Surveillance Exploration

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We consider the problem of patrolling a known environment while maintaining surveillance of a target.

1 Surveillance

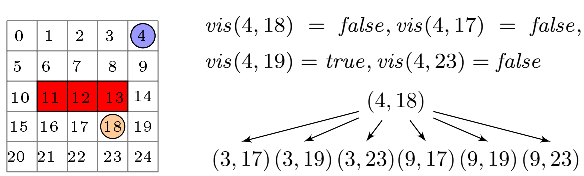
We begin by defining a formal model for describing surveillance strategy synthesis problems, in the form of a two-player game between an agent and a target, in which the agent has only partial information about the target's location.

1.1 Surveillance Game Structures

We define a *surveillance game structure* to be a tuple $G = (S, s^{\mathsf{init}}, T, vis)$, with the following components:

- $S = L_a \times L_t$ is the set of states, with L_a the set of locations of the agent, and L_t the locations of the target;
- $s^{\mathsf{init}} = (l_a^{\mathsf{init}}, l_t^{\mathsf{init}})$ is the initial state;
- $T \subseteq S \times S$ is the transition relation describing the possible moves of the agent and the target; and
- $vis : S \to \mathbb{B}$ is a function that maps a state (l_a, l_t) to *true* iff *position l_t is in the area of sight of l_a.*

The transition relation T encodes the one-step move of both the target and the agent, where the target moves first and the agent moves second. For a state (l_a, l_t) we define $succ_t(l_a, l_t)$ as the set of successor locations of the target:
$succ_t(l_a, l_t) = \{l'_t \in L_t \mid \exists l'_a.\ ((l_a, l_t), (l'_a, l'_t)) \in T\}$.



(a) Surveillance arena (b) Transitions from the initial state

Figure 1: A simple surveillance game on a grid arena. Obstacles are shown in red, the agent (at location 4) and the target (at location 18) are coloured in blue and orange respectively.

$$\begin{tikzcd}[column sep=small]
& (4,\{18\}) \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & & \\
(3,\{19\}) & (3,\{17,23\}) & (9,\{19\}) & (9,\{17,23\})
\end{tikzcd}$$

Figure 2: Transitions from the initial state in the belief-set game from Example ?? where $vis(4, 17) = vis(4, 23) = false$.

We extend $succ_t$ to sets of locations of the target by stipulating that the set $succ_t(l_a, L)$ consists of all possible successor locations of the target for states in $\{l_a\} \times L$. Formally, let $succ_t(l_a, L) = \bigcup_{l_t \in L} succ_t(l_a, l_t)$.
For a state (l_a, l_t) and a successor location of the target l'_t, we denote with $succ_a(l_a, l_t, l'_t)$ the set of successor locations of the agent, given that the target moves to l'_t:
$succ_a(l_a, l_t, l'_t) = \{l'_a \in L_a \mid ((l_a, l_t), (l'_a, l'_t)) \in T\}$.
We assume that, for every $s \in S$, there exists $s' \in S$ such that $(s, s') \in T$, that is, from every state there is at least one move possible (this might be staying in the same state). We also assume that when the target moves to an invisible location, its position does not influence the possible one-step moves of the agent. Formally, we require that if $vis(l_a, l'_t) = vis(l_a, \widehat{l'_t}) = false$, then $succ_a(l_a, l_t, l'_t) = succ_a(l_a, \widehat{l_t}, \widehat{l'_t})$ for all target locations $l_t, l'_t, \widehat{l_t}, \widehat{l'_t} \in L_t$. This assumption is natural in the setting when the agent can move in one step only to locations that are in its sight.

1.2 Belief-Set Game Structures

In surveillance strategy synthesis we need to state properties of, and reason about, the information which the agent has, i.e. its *belief* about the location of the target. To this end, we can employ a powerset construction which is commonly used to transform a partial-information game into a perfect-information one, by explicitly tracking the knowledge one player has as a set of possible states of the other player.
Given a set B, we denote with $\mathcal{P}(B) = \{B' \mid B' \subseteq B\}$ the powerset (set of all subsets) of B.
For a surveillance game structure $G = (S, s^{\mathsf{init}}, T, vis)$ we define the corresponding *belief-set game structure* $G_{\mathsf{belief}} = (S_{\mathsf{belief}}, s^{\mathsf{init}}_{\mathsf{belief}}, T_{\mathsf{belief}})$ with the following components:

- $S_{\mathsf{belief}} = L_a \times \mathcal{P}(L_t)$ is the set of states, with L_a the set of locations of the agent, and $\mathcal{P}(L_t)$ the set of *belief sets* describing information about the location of the target;
- $s^{\mathsf{init}}_{\mathsf{belief}} = (l_a^{\mathsf{init}}, \{l_t^{\mathsf{init}}\})$ is the initial state;
- $T_{\mathsf{belief}} \subseteq S_{\mathsf{belief}} \times S_{\mathsf{belief}}$ is the transition relation where $((l_a, B_t), (l'_a, B'_t)) \in T_{\mathsf{belief}}$ iff $l'_a \in succ_a(l_a, l_t, l'_t)$ for some $l_t \in B_t$ and $l'_t \in B'_t$ and one of these holds:
  (1) $B'_t = \{l'_t\}$, $l'_t \in succ_t(l_a, B_t)$, $vis(l_a, l'_t) = true$;
  (2) $B'_t = \{l'_t \in succ_t(l_a, B_t) \mid vis(l_a, l'_t) = false\}$.

Condition (1) captures the successor locations of the target that can be observed from the agent's current position l_a. Condition (2) corresponds to the belief set consisting of *all possible successor locations of the target not visible from l_a*.
Based on T_{belief}, we can define the functions $succ_t : S_{\mathsf{belief}} \to \mathcal{P}(\mathcal{P}(L_t))$ and $succ_a : S_{\mathsf{belief}} \times \mathcal{P}(L_t) \to \mathcal{P}(L_a)$ similarly to the corresponding functions defined for G.
A *run* in G_{belief} is an infinite sequence $s_0, s_1, \ldots$ of states in S_{belief}, where $s_0 = s^{\mathsf{init}}_{\mathsf{belief}}$, $(s_i, s_{i+1}) \in T_{\mathsf{belief}}$ for all i.

A *strategy for the target in* G_{belief} is a function $f_t : S^+_{\mathsf{belief}} \to \mathcal{P}(L_t)$ such that $f_t(\pi \cdot s) = B_t$ implies $B_t \in succ_t(s)$ for every $\pi \in S^*_{\mathsf{belief}}$ and $s \in S_{\mathsf{belief}}$. That is, a strategy for the target suggests a move resulting in some belief set reachable from some location in the current belief.
A *strategy for the agent in* G_{belief} is a function $f_a : S^+_{\mathsf{belief}} \times \mathcal{P}(L_t) \to S_{\mathsf{belief}}$ such that $f_a(\pi \cdot s, B_t) = (l'_a, B'_t)$ implies $B'_t = B_t$ and $l'_a \in succ_a(s, B_t)$ for every $\pi \in S^*_{\mathsf{belief}}$, $s \in S_{\mathsf{belief}}$ and $B_t \in \mathcal{P}(L_t)$. Intuitively, a strategy for the agent suggests a move based on the observed history of the play and the current belief about the target's position.
The outcome of given strategies f_a and f_t for the agent and the target in G_{belief}, denoted $outcome(G_{\mathsf{belief}}, f_a, f_t)$, is a run $s_0, s_1, \ldots$ of G_{belief} such that for every $i \geq 0$, we have $s_{i+1} = f_a(s_0, \ldots, s_i, B^i_t)$, where $B^i_t = f_t(s_0, \ldots, s_i)$.

1.3 Temporal Surveillance Objectives

Since the states of a belief-set game structure track the information that the agent has, we can state and interpret surveillance objectives over its runs. We now formally define the surveillance properties in which we are interested.
We consider a set of *surveillance predicates* $\mathcal{SP} = \{p_k \mid k \in \mathbb{N}_{>0}\}$, where for $k \in \mathbb{N}_{>0}$ we say that a state (l_a, B_t) in the belief game structure satisfies p_k (denoted $(l_a, B_t) \models p_k$) iff $|\{l_t \in B_t \mid vis(l_a, l_t) = \mathit{false}\}| \leq k$. Intuitively, p_k is satisfied by the states in the belief game structure where the size of the belief set does not exceed the threshold $k \in \mathbb{N}_{>0}$.
We study surveillance objectives expressed by formulas of linear temporal logic (LTL) over surveillance predicates. The LTL surveillance formulas are generated by the grammar
$\varphi := p \mid \mathit{true} \mid \mathit{false} \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \bigcirc \varphi \mid \varphi \,\mathcal{U}\, \varphi \mid \varphi \,\mathcal{R}\, \varphi$,
where $p \in \mathcal{SP}$ is a surveillance predicate, $\bigcirc$ is the *next* operator, $\mathcal{U}$ is the *until* operator, and $\mathcal{R}$ is the *release* operator. We also define the derived operators *finally*: $\Diamond \varphi = \mathit{true}\,\mathcal{U}\,\varphi$ and *globally*: $\Box \varphi = \mathit{false}\,\mathcal{R}\,\varphi$.
LTL formulas are interpreted over (infinite) runs. If a run ρ satisfies an LTL formula φ, we write $\rho \models \varphi$. The formal definition of LTL semantics can be found in [?]. Here we informally explain the meaning of the formulas we use.
Of special interest will be surveillance formulas of the form $\Box p_k$, termed *safety surveillance objective*, and $\Box\Diamond p_k$, called *liveness surveillance objective*. Intuitively, the safety surveillance formula $\Box p_k$ is satisfied if at each point in time the size of the belief set does not exceed k. The liveness surveillance objective $\Box\Diamond p_k$, on the other hand, requires that infinitely often this size is below or equal to k.

1.4 Incorporating Task Specifications

We can integrate LTL objectives not related to surveillance, i.e., *task specifications*, by considering, in addition to $\mathcal{SP}$, a set $\mathcal{AP}$ of atomic predicates interpreted over states of G. In order to define the semantics of $p \in \mathcal{AP}$ over states of G_{belief}, we restrict ourselves to predicates observable by the agent. Formally, we require that for $p \in \mathcal{AP}$, and states (l_a, l'_t) and (l_a, l''_t) with $vis(l_a, l'_t) = vis(l_a, l''_t) = \mathit{false}$ it holds that $(l_a, l'_t) \models p$ iff $(l_a, l''_t) \models p$. One class of such predicates are those that depend only on the agent's position.

1.5 Surveillance Synthesis Problem

A *surveillance game* is a pair (G, φ), where G is a surveillance game structure and φ is a surveillance objective. A *winning strategy for the agent for* (G, φ) is a strategy f_a for the agent in the corresponding belief-set game structure G_{belief} such that for every strategy f_t for the target in G_{belief} it holds that $outcome(G_{\mathsf{belief}}, f_a, f_t) \models \varphi$. Analogously, a *winning strategy for the target*

for (G, φ) is a strategy f_t such that, for every strategy f_a for the agent in G_{belief}, it holds that *outcome*$(G_{\mathsf{belief}}, f_a, f_t) \not\models \varphi$.
Surveillance synthesis problem: Given a surveillance game (G, φ), compute a winning strategy for the agent for (G, φ), or determine that such a strategy does not exist.
It is well-known that two-player perfect-information games with LTL objectives over finite-state game structures are determined, that is exactly one of the players has a winning strategy [**?**]. This means that the agent does not have a winning strategy for a given surveillance game, if and only if the target has a winning strategy for this game. We refer to winning strategies of the target as *counterexamples.*

2 Patrol

Let $\mathcal{X}$ be the set of all possible environment configurations. Each $\Omega \in \mathcal{X}$ is an open set representing the free space and Ω^C is a closed set consisting of a finite number of connected components. Let $\mathcal{O} = \{x_i\}_{i=0}^k$ be the sequence of vantage points. For each vantage point, the operator $\mathcal{P}_{x_i}\Omega$ is a projection of Ω along x_i. Then $\mathcal{P}_{x_i}\Omega$ is a set of range measurements defined on the unit sphere. The back projection $\mathcal{Q}$ maps the range measurements to the visibility set $\mathcal{V}_{x_i}\Omega := \mathcal{Q}(\mathcal{P}_{x_i})\Omega$; that is, points in this set are visible from x_i. As more range measurements are acquired, the environment can be approximated by the *cumulatively visible set* Ω_k:

$$\Omega_k = \bigcup_{i=0}^{k} \mathcal{V}_{x_i}\Omega \ .$$

By construction, Ω_k admits partial ordering: $\Omega_{i-1} \subset \Omega_i$. For suitable choices of x_i, it is possible that $\Omega_n \to \Omega$, (say, in the Hausdorff distance). We aim at determining a *minimal number of vantage points* from which every point in Ω can be seen.

2.1 A Greedy Approach

We consider a greedy approach which sequentially determines a new vantage point, x_{k+1}, based on the information gathered from all previous vantage points, $x_0, x_1, \cdots, x_k$. The strategy is greedy because x_{k+1} would be a location that *maximizes the information gain.*
If the environment Ω is known, we define the *gain* function

$$g(x; \Omega_k, \Omega) := |\mathcal{V}_x\Omega \cup \Omega_k| - |\Omega_k|,$$

i.e. the volume of the region that is visible from x but not from $x_0, x_1, \cdots, x_k$.
Define Γ as the feasible set (satisfying the surveillance requirements). Then, for patrolling, we consider:

$$x_{k+1} = \arg\max_{x \in \Gamma} g(x; \Omega_k, \Omega). \tag{1}$$

In other words, the next vantage point should be the point in the feasible set that maximizes the newly surveyed area.