E3) A

$$f = \begin{pmatrix} x(k) + (\delta_d + v_d)\cos\theta(k) \\ y(k) + (\delta_d + v_d)\sin\theta(k) \\ \theta(k) + \delta_\theta + v_\theta \end{pmatrix} = \begin{pmatrix} x(k) + \delta_d\cos\theta(k) + v_d\cos\theta(k) \\ y(k) + \delta_d\sin\theta(k) + v_d\sin\theta(k) \\ \theta(k) + \delta_\theta + v_\theta \end{pmatrix}$$

$$F_x = \frac{\partial f}{\partial x}\bigg|_{v=0} = \begin{bmatrix} 1 & -v_d\sin\theta(k) & -\delta_d\sin\theta_v \\ 0 & v_d\cos\theta(k) & \delta_d\cos\theta_v \\ 0 & 0 & 1 \end{bmatrix}\Bigg|_{v=0}$$

$$\begin{bmatrix} 1 & 0 & -\delta_d\sin\theta_v \\ 0 & 0 & \delta_d\cos\theta_v \\ 0 & 0 & 1 \end{bmatrix}$$

$$F_v = \frac{\partial f}{\partial v}\bigg|_{v=0} \begin{bmatrix} \cos\theta_v & 0 & 0 \\ \sin\theta_v & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad \delta_d = 1$$

$$h = \begin{pmatrix} \sqrt{(y_i-y_v)^2 + (x_i-x_v)^2} \\ \tan^{-1}(y_i-y_v)/(x_i-x_v) - \theta_v \end{pmatrix} + \begin{pmatrix} w_r \\ w_\beta \end{pmatrix}$$

$$\frac{1}{1+\left(\frac{y_i-y_v}{x_i-x_v}\right)^2}$$

$$\frac{1}{2\sqrt{(y_i-y_v)^2+(x_i-x_v)^2}} \rightarrow x_i^2 - 2x_vx_i + x_v^2$$

No theta

$$\rightarrow w_r + \sqrt{(y_i-y_v)^2 + (x_i-x_v)^2}$$

$$w_\beta + \tan^{-1}(y_i-y_v)/(x_i-x_v) - \theta_v$$

$$\begin{bmatrix} 1 & 0 & \cancel{0} \\ 0 & 1 & \cancel{0} \end{bmatrix}$$

$$H_x = \frac{\partial h}{\partial x}\bigg|_{w=0}$$

$$\boxed{x = [x_v, y_v, \theta_v]}$$

$$\boxed{H_x = \begin{bmatrix} \frac{-x_i - x_v}{r} & \frac{y_i - y_v}{r} & 0 \\ \frac{y_i - y_v}{r^2} & \frac{-x_i - x_v}{r^2} & -1 \end{bmatrix}}$$

$$\boxed{H_w = \frac{\partial h}{\partial w}\bigg|_{w=0} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

E3 cont.)

$$y(x,z) = \begin{pmatrix} x \\ g(x,v) \end{pmatrix} \begin{matrix} n \\ 2 \end{matrix}$$

$n = 2m+3$

$m = \#$ landmarks

$$Y_x = \frac{\partial y}{\partial x} = \begin{bmatrix} \frac{\partial x}{\partial x} \\ \frac{\partial g}{\partial x} \end{bmatrix} = \begin{bmatrix} I_{n\times n} \\ 0_{2\times n} \end{bmatrix}$$

$$Y_z = \frac{\partial y}{\partial z} = \begin{bmatrix} \frac{\partial x}{\partial z} \\ \frac{\partial g}{\partial z} \end{bmatrix} = \begin{bmatrix} 0_{n\times 2} \\ G_z \end{bmatrix} \qquad G_z = 2\times 2$$

$$Y_x P Y_x^T + Y_z W Y_z^T$$

$$= \begin{bmatrix} P & 0_{n\times 2} \\ 0_{2\times n} & 0_{2\times 2} \end{bmatrix} + \begin{bmatrix} 0_{n\times n} & 0_{n\times 2} \\ 0_{2\times n} & G_z W G_z^T \end{bmatrix} \qquad P = n\times n$$

$$= \begin{bmatrix} P & 0_{n\times 2} \\ 0_{2\times n} & G_z W G_z^T \end{bmatrix}$$

$\downarrow$

Insertion

$$\begin{bmatrix} I_{n\times n} & 0_{n\times 2} \\ 0_{2\times n} & G_z \end{bmatrix} \begin{bmatrix} P & 0_{n\times 2} \\ 0_{2\times n} & W \end{bmatrix} \begin{bmatrix} I_{n\times n} & 0_{n\times 2} \\ 0_{2\times n} & G_z \end{bmatrix}^T$$

$$Y_x = \begin{bmatrix} \frac{\partial x_v}{\partial x_v} & \frac{\partial x_v}{\partial x_m} \\ \frac{\partial x_m}{\partial x_v} & \frac{\partial x_m}{\partial x_m} \\ \frac{\partial g}{\partial x_v} & \frac{\partial g}{\partial x_m} \end{bmatrix} = \begin{bmatrix} I_{3\times 3} & 0_{3\times 2m} \\ 0_{2m\times 3} & I_{2m\times 2m} \\ G_{xv} & 0_{2\times 2m} \end{bmatrix} = \begin{bmatrix} I_{n\times n} \\ G_x \end{bmatrix} n$$

$G_x = 2\times n$

$$Y_z = \begin{pmatrix} 0_{n\times 2} \\ G_z \end{pmatrix}$$

$Y_\alpha P Y_\alpha^T + Y_z W Y_z^T$ Substitute Jacobians

$$\begin{bmatrix} I_{n\times n} & 0_{n\times 2} \\ G_\alpha & G_z \end{bmatrix} \begin{bmatrix} P & 0_{n\times 2} \\ 0_{2\times n} & W \end{bmatrix} \begin{bmatrix} I_{n\times n} & 0_{n\times 2} \\ G_\alpha & G_z \end{bmatrix}^T$$

Insertion

$$M \begin{pmatrix} P & 0 \\ 0 & W \end{pmatrix} M^T \qquad M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$\begin{bmatrix} APA^T + BWB^T & APC^T + BWD^T \\ CPA^T + DWB^T & CPC^T + DWD^T \end{bmatrix}$$

$$\begin{bmatrix} P & 0 \\ 0 & G_z W G_z^T \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & G_z \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & W \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & G_z \end{bmatrix}^T$$