

Unit-3)

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Q-1) Find optimal knapsack using greedy strategy
 $n=7, m=15$

$$O = \{1, 2, 3, 4, 5, 6, 7\}$$

$$\text{Weight} = \{2, 3, 5, 7, 1, 4, 1\}$$

$$\text{Profit} = \{10, 5, 15, 7, 6, 18, 3\}$$

Step-1): Compute for unit weight cost
$$p(i) = \frac{p(i)}{w(i)}$$

O	O ₁	O ₂	O ₃	O ₄	O ₅	O ₆	O ₇
w	2	3	5	7	1	4	1
p	10	5	15	7	6	18	3
$p(i) = \frac{p(i)}{w(i)}$	5	1.6	3	1	6	4.5	3

Step-2) Sort or arrange object in descending order with ratio

O	O ₅	O ₁	O ₆	O ₃	O ₇	O ₂	O ₄
$p(i) = \frac{p(i)}{w(i)}$	6	5	4.5	3	3	1.6	1
p(i)	6	10	18	15	3	5	7
w(i)	1	2	4	5	1	3	7

Step-3): Pack item/objects into greedy strategy
 $i \leftarrow 1$

~~while (w > 0)~~ while (w > 0)
do amt $\leftarrow \min(w, w(i))$
solution(i) \leftarrow amt
 $i \leftarrow i++$
return solution

\uparrow $m=15$ \downarrow	O_2	2	15
	O_7	1	13
	O_3	5	12
	O_6	4	7
	O_1	2	3
	O_5	1	1

$$\text{Total Profit} = (6 \times 1) + (5 \times 2) + (4.5 \times 4) + (3 \times 5) + (3 \times 1) + (1.6 \times 2)$$

$$\boxed{\text{Total Profit} = 55.2}$$

Q-2) Solve following using partial Knapsack.
 $p = \{10, 15, 20, 16, 9\}$
 $w = \{2, 8, 6, 5, 3\}$ $\eta = 5, M = 15$

Step-1) Compute per unit weight cost

I	I_1	I_2	I_3	I_4	I_5
w	2	8	6	5	3
p	10	15	20	16	9
p/w	5	1.8	3.3	3.2	3

Step-2) Sort Items in descending order with ratio

I	I_1	I_3	I_4	I_5	I_2
p/w	5	3.3	3.2	3	1.8
p	10	20	16	9	15
w	2	6	5	3	8

Step-3) Pack items into knapsack into greedy strategy

$i \leftarrow 1$

while ($w > 0$)

do $amt \leftarrow \min(w, w(i))$

$solution(i) \leftarrow amt$

$i++$

return solution

$m=15$ \uparrow \downarrow	I ₅	2	(15)
	I ₄	5	13
	I ₃	6	8
	I ₂	2	2
	I ₁	2	2

$$\text{Total Profit} = (5 \times 2) + (3 \cdot 3 \times 6) + (3 \cdot 2 \times 5) + (3 \times 2)$$

$$\boxed{\text{Total Profit} = 51.8}$$

Q-4) Find the optimal solution to the fractional knapsack if the knapsack capacity $w=60\text{kg}$.

Item	I ₁	I ₂	I ₃	I ₄	I ₅
weight	5	10	15	22	25
Cost	30	40	45	77	90

Step-1 Compute per unit weight cost

Item	I_1	I_2	I_3	I_4	I_5
w	5	10	15	22	25
p	30	40	45	77	90
p/w	6	4	3	3.5	3.6

Step-2 Sort them in Descending order of ratio (p/w)

Item	I_1	I_2	I_3	I_5	I_4
p/w	6	4	3	3.6	3.5
p	30	40	45	90	77
w	5	10	15	25	22

Step-3 Pack Item into Knapsack into Greedy Strategy

$i \leftarrow 1$

while ($w > 0$)

do $\text{amt} \leftarrow \min(w, w_i)$

$\text{solution}[i] \leftarrow \text{amt}$

$i++$

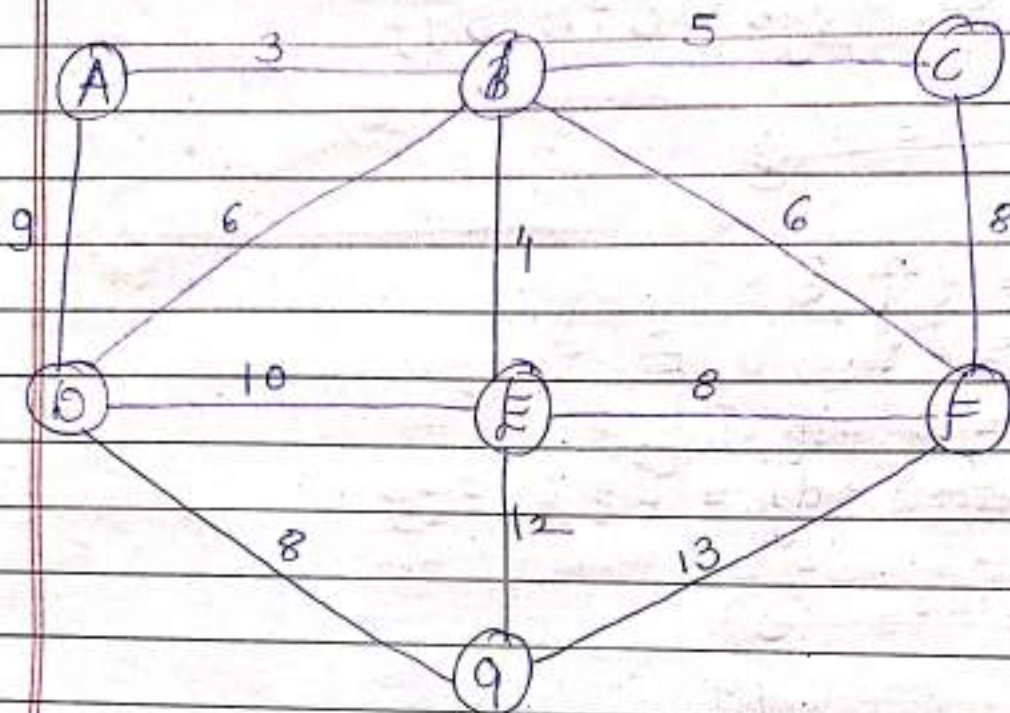
return solution

\uparrow	I_4	5	
	I_5	25	60
$w=15$	I_3	15	55
	I_2	10	30
\downarrow	I_1	5	15
			5

$$\begin{aligned} \text{Total Profit} &= (6 \times 5) + (4 \times 10) \\ &+ (3 \times 15) + (3.6 \times 25) + \\ &+ (3.5 \times 5) \end{aligned}$$

$$\boxed{\text{Total Profit} = 222.5}$$

Q-5) what is Minimum Cost Spanning Tree?
Find Minimum Cost spanning Tree for following using Prim's algorithm?



Ans: A Minimum Cost Spanning Tree is like the most efficient way to connect a bunch of places together with roads.

A Minimum Cost Spanning Tree (MST) or a minimum weight spanning tree for a weighted, connected, undirected graph is a spanning tree with a less weight of every other spanning tree.

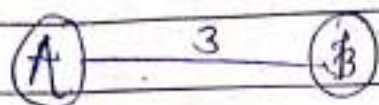
→ Step 1

$$S = \{A\}$$

$$R = \{B, C, D, E, F, G\}$$

$$A = \{ \}$$

$$\text{lightest edge} = \{ \{A, B\} \}$$



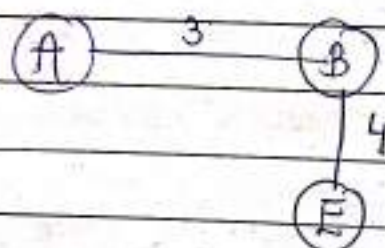
Step 2

$$S = \{A, B\}$$

$$R = \{C, D, E, F, G\}$$

$$A = \{ \{A, B\} \}$$

$$\text{lightest edge} = \{ \{B, E\} \}$$



Step 3

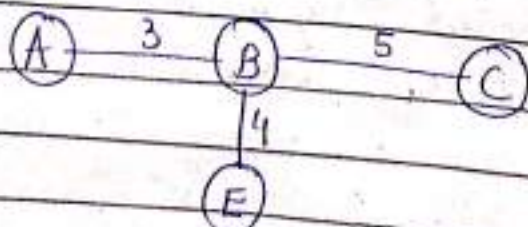
$$S = \{A, B, E\}$$

$$R = \{C, D, F, G\}$$

$$A = \{ \{A, B\}, \{B, E\} \}$$

$$\text{lightest edge} = \{ \{B, C\} \}$$

Step 4

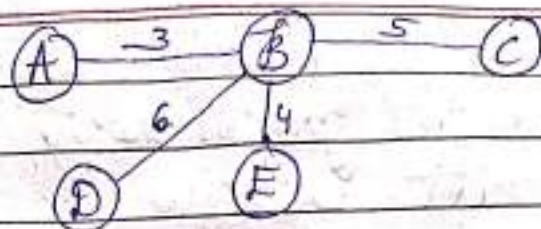


$$S = \{A, B, E, C\}$$

$$R = \{D, F, G\}$$

$$A = \{ \{A, B\}, \{B, E\}, \{B, C\} \}$$

$$\text{lightest edge} = \{ \{B, D\} \}$$

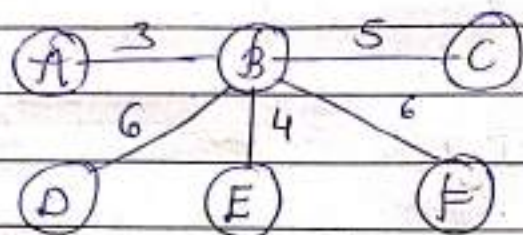
Step-5)

$$S = \{A, B, C, D, E\}$$

$$R = \{F, G\}$$

$$A = \{\{A, B\}, \{B, E\}, \{B, C\}, \{B, D\}\}$$

$$\text{lightest edge} = \{\{B, F\}\}$$

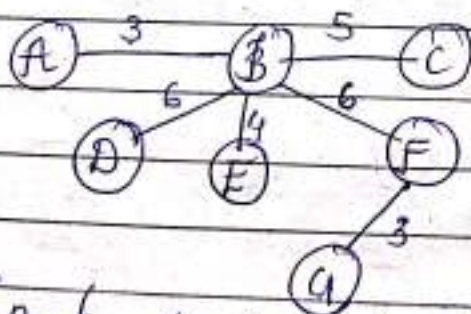
Step-6)

$$S = \{A, B, C, D, E, F\}$$

$$R = \{G\}$$

$$A = \{\{A, B\}, \{B, E\}, \{B, C\}, \{B, D\}, \{B, F\}\}$$

$$\text{lightest edge} = \{\{F, G\}\}$$

Step-7)

$$S = \{A, B, E, C, D, F, G\}$$

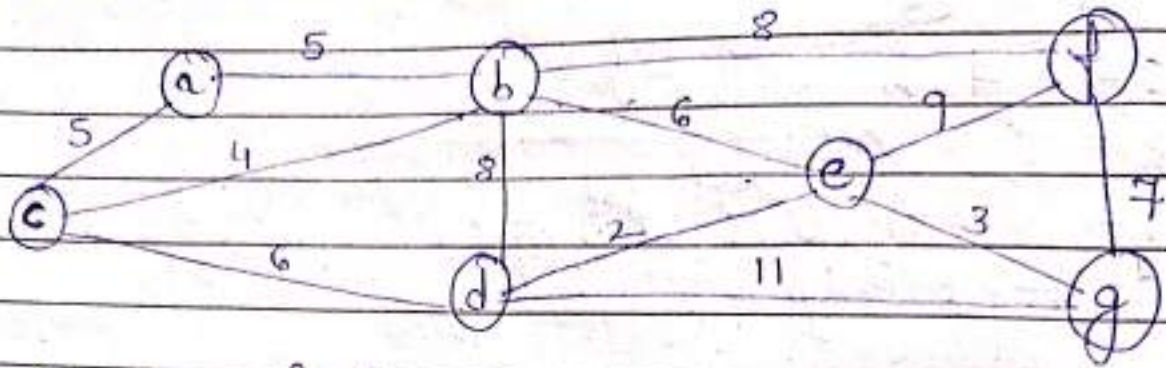
$$R = \{\}$$

$$A = \{\{A, B\}, \{B, E\}, \{B, C\}, \{B, D\}, \{B, F\}, \{F, G\}\}$$

$$\text{Find Cost! } w(A, C) + w(B, E) =$$

$$\text{Min Cost Spanning Tree} = \underline{\underline{27}}$$

Ques → 6) Obtain Minimum Cost Search Spanning with its given undirected graph using Prim's algo Vertex 'a' as a root vertex.

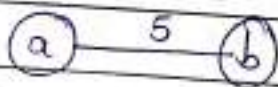


Step-1) $S = \{a\}$

$K = \{b, c, d, e, f, g\}$

$A = \{ \}$

Lightest edge = $\{ \{a, b\} \}$

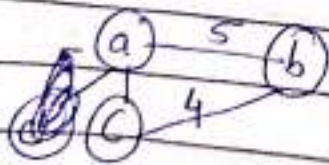


Step-2) $S = \{a, b\}$

$K = \{c, d, e, f, g\}$

$A = \{ \{a, b\} \}$

Lightest edge = $\{ \{b, c\} \}$

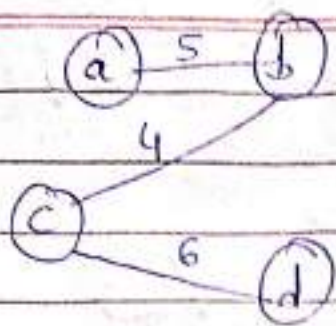


Step-3) $S = \{a, b, c\}$

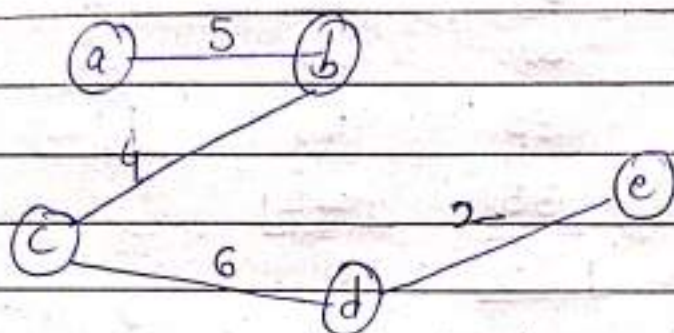
$K = \{d, e, f, g\}$

$A = \{ \{a, b\}, \{b, c\} \}$

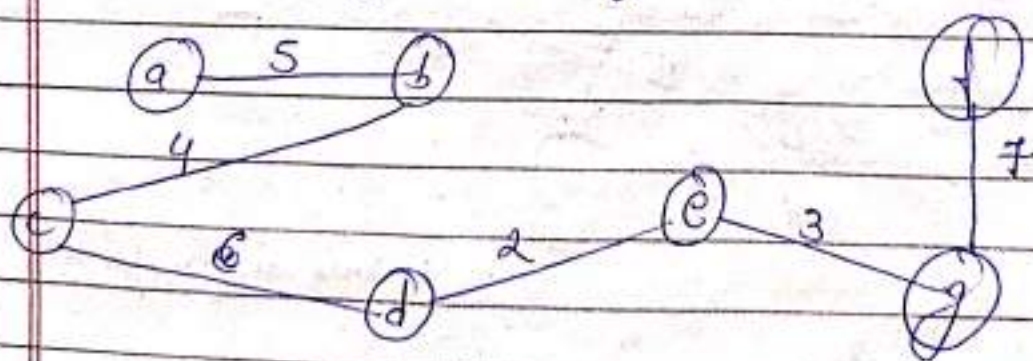
Lightest edge = $\{ \{c, d\} \}$



Step-4) $S = \{a, b, c, d\}$
 $R = \{e, f, g\}$
 $A = \{\{a, b\}, \{b, c\}, \{c, d\}\}$
 Lightest edge = $\{d, e\}$



Step-5) $S = \{a, b, c, d, e\}$
 $R = \{f, g\}$
 $A = \{\{a, b\}, \{b, c\}, \{c, d\}, \{d, e\}\}$
 Lightest edge = $\{e, g\}$



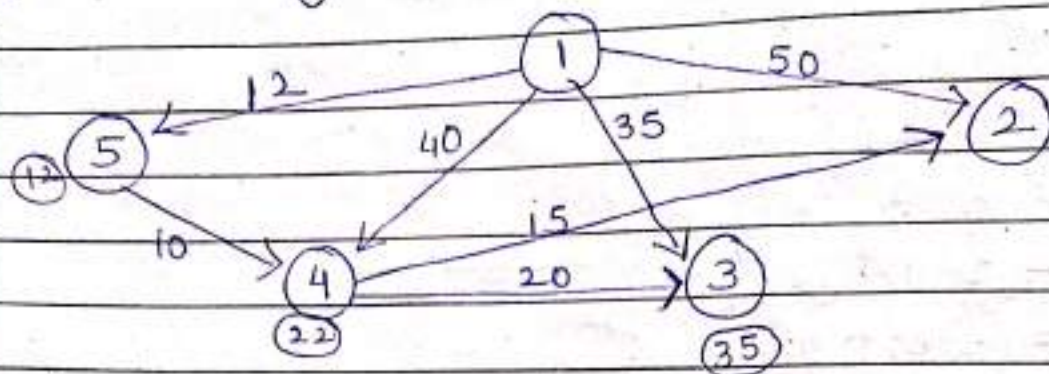
Min Cost of Spanning Tree = 27

Dijkstra's Algo

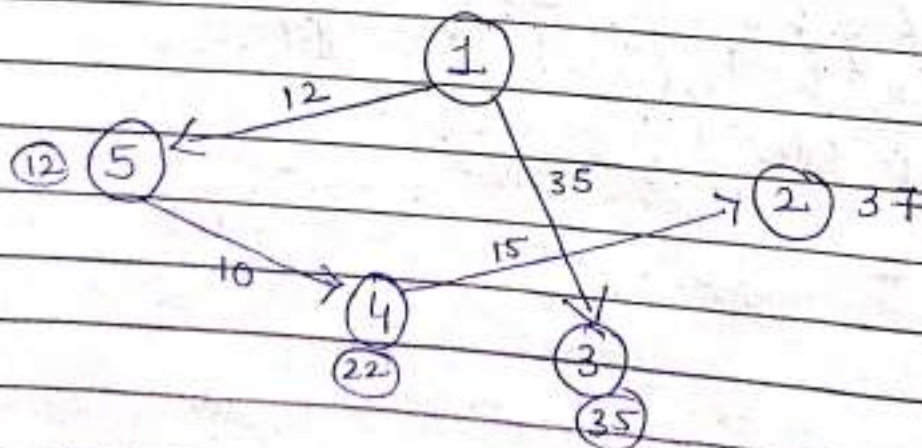
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Q-8) Write Greedy Based single source shortest path algorithm



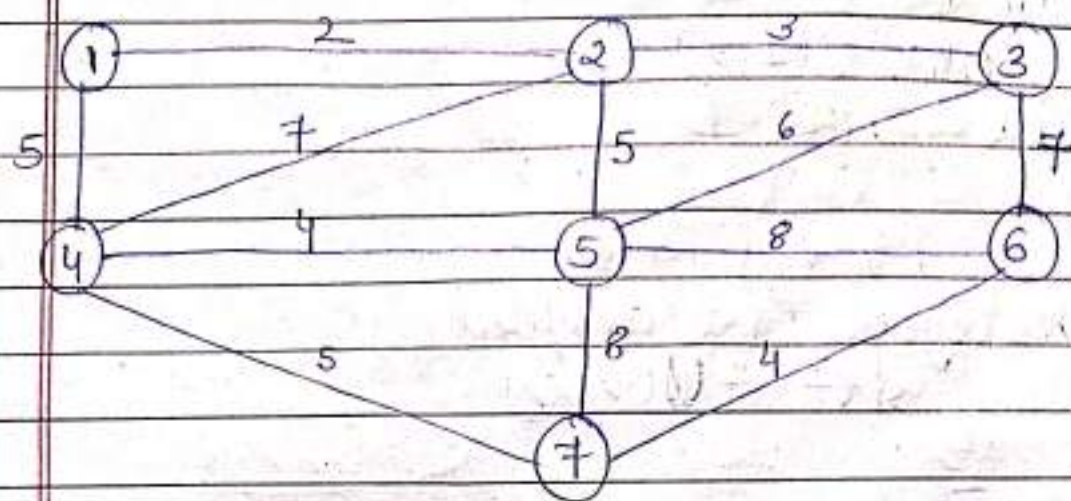
Source	Destination				
1	2	3	4	5	
1	∞	∞	∞	∞	
1, 5	50	35	40	12	
1, 5, 4	50	35	22		
1, 5, 4, 3	37	35			
1, 5, 4, 3, 2	37	1			



Vertex	$d(v)$
1	0
2	37
3	35
4	22
5	12

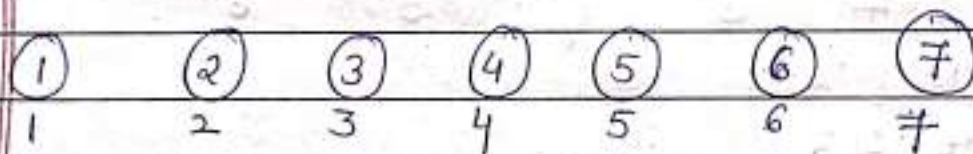
* This is graph

Ques-9) Krushal's Algorithm to find minimum Spanning Tree.



Solⁿ Step-1) Make set of N vertices

① $A \leftarrow \emptyset$



Step-2) Sort edges in increasing order of weight

edges	weight
1 2	2
2 3	3
4 5	4
6 7	4
2 5	5
4 7	5
1 4	5
3 5	6
3 6	7
4 2	7
5 7	8
5 6	8

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Step-3) Construction of Minimal Spanning Tree,

→ for edge 12

$$(u, v) = (1, 2)$$

$$\text{FIND-SET}(1) = 1$$

$$\text{" } (2) = 2$$

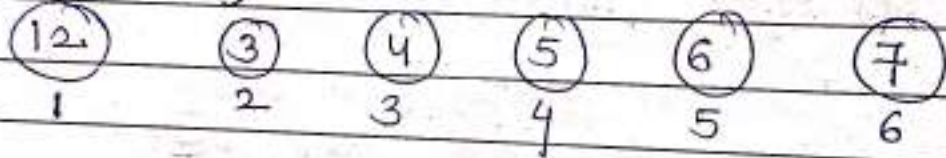
$$1 \neq 2 \text{ (True)}$$

No cycle (1,2) accepted

$$A \leftarrow A \cup (v, v)$$



Union (u, v)



→ for edge 23

$$(u, v) = (2, 3)$$

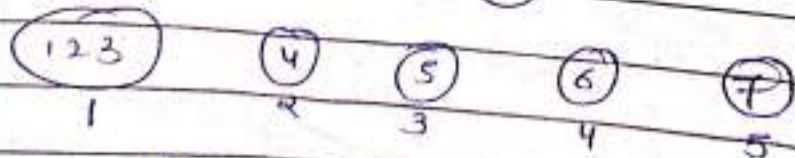
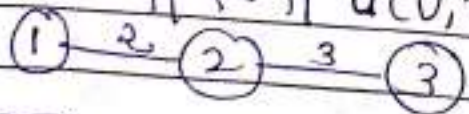
$$\text{FD}(2) = 1$$

$$\text{FD}(3) = 2$$

$$1 \neq 2 \text{ (True)}$$

No cycle (2,3) accepted

$$A \leftarrow A \cup (v, v)$$



→ for edge 45

$$(u, v) = (4, 5)$$

$$\text{FS}(4) = 2$$

$$\text{FS}(5) = 3$$

$$2 \neq 3 \text{ (True)}$$

No cycle (2,3) accepted

→ for edge 67

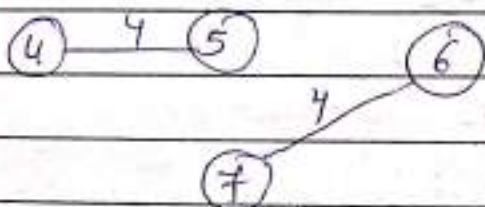
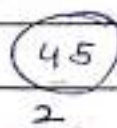
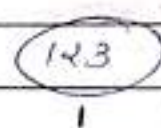
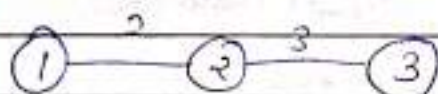
$$(u, v) = (6, 7)$$

$$F(6) = 3$$

$$F(7) = 4$$

$3 \neq 4$ No cycle

67 edge is accepted



→ for edge 25

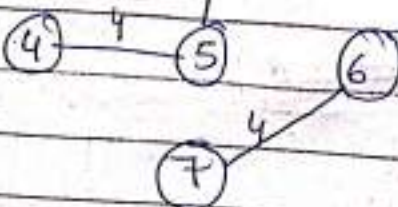
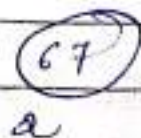
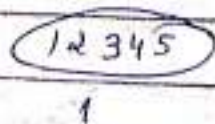
$$(u, v) = (2, 5)$$

$$F_S(2) = 1, F_S(5) = 2$$

$$1 \neq 2$$

(True)

25 is accepted



→ for edge 47

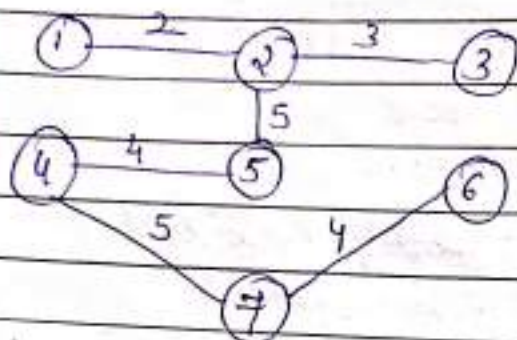
$$(u, v) = (4, 7)$$

$$f_s(4) = 1$$

$$f_s(7) = 2$$

$1 \neq 2$ (True) No cycle

47 is accepted



union

1 2 3 4 5 6 7

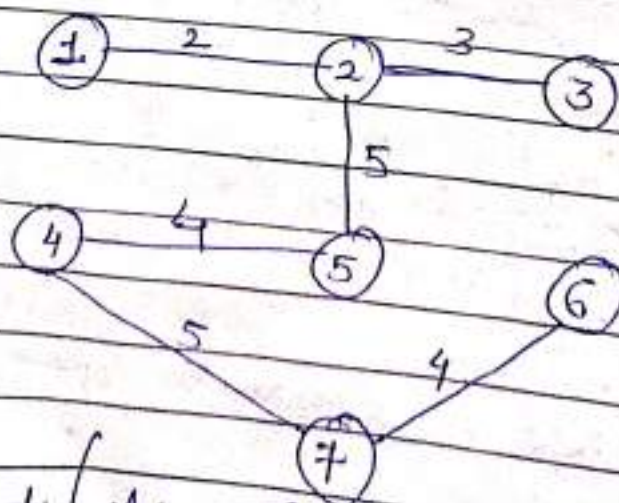
→ for edge 14

$$f_s(1) = 1, f_s(4) = 1$$

$$1 \neq 1 \text{ (false)}$$

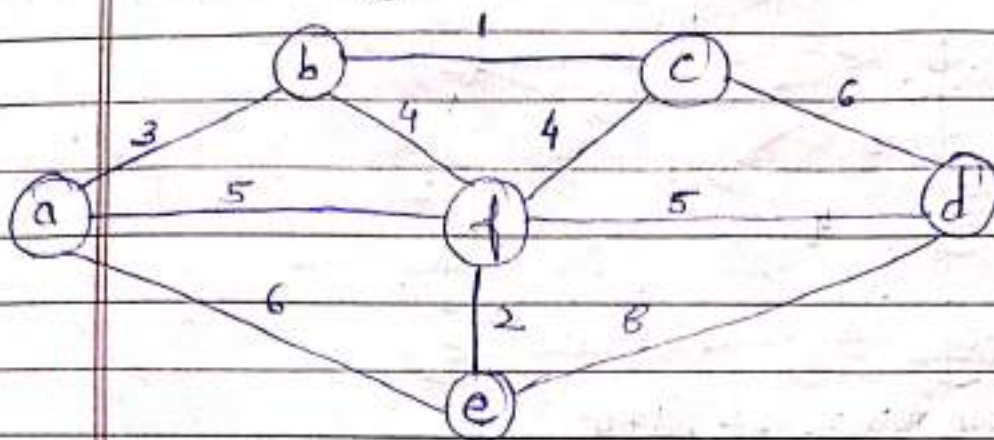
14 rejected

Similarly all the remaining edges are rejected
so final MST.



Find cost of Minimal Cost of spanning Tree = 23

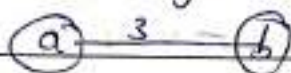
Q-10) Kruskal's algorithm determine Minimum Cost of Spanning Tree



Solⁿ:

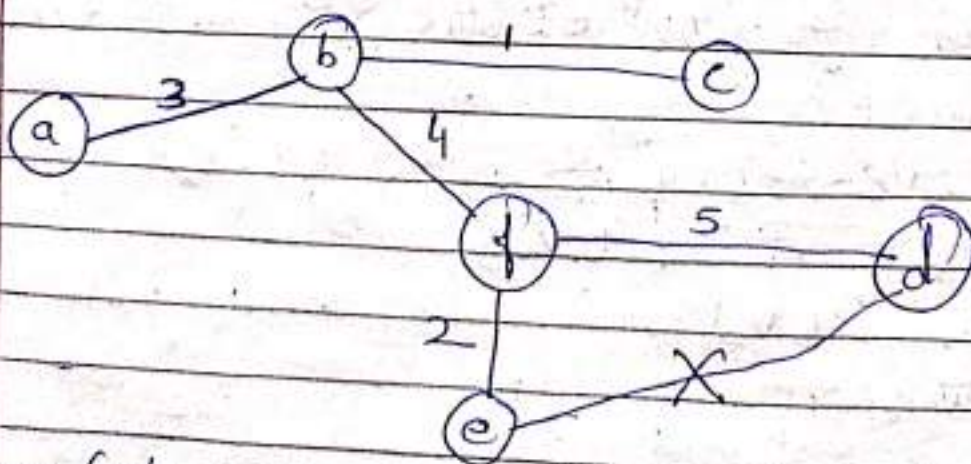
(A)

$S = \{a\}$, $K = \{b, c, d, e, f\}$
 $A = \{\}$ lightest edge = $\{a, b\}$



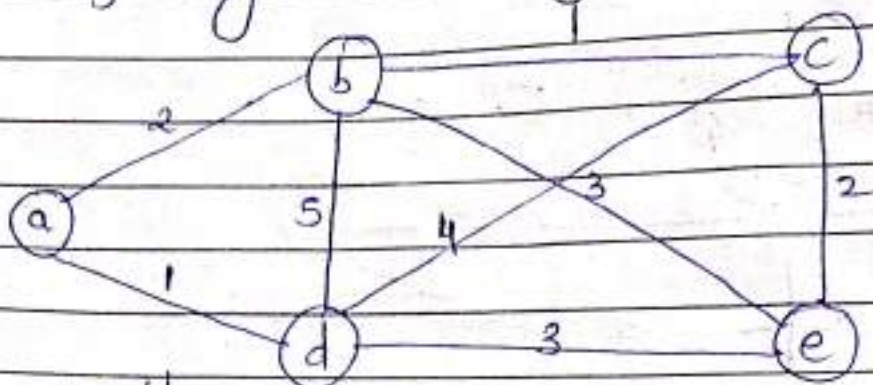
Final step:

$S = \{a, b, c, d, e, f\}$, $K = \{\}$
 $A = \{\{a, b\}, \{b, c\}, \{b, f\}, \{f, d\}, \{f, e\}\}$



Minimum Cost = 15
 of Spanning Tree

Q-11) Write Kruskal's Algorithm to generate Spanning Tree. Apply algo on Graph.



Write Complexity of Algo

Ans: Kruskal's Algorithm:

Step-1) 1) Make set of n vertices
 $a \in \phi$

2) for each vertex $v \in V(G)$

3) Do make set (v)

Step-2) Sort the edges E into Increasing / non-decreasing order by weight w

Step-3) Construct the MST by Merging of Trees

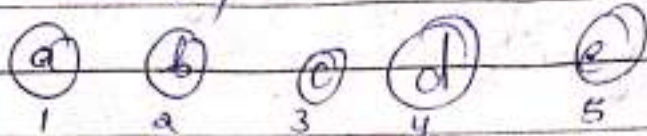
(1) For each edge $(u,v) \in E$, taken in non-decreasing order by weight.

(2) Do if $\text{FIND-SET}(u) \neq \text{FIND-SET}(v)$

(3) Then $A \leftarrow A \cup \{(u,v)\}$

(4) union (u,v)

(5) returns A

Step-1) $a \leftarrow 0$ Step-2)

sort edges

edges

ad

bc

de

ec

weight

1

1

2

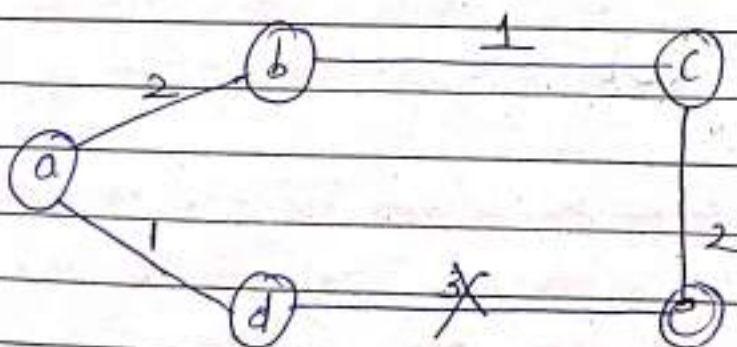
2

3

3

4

5

Steps -> kor lena!

ab cde

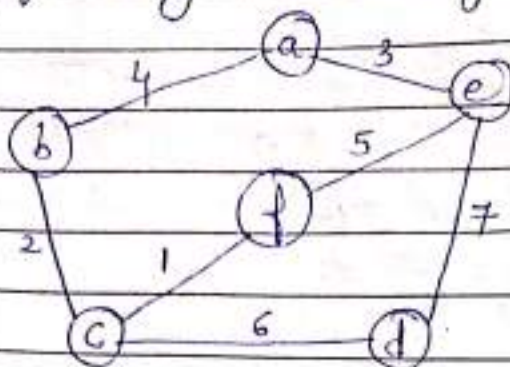
Minimal Cost of Spanning Tree = 6

Time Complexity = $O(E \log V)$

edges

no. of vertices

- (12) Obtain Minimum Spanning Tree for given undirected graph using Kruskal's algorithm.

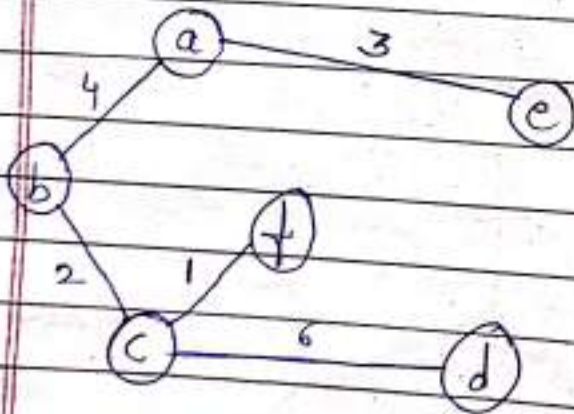


Soln:

$$S = \{a, b, c, d, e, f\}$$

$$k = 1$$

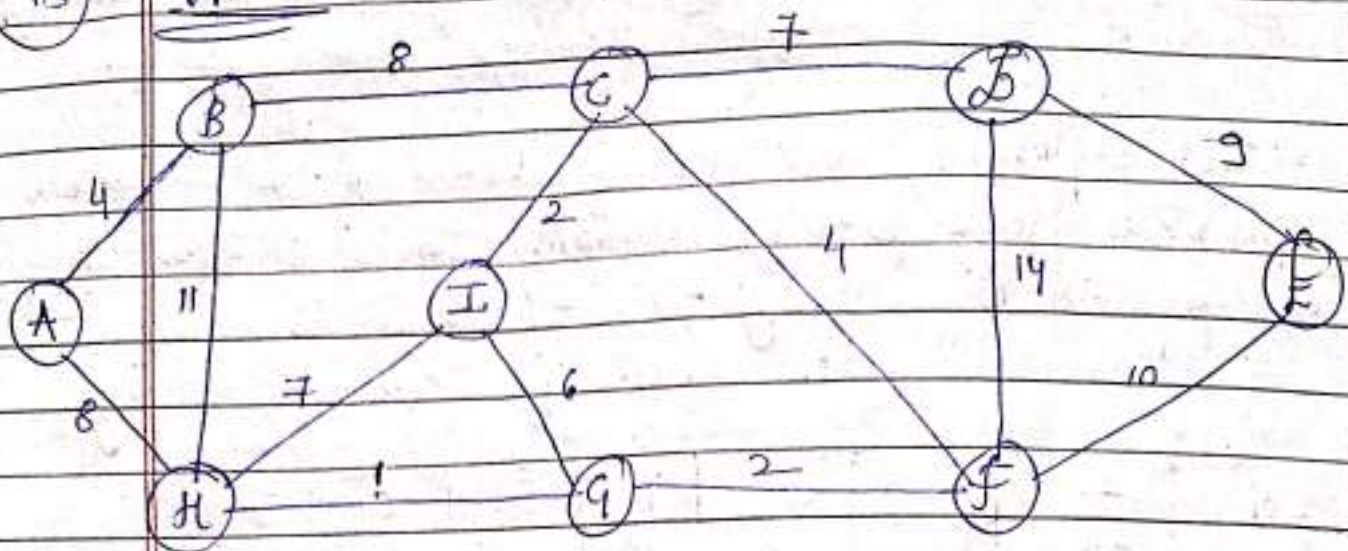
$$A = \{ \{a, b\}, \{a, e\}, \{b, c\}, \{c, f\}, \{c, d\} \}$$



Cost of minimal Spanning Tree = 16

1.3

PRIMS



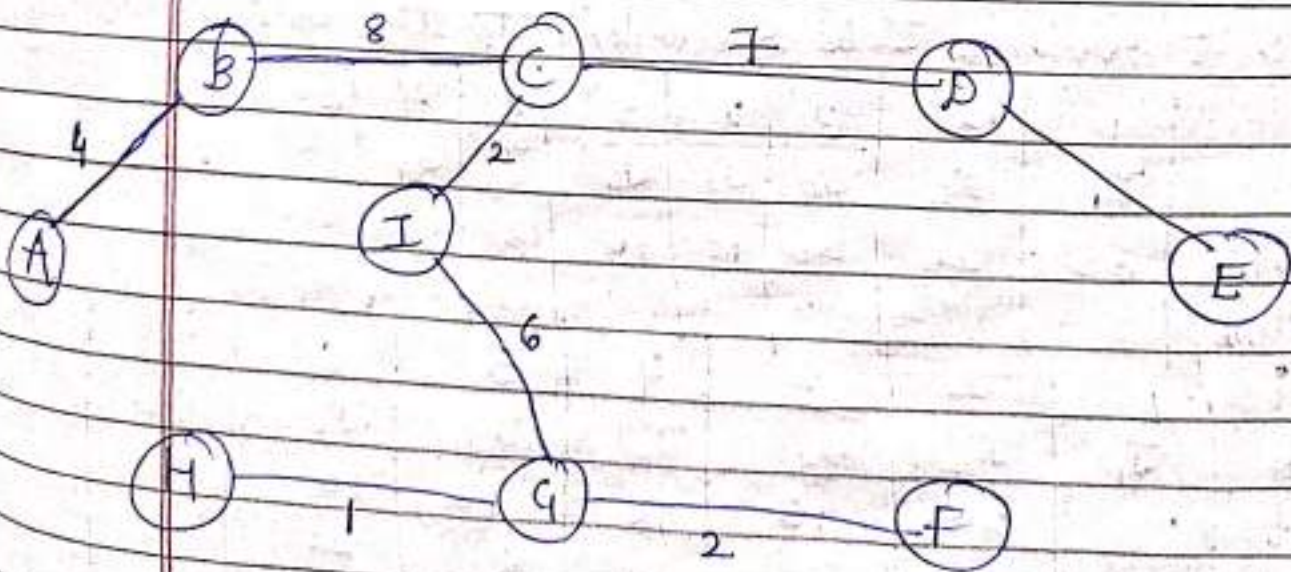
$S = \{A, B, C, D, E, F, G, H, I\}$

$K = \{6\}$

$A = \{ \}$

\emptyset

lightest edge = $\{ \}$



Ques 4: Explain Job Sequencing Approach. Find best possible Sequence

Ans Job Sequencing is a scheduling technique used in various industries to prioritize & manage tasks or jobs. It typically involves assigning priorities to jobs based on certain criteria and then scheduling them accordingly.

Job	1	2	3	4	5	6	7
Gain	35	20	18	16	12	10	8
Deadline	3	1	3	4	2	2	1

Step 1) Label the Jobs accordingly

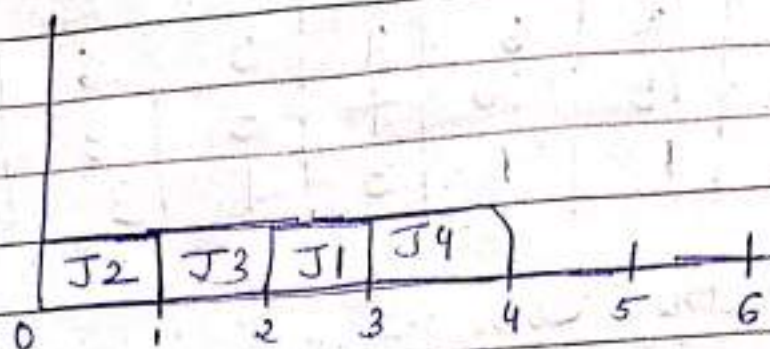
Job	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆	J ₇
Gain	35	20	18	16	12	10	8
Deadline	3	1	3	4	2	2	1

Step 2) Sort / Arrange Jobs According to the Gain in descending order

i	1	2	3	4	5	6	7	Order
Job	J ₁	J ₂	J ₆	J ₃	J ₄	J ₅	J ₇	
Gain	35	20	20	18	16	12		
Deadline								

i	1	2	3	4	5	6	7
Job	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆	J ₇
Gain / Profit	35	20	18	16	12	10	8
Deadline	3	1	3	4	2	2	1

Step 3 Finding set of Job Containing flexible Job to get a maximum profit



J	Assigned slots	Job considered	Action	Profit
	None	J1	Assign [2,3]	0
J1	[2,3]	J2	Assign [0,1]	35
J1, J2	[2,3][0,1]	J3	Assign [1,2]	35 + 20 =
J1, J2, J3	[2,3][0,1][1,2]	J4	Assign [3,4]	55 + 10 =
J1, J2, J3	[2,3][0,1][1,2]	J5	Assign [3,4]	73
			Reject	73

$J_1 + J_2 + J_3 + J_4$

89

Q-16) Explain Job Scheduling approach find the optimal schedule for following Job with $n=6$.

Job(n)	1	2	3	4	5	6
Gain/Profit	20	15	10	7	5	3
Deadline	3	1	1	3	1	3

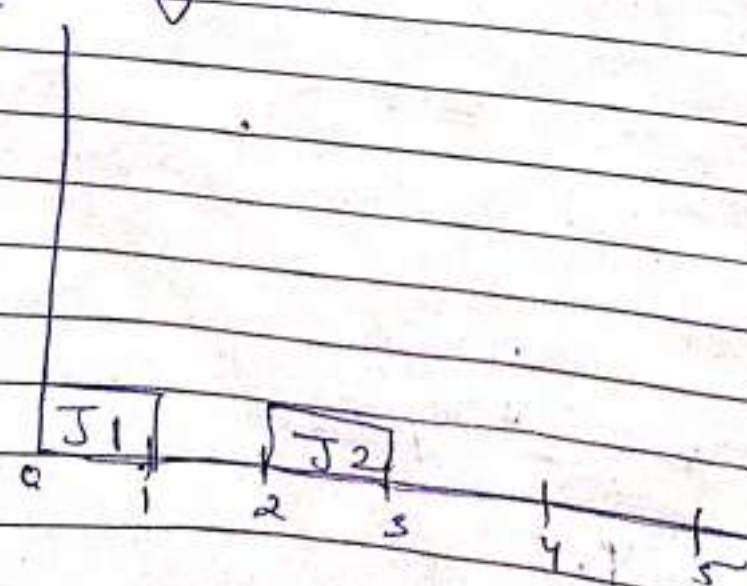
Soln Step-1) Label the Jobs

Job(n)	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
Profit	20	15	10	7	5	3
Deadline	3	1	1	3	1	3

Step-2) Sort in Decreasing order of Profit-

i	1	2	3	4	5	6
Job(n)	J ₁	J ₂	J ₃	J ₄	J ₅	J ₆
Profit	20	15	10	7	5	3
Deadline	3	1	1	3	1	3

Step-3) finding set



J	Assign Slots	Job Considered	Action	Profit
0	None	J1	Assign [2,3]	0
J1	[2,3]	J2	Assign [0,1]	0 + 20 = 20
J1, J2	[2,3] [0,1]	J3	Assign reject	20 + 15 = 35
J1, J2	[2,3] [0,1]	J4	reject	35
		J5	reject	
		J6	reject	
Total Profit = p(J1) + p(J2)				
= 35				

Ques-15) Explain Greedy Algorithm for scheduling problem with Deadline.

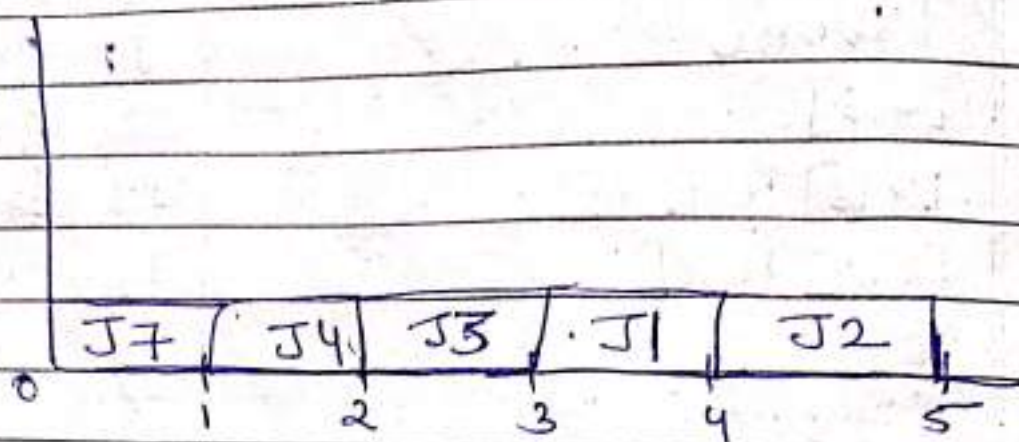
Job	J1	J2	J3	J4	J5	J6	J7	J8
Profit	102	91	39	55	25	18	40	05
Deadline	4	5	3	2	1	5	1	2

Step-1) Label the Jobs

Step-2) Arrange (sort) the Jobs in the Decreasing order of Profit.

i	1	2	3	4	5	6	7	8
Job	J1	J2	J4	J7	J3	J5	J6	J8
Profit	102	91	55	40 40	39	25	18	05
Deadline	4	5	2	1	3	1	5	2

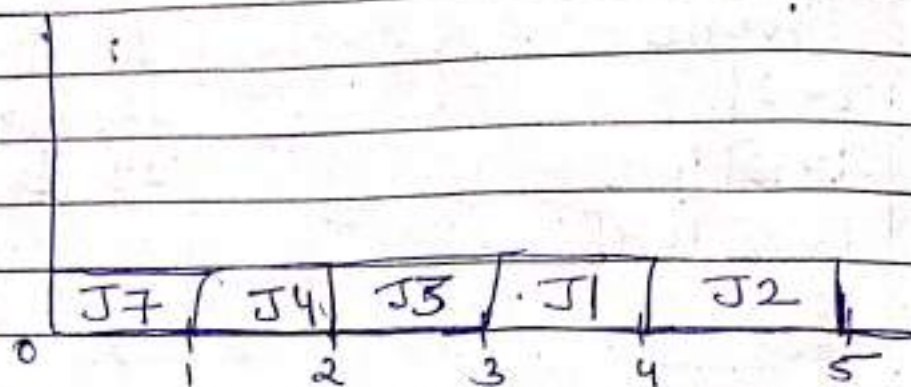
3) Finding set of Job Containing flexible Job to get of maximum profit



Assigned slots	Job Considered	Action	Profit
None	J1	Assign [3, 4]	0
[3, 4]	J2	Assign [4, 5]	102
[3, 4] [4, 5]	J4	Assign [1, 2]	102+91
[3, 4] [4, 5] [1, 2]	J7	Assign [0, 1]	102+91+
[3, 4] [4, 5] [1, 2]	J3	Assign [2, 3]	102+91+40
[3, 4] [4, 5] [1, 2] [2, 3]	J5	Reject	102+91+40+19
S1 S1 S1 S1 S1	Reject	hoga	

$$\begin{aligned}
 \text{Total Profit} &= w(J_1) + w(J_2) + w(J_3) \\
 &\quad + w(J_4) + w(J_4) \\
 &= 327
 \end{aligned}$$

Step-3 Finding set of Job Containing flexible Job to get of maximum profit



J	Assigned slot	Job Considered	Action	Profit
ϕ	None	J1	Assign [3,4]	0
J1	[3,4]	J2	Assign [4,5]	102
J2	[3,4] [4,5]	J4	Assign [1,2]	102+91
J4	[3,4] [4,5] [1,2]	J7	Assign [0,1]	102+91+40
J7	[3,4] [4,5] [1,2]	J3	Assign [2,3]	102+91+40+40
J3	[3,4] [4,5] [1,2] [2,3]	J5	Reject	40+91
			Reject hogi	

$$\text{Total Profit} = W(J_1) + W(J_2) + W(J_3) + W(J_4) + W(J_4)$$

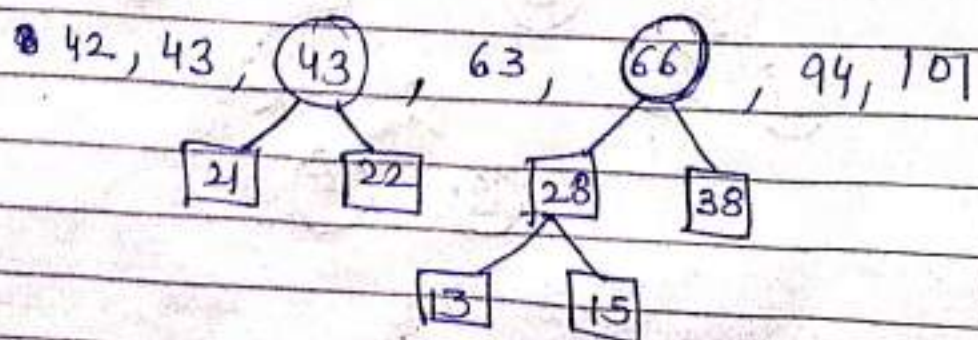
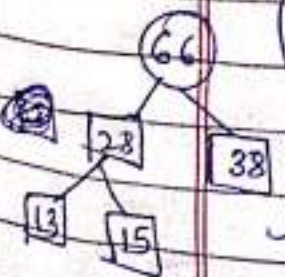
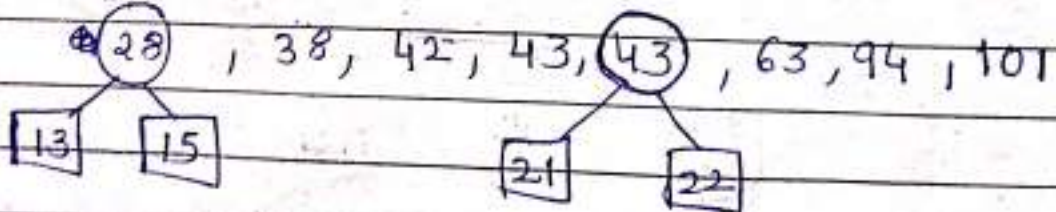
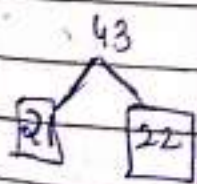
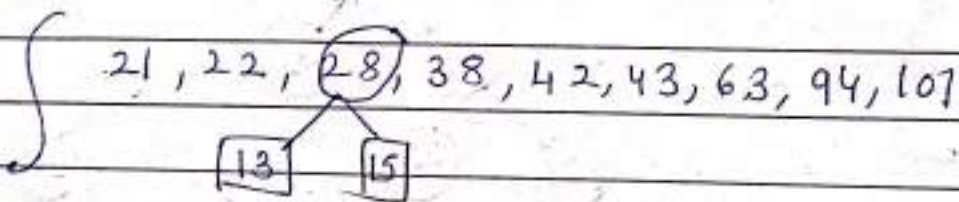
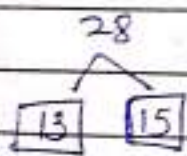
$$= 327$$

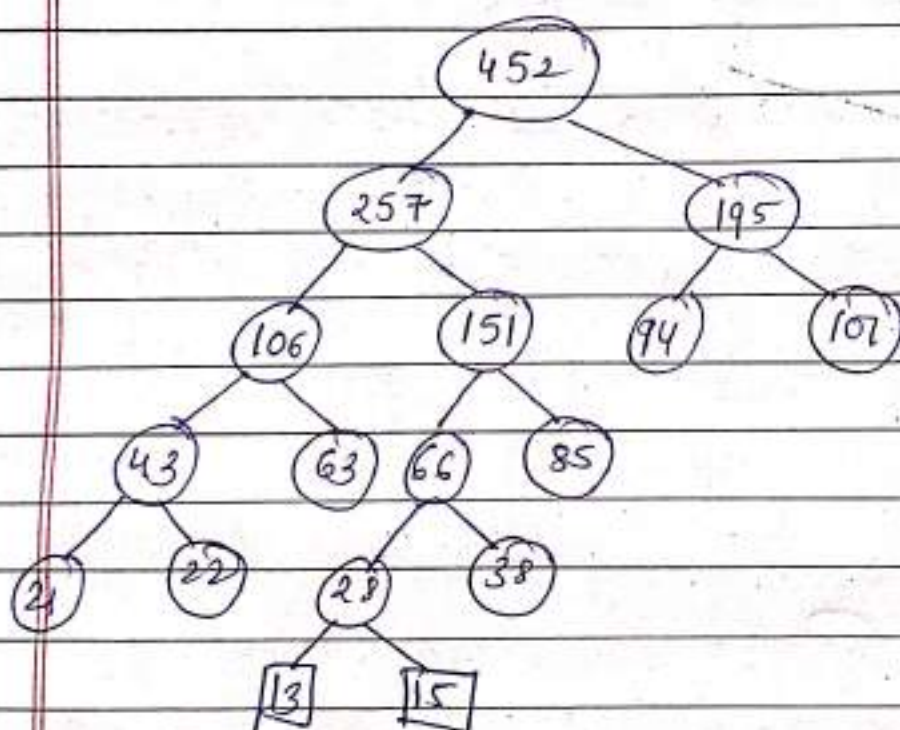
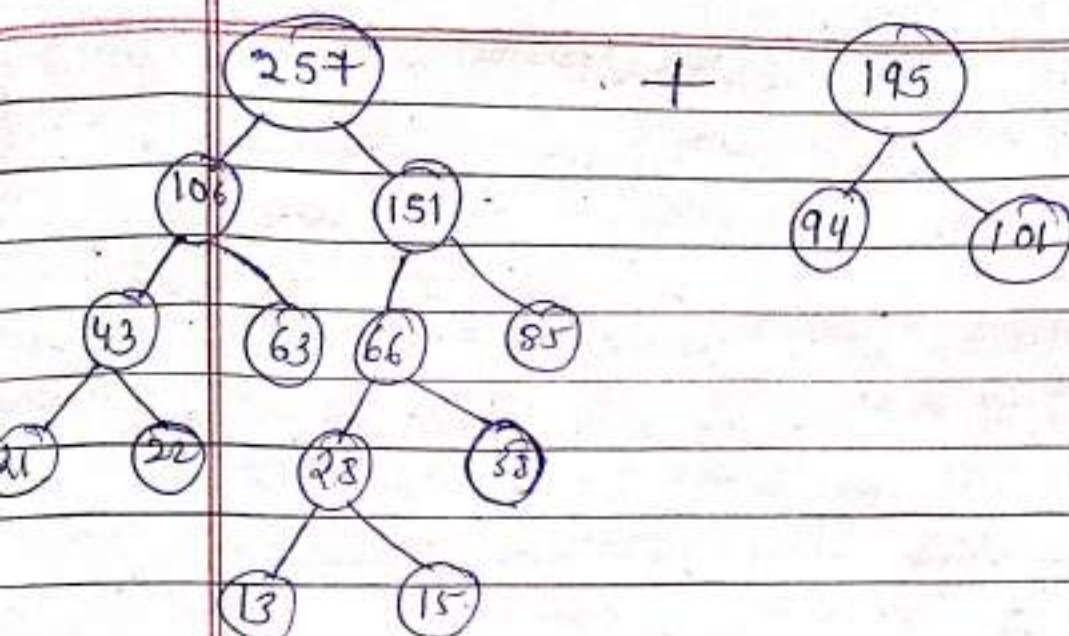
Ques-19) what is Optimal Merge Pattern? Implement optimal binary merge pattern for ten files whose length are 38, 42, 22, 15, 94, 63, 101, 43, 13, 21.

Ans: The "Optimal merge pattern" is a concept in Computer Science, specifically in context of Sorting & Merging data. It's a technique used for efficiently merging multiple sorted sequences into a single sorted sequence, often used in external sorting & Database Management Systems.

Step-1 Sort in Ascending order

13, 15, 21, 22, 38, 42, 43, 63, 94, 101





$$\begin{aligned} \text{Total values} &= 452 + 257 + 195 + 106 + 151 + 43 \\ &\quad + 66 + 85 + 28 = 1383 \end{aligned}$$

7 The