DEARBORN

COLLEGE OF ENGINEERING
& COMPUTER SCIENCE

Project 3 – Applied Queuing Method in Taco Bell Drive Through

IMSE 500: Models of Operations Research
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Background

Queuing theory is the discipline of knowledge within the field of operations research. Queuing theory is about the waiting line, which is a part of everyday life. We queue or wait in line to get served in morning coffee drive through, supermarkets or fast food restaurants, etc. (Kavitha; Palaniammal, 2014).

Queuing theory according to Dharmawirya and Adi (2011) was particularly suitable to be applied in a portion of fast food or drive-through settings since it has an associated queue or waiting line was customers who cannot be served immediately have to queue for service. Taco Bell is a chain of fast food restaurant selling fast food.

This paper studies the application of queuing theory in two different Taco Bell's Drive-through queues. in Michigan - Troy, Farmington Hills. Drive-through operates in a manner that customers can take away their orders immediately after payment or wait in the car for some minutes in the queue. However, clients suffer unnecessary delays, especially during peak periods.

This shows a need for a numerical model for Taco Bell's management to understand the situation better. The goal of the study is to solve the problem of waiting during peak hours in Taco Bell drive-through by decreasing customers' waiting time by modeling a queuing theory to simulate the waiting lines. It is intended to show that queuing theory satisfies the model when tested with a real –case scenario. The paper analyzes the data collected from two different Taco Bell's drive-through at different peak hours- morning (10:00 am), lunch(12:00 pm) and evening(4:00 pm) the Taco Bell's Model. This is followed by the proposed queuing model and results.

Figure: Drive through Taco Bell (LHS) Drive through during peak hours(RHS)

Problem Statement

Drive-thru restaurants as often as possible face the issue of long holding up lines amid peak hours particularly amid advancements, lunch and dinner time. Unfit to be served promptly, customers need to sit tight for their turn, frequently fretfully. The individuals who are reluctant to hang tight for quite a while may pick to leave. We have watched that purchasers would possess an alternate anticipated hanging tight energy for various businesses, where customers with a higher estimation of time would lean toward the more expensive stores which have lower holding up time/waiting time.

Long holding up lines of cars with customers wanting to purchase food through the Drive Through windows at various Taco Bell food outlets in a typical wonder amid noon. Long holding up lines are most certainly not ideal since customers may turn away (balk away to contenders/competitors), and this implies lost business. The use of simulation for taking care of solving waiting for lines will deliver some critical data that will help the board in basic leadership in order to decrease customers holding/waiting times, henceforth ensuring the recoiling of customers to contenders/competitors.

Research on quick service restaurants showed that the long line was the consequences of long planning time in the kitchen. Thus improving sustenance planning time is essential. The accomplishment of drive-through eateries depends basically on overseeing nourishment planning time/food preparation time, queue/line length and holding up time/waiting time. In this exploration, we expect to dissect the line procedure at a nearby Taco Bell outlet. Customers arriving time, holding up/waiting time, serving time and departure time amid lunch hour and evenings are observed/monitored to give the critical data for this study.

Objectives

To study the queuing phenomenon at a fast food restaurant chain. We set and accomplished the following objectives.

- ❏ Study and record the current queuing infrastructure at Taco Bell Drive-Through.
- ❏ Analyze arrival and service data of 2 hours at 2 different locations.
- ❏ Determine and verify the arrival and service patterns of queuing model.
- ❏ Demonstrate the effectiveness of implementing a multi-server queuing model instead of a single server model through the performance parameters.

Assumptions

- ❏ Unlimited input source.
- ❏ Poisson arrival pattern.
- ❏ Customers may arrive and be served as single.
- ❏ Customers might engage in Balking.
- ❏ M/M/1/∞/FCFS
    a. Poisson Arrival Pattern
    b. General Service Pattern
    c. 1 server
    d. Infinite capacity
    e. First come First Served

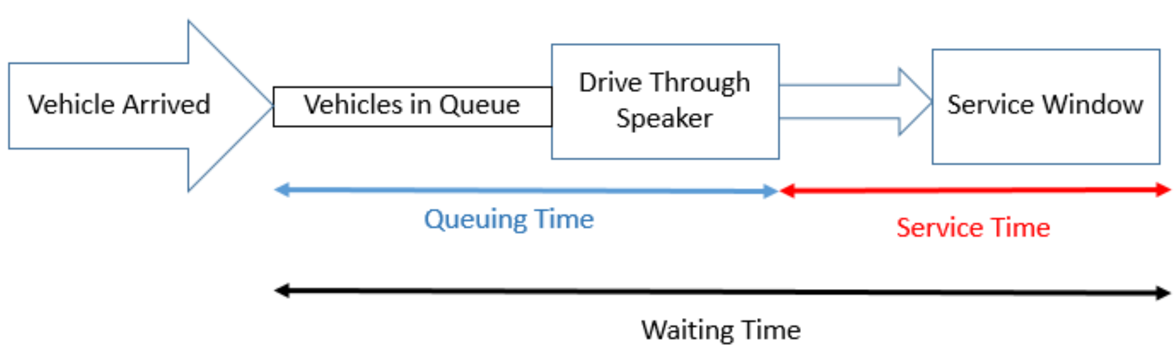
Data Collection and Validation (Methods and sources of data) :

We collected data by noting the time of arrival of the car at a token booth, the time at which it places an order, and the time it gets served on. The time between the ordering booth and the token booth is treated as waiting time. The time between service booth and order booth is treated as service time. We collect this data for 2 hours at different taco-bell restaurants at a different time of the day. So data collected is independent of each other.

The Queuing model and its solution

The Queuing model of two different Taco Bell locations was studied. It was first assumed that both Queuing models followed an M/M/1/∞/FCFS as described in the assumptions portion of this report and the Queuing diagram can be seen below. However, it was important to analyze the data to confirm the Queue at those Taco Bell locations followed this assumed queuing model.

Diagram 1: Taco Bell Queuing Model



First, the data was analyzed after it was taken at two different Taco Bell locations. The first analysis was done to confirm a Poisson arrival pattern. This was done by dividing the data into intervals of 5 min and recording the number of vehicles that arrived during those intervals. Using the Poisson Distribution shown below, we managed to get the expected values and then ran a Chi-Square test with a significance level of 5% to determine if, in fact, the arrival pattern followed a Poisson distribution. Our findings for all two locations can be seen in tables 1 and 2 in the Appendix. The summary of the Chi-Square test for the arrival pattern can be seen in table 3 below.

Poisson Probability Distribution Function (λ varies per location)

$$p(x) = \frac{\lambda^{x}}{x!} e^{-\lambda}$$

Table 3: Chi-Square Test Results for arrival pattern

Chi-Square Test	Degrees of freedom (k-1-p): k = 6, p = 1	Computed Chi-Square Value	Critical Chi-Square Value (α = 0.05)	Reject Null Hypothesis: Y/N
Location 1	4	8.62	9.488	N
Location 2	4	9	9.488	N

After confirming the arrival pattern followed the Poisson probability distribution. The team moved on to determine if the service times followed an exponential distribution by using the following probability equation shown below. Similar to the arrival pattern, the data was run through a Chi-Square Test to determine if the service times were exponential in nature. Results for the calculation of the Chi-Square value can be seen in tables 4 to 5 in the Appendix. Furthermore, the conclusion of the Chi-Square test with a significance level of 5% can be seen in table 6 shown below.

Exponential Probability Function (μ varies per location)

$$F(x) = 1 - e^{-\mu x}$$

Table 6: Chi-Square Test Results for Service Times

Chi-Square Test	Degrees of freedom (k-1-p): k = 4, p = 1	Computed Chi-Square Value	Critical Chi-Square Value (α = 0.05)	Reject Null Hypothesis: Y/N
Location 1	2	6	6	N
Location 2	2	2.58	6	N

With this analysis, the team was able to confirm that the Taco Bell Locations in question did follow an M/M/1/∞/FCFS Queuing Model. Using the Excel solver shown in the appendix. The team was able to solve the Queuing model equations shown below. Where Po = Probability of system being empty, Pw = Probability of Waiting, Pn = Probability of a number of customers in the system, W = Average time in the system, Wq = Average time in queue, L = Average number in the system, and Lq = Average number in the queue. The solution to these equations can be seen in table 7 below.

Queuing Model M/M/1/∞/FCFS Relationship equations

$$P_o = 1 - \lambda/\mu$$
$$P_w = \lambda/\mu$$
$$P_n = (\lambda/\mu)^n (1 - \lambda/\mu)$$
$$W = 1/(\mu - \lambda)$$
$$W = W_q + 1/\mu$$
$$L = \lambda W$$
$$L_q = \lambda W_q$$

Table 7: Performance metrics for Taco Bell Location 1 & 2

Metric	Location 1	Location 2

λ	**1.52**	**1.56**
μ	**1.68**	**1.94**
Po	**0.095**	**0.196**
Pw	**0.852**	**0.684**
W	**6.25**	**2.632**
Wq	**5.655**	**2.116**
L	**9.5**	**4.105**
Lq	**8.595**	**3.301**
Pn	**See Appendix**	

Conclusion

The Queuing model currently has an average waiting time of about 3 and 6 min for Location 1 and 2 respectively. Therefore, our conclusion is that there is currently no need for alterations to the Queuing model. However, in the future, if due to population growth in the area, there could potentially be a significant increase in waiting time and a second server, or in our case a second order window, should be advantageous.

Recommendations

Our recommendation is to maintain the current queuing system as it has low waiting times for customers to be processed. However, we analyzed the queuing system with a count of 2 servers to determine if there would be any impact on the waiting time. Clearly, there is an advantage for utilizing a secondary server due to the drop of waiting time in system down to less than a minute as it can be seen in table 8.

Diagram 2: Taco Bell Queuing Model (Two Servers)

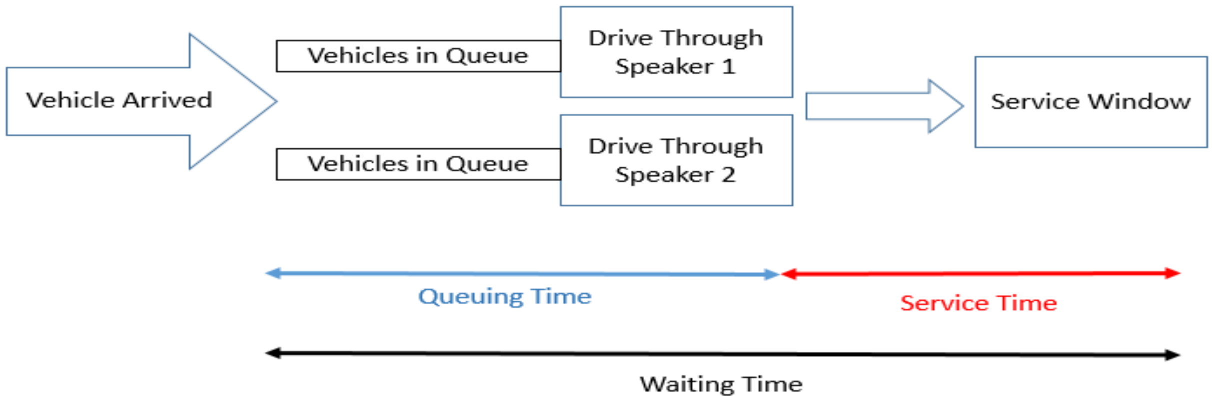


Table 8: Performance metrics for Taco Bell Location 1 & 2 (Two Servers)

Metric	Location 1	Location 2
λ	1.52	1.56
μ	1.68	1.94
Po	0.377	0.426
Pw	0.268	0.197
W	0.748	0.615
Wq	0.153	0.099
L	1.138	0.959
Lq	0.233	0.155
Pn	See Appendix	

References

1. QueuingTheory-<https://ucltrafficproject.wordpress.com/classical-traffic-flow-theory/queuing-theor y/>
2. Queue'sWaitingtime,LangWang,ConghuanXu<https://dornsife.usc.edu/assets/sites/406/docs/505b/Project_Queue-5.pdf>
3. Bai,JSo,KCTang,C,"Aqueueingmodelformanagingsmallprojectsunder uncertainties", European Journal of Operational Research, Volume 253 No(3),2016-09-16, doi: 10.1016/j.ejor.2016.02.052 <https://escholarship.org/content/qt54c0w1rx/qt54c0w1rx.pdf>

Appendix

Table 1: Taco Bell Location 1 Arrival Data Table (λ=1.52)

# of Cars per 5 min	Poisson Probability (λ=1.52)	Observed (O)	Expected (E)	Diff (O-E)	Chi-Square Value (O-E)^2/E
0	0.218	7	3.48	3.52	3.55
1	0.332	2	5.31	-3.31	2.06
2	0.253	2	4.05	-2.05	1.04
3	0.129	3	2.06	0.94	0.43
4	0.049	1	0.78	0.22	0.06
≥ 5	0.020	1	0.32	0.68	1.48
Sum		16	16		8.62

Table 2: Taco Bell Location 2 Arrival Data Table (λ=1.56)

# of Cars per 5 min	Poisson Probability (λ=1.56)	Observed (O)	Expected (E)	Diff (O-E)	Chi-Square Value (O-E)^2/E
0	0.210	1	2.52	-1.52	0.91
1	0.328	1	3.93	-2.93	2.18
2	0.256	4	3.07	0.93	0.28
3	0.133	4	1.60	2.40	3.60
4	0.052	1	0.62	0.38	0.23
≥ 5	0.022	1	0.26	0.74	2.11
Sum		12	12		9

Table 4: Taco Bell Location 1 Service Data Table (μ = 1.68)

Service time	F(x) = 1 - e^(-μx) (μ = 1.68)	Observed (O)	Expected (E)	Diff (O-E)	Chi-Square Value (O-E)^2/E
1	0.402	16	14.081	1.918	0.26
2	0.240	7	8.416	-1.416	0.24
3	0.144	8	5.030	2.970	1.75
4	0.086	4	3.006	0.994	0.33
Sum		35			2.58

Table 5 : Taco Bell Location 2 Service Data Table (μ = 1.94)

Service time	F(x) = 1 - e^(-μx) (μ = 1.94)	Observed (O)	Expected (E)	Diff (O-E)	Chi-Square Value (O-E)^2/E
1	0.449	15	11.21	3.786	1.28
2	0.247	5	6.18	-1.184	0.23
3	0.136	3	3.41	-0.410	0.05
4	0.075	2	1.88	0.120	0.01
Sum		25			1.56

Queuing Model Solution: Taco Bell Location 1

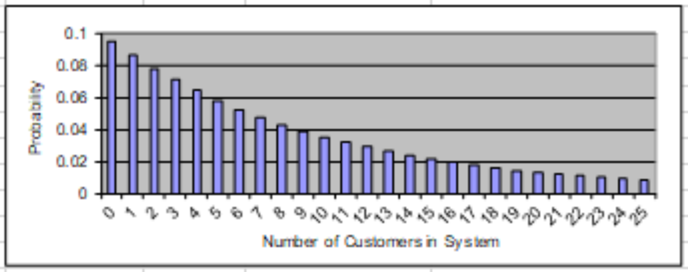
Template for the M/M/s Queueing Model

	Data	
λ =	1.52	(mean arrival rate)
μ =	1.68	(mean service rate)
s =	1	(# servers)
Pr(W > t) =	0.852144	
when t =	1	
Prob(W_q > t) =	0.770987	
when t =	1	

	Results
L =	9.5
L_q =	8.595238095
W =	6.25
W_q =	5.654761905
ρ =	0.904761905

n	P_n
0	0.095238095
1	0.0861678
2	0.077961343
3	0.070536453
4	0.063818696
5	0.057740725
6	0.052241608
7	0.047266217
8	0.042764673
9	0.038691847
10	0.035006909
11	0.031672917
12	0.028656449
13	0.025927264
14	0.023458
15	0.021223905
16	0.019202581
17	0.017373764
18	0.015719119
19	0.01422206
20	0.012867578
21	0.011642095
22	0.010533324
23	0.00953015
24	0.008622517
25	0.007801325

Range Name	Cells
L	G4
Lambda	C4
Lq	G5
Mu	C5
n	F13:F38
P0	G13
Pn	G13:G38
Rho	G10
s	C6
Time1	C9
Time2	C12
W	G7
Wq	G8



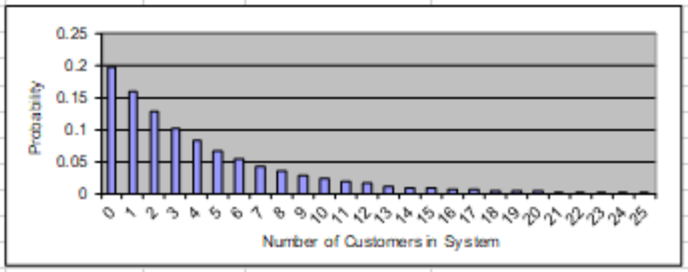
Queuing Model Solution: Taco Bell Location 2

Template for the M/M/s Queueing Model

	Data	
λ =	1.56	(mean arrival rate)
μ =	1.94	(mean service rate)
s =	1	(# servers)
Pr(W > t) =	0.683861	
when t =	1	
Prob(W_q > t) =	0.549909	
when t =	1	

	Results
L =	4.105263158
L_q =	3.301139447
W =	2.631578947
W_q =	2.11611503
ρ =	0.804123711

Range Name	Cells
L	G4
Lambda	C4
Lq	G5
Mu	C5
n	F13:F38
P0	G13
Pn	G13:G38
Rho	G10
s	C6
Time1	C9
Time2	C12
W	G7
Wq	G8



n	P_n
0	0.195876289
1	0.157508768
2	0.126656535
3	0.101847523
4	0.081898008
5	0.06585613
6	0.052956476
7	0.042583558
8	0.034242449
9	0.027535166
10	0.022141679
11	0.017804649
12	0.014317141
13	0.011512752
14	0.009257677
15	0.007444318
16	0.005986152
17	0.004813607
18	0.003870736
19	0.00311255
20	0.002502875
21	0.002012621
22	0.001618397
23	0.001301391
24	0.001046479
25	0.000841499

Queuing Model Solution: Taco Bell Location 1 (Two Server)

Template for the M/M/s Queueing Model

	Data	
λ =	1.52	(mean arrival rate)
μ =	1.68	(mean service rate)
s =	2	(# servers)
Pr(W > t) =	0.267914	
when t =	1	
Prob(W_q > t) =	0.044757	
when t =	1	

	Results
L =	1.137562368
L_q =	0.232800462
W =	0.748396294
W_q =	0.153158198
ρ =	0.452380952

n	P_n
0	0.37704918
1	0.341139735
2	0.154325118
3	0.069813744
4	0.031582408
5	0.01428728
6	0.006463293
7	0.002923871
8	0.001322703
9	0.000598366
10	0.000270689
11	0.000122455
12	5.53962E-05
13	2.50602E-05
14	1.13367E-05
15	5.12853E-06
16	2.32005E-06
17	1.04955E-06
18	4.74794E-07
19	2.14788E-07
20	9.7166E-08
21	4.3956E-08
22	1.98849E-08
23	8.99554E-09
24	4.06941E-09
25	1.84092E-09

Range Name	Cells
L	G4
Lambda	C4
Lq	G5
Mu	C5
n	F13:F38
P0	G13
Pn	G13:G38
Rho	G10
s	C6
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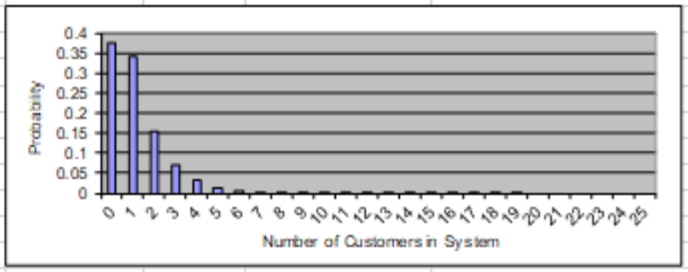
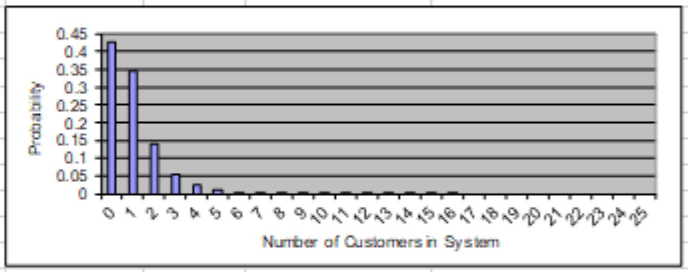
Time2	C12
W	G7
Wq	G8



Queuing Model Solution: Taco Bell Location 2 (Two Server)

Template for the M/M/s Queueing Model

	Data	
λ =	1.56	(mean arrival rate)
μ =	1.94	(mean service rate)
s =	2	(# servers)
Pr(W > t) =	0.197187	
when t =	1	
Prob(W_q > t) =	0.022661	
when t =	1	



	Results
L =	0.959178499
L_q =	0.155054788
W =	0.614858012
W_q =	0.099394096
ρ =	0.402061856

n	P_n
0	0.426470588
1	0.342935112
2	0.137881128
3	0.055436742
4	0.022288999
5	0.008961556
6	0.0036031
7	0.001448669
8	0.000582455
9	0.000234183
10	9.4156E-05
11	3.78565E-05
12	1.52207E-05
13	6.11965E-06
14	2.46048E-06
15	9.89264E-07
16	3.97745E-07
17	1.59918E-07
18	6.4297E-08
19	2.58514E-08
20	1.03939E-08
21	4.17897E-09
22	1.68021E-09
23	6.75546E-10
24	2.71611E-10
25	1.09205E-10

Range Name	Cells
L	G4
Lambda	C4
Lq	G5
Mu	C5
n	F13:F38
P0	G13
Pn	G13:G38
Rho	G10
s	C6
Time1	C9
Time2	C12
W	G7
Wq	G8