

## Sheet 5

1) What is backtracking?

Backtracking is a problem-solving algorithmic technique that involves finding a "sol" incrementally by trying different options & undoing them if they lead to dead end.

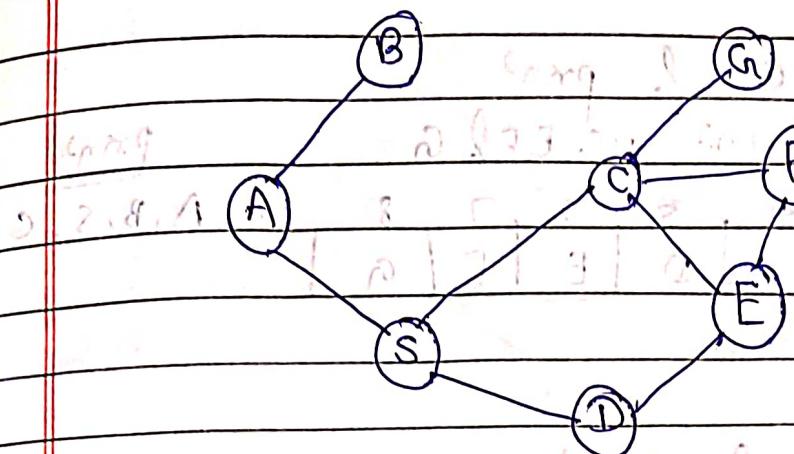
Commonly used in situations where we need to explore multiple possibilities to solve a problem.

e.g. searching path in maze  
solving sudoku puzzle.

When dead end is reached, the algorithm backtracks to previous decision point and explores a different path until a "sol" is found or all possibilities have been exhausted.

Types of Backtracking problems:-

- (1) Decision problems
- (2) Optimization problems
- (3) Enumeration problems



BFS may also be done by level wise.

Let here starting vertex  $\textcircled{A}$ .

Step 1: Insert starting vertex  $\textcircled{A}$  in queue.

1	2	3	4	5	6	7	8
A							

Step 2: Remove  $\textcircled{A}$  & print it.

Insert its adjacent vertex i.e.  $\textcircled{B}, \textcircled{S}$ .

1	2	3	4	5	6	7	8	Print
		B	S					A

Step 3: Remove  $\textcircled{B}$  & print it.

Insert its adjacent vertex None.

1	2	3	4	5	6	7	8	Print
				S				A, B

Step 4: Remove  $\textcircled{S}$  & print it.

Insert its adjacent vertex  $\textcircled{C}, \textcircled{D}$ .

1	2	3	4	5	6	7	8	Print
				C	D			A, B, S

Step 5

remove C & print  
insert adjacent i.e. E, F & G

1	2	3	4	5	6	7	8
				D	E	F	G

print

(A) A, B, S, C

Step 6

remove D & print

insert adjacent if already present

1	(A)	2	3	4	5	6	7	8
				E	F	G	I	J

A, B, S, C

Step 7

remove E & print

insert adjacent if already present

1	2	3	4	5	6	7	8
				F	G	I	J

A, B, S, C, D, E

Step 8

remove F & print

insert adjacent if already present

1	2	3	4	5	6	7	8
				I	J	G	H

A, B, S, C, D, E, F

Step 9

remove G & print

insert adjacent if already present

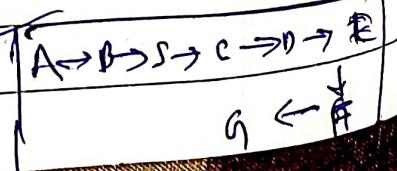
1	2	3	4	5	6	7	8
				I	J	H	K

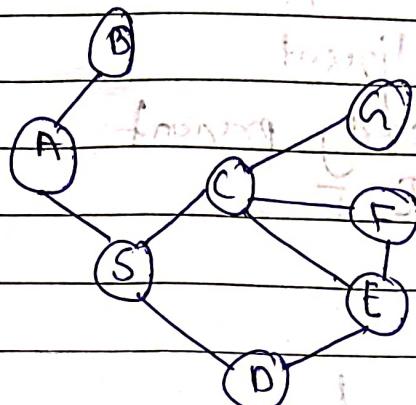
print

(A) A, B, S, C, D, E, F, G

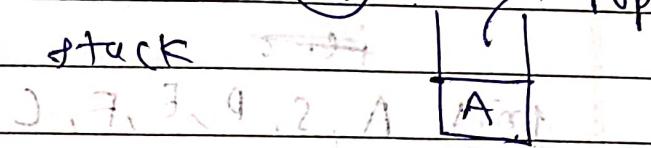
Here, queue is empty.

So, search is complete



DFS

Stacked, Here initial vertex is S.  
Step 1: Insert A in stack



Step 2: ~~insert its adj~~

pop to print element (A) & insert its adjacent in stack  
 point puped element

					<u>A</u>	→ pop
					<u>S</u>	
					<u>B</u>	

Step 3: ~~print~~ A & S in stack

					<u>D</u>	→ pop
					<u>C</u>	
					<u>B</u>	

print A, S in stack

Step 4: pop (D) & insert adj

print A, S, D

					<u>E</u>	→ pop
					<u>C</u>	
					<u>B</u>	

print A, S, D

Step 5: pop (E) & insert adj

print A, S, D, E

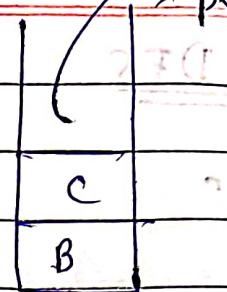
					<u>F</u>	→ pop
					<u>C</u>	
					<u>B</u>	

Step 6: pop (F) &

insert adjacent

i.e. already present

print A, S, D, E, F

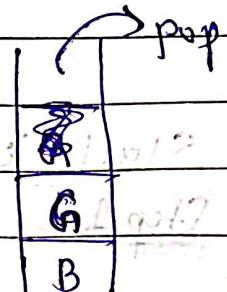


Step 7: pop (C) &

insert adj. of its adjacent

i.e. adjacent in A list

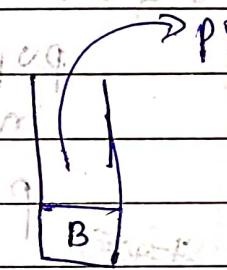
print A, S, D, E, F, C



Step 8: pop (G) &

insert adj. i.e. ribbon di chain

print A, S, D, E, F, C, G



Step 9: pop (B) &

insert adjacent more & then

Here stack is empty

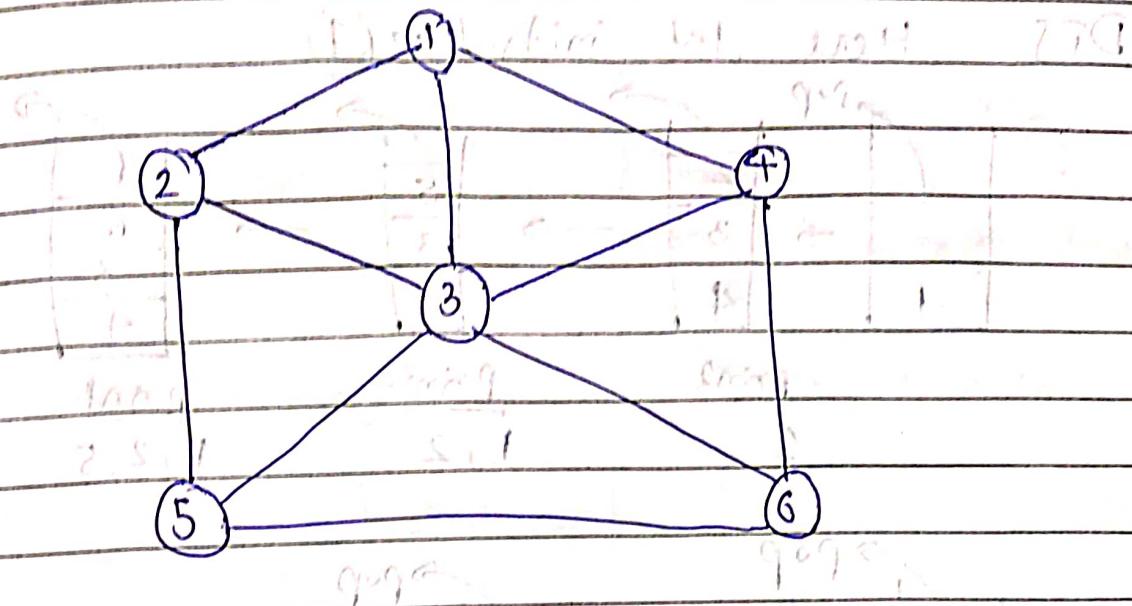
empty

print: A, S, D, E, F, C, G, B

Hence traversal is completed

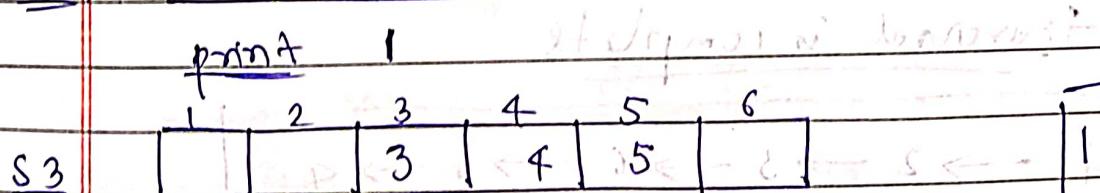
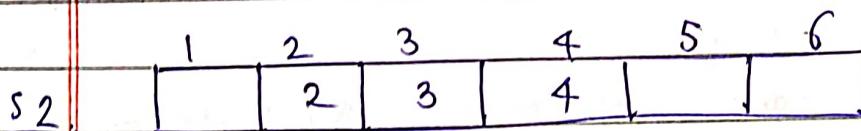
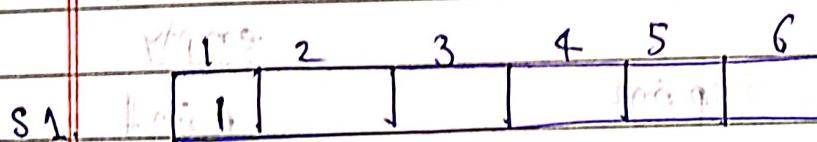
A → S → D → E → F → C → G → B

(Q 2)

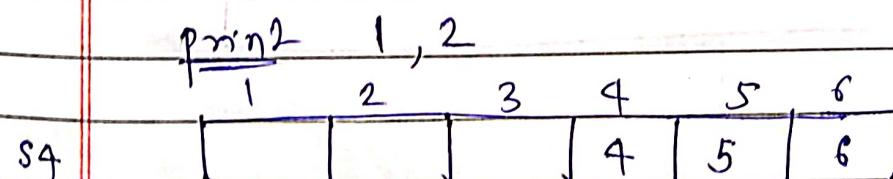


BFS Let, Here initial ①

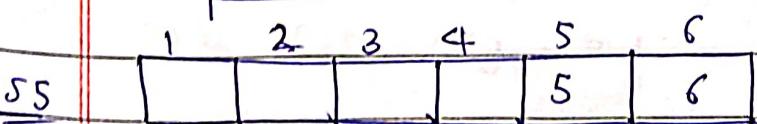
Directed  
kata



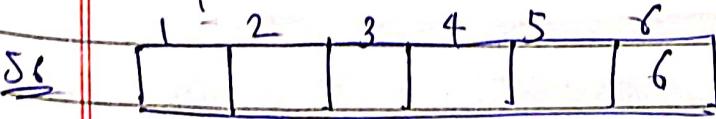
$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$   
 $6 \leftarrow 5$



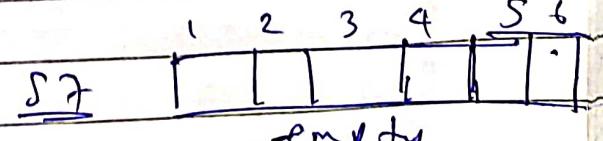
print 1, 2, 3



print 1, 2, 3, 4



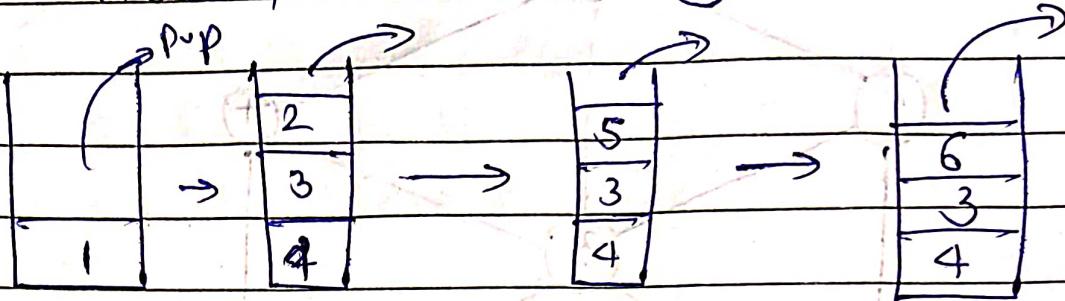
print 1, 2, 3, 4, 5



empty

print 1, 2, 3, 4, 5, 6

DFS Here, let initial is ①

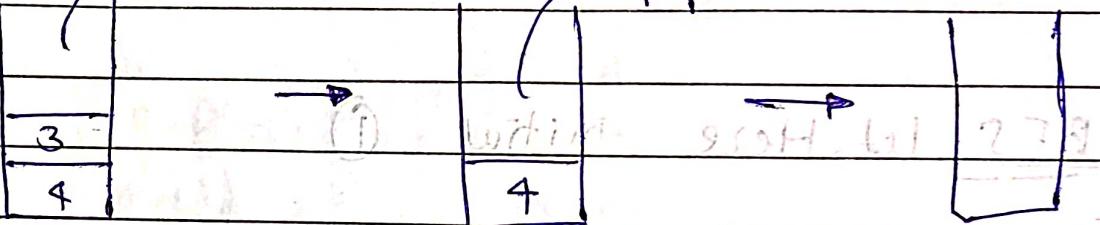


print2  
1

print  
1, 2

print  
1, 2, 5

Pop



print

1, 2, 5, 6

print2

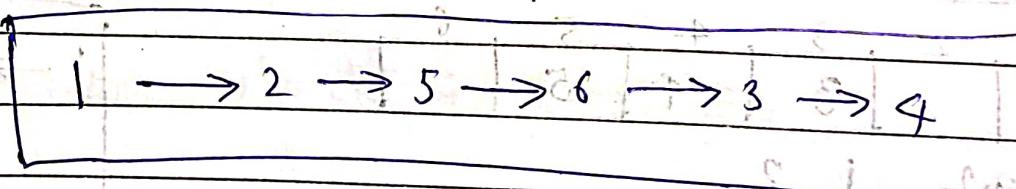
1, 2, 5, 6, 3

empty

print1

1, 2, 5, 6, 3, 4

Traversal is complete

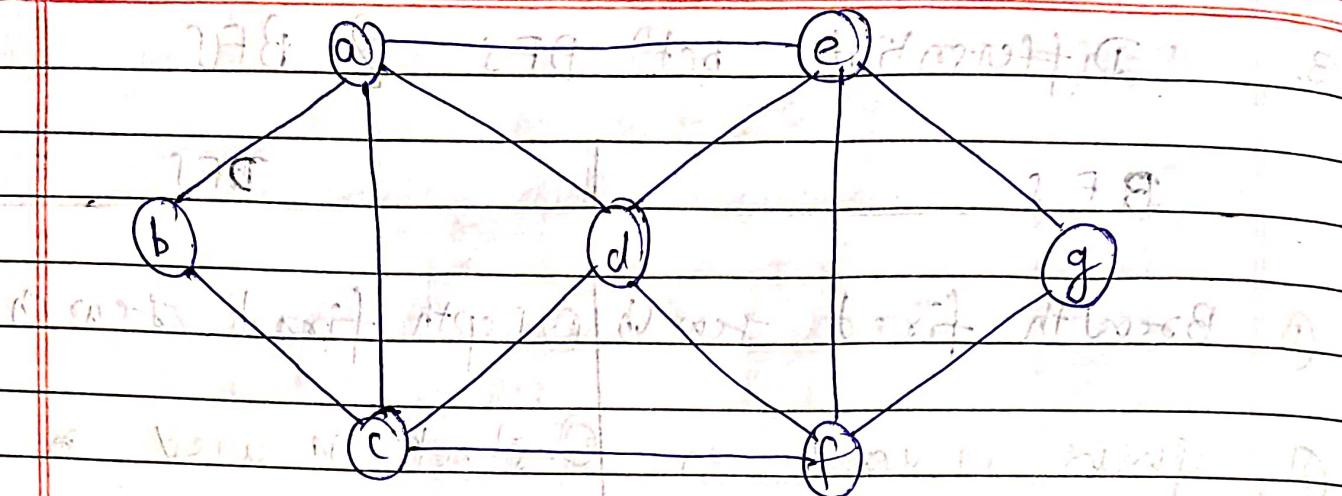


## Differentiate bet' DFS & BFS

3.

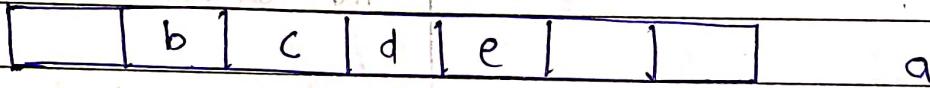
**BFS****DFS**

① Breadth first search	① Depth first search
② Queue is used in BFS	② Stack is used in DFS
③ for finding shortest path.	③ with depth, through nodes, as far as possible with no unvisited nearby nodes.
④ Concept of FIFO	④ Concept of LIFO
⑤ more suitable for searching vertices closer to given source.	⑤ more suitable when there are nodes away from src.
⑥ not suitable for decision making tree, because it consider all neighbours first	⑥ more suitable for decision, game & puzzle problems.
⑦ Time complexity $O(V+E)$	⑦ $O(V+E)$
⑧ Space complexity is high.	⑧ lesser complexity.
⑨ Speed is slow require more memory Optimal for finding shortest path.	⑨ speed is fast require less memory. not optimal for finding shortest path.
⑩	⑩
⑪	⑪

BFSLet a is initial node without any

action array [1 2 3 4 5 6] level and bid print  
 start from a [ ] [ ] [ ] [ ] [ ] [ ] [ ]

list of edges between one

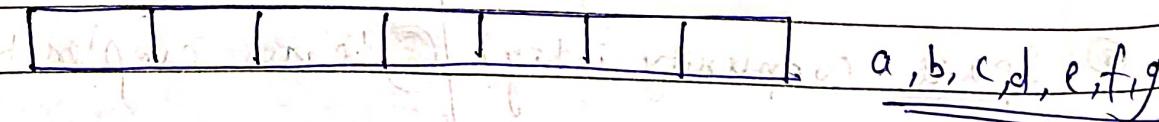
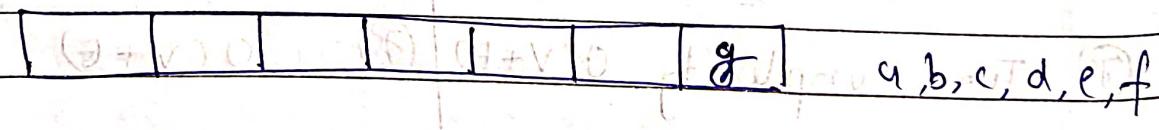


0 → 1 → 2 → 3 → 4 → 5 → 6 level and bid print  
 [ ] [ ] [ ] [ ] [ ] [ ] [ ] a, b

start next action 3 [ ] [ ] [ ] [ ] [ ] [ ] [ ]  
 [ ] [ ] [ ] [ ] [ ] [ ] [ ] a, b, c

initially ~ 1 visitin count 1, ex for 1 with (a, b, c, d)

initially ~ 2 visitin count 1, ex for 2 with (a, b, c, d, e)

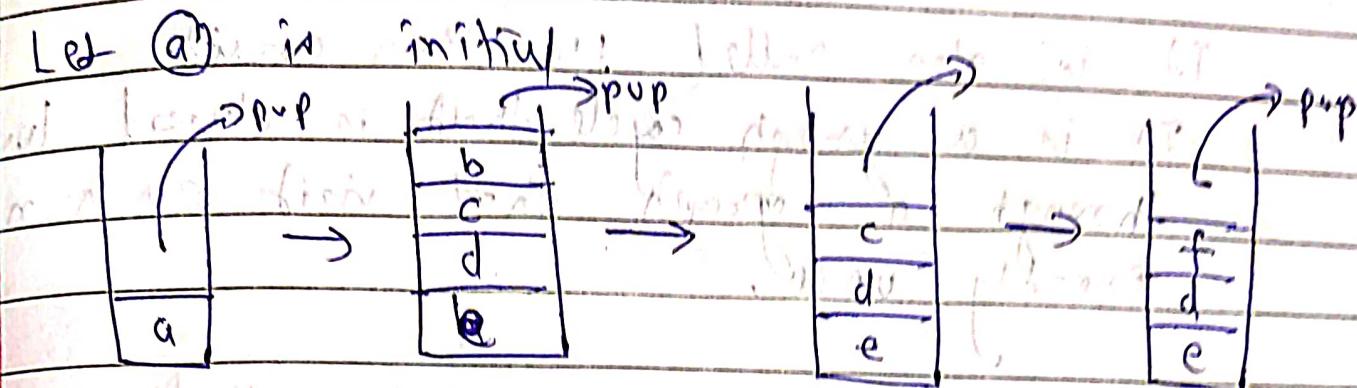


empty queue with no head

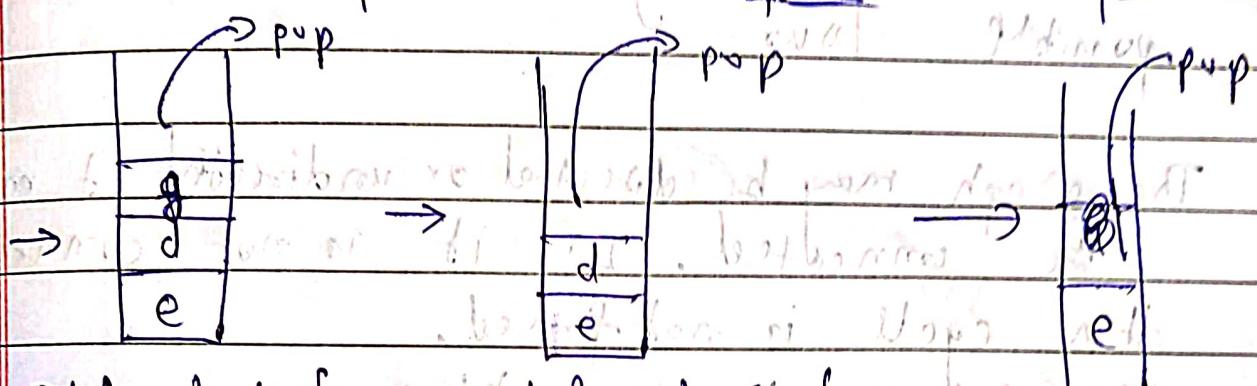
$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g$

DFS

Let a is initial node of graph



print a, b, c



print a, b, c, f    print a, b, c, f, g    print a, b, c, f, g, d

Now we are finished so stop searching

Because all nodes are visited with vis symbol

so no more nodes are available

so we can't search further

so we can't search further

empty stack

print a, b, c, f, g, d, e

Here traversal is completed

$a \rightarrow b \rightarrow c \rightarrow f \rightarrow g \rightarrow d \rightarrow e$

## Q4. What is Hamiltonian cycle?

Vishal

It is also called Hamilton circuit.

It is a graph cycle that is closed loop through a graph that visit each node exactly once.

In hamiltonian cycle, we have to find the all possible tour.

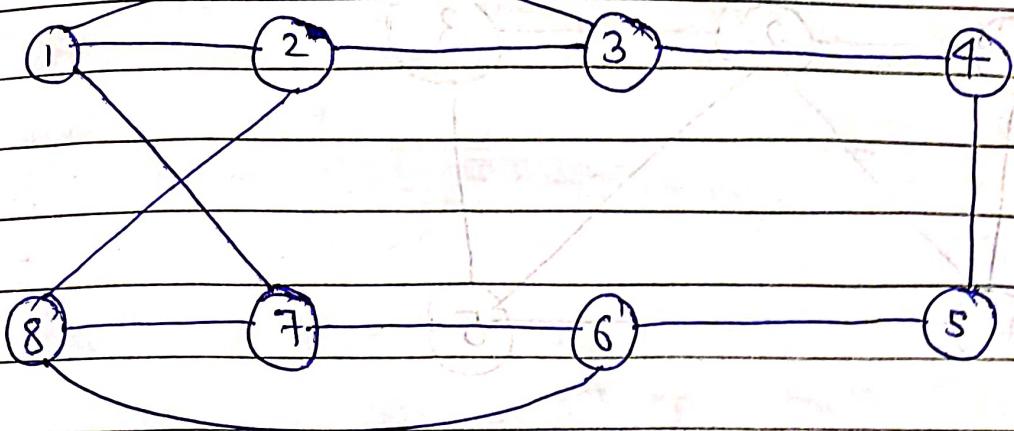
The graph may be directed or undirected if must be connected. If it is not connected then cycle is not formed.

This is np-hard problem. F. d. o. b.

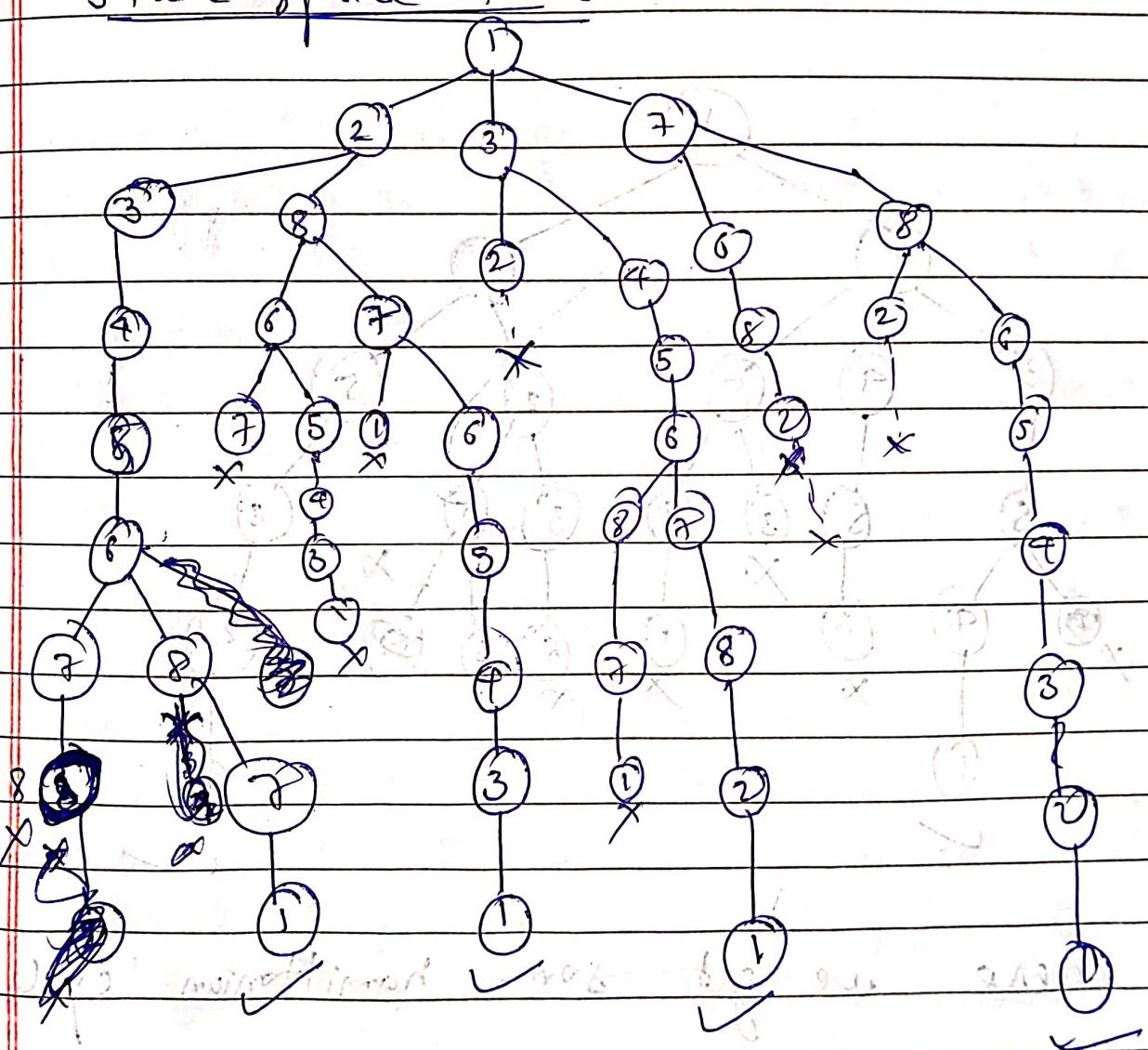
Hamiltonian cycle or circuit in a graph  $G$  is a cycle that visits every vertex of  $G$  exactly once and returns to the starting vertex.

### Use of hamiltonian cycle:

- finding optimal routes in transportation network
- Circuit design
- graph theory research
- in computer graphic
- mapping genomes



## State space tree



Here we get 1 generic path,

## Hamiltonian cycles

1 - 2 - 3 - 4 - 5 - 6 - 8 - 7 - 1

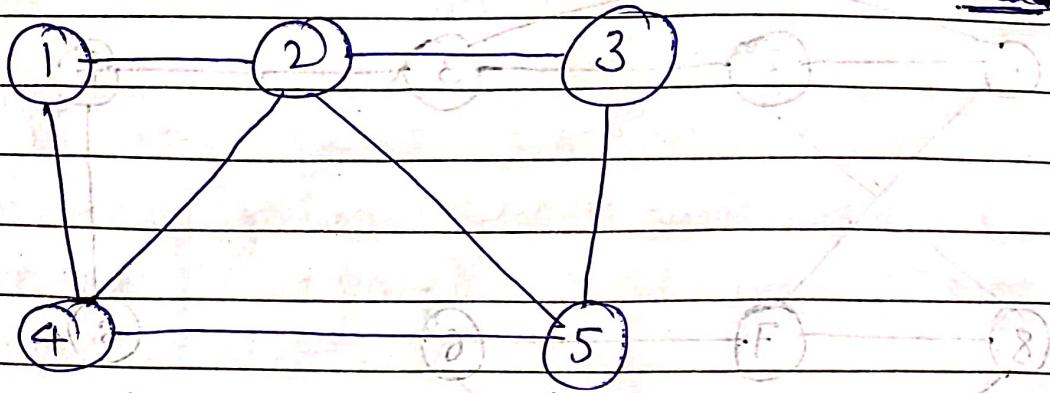
1 — 2 — 8 — 7 — 6 — 5 — 4 — 3 — 1

1 - 3 - 4 - 5 - 6 - 7 - 8 - 2 - 1

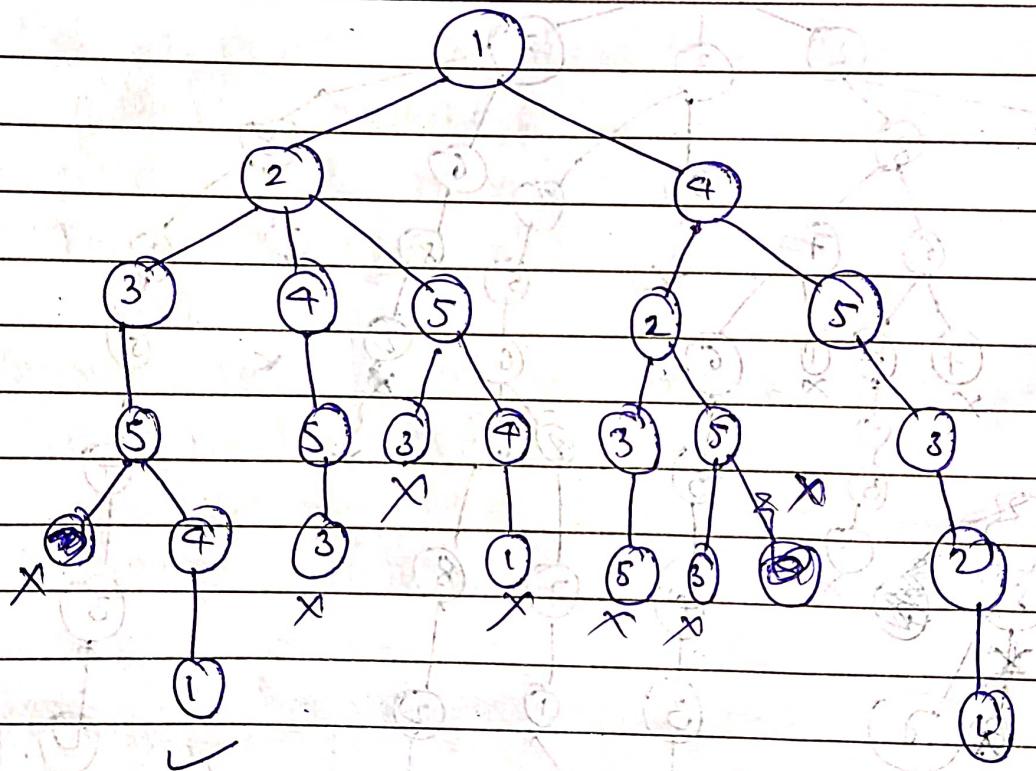
1 — 7 — 8 — 6 — 5 — 4 ← 3 — 2 — 1

Date: \_\_\_ / \_\_\_ / \_\_\_

φ 5.



## State space tree



Here we get some hamiltonian cycle.

$$1 - 2 \frac{1}{3} - 3 \frac{5}{6} - 4 \frac{1}{4} = 1$$

1-4-5-3-2-1 not on H ✓

$$1 + 4 - 8 + 2 - 2 + 4 - 8 - 8 = 1$$

$\hat{P} = \hat{E} + \hat{C} + \hat{D} + \hat{B} + \hat{A} + \hat{F} + \hat{R} + \hat{S} + \hat{I}$

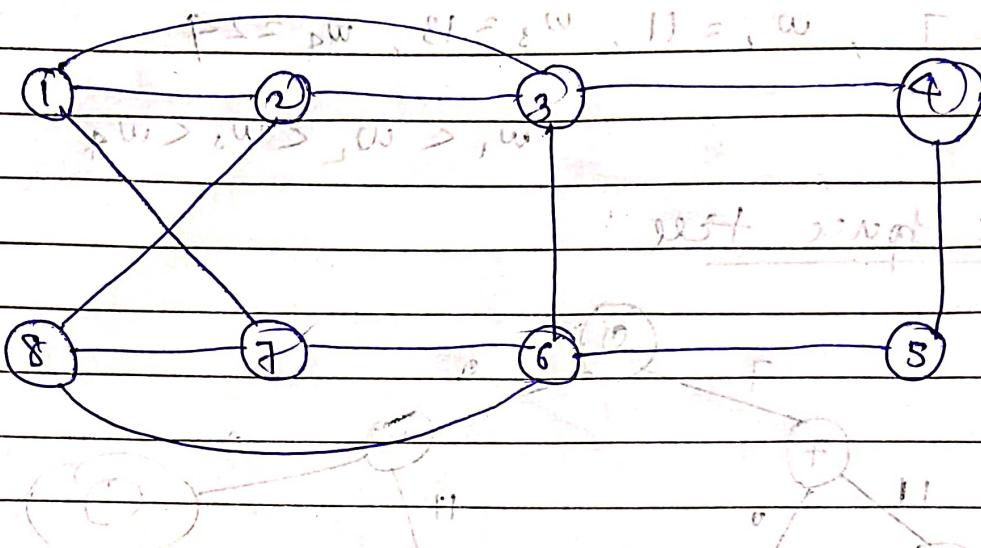
$1 + s + s^2 + \dots + s^n$

96)

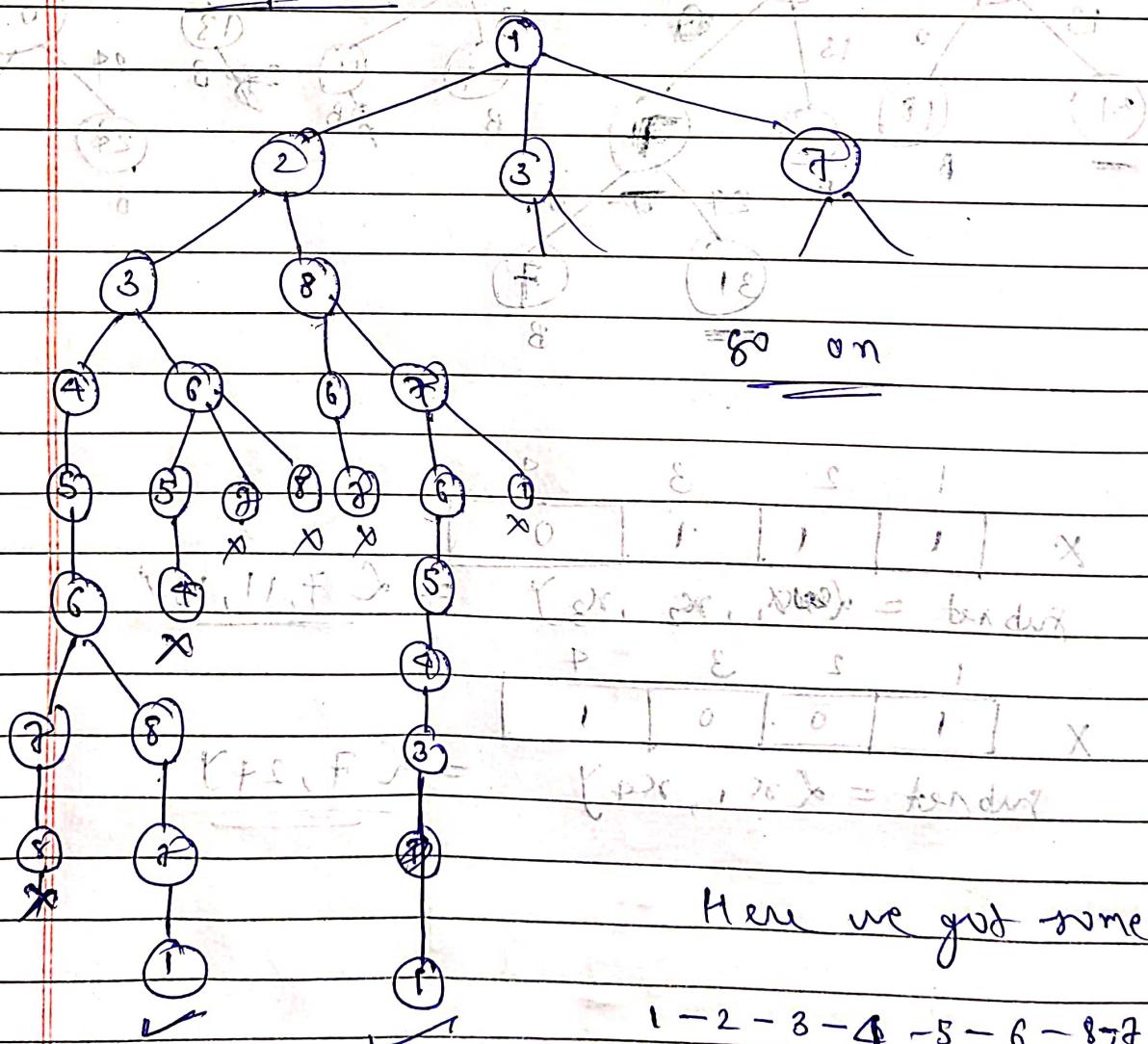
## Hamiltonian

$$(F, \mathcal{PS}, \mathcal{EI}, \mathcal{W}) = w$$

$$1 \cdot \epsilon = m$$

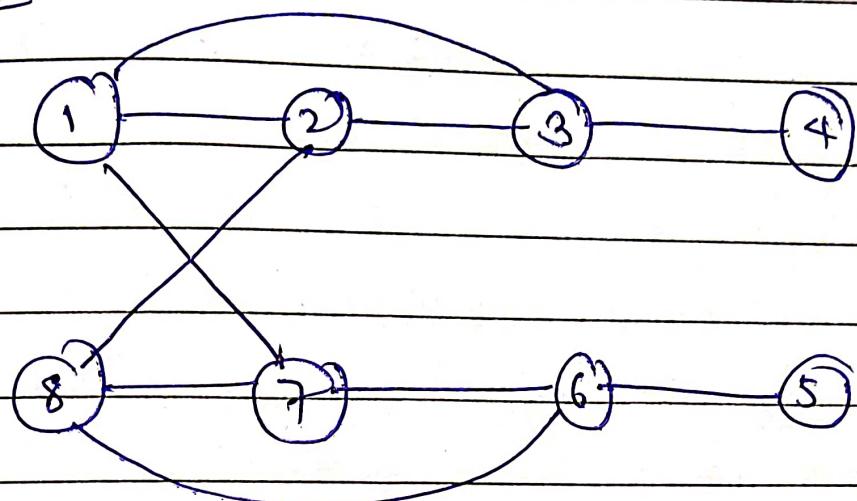


## State space tree



Here we got some paths!

$$\begin{array}{ccccccccc} 1 & - & 2 & - & 8 & - & \underline{\underline{4}} & - & 5 & - & 6 & - & 8 & \cancel{7} & - & 1 \\ 1 & - & 2 & - & 8 & - & \cancel{7} & - & 6 & - & 5 & - & 4 & - & 3 & - & 1 \end{array}$$

Q7

ye nahi hoga hamiltonian cycle

because wo connected nahi ha ,

Q8. What is 8-Queen problem? Explain implicit & explicit constraints associated with this problem. Give at least two sol<sup>n</sup> for problem.

$\Rightarrow$  Backtracking approach for 8-Queen problem  
 - it is a recursive approach for solving any problem where we have search among all possible sol<sup>n</sup>, following some constraints.

Given an 8-Queen problem, we must place 8 queens from the  $8 \times 8$  chessboard such that none of them attack one another. Here no two queens in same row, column, or diagonal.

More generally, n Queen problem places n queen on  $n \times n$  chessboard.

Here, we suppose that the queen "i" is to be placed in first row, only but we can say that 1 queen is placed in first row only but can have columns from 1, 2, ..., n so that they satisfy all explicit & implicit constraints.

All sol<sup>n</sup> to 8-Queen problem are represented as 8-tuples  $(x_1, x_2, x_3, \dots, x_8)$  where  $x_i$  is the column on which queen "i" is to be placed.

Developed 8 nodes group - 8 in total  
 where Explicit constraints are 6 based  
 those to avoid nodes with same  
 The explicit constraint using this  
 formula are  $S_i = \{1, 2, 3, \dots, 7, 8\}$   
 where  $1 \leq i \leq 8$  and  $i \neq j$   
 Therefore, solution space consists of  $8 \times 8$   
 non-repeating 8-tuples.

Considering group - 8, no avoid  
 therefore Implicit constraints & could have in  
 two diagonal with each other  
 No two  $x_{ij}$ 's can be same. In two groups  
 commutes being same, even same column or  
 in same diagonal so each pair, it should  
 be possible different.

### 8 green problem soln

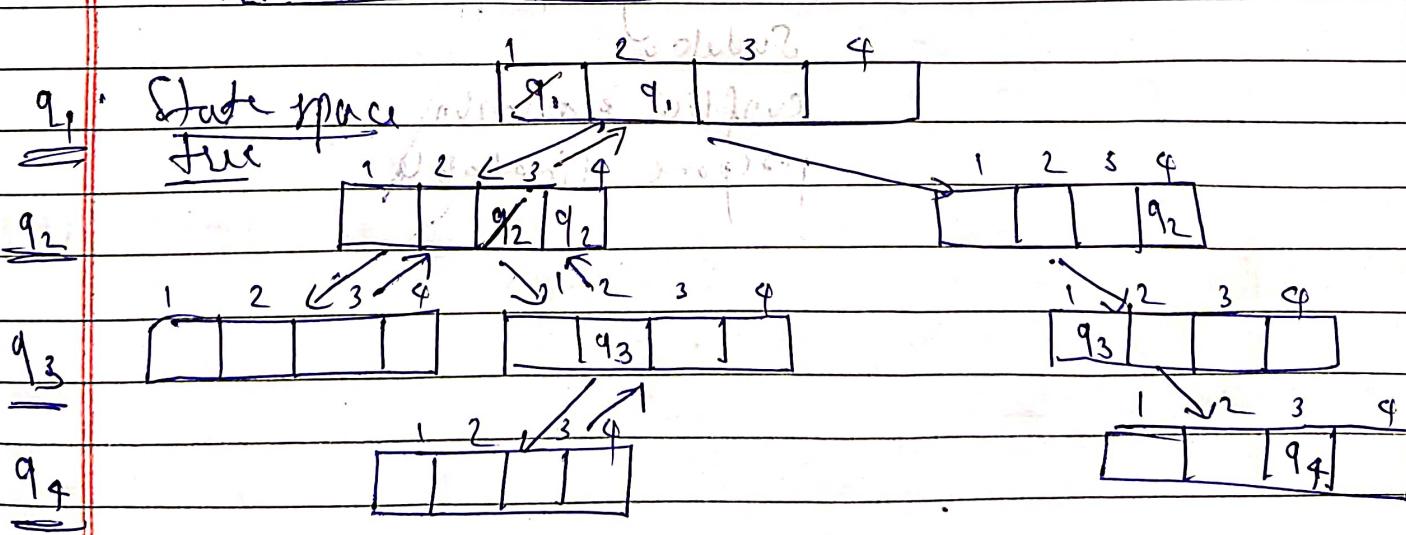
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8
9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.1
9.3	9.4	9.5	9.6	9.7	9.8	9.1	9.2
9.4	9.5	9.6	9.7	9.8	9.1	9.2	9.3
9.5	9.6	9.7	9.8	9.1	9.2	9.3	9.4
9.6	9.7	9.8	9.1	9.2	9.3	9.4	9.5
9.7	9.8	9.1	9.2	9.3	9.4	9.5	9.6
9.8	9.1	9.2	9.3	9.4	9.5	9.6	9.7

91	(red)	afforded	9,	
		92		afforded
	93			
94	600	afforded	1600	1600
		95		6
	96			
97				
	98	value	100	100
	99		10	10

for 4-5 greenish and blueish color in wings  
eggs 2nd & 3rd of 9 instars 17 and 19

ey.

1	9 <sub>1</sub>	9 <sub>2</sub>	9 <sub>3</sub>	9 <sub>4</sub>	9 <sub>5</sub>	9 <sub>6</sub>
2				9 <sub>2</sub>		
3	9 <sub>3</sub>					
4			9 <sub>4</sub>			



(q10) What is graph coloring?

Graph coloring refers to the problem of coloring vertices of a graph in such a way that no two adjacent vertices have the same color.

This is also called the vertex coloring problem. If coloring is done using at most  $m$  colors, it is called  $m$ -coloring.

Application — map coloring

scheduling tasks

schedules

conflict resolution

prepare timetable

(1)

(2)

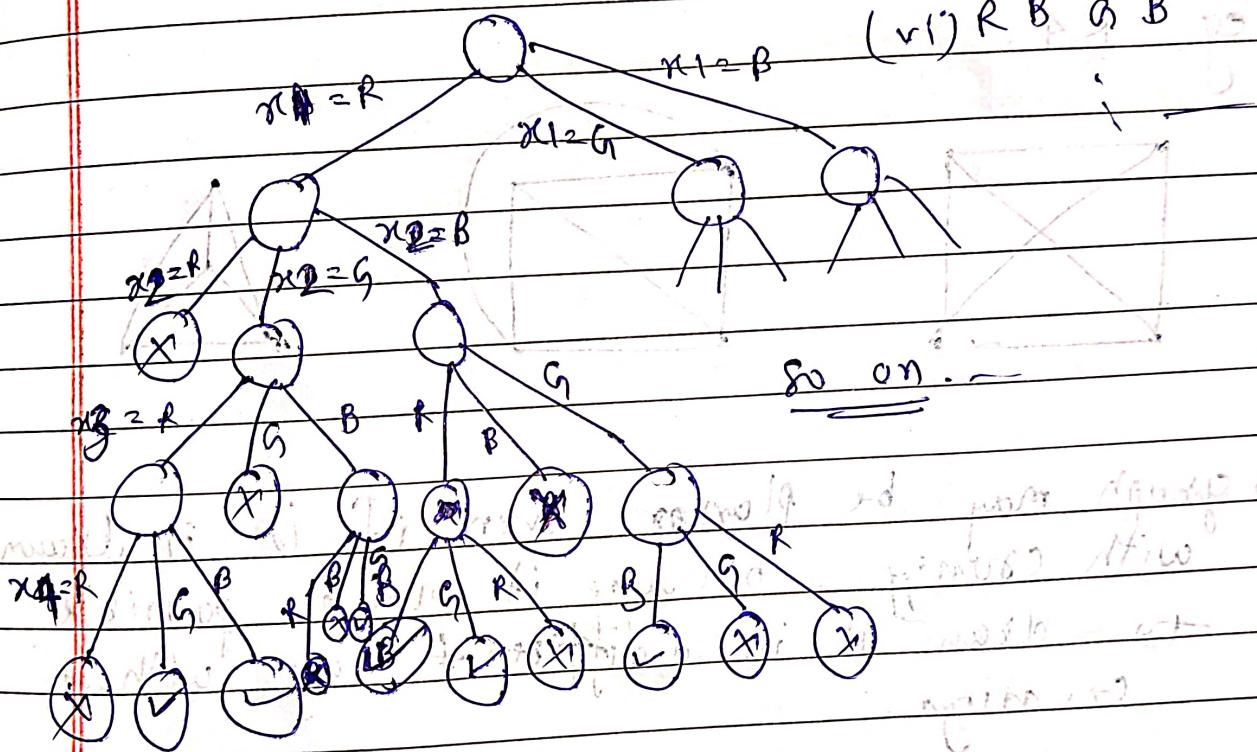
(4)

(3)

Let

 $\text{color} = (R, G, B)$ 

- state space tree
- (i)  $R \rightarrow A$
  - (ii)  $R \rightarrow G \rightarrow R \rightarrow S$
  - (iii)  $R \rightarrow G \rightarrow B \rightarrow G$
  - (iv)  $R \rightarrow B \rightarrow R \rightarrow S$
  - (v)  $R \rightarrow B \rightarrow R \rightarrow B$
  - (vi)  $R \rightarrow B \rightarrow G \rightarrow B$



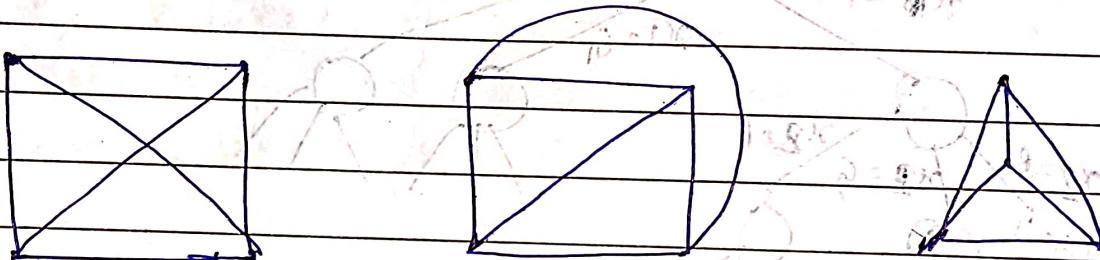
Q11)isme kitne color or konne diye hee nahi?

(Q12)

Similar to que 10,Planar Graph

A graph is said to be planar if it can be drawn in a plane such that no edge cross.

- (a) If it can be drawn on a plane
- (i) without any edges crossing.
- (ii) with some crossings.

eg.  $K_4$ 

A graph may be planar even if it is drawn with crossings, because it may be possible to draw it in a different way without crossings.

Q16) Apply backtracking alg. to solve instance of sum of subset problem.  $S = \{1, 3, 4, 5\}$ ,  $m = 10$ . find all possible subset of  $S$  that generates  $\text{sum} = m$  & draw solution space tree for same.

~~Ans~~  $\Rightarrow$

This problem will be solved using space state tree with help of backtracking technique.

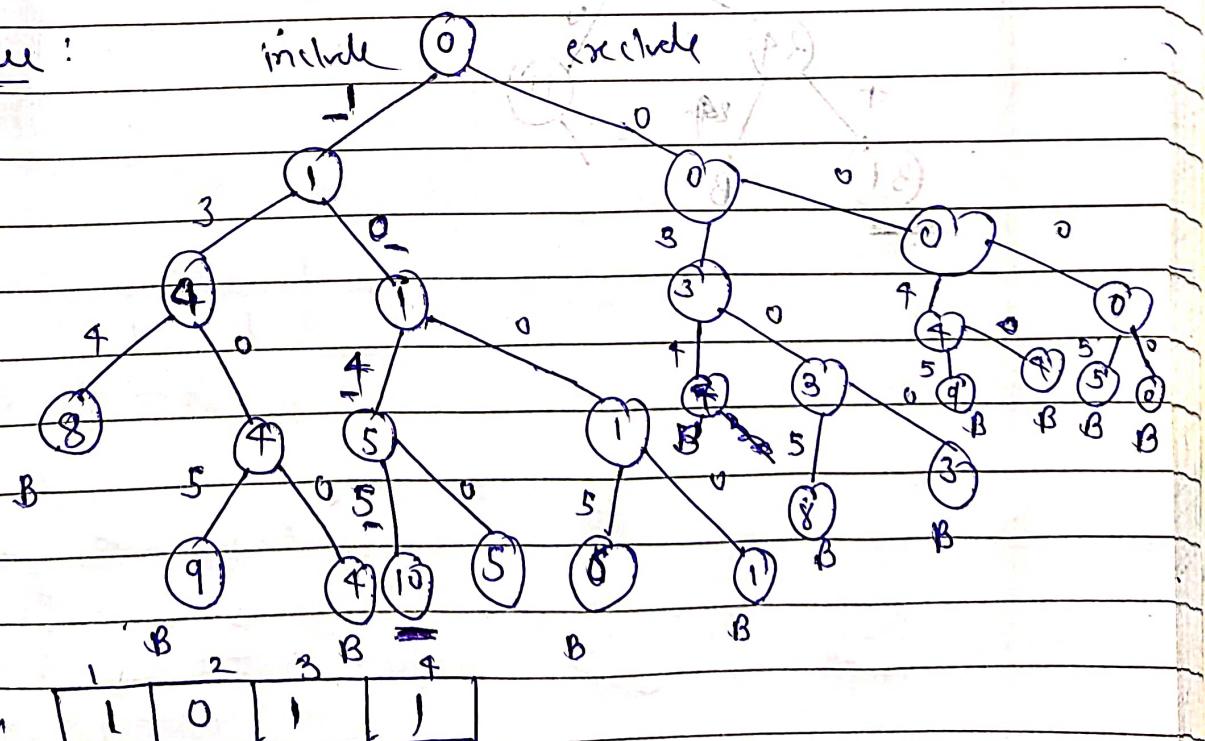
It involves determining whether or not a subset from list of integers can sum to a target value.

$$\text{Boundary func: } \sum_{i=1}^k w_i x_i + w_k + 1 \leq m$$

$$\sum_{i=1}^k w_i x_i + \sum_{i=k+1}^n w_i > m$$

$$\text{Let, } w_1 = 1, w_2 = 3, w_3 = 4, w_4 = 5, m = 10$$

State Space tree:



x	1	0	1	1
---	---	---	---	---

$$\text{subset} = \{x_1, x_3, x_4, y\} = \{1, 4, 5\}$$

15

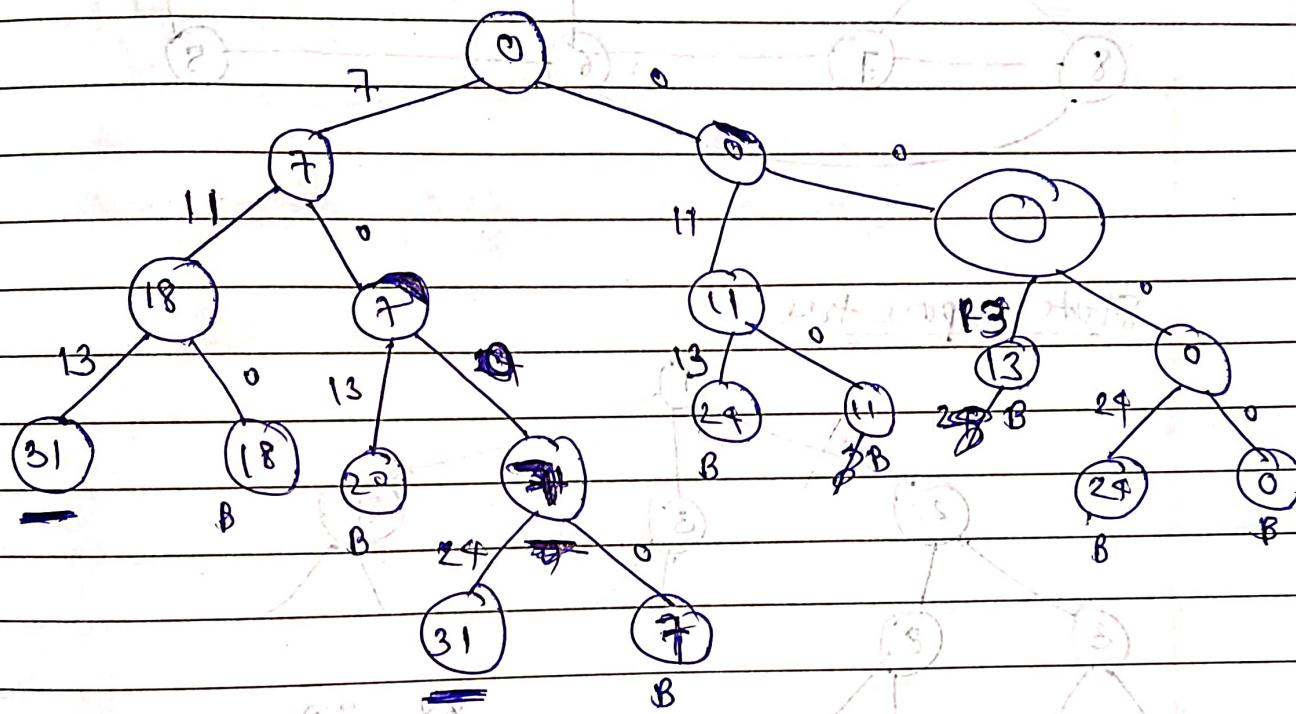
$$W = \{11, 13, 24, 7\}$$

$$m = 31$$

$$w_1 = 7, w_2 = 11, w_3 = 13, w_4 = 24$$

$$w_1 < w_2 < w_3 < w_4$$

state space tree:



1	2	3	4
x	1	1	1

$$\text{subset} = \{x_1, x_2, x_3\} = \underline{\{7, 11, 13\}}$$

1	2	3	4
x	1	0	0

$$\text{subset} = \{x_1, x_4\} = \underline{\{7, 24\}}$$

using state space tree method

$$1 + 4 + 8 + 3 + 2 + 1 + 8 + 5 = 27$$

$$1 + 5 + 4 + 2 + 3 + 4 + 8 + 5 = 27$$