

## Sheet 5

1) What is backtracking?

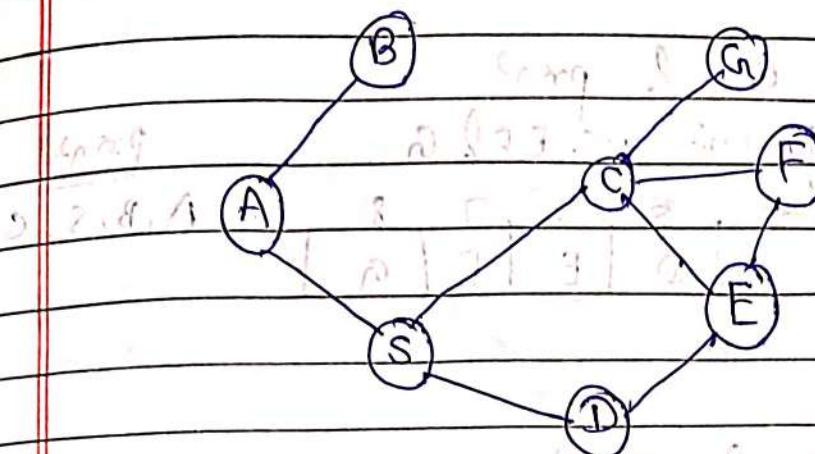
Backtracking is a problem-solving algorithmic technique that involves finding a "sol" incrementally by trying different options & undoing them if they lead to dead end.

Commonly used in situations where we need to explore multiple possibilities to solve a problem.  
eg searching path in maze  
solving sudoku puzzle.

When dead end is reached, the algorithm backtracks to previous decision point and explores a different path until a "sol" is found or all possibilities have been exhausted.

Types of Backtracking problems:-

- ① Decision problems
- ② Optimization problems
- ③ Enumeration problems



BFS may also be applied to this graph.

Let here starting vertex  $\textcircled{A}$ :

Step 1: Insert starting vertex A in queue.

1 2 3 4 5 6 7 8



Step 2: Remove A & print it.

A inserted its adjacent vertex i.e. B, S

1 2 [3] 4 5 6 7 8 | Print

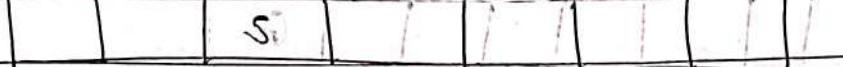


A

Step 3: Remove B & print it.

A inserted its adjacent vertex None

1 2 3 4 5 6 7 8 | Print



A, B

Step 4: Remove S & print it

insert its adjacent. i.e. C & D

1 2 3 4 5 6 7 8 | Print



A, B, S

Step 5

remove C & print  
insert adjacent i.e. E, F & G

1	2	3	4	5	6	7	8
				D	E   F   G		

print

(A, B, S, C)

Step 6

remove D & print

insert adjacent if already present

1	(A, B, S, C)	2	3	4	5	6	7	8
					E   F   G			

print

A, B, S, C

Step 7

remove E & print

insert adjacent if already present

1	2	3	4	5	6	7	8
					F   G		

print

A, B, S, C, D, E

Step 8

remove F & print

insert adjacent if already present

1	2	3	4	5	6	7	8
					G		

print

A, B, S, C, D, E, F

Step 9

remove G & print

insert adjacent

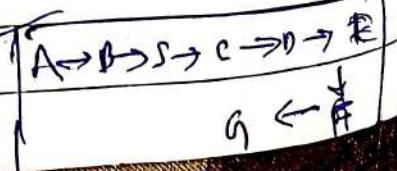
1	2	3	4	5	6	7	8

print

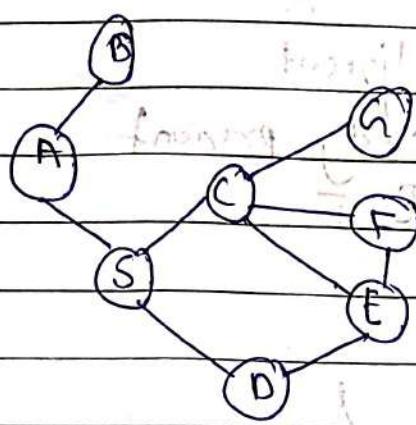
A, B, S, C, D, E, F

Here, queue is empty.

So, search is complete



PES



Stacked, Here initial vertex is A  
Step 1: Insert A in stack

stack 5-9  
3, 7, 3, 9, 2, A

Step 2: ~~insert its obj~~

pop to print element A

inner 2 its adjacent in stack

print puped element

	$p_{\text{app}}$
S	
B	

13

359

Mr. J. T. J. Q. 2. A friend

Step 3: pop topmost element (S) &

insert it as follows

D  
C  
B

print A,S in code unit

Step 4: pup ① & inner 2 only

print A,S,D 2 3,3 4,2 J

E  
C  
B

Step 5: pup E Linnend adj

print A, S, D, E

F  
C  
B

Date: 1/1

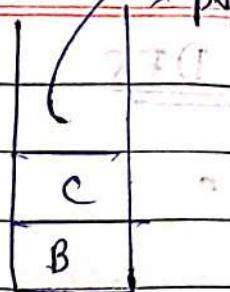
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Step 6: pop (F) &

insert adjacent

i.e. already present

print A, S, D, E, F



Step 7: pop (C) &

insert its adjacent

~~it's~~ adjacent in A case

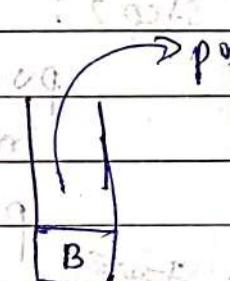
print A, S, D, E, F, C



Step 8: pop (G) &

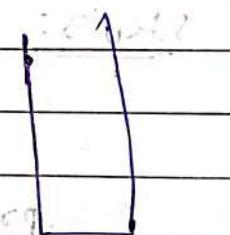
insert adj. i.e. none in case

print A, S, D, E, F, C, G



Step 9: Pop (B) &

insert adj. i.e. none in case



Here stack is empty.

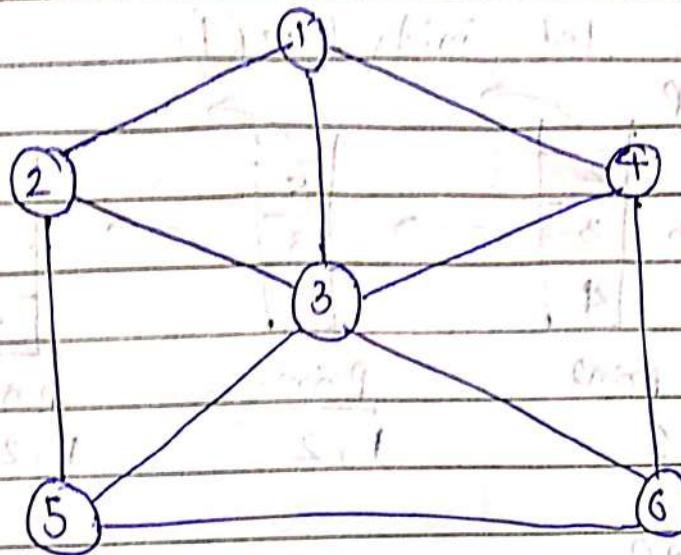
empty stack

print: A, S, D, E, F, C, G, B

Hence traversal is completed

A → S → D → E → F → C → G → B

(Q 2)

BFS Let, Here initial ①Directed  
kata

	1	2	3	4	5	6
S1	1					

	1	2	3	4	5	6
S2		2	3	4		

	1	2	3	4	5	6
S3			3	4	5	

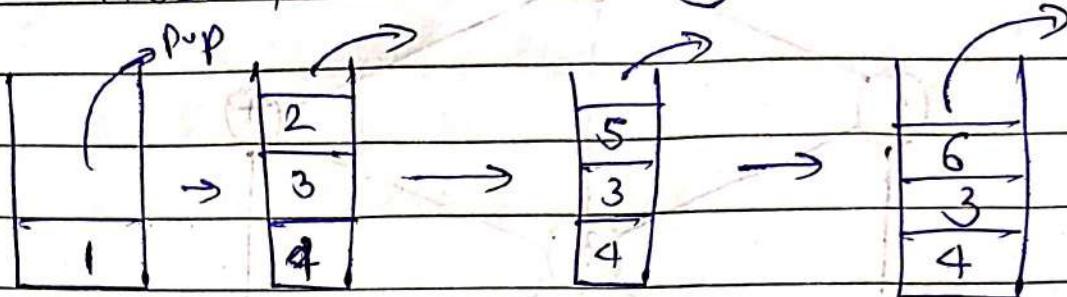
	1	2	3	4	5	6
S4				4	5	6

	1	2	3	4	5	6
S5					5	6

	1	2	3	4	5	6
S6					6	

print2 1, 2, 3, 4, 5, 6

DFS Here, let initial is ①



print2

1

print

1, 2

print

1, 2, 5

pop

pop

print

1, 2, 5, 6

print2

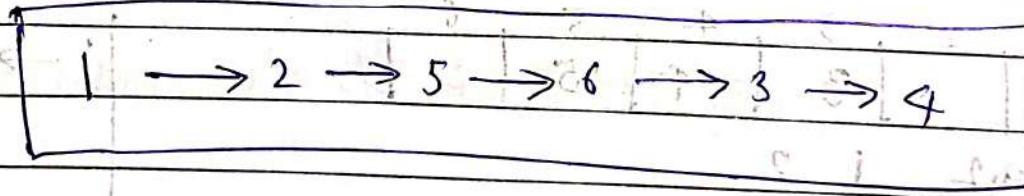
1, 2, 5, 6, 3

empty

print1

1, 2, 5, 6, 3, 4

Traversal is complete

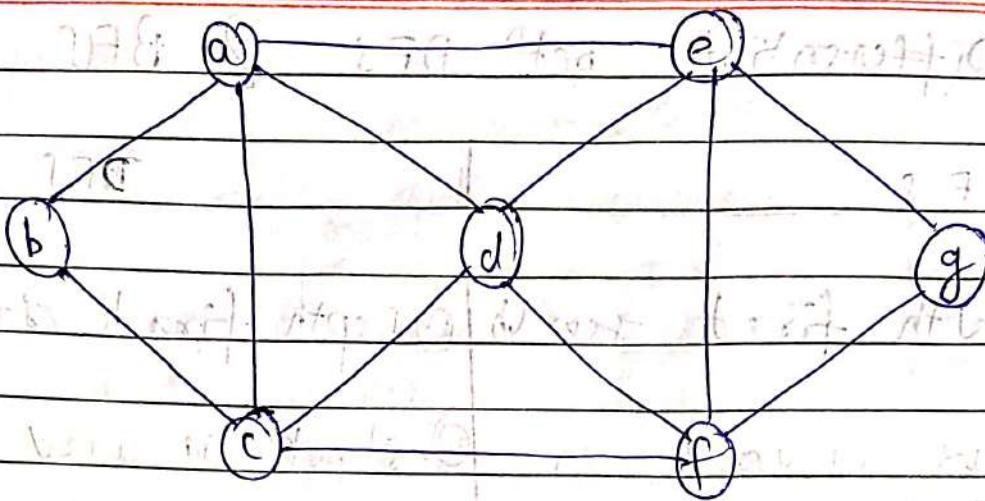
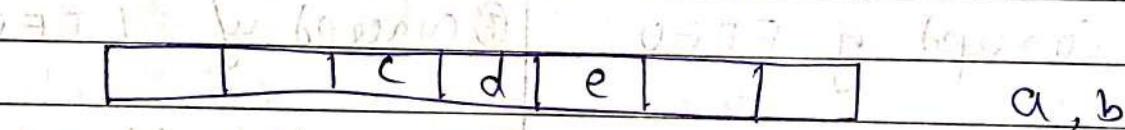
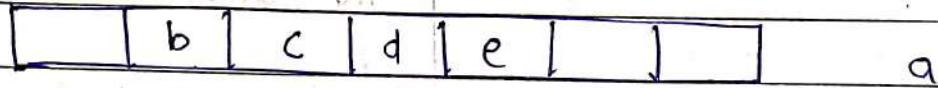
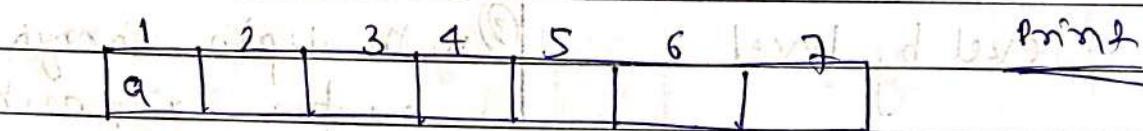


## Differentiate betw' DFS & BFS

BFS

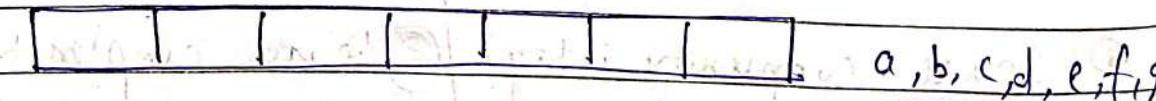
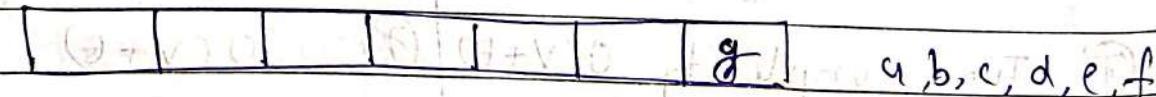
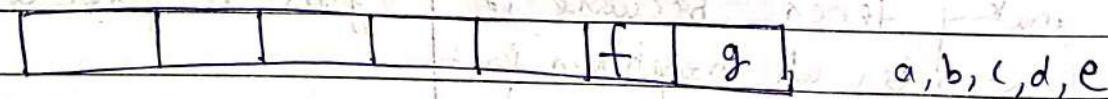
BFS

BFS	BFS
① Breadth first search	① Depth first search
② queue is used in BFS	② stack is used
③ for finding shortest path.	③ with depth, through nodes, as far as possible with no unvisited nearby nodes.
④ Concept of FIFO	④ Concept of LIFO
⑤ more suitable for searching vertices closer to given source	⑤ more suitable when there are far away from src.
⑥ not suitable for decision making tree, because it consider all neighbour first	⑥ more suitable for decision, game & puzzle problems.
⑦ Time complexity $O(V+E)$	⑦ $O(V+E)$
⑧ Space complexity is high.	⑧ lesser complexity.
⑨ Speed is slow	⑨ speed is fast
⑩ require more memory	⑩ require less memory.
⑪ optimal for finding shortest path.	⑪ not optimal for finding shortest path.

BFSLet a is initial

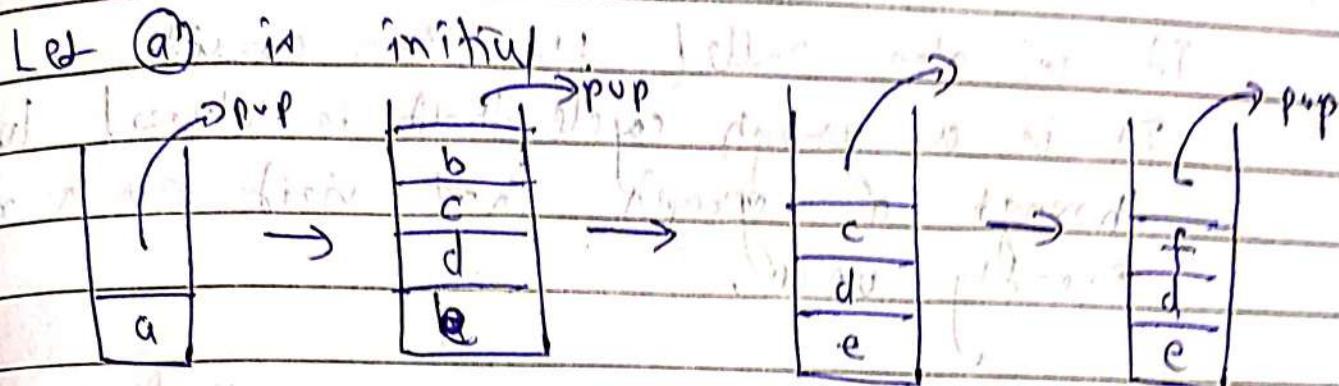
small nodes selection - remaining nodes after selection -  $a, b, c$   
which are not yet visit and  $d, e, f$  are only  $a, b, c$

remaining nodes - visit and  $e, f$  -  $a, b, c, d$

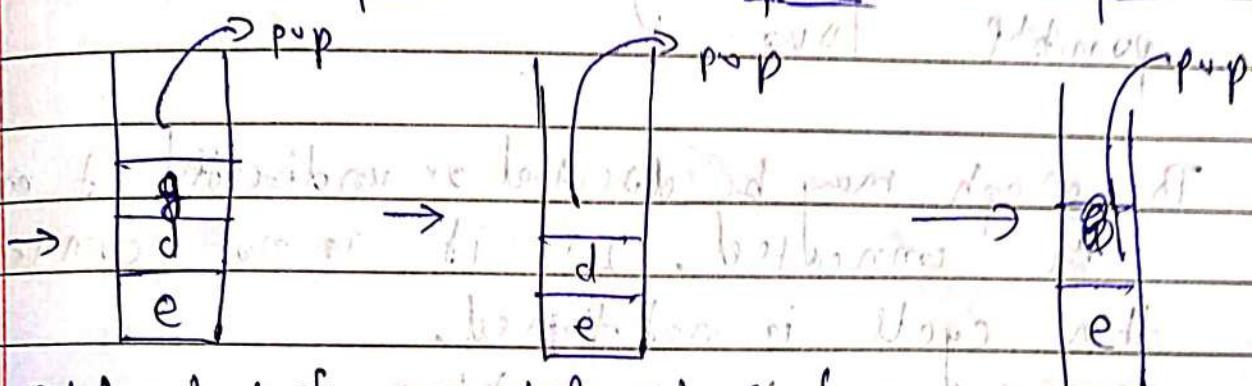


empty queue

$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow f \rightarrow g$

DFS

print a, print b and print a, b, c



print a, b, c, f, print a, b, c, f, g, print a, b, c, f,

g, d

Now all the children of 'a' are stored in stack

Now we can print 'a' and then pop it from stack

Now we can print 'b' and then pop it from stack

Now we can print 'c' and then pop it from stack

empty stack

print a, b, c, f, g, d, e

Here traversal is completed.

$a \rightarrow b \rightarrow c \rightarrow f \rightarrow g \rightarrow d \rightarrow e$

## Q4. What is Hamiltonian cycle?

Vishal

It is also called Hamilton circuit.

It is a graph cycle that is closed loop through a graph that visit each node exactly once.

In hamiltonian cycle, we have to find the all possible tour.

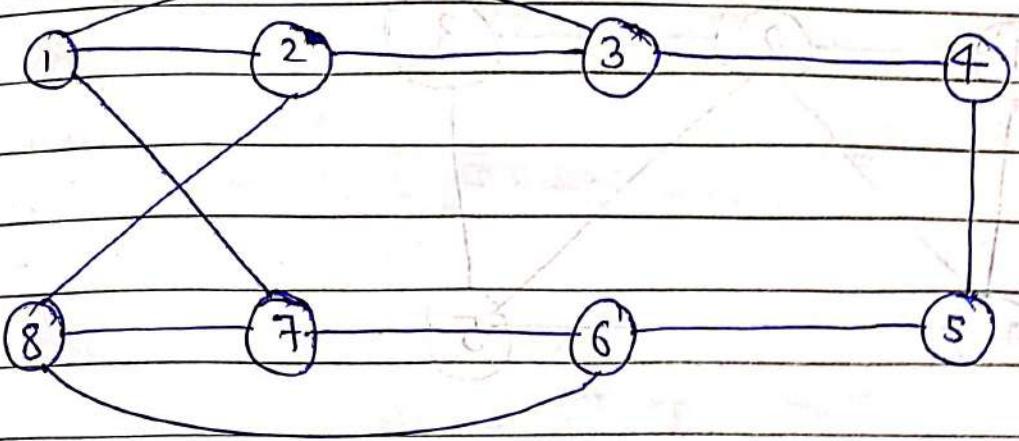
The graph may be directed or undirected if must be connected. If it is not connected then cycle is not formed.

This is NP-hard problem.

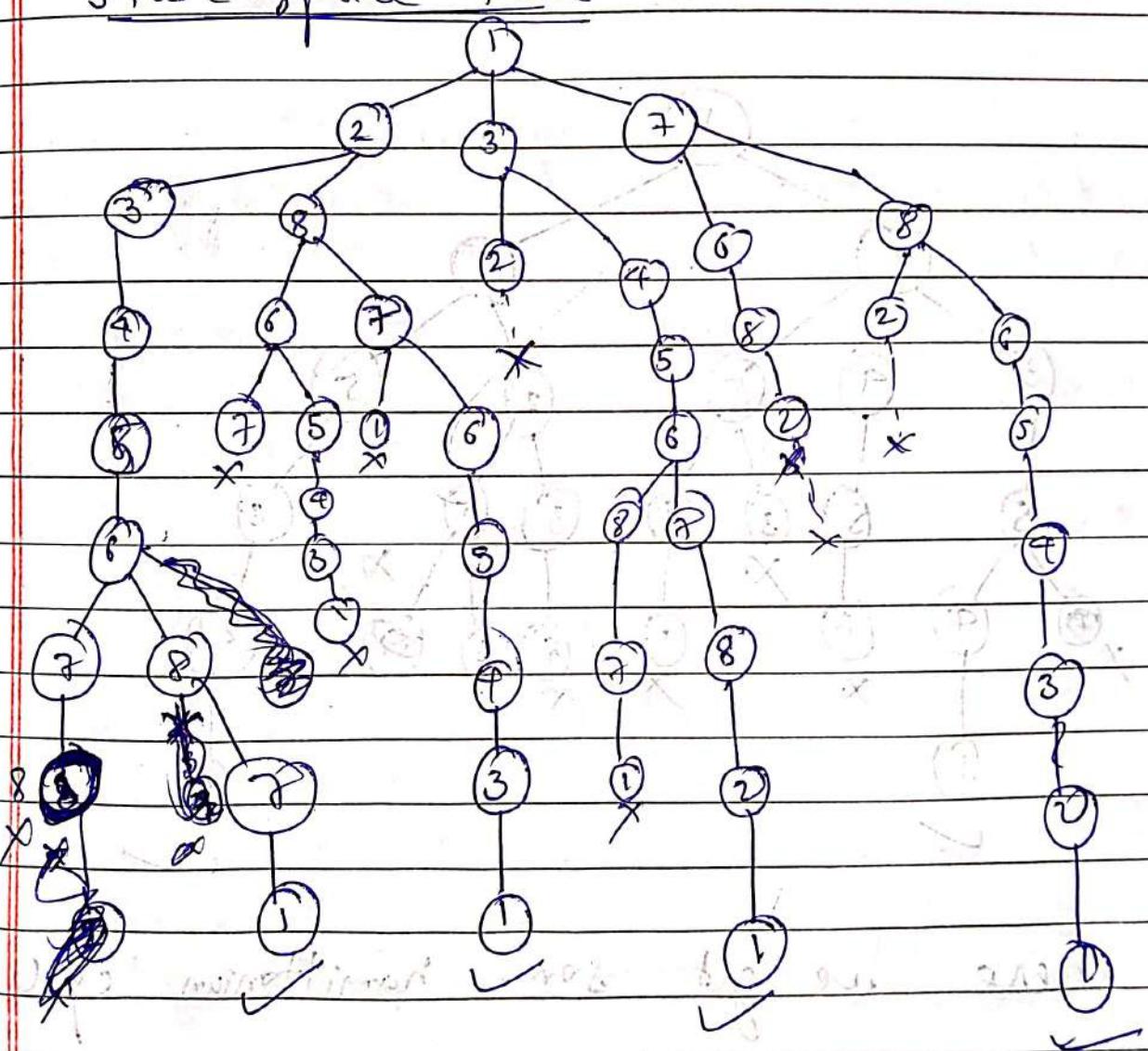
Hamiltonian cycle or circuit in a graph  $G$  is a cycle that visits every vertex of  $G$  exactly once and returns to the starting vertex.

### Use of hamiltonian cycle:

- finding optimal routes in transportation networks.
- Circuit design
- graph theory research
- in computer graphics
- mapping genomes.



## State space tree



Here we get 1 source path,

## Hamiltonian cycles

1 - 2 - 3 - 4 - 5 - 6 - 8 - 7 - 1

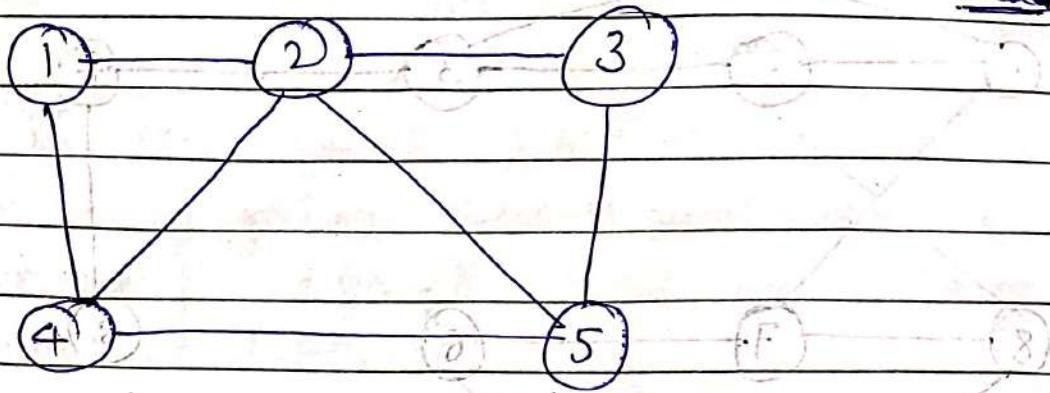
1 — 2 — 8 — 7 — 6 — 5 — 4 — 3 — 1

1 - 3 - 4 - 5 - 6 - 7 - 8 - 2 - 1

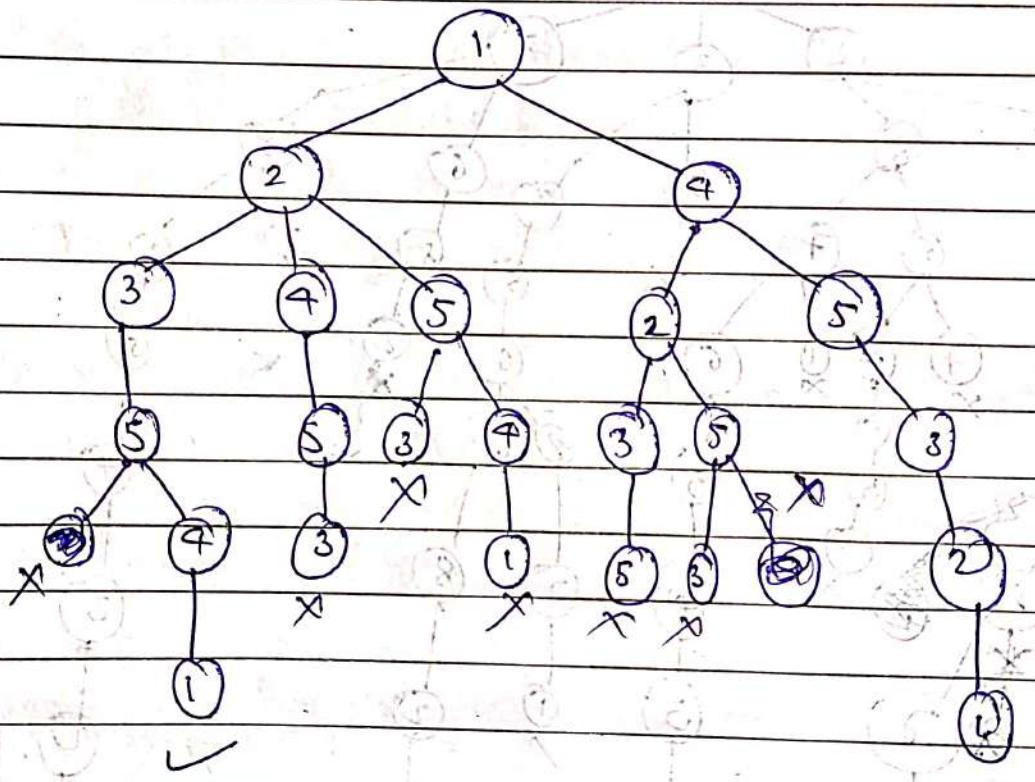
1 — 7 — 8 — 6 — 5 — 4 — 3 — 2 — 1

Date: \_\_\_\_\_ / \_\_\_\_\_ / \_\_\_\_\_

q 5.



## State space tree



Here we get some hamiltonian cycle

1 - 2 — 3-5-4 - 1

1-4-5-3-2-

$$1 - s - e - d - c - b - a - r - s = 0$$

$$1 + 2 + 3 + 2 + 3 + 4 + 2 + 3 + 1$$

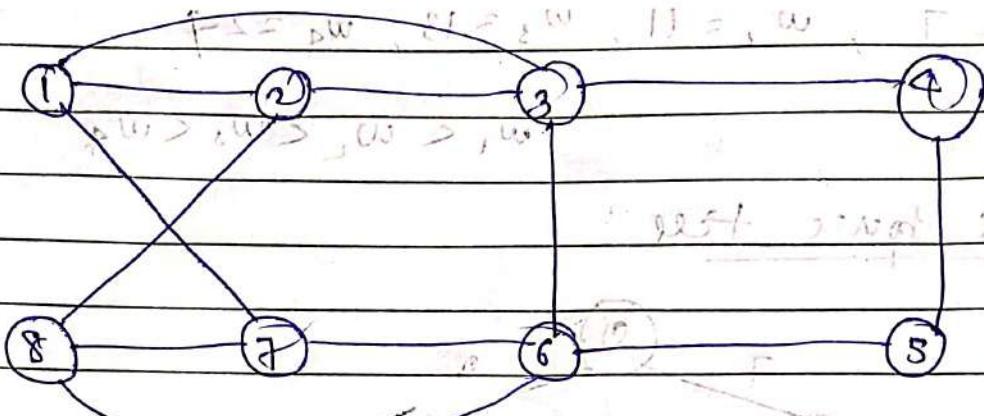
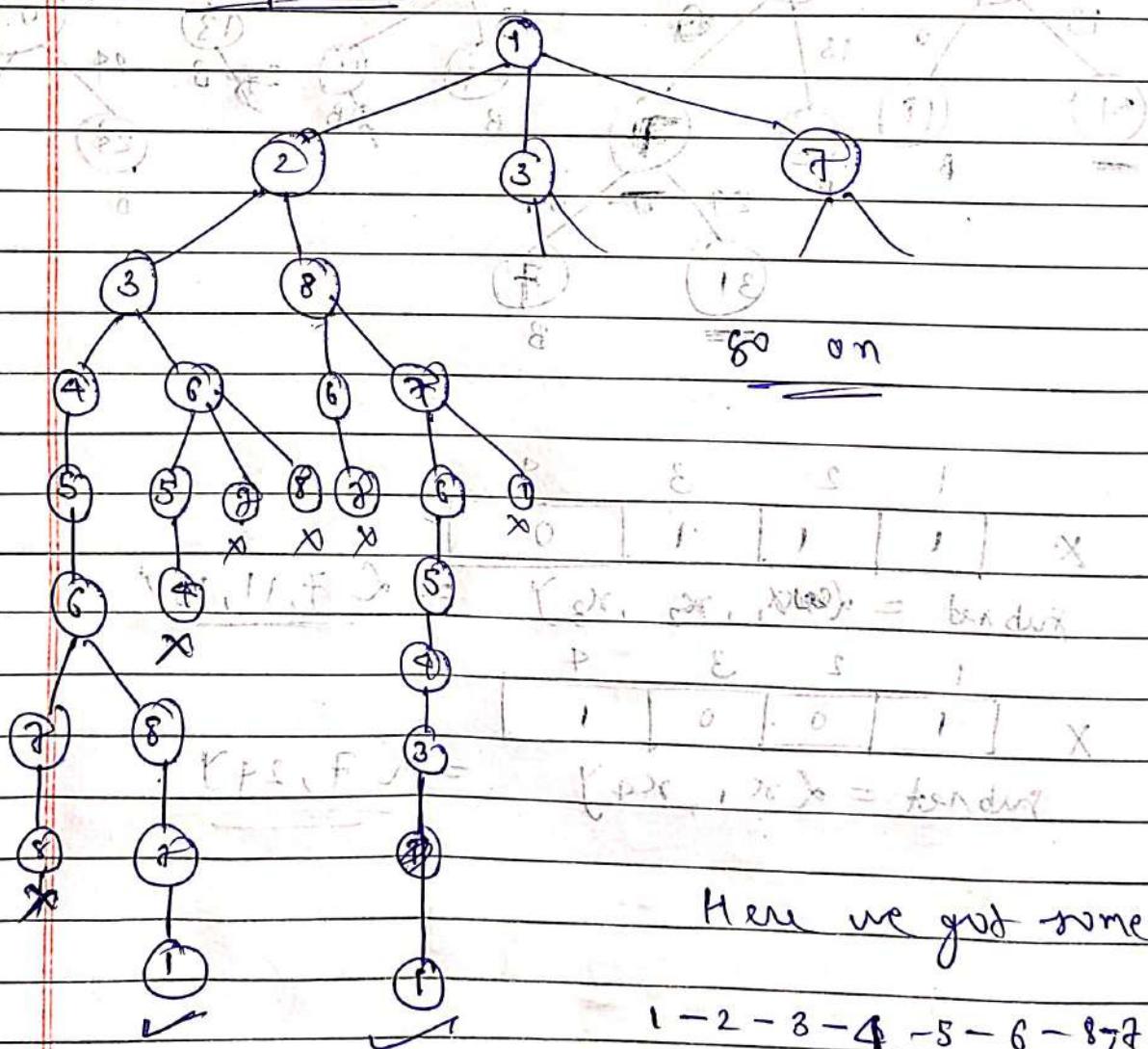
$$1 - \theta = \pi T + 2\pi - \theta = \pi n,$$

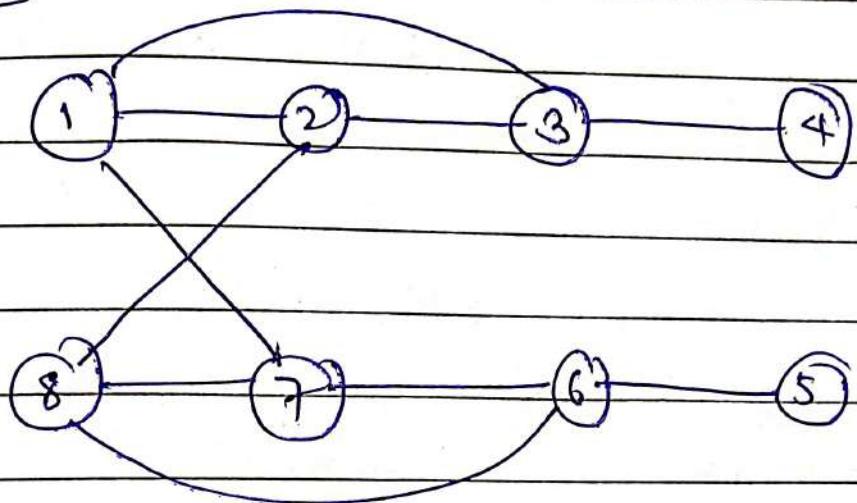
Q6)

## Hamiltonian

F.P.S.E, H = W

L.E = M

State space tree

Q7

ye nahi hoga hamiltonian cycle

because wo connected nahi ha,

Q8. What is 8-queen problem? Explain implicit & explicit constraints associated with this two sol<sup>n</sup> for problem. Give at least one sol<sup>n</sup> for problem.

$\Rightarrow$  Backtracking approach for 8-queen problem  
— it is a recursive approach for solving any problem where we have search among all possible sol<sup>n</sup>, following some constraints.

Given an 8-queen problem, we must place 8 queens on the 8x8 chessboard such that none of them attack one another.

Here no two queens in same row, column, or diagonal.

More generally, n queen problem places n queen on nxn chessboard.

Here, we suppose that the queen "i" is to be placed in first row, only we can say that 1 queen is placed in first row only but can have columns from 1, 2, ..., n so that they satisfy all explicit & implicit constraints.

All sol<sup>n</sup> to 8-queen problem are represented as 8-tuples  $(x_1, x_2, x_3, \dots, x_8)$  where  $x_i$  is the column on which queen "i" is to be placed.

developed 8 methods with 3 in 1970

Explicit constraints are based on the fact that each position is filled with one

The explicit constraints using this formula are  $S_i = \{1, 2, 3, \dots, 7, 8\}$

where  $1 \leq i \leq 8$  which is 8.

Therefore, solution space consists of  $8 \times 8$  8-tuples. Total number will be

using formula - 8, 00, 000

Implicit constraints & result from the

two double conditions can be

No two  $x_{ij}$ 's can be same. i.e. two green

columns be in same row, same column or

in same diagonal so each row, column

and diagonal must be different.

### 8 green problem soln

and off-diagonal cells must be empty.

First row constraint was fixed at 1, 2, 3, 4, 5, 6, 7, 8

Second row was 1, 2, 3, 4, 5, 6, 7, 8

Third row was 1, 2, 3, 4, 5, 6, 7, 8

Fourth row was 1, 2, 3, 4, 5, 6, 7, 8

Fifth row was 1, 2, 3, 4, 5, 6, 7, 8

Sixth row was 1, 2, 3, 4, 5, 6, 7, 8

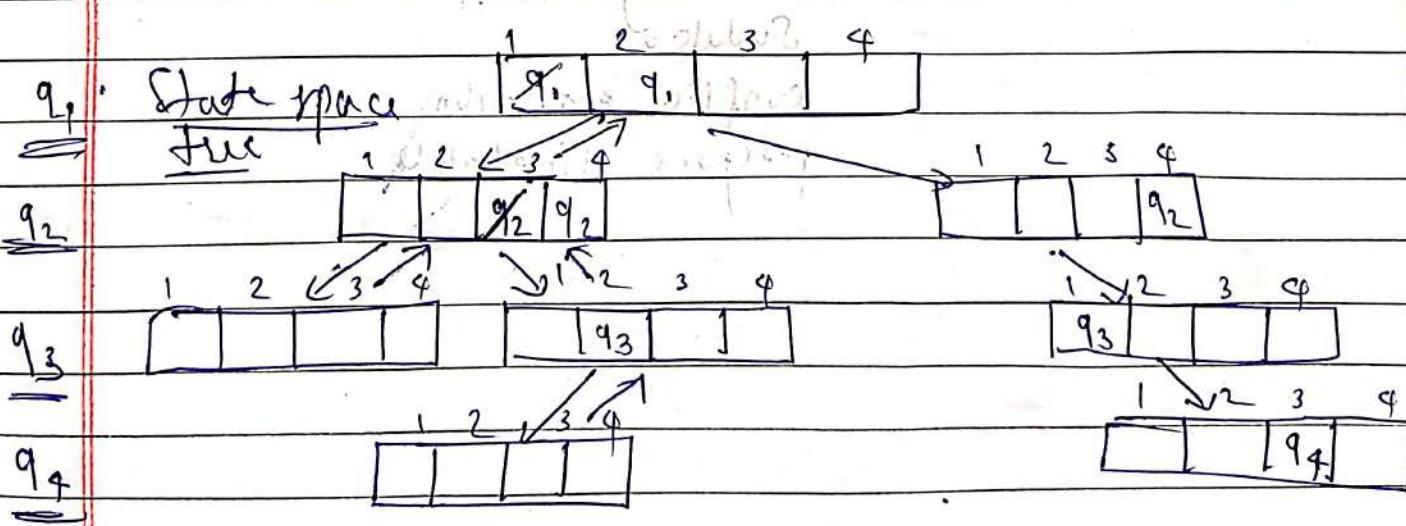
Seventh row was 1, 2, 3, 4, 5, 6, 7, 8

Eighth row was 1, 2, 3, 4, 5, 6, 7, 8

91	(reduced) forward	9,	
		92	initial
		93	
94	for action training	94	6
	action 95	95	
		96	
97		97	
		98	initial
		99	6

9. similarly all the other vertices except  
one will have 3 edges so the vertices which  
only 3-green be placed for 3m-green.

for 4-green and yellow in right  
eg. 1 2 3 4 5 6 7 8 9



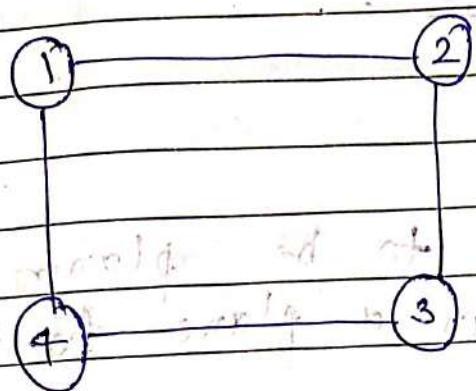
(q10) What is graph coloring?

Graph coloring refers to the problem of coloring vertices of a graph in such a way that no two adjacent vertices have same color.

This is also called the vertex coloring problem. If coloring is done using at most  $m$  colors, it is called  $m$ -coloring.

Application — map coloring  
scheduling tasks  
Sudoku

conflict resolution  
generate timetable



Let

$$\text{color} = (R, G, B)$$

state space tree

(i) R G R G

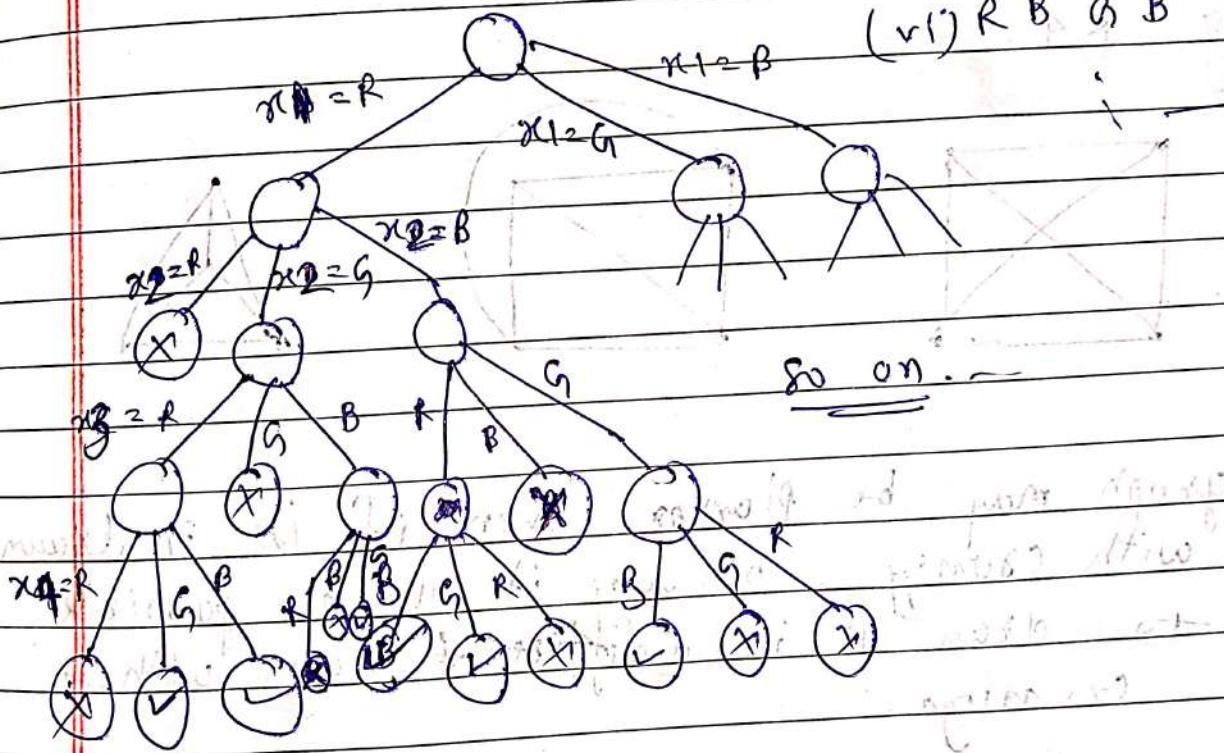
(ii) R G R B

(iii) R G B G

(iv) R B R G

(v) R B R B

(vi) R B G B



so on.

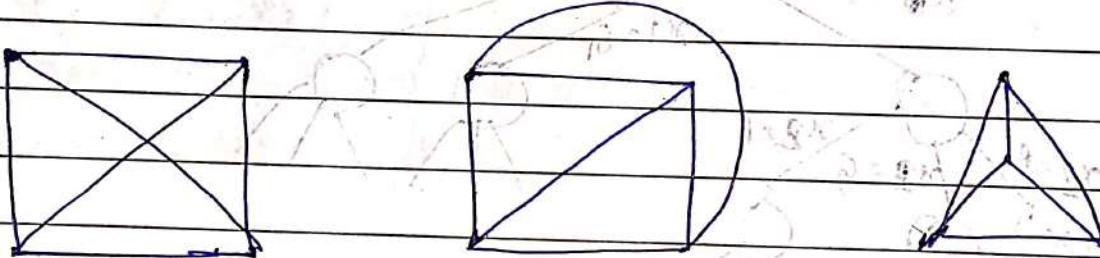
(ii)isme kitne color or konne diye hee nahi?

(Q12)

Similar to que 10.Planar graph

A graph is said to be planar if it can be drawn in a plane such that no edge cross.

- (i) If it can be drawn on a plane without any edges crossing.
- (ii) If it can be drawn on a plane without any edges crossing with some edges overlapping.

eg.  $K_4$ 

A graph may be planar even if it is drawn with crossings, because it may be possible to draw it in a different way without crossings.

Editor and Spib: SMOY in color and black (10)

(g) 16) Apply backtracking algo. to solve instance of sum of subset problem.  $S = \{1, 3, 4, 5\}$ ,  $m = 10$ . find all possible subset of  $S$  that generates  $\text{sum} = m$ . draw solution space tree for same.

$\Rightarrow$

This problem will be solved using space state tree with help of backtracking technique.

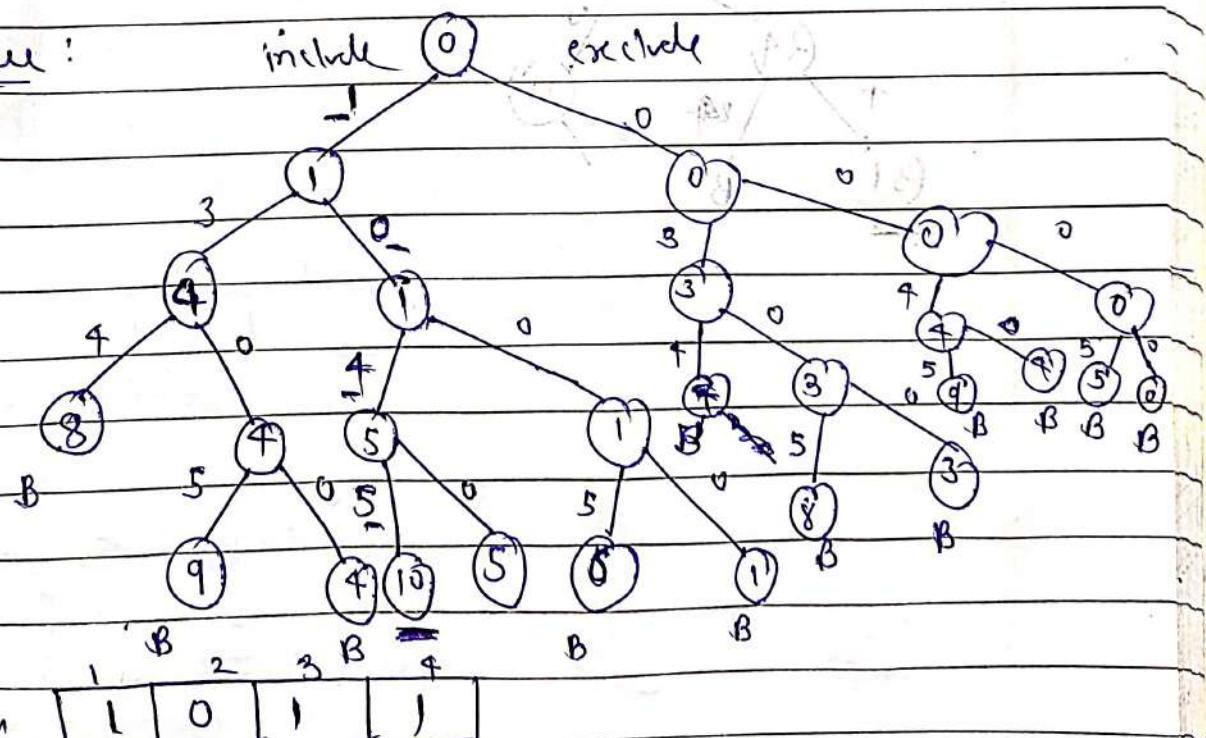
It involves determining whether or not a subset from list of integers can sum to a target value.

Bounding func:  $\sum_{i=1}^k w_i x_i + w_k + l \leq m$

$$\sum_{i=1}^k w_i x_i + \sum_{i=k+1}^m w_i > m$$

Let,  $w_1 = 1$ ,  $w_2 = 3$ ,  $w_3 = 4$ ,  $w_4 = 5$ ,  $m = 10$

state  
Space tree



X | | - | |

$$\text{subset} = \{x_1, x_3, x_4, y\} = \{1, 4, 5\}$$

15

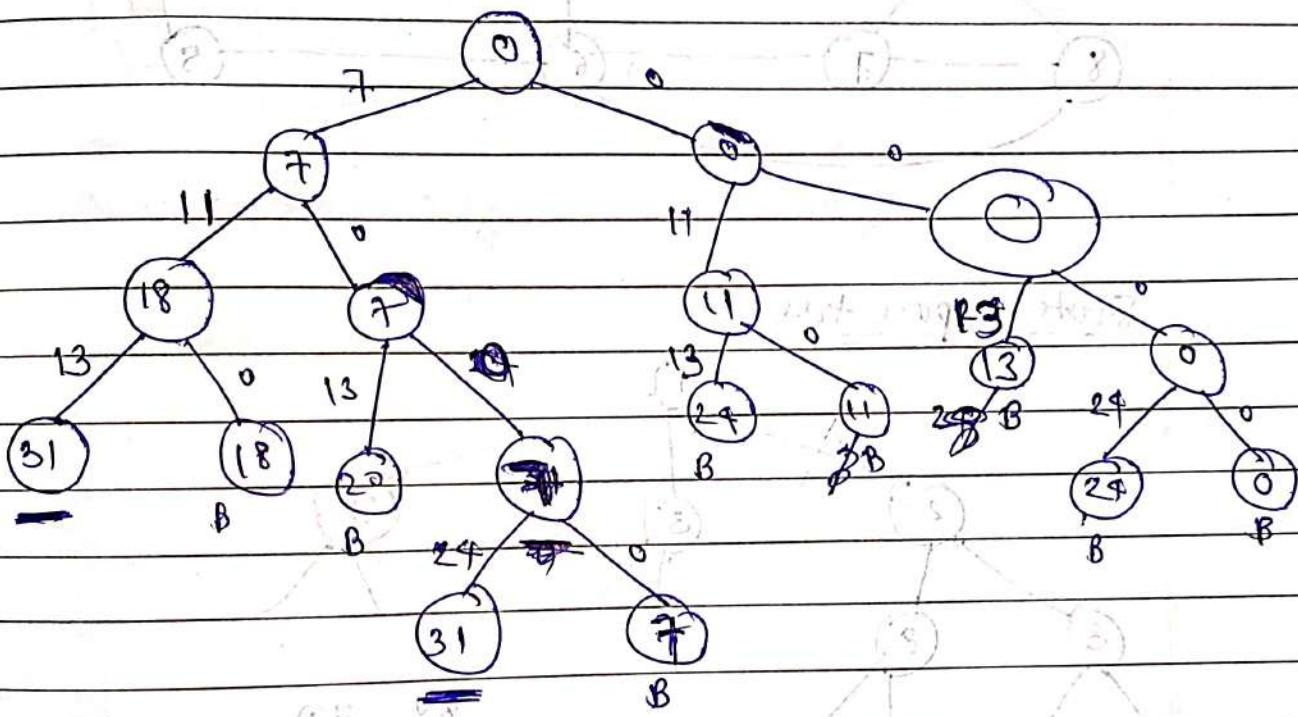
$$W = \{11, 13, 24, 7\}$$

$$m = 31$$

$$w_1 = 7, w_2 = 11, w_3 = 13, w_4 = 24$$

$$w_1 < w_2 < w_3 < w_4$$

state space tree:



1	2	3	4
x	1	1	1

$$\text{subset} = \{x_1, x_2, x_3\} = \underline{\{7, 11, 13\}}$$

1	2	3	4
x	1	0	0

$$\text{subset} = \{x_1, x_4\} = \underline{\{7, 24\}}$$

$$1 - 4 + 8 - 3 - 2 - 1 - 8 - 5 = 1$$

$$1 - 6 + 4 - 2 - 3 - 4 - 8 - 5 = 1$$