

Now we know that $\text{fib}(0) = 0$ and $\text{fib}(1) = 1$ we can substitute these values in the above tree diagram, then $\text{fib}(2) = \text{fib}(1) + \text{fib}(0) = 0 + 1 = 1$.

Now $\text{fib}(3) = \text{fib}(2) + \text{fib}(1) = 1 + 1 = 2$

$\text{fib}(4) = \text{fib}(3) + \text{fib}(2) = 2 + 1 = 3$

$\text{fib}(5) = \text{fib}(4) + \text{fib}(3) = 3 + 2 = 5$

Q3: given two integers a and b. Find the value of a^b using recursion.

Input 1: $a = 3, b = 4$

Output 1: 81

Solution :

Code: [LP_Code3.java](#)

Output:

Required answer is 81

Approach :

- Create a recursive function say `power(int a, int b)`
- Base condition : if $(b == 0)$ return 1 because we know anything to the base 0 = 1.
- Otherwise, return $(a * \text{power}(a, b - 1))$

Efficient approach:

Lets see a better approach on how we can improve the time complexity of our solution, lets define a function `func` which will take two numbers a and b as input and will return a^b . Now this function can be written in this way `func(a, b) = func(a, b / 2) * func(a, b / 2)`; Like humans do in real life for example $a=2, b=4$, so for calculating $2^4 =$ we can find (2^2) , take square of it in case b is even. In case b is odd, for example $a=2, b=5$, so for calculating $2^5 =$ we can find (2^2) , take square of it and multiply it with 2 like $(2^5) = (2) * (2^2) * (2^2)$.

`power(a, b) = power(a, b / 2) * power(a, b / 2); // if b is even`

`func(a, b) = a * power(a, (b-1) / 2) * power(a, (b-1) / 2); // if b is odd`

The generated recursion tree when above recurrence relation is implemented to calculate `power(3, 4)` is shown below.

As you can observe, `func(2,2)`, `func(2,1)` are being computed multiple times. To avoid these redundant computations, we store the result in a result variable while making the recursive call and use this variable value subsequently.

Code : [LP_Code4.java](#)

Output :

Required answer is 81

Approach :

- This function has a base case: $b = 0$.
- This function has two recursive calls. Only one of them is made in any given call.
- Let's first consider the case when b is even: In that case, the recursive call is made with $b / 2$. I will store the value return by `power(a, b / 2)` in result variable and will return square of result variable i.e $(\text{result} * \text{result})$.