Modelling Surface Acoustic Wave Driven Flows over Topography

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Want to numerically simulate the flow of a fluid driven primarily by high frequency surface acoustic waves over a surface that may include topography and may be inclined.

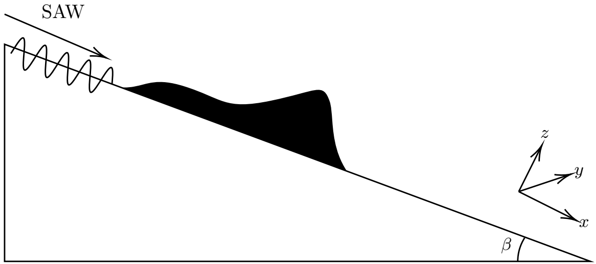

Figure: A simplified sketch of a fluid flowing down an inclined plane with no topography.

We let $s(x, y)$ describe the topography of the surface, $h(x, y, t)$ be the film thickness relative to $s(x, y)$ at a time t, and $\phi(x, y, t) = s(x, y) + h(x, y, t)$ be the height of the free surface at a time t.

For completeness, we derive an equation to model fluids in a 3-D system, but we perform numerical simulations on the simplified 2-D case.

We start with the Incompressible Navier-Stokes Equation

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\,\mathbf{u}\right)=-\nabla p+\mu\nabla^2\mathbf{u}+\rho g\sin\beta\mathbf{i}-\rho g\cos\beta\mathbf{k}$$
$$-\rho J e^{2k_i(x+\alpha_1 z)}\mathbf{i}-\rho J\alpha_1 e^{2k_i(x+\alpha_1 z)}\mathbf{k} \quad (1)$$

where $\mathbf{u}$ = Fluid velocity, p = Fluid pressure, ρ = Fluid density, μ = Fluid viscosity, k_i = Attenuation coefficient, α_1 = Geometric constant, and $J=\left(1+\alpha_1^2\right)A^2\omega^2 k_i$ is a constant we define to consolidate terms.

The first two vector terms in the x and z direction represent the in plane and out of plane components of gravity, while the last two vector terms represent the in plane and out of plane components of the SAW forcing.

The Lubrication Approximation assumes we are dealing with thin films and allows us to ignore the inertial terms of the Navier-Stokes equation (LHS) as well as the in plane derivatives and normal component of **u**. Hence, Eq. (1) reduces to

$$\begin{aligned} \nabla_2 p &= \mu \frac{\partial^2 \mathbf{v}}{\partial z^2} + \rho g \sin\beta \mathbf{i} - \rho J e^{2k_i(x+\alpha_1 z)} \mathbf{i} \\ \frac{\partial p}{\partial z} &= -\rho g \cos\beta - \rho J \alpha_1 e^{2k_i(x+\alpha_1 z)} \end{aligned} \tag{2}$$

where $\nabla_2 = (\partial_x, \partial_y)$ and $\mathbf{v} = (u, v)$.

Boundary Conditions