admits partial ordering: $\Omega_{i-1} \subset \Omega_i$. For suitable choices of x_i, it is possible that $\Omega_n \to \Omega$, (say, in the Hausdorff distance). We aim at determining a *minimal number of vantage points* from which every point in Ω can be seen.

2.1 A Greedy Approach

We consider a greedy approach which sequentially determines a new vantage point, x_{k+1}, based on the information gathered from all previous vantage points, $x_0, x_1, \cdots, x_k$. The strategy is greedy because x_{k+1} would be a location that *maximizes the information gain.*
If the environment Ω is known, we define the *gain* function

$$g(x; \Omega_k, \Omega) := |\mathcal{V}_x\Omega \cup \Omega_k| - |\Omega_k|,$$

i.e. the volume of the region that is visible from x but not from $x_0, x_1, \cdots, x_k$.
Define Γ as the feasible set (satisfying the surveillance requirements). Then, for patrolling, we consider:

$$x_{k+1} = \arg\max_{x \in \Gamma} g(x; \Omega_k, \Omega). \tag{1}$$

In other words, the next vantage point should be the point in the feasible set that maximizes the newly surveyed area.