Unit :- 04

PDA :-

PDA :-
$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

δ :- It is a mapping from $Q \times (\Sigma \cup \{\epsilon\}) \times \Gamma^*$ to the finite subset of $Q \times \Gamma^*$.

Q1. Design PDA for CFL.

$L = \{ wcw^R \mid w \in (a,b)^* \}$

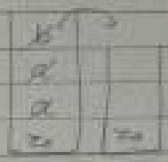
let $w = aab$
$w^R = baa$

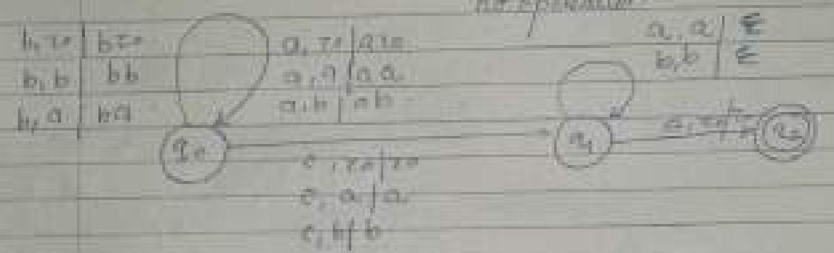
separator

$\underbrace{a\,ab}_{push}$ $\overset{\downarrow}{c}$ $\underbrace{baa}_{pop}$ ϵ

w w^R

no operation





push - 2rd

let we consider a string

$\underbrace{abbaa}_{w}\ \overset{\downarrow}{c}\ \underbrace{aabba}_{w^R}$

Accept

$$\delta(q_0, a, z_0) = (q_0, az_0)$$
$$\delta(q_0, b, a) = (q_0, ba)$$
$$\delta(q_0, a, b) = (q_0, ab)$$
$$\delta(q_0, b, z_0) = (q_0, bz_0)$$
$$\delta(q_0, b, b) = (q_0, bb)$$
$$\delta(q_0, b, a) = (q_0, ba)$$

} push operation for 'w' i/p

$$\delta(q_0, c, z_0) = (q_1, z_0)$$
$$\delta(q_0, c, a) = (q_1, a)$$
$$\delta(q_0, c, b) = (q_1, b)$$

} skip operation for 'c' input

$$\delta(q_1, a, a) = (q_1, \epsilon)$$
$$\delta(q_1, b, b) = (q_1, \epsilon)$$

} pop operation

$$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$$

skip/ No operation for ϵ-input [final state] acceptance

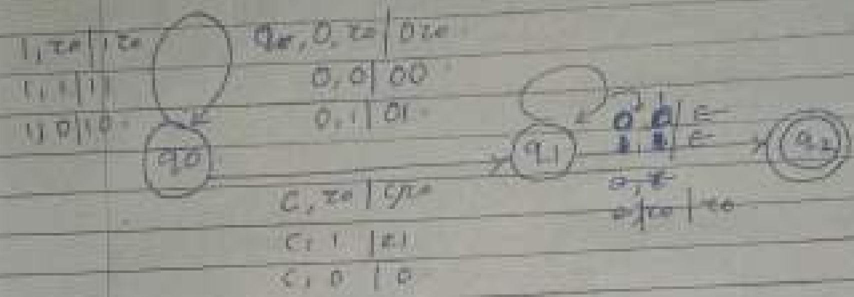
As the string is accepted because the stack become empty

Q1. $L = \{wcw^R \mid w \in (0+1)^*\}$

Sol^n: Let we consider

$w = 0011c1100$

$\therefore L = 0011c1100$



$(q_0, 0, z_0) = (q_0, 0z_0)$
$(q_0, 0, 0) = (q_0, 00)$
$(q_0, 0, 1) = (q_0, 01)$
$(q_0, 1, z_0) = (q_0, 1z_0)$
$(q_0, 1, 1) = (q_0, 11)$
$(q_0, 1, 0) = (q_0, 10)$
} push

$(q_0, c, z_0) = (q_1, z_0)$
$(q_0, c, 0) = (q_1, 0)$
} pop

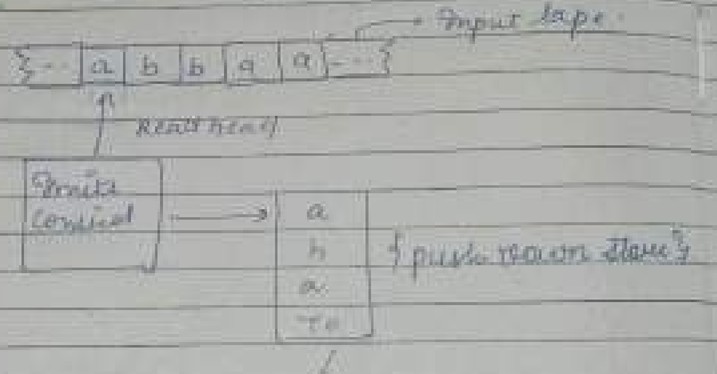
$(q_0, c, 1) = (q_1, 1)$

$(q_1, a, a) = (q_1, \epsilon)$
$(q_1, b, b) = (q_1, \epsilon)$
$(q_1, \epsilon, z_0) = (q_2, z_0)$ } skip operation for i/p (final state)

Q2. Explain the model of PDA and its acceptance by stack and acceptance by final state.

Ans:- i] Push down automata is an extension of finite automata with control of both an input tape and a stack.

ii] PDA has input tape, finite control and a stack.



Push down store

$\delta :- Q \times (\Sigma \times \Gamma) \rightarrow Q \times \Gamma^*$
$\delta(q_0, a, a) = (q_0, aa)$ push
$\delta(q_0, a, a) = (q_0, \epsilon)$ pop

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

Q = It is the set of states

Σ = { set input symbols }

Γ = It is a set of stack symbols

q_0 = It is initial state in Q

F = is a set of final states. $F \subseteq Q$

z_0 = It is initial symbol of stack. It is in Γ

δ :- It is a mapping. $Q \times \Sigma \times \Gamma \to Q \times \Gamma^*$

- meanings & Representation :-

$\delta(q_0, a, z_0) = (q_0, az_0) = \{(P_1, \gamma_1), (P_2, \gamma_2) \ldots (P_n, \gamma_n)\}$

the meaning of the statement in PDA is in state q_0 reading the input symbol a with z_0 the topmost symbol on PDA can enter any state q_i and replace z_0 by γ_i to advance the input head by 1 symbol.

i) Acceptance by stack :-

Let $A = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$ be a PDA. The set of string N(A) N(A) accepted by null stack is defined by

$\{ w \in \Sigma^* \mid (q_0, w, z_0) \vdash^* (q, \epsilon, \epsilon)$ for some $q \in Q \}$

$$w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q_f, \epsilon, \alpha)$$
$$\vdash^* (q_n, \epsilon, \epsilon)$$

(i) Acceptance by final state :-
Let $A = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$$T(A) = \{ w \in \Sigma^* \mid (q_0, w, Z_0) \vdash^* (q_f, \epsilon, \alpha)$$

for some $\{ q_f \in F$ & $\alpha \in \Gamma^* \}$

- The language accepted by PDA is a set of all input for which some sequence of move cause the PDA to enter a final state.
- —"— to empty its stack.

Q2. Design an NPDA for $L = \{ ww^R \mid w \in \{a,b\}^* \}$ even Palindrome

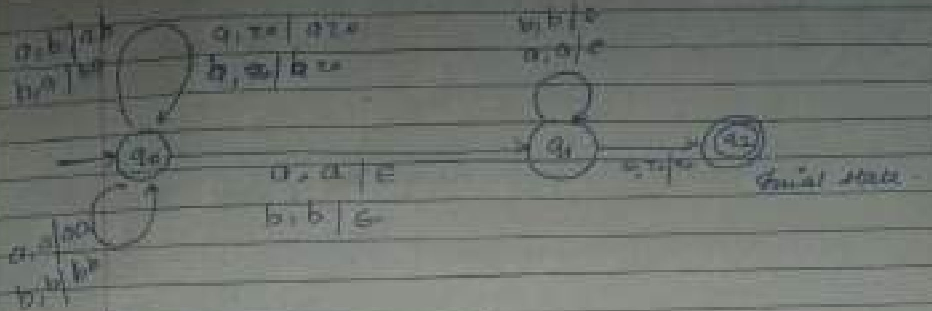
Sol^n let $w = abab$
$w^R = baba$

$L = \{ \epsilon, aa, bb, abba, baab, aabbbbaa, ababbaba$

push pop
a a a a

push pop
ww^R = a ab | b a a

There is no way of finding the center position anyone center position is fixed no on PDA.



$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, b, z_0) = (q_0, bz_0)$
$\delta(q_0, a, a) = (q_0, aa)\ (q_1, \epsilon)$ } NPDA : Multiple
$\delta(q_0, b, b) = (q_0, bb)\ (q_1, \epsilon)$ } moves under a situation.
$\delta(q_0, a, b) = (q_0, ab)$
$\delta(q_0, b, b) = (q_0, bb)$
$\delta(q_1, a, a) = (q_1, \epsilon)$
$\delta(q_1, b, b) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_2, z_0)$

<u>Instantaneous description</u> for string aaaa.

aa | aa

$(q_0, aaaa, z_0) = (q_0, aaa, az_0)$ { before center }
$= (q_0, aa, aaz_0)$ { after center }
$= (q_0, a, az_0)$
$= (q_0, az_0)$

CFG - (UTPD)

Q4] Convert the given PDA to CFG :-

Soln:
$\delta(q_0, a, z_0) \rightarrow (q_0, x z_0)$
$\delta(q_0, a, x) \rightarrow (q_0, xx)$
$\delta(q_0, b, x) \rightarrow (q_1, \epsilon)$
$\delta(q_1, b, x) \rightarrow (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) \rightarrow (q_1, \epsilon)$

For all states :-

$S \longrightarrow [q_0, z_0, q_0]$ } starting with
$S \longrightarrow [q_0, z_0, q_1]$ } initial state

Production rules.

For first move :- $\delta(q_0, a, z_0) \rightarrow (q_0, x z_0)$

For move :- $\delta(q_0, a, z_0) = (q_0, x z_0)$

$[q_0, z_0, q_0] \longrightarrow a\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_0]$
$[q_0\ z_0\ q_0] \longrightarrow a\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_0]$
$[q_0\ z_0\ q_1] \longrightarrow a\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_1]$
$[q_0\ z_0\ q_1] \longrightarrow a\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_1]$

For move $\delta(q_0, a, x) \longrightarrow (q_0, xx)$

$[q_0\ x\ q_0] \rightarrow a\ [q_0\ x\ q_0]\ [q_0\ x\ q_0]$
$[q_0\ x\ q_0] \rightarrow a\ [q_0\ x\ q_1]\ [q_1\ x\ q_0]$
$[q_0\ x\ q_1] \rightarrow a\ [q_0\ x\ q_0]\ [q_0\ x\ q_1]$
$[q_0\ x\ q_1] \rightarrow a\ [q_0\ x\ q_1]\ [q_1\ x\ q_1]$

- For move $\delta(q_0, b, x) \rightarrow (q_1, \epsilon)$

$\delta(q_0, b, x) \longrightarrow (q_1, \epsilon)$

$[q_0 \; x \; q_0] \longrightarrow b \; [q_1$

- For move $(q_1, b, x) \longrightarrow (q_1, \epsilon)$

$\delta(q_1, b, x) \longrightarrow (q_1, \epsilon)$

$[q_1 \; x \; q_1] \longrightarrow b\epsilon$

- For move $(q_1, \epsilon, z_0) \rightarrow (q_1, \epsilon)$

$[q_1 \; z_0 \; q_1] \longrightarrow \epsilon$

Rename :-

$[q_0 \; z_0 \; q_0] = A$ $[q_0, x, q_0] = E$

$[q_0 \; z_0 \; q_1] = B$ $[q_0, x, q_1] = F$

$[q_1 \; z_0 \; q_0] = C$ $[q_1, x, q_1] = H$

$[q_1 \; z_0 \; q_1] = D$ $[q_1, x, q_0] = G$

$S \longrightarrow A \mid B \mid$

$S \rightarrow B$

$A \longrightarrow$

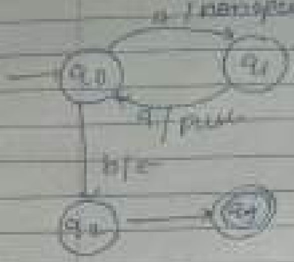
Q5. Construct PDA for $L = \{a^{2n} b^n \mid n \geq 0\}$.

Soln- let n=1, 2, 3

$L = \{aab,\ aaaabb,\ aaaaaabbb,\ \ldots\}$

So $\underline{aa}\,\underline{aa}\,\underline{aa}\ \underline{bbb}$

$\cancel{a}$
a
a
z_0

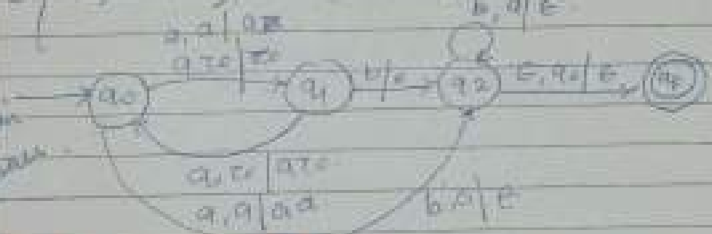


$\delta(q_0, a, z_0) \rightarrow (q_1, az_0)$ - (no op.)
$\delta(q_1, a, z_0) \rightarrow (q_0, az_0)$ push
$\delta(q_0, a, a) \rightarrow (q_1, a)$ - (no op
$\delta(q_1, a, a) \rightarrow (q_1, aa)$. push

$\delta(q_0, b, a) \rightarrow (q_2, \epsilon)$
$\delta(q_2, b, a) \rightarrow (q_2, \epsilon)$
top of stack

$\delta(q_2, \epsilon, z_0) \rightarrow (q_f, \epsilon)$

Transition diagram



let w = aaaabb

$(q_0, aaaabb, z_0) \vdash (q_0, aaabb, z_0)$

$\vdash (q_0, aabb, az_0)$

$\vdash (q_1, abb, az_0)$

$\vdash (q_0, bb, aaz_0)$

$\vdash (q_0, bb, az_0)$

$\vdash (q_f, \epsilon, z_0)$

$\vdash (q_f, z_0)$

$(q_0, aaaabb, z_0) \vdash^* (q_f, z_0)$.

Hence the string is accepted by PDA.

Q6. Convert the following CFG into PDA.

(1) $E \to aAB \mid a$

$A \to BA \mid a$

$B \to Ead \mid c$

Solⁿ:- ~~PDA~~ will be

$\delta(q, a, E) \to \delta(q, A)$

- For non terminal symbol

$A \to \alpha$

$\delta(q, \epsilon, A) \to (q, \alpha)$

- For terminal 'a'

$\delta(q, a, a) \to (q, \epsilon)$

$V = \{E, A, B\} \rightarrow$ {non terminal}
$\Sigma = \{a, c, d\} \rightarrow$ {terminal}

PDA

$\delta(q, \epsilon, E) \rightarrow \{(q, aAB), (q, a)\}$

$\delta(q, \epsilon, A) \rightarrow \{(q, BA), (q, a)\}$

$\delta(q, \epsilon, B) \rightarrow \{(q, Bad), (q, c)\}$

$\delta(q, a, a) \rightarrow (q, \epsilon)$

$\delta(q, c, c) \rightarrow (q, \epsilon)$

$\delta(q, d, d) \rightarrow (q, \epsilon)$

Q 8. Construct PDA for given grammar :-

$E \rightarrow +EE \mid *EE \mid \$TF$

$T \rightarrow +T \mid T$

$F \rightarrow *F \mid T$

- $A \rightarrow \alpha$
- $\delta(q, \epsilon, A) \rightarrow (q, \alpha)$
- For terminal a

$(q, +, E) \rightarrow \{(q, EE)\}$
$(q, *, E) \rightarrow \{(q, EE)\}$
$(q, \$, E) \rightarrow \{(q, TF)\}$
$(q, +, T) \rightarrow \{\{q, T\}\}$
$(q, +, T) \rightarrow (\{q, \epsilon\})\}$
$(q, *, F) \rightarrow \{\{q, F\}\}$
$(q, *, F) \rightarrow (\{q, \epsilon\})\}$

Q3. Construct CFG from following PDA

$\delta(q_0, 1, z_0) \rightarrow (q_0, xz_0)$

$\delta(q_0, 1, x) \rightarrow (q_0, xx)$

$\delta(q_0, 0, x) \rightarrow (q_1, x)$

$\delta(q_0, \epsilon, z_0) \rightarrow (q_0, \epsilon)$

$\delta(q_1, 1, x) \rightarrow (q_1, \epsilon)$

$\delta(q_1, 0, z_0) \rightarrow (q_0, z_0)$

$S \rightarrow [q_0\ z_0\ q_0]$

$S \rightarrow [q_0\ z_0\ q_1]$

Soln. • for move $\delta(q_0, 1, z_0) = (q_0, xz_0)$

$[q_0\ z_0\ q_0] \rightarrow 1\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_0]$

$[q_0\ z_0\ q_0] \rightarrow 1\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_0]$

$[q_0\ z_0\ q_1] \rightarrow 1\ [q_0\ x\ q_0]\ [q_0\ z_0\ q_1]$

$[q_0\ z_0\ q_1] \rightarrow 1\ [q_0\ x\ q_1]\ [q_1\ z_0\ q_1]$

• for move $\delta(q_0, 1, x) \rightarrow (q_0, xx)$

$[q_0\ x\ q_0] \rightarrow 1\ [q_0\ x\ q_0]\ [q_0\ x\ q_0]$

$[q_0\ x\ q_0] \rightarrow 1\ [q_0\ x\ q_1]\ [q_1\ x\ q_0]$

$[q_0\ x\ q_1] \rightarrow 1\ [q_0\ x\ q_0]\ [q_0\ x\ q_1]$

$[q_0\ x\ q_1] \rightarrow 1\ [q_0\ x\ q_1]\ [q_1\ x\ q_1]$

- for move $(q_0, 0, x) \rightarrow (q_1, x)$

$[q_0 \; x \; q_0] \rightarrow 0 \, [q_1 \; x \; q_0]$
$[q_0 \; x \; q_1] \rightarrow 0 \, [q_1 \; x \; q_1]$

- for move $(q_0, \epsilon, z_0) \rightarrow (q_0, \epsilon)$

$[q_0, z_0, q_0] \rightarrow [q_0, \epsilon$

- for move $(q_1, 1, x) \rightarrow (q_1, \epsilon)$

$[q_1 \; x \; q_1] \rightarrow 1$

- for move $(q_1, 0, z_0) \rightarrow (q_0, z_0)$

$[q_1 \; z_0 \; q_1] \rightarrow 0 \, [q_0$

- for move $(q_1, 0, z_0) \rightarrow (q_0, z_0)$

$[q_1 \; \cancel{z_0}] \rightarrow 0 \, [q_0 \; z_0 \; q_0] \, [q_0$
$[q_1 \; z_0 \; q_0] \rightarrow 0 \, [q_0 \; z_0 \; q_1]$

Remarks:-

Q10:- diff b/w NPDA & DPDA

	NPDA	DPDA
Determinism	Can have multiple possible transitions for a given state and input symbol	Has at-most one transition for a given state and input symbol.
Stack	Can have multiple possible stack operations.	Has a unique stack operation.
Acceptance	Accepts a string if at least one computation path leads to a accepting state	accepts a string if the only computation path leads to a accepting state.
Complexity	Generally more expressive but potentially slower due to non-determinism	Typically simpler and faster due to determinism
Design	allows for more flexibility in handling diff scenarios during computation.	More restrictive but easier to analyze and implement.
Parallelism	can explore diff. computation paths simultaneously.	Lacks the ability to explore multiple paths simultaneously
Closure prop	Closure under union. Concatenation, Kleene star. accepted by NPDA	— | — are not necessarily closed.

Q11. construct PDA for following
$L = \{a^n b^m c^m d^n \mid n, m \geq 1\}$

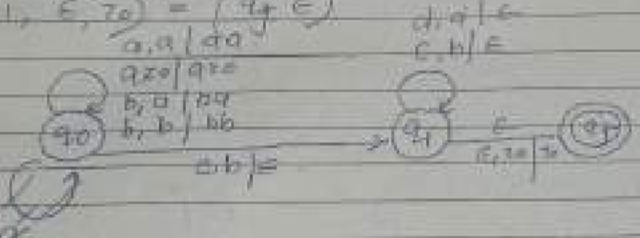
let $n=2, m=3$
aabbbcccdd

$L = \{$ ~~abbbcccdd~~, $\underset{n,m=1}{abcd}$, abbccd, ---- $\}$

c – pop b
d – pop a

$\delta(q_0, a, z_0) = (q_0, az_0)$
$\delta(q_0, a, a) = (q_0, aa)$
$\delta(q_0, b, a) = (q_0, ba)$
$\delta(q_0, b, b) = (q_0, bb)$
$\delta(q_0, c, b) = (q_1, \epsilon)$

$\delta(q_1, c, b) = (q_1, \epsilon)$
$\delta(q_1, d, a) = (q_1, \epsilon)$
$\delta(q_1, \epsilon, z_0) = (q_f, \epsilon)$



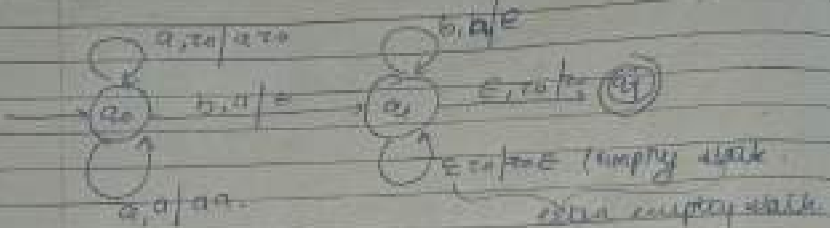
let $(q_0, aabbbcccdd, z_0) = (q_0, abbbcccdd, az_0)$
$= (q_0, bbbcccdd, aaz_0)$
$= (q_0, bbcccdd, baaz_0)$
$= (q_0, bcccdd, bbaaz_0)$
$= (q_1, cccdd, bbbaaz_0)$
$= (q_1, ccdd, bbaaz_0)$
$= (q_1, \epsilon, z_0) = (q_f, z_0)$

Q12. $L = \{a^n b^n, n \geq 1\}$ construct PDA [illegible]
& to CFG

$L = \{ab, aabb, aaabbb, \ldots\}$

~~$q_0 / a, \delta(q_0, a, z_0) = (q_0, az_0)$~~

~~$\delta(q_0, a, a) = (q_0, aa)$~~
~~$\delta(q_0, b, a) = (q_0, \epsilon)$~~



$\delta(q_0, a, z_0) = (q_0, az_0)$ push
$\delta(q_0, a, a) = (q_0, aa)$ push
$\delta(q_0, b, a) = (q_1, \epsilon)$ pop
$\delta(q_1, b, a) = (q_1, \epsilon)$ pop
$\delta(q_1, \epsilon, z_0) = (q_f, z_0)$ skip
~~$\delta(q_1,$~~

PDA → final state
→ empty stack

$\delta(q_1, \epsilon, z_0) = (q_1, \epsilon)$
Acceptance by empty stack.

Q13. Design a PDA from CFG.

$S \longrightarrow aSa \mid aSb \mid a$

Sol^n

$(q, a, S) \longrightarrow (q, aSa)$

(q, ϵ, a)

1. $\delta(q_0, \epsilon, \epsilon) = (q_0, \epsilon)$
2. $\delta(q_0, \epsilon, S) = (q_0, aSa)$
3. $\delta(q_0, \epsilon, S) = (q_0, bSb)$
4. $\delta(q_0, \epsilon, S) = (q_0, \epsilon)$
5. $\delta(q_0, a, a) = (q_1, \epsilon)$
6. $\delta(q_1, b, b) = (q_2, \epsilon)$
7. $\delta(q_2, \epsilon, \epsilon) = (q_3, \epsilon)$