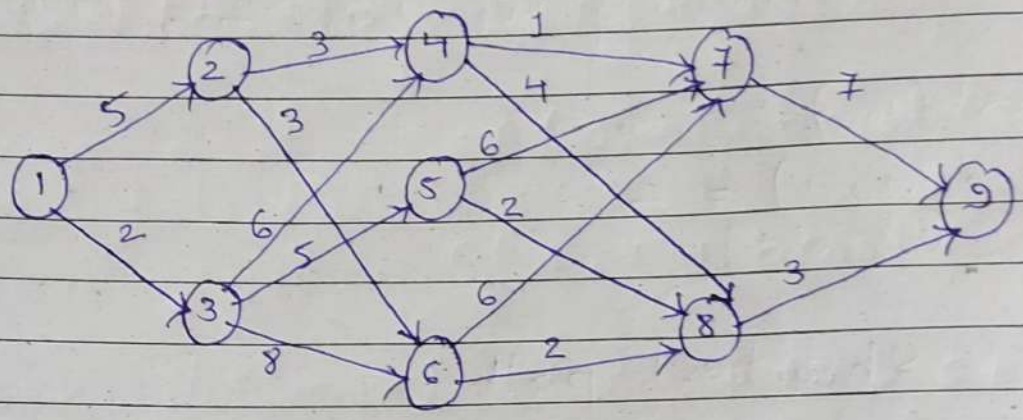
UNIT 4 :-

Q.1) Solve using Backward multistage graph



Soln :- Backward Approach :-

$$bCost(i,j) = \min \{ c(l,j) + bCost(i-1,l) \}$$

* Stage 1 :-

$bCost(1,1) = 0$

~~d (1,1) =~~

~~b cost (=~~

Vertex	1	2	3	4	5	6	7	8	9
Cost	0	5	2	8	7	8	9	9	12
distance	1	1	1	3	3	2	4	5	8

* Stage 2 :-

$$bCost(2,2) = \min \{ c(1,2) + bCost(1,1) \}$$

$bCost(2,2) = \min \{ 5 + 0 \} = 5$

$d(2,2) = 1$

$bCost(2,3) = min\{c(1,3) + bCost(1,1)\}$

$= min\{2 + 0\}$

$bCost(2,3) = 2$

$d(2,3) = 1$

* Stage 3:-

$bCost(3,4) = min\{c(2,4) + bCost(2,2), c(3,4) + bcost(2,3)\}$

$= min\{3 + 5, 6 + 2\}$

$bcost(3,4) = min\{8, 8\} = 8$

$d(3,4) = 2$ or 3

$bCost(3,5) = min\{c(3,5) + bcost(2,3)\}$

$bcost(3,5) = \cancel{min\{}\ \cancel{5+}\ min\{5 + 2\} = 7$

$d(3,5) = 3$

$bCost(3,6) = min\{c(2,6) + bcost(2,2), c(3,6) + bCost(2,3)\}$

$= min\{3 + 5, 8 + 2\}$

$bcost(3,6) = min\{8, 10\} = 8$

$d(3,6) = 2$

* Stage 4:-

$bCost(4,7) = \min\{ c(4,7) + bcost(3,4),$

$c(5,7) + bcost(3,5),$

$c(6,7) + bcost(3,6) \}$

$= \min\{ 1+8, 6+7, 6+8 \}$

$bcost(4,7) = \min\{ 9, 13, 14 \} = 9$

$d(4,7) = 4.$

$bcost(4,8) = \min\{ c(4,8) + bcost(3,4),$

$c(5,8) + bcost(3,5),$

$c(6,8) + bcost(3,6) \}$

$= \min\{ 4+8, 2+7, 2+8 \}$

$= \min\{ 12, 9, 10 \}$

$bcost(4,8) = 9$

$d(4,8) = 5.$

* Stage 5 :-

$$bcost(5,2) = \min\{ c(7,2) + bcost(6,7),\ c(8,2) + bcost(4,8) \}$$
$$= \min\{ 7+2,\ 3+2 \}$$
$$= \min\{ 16,\ 12 \}$$
$$bcost(5,2) = 12$$

$$d(5,2) = 8.$$

Shortest path

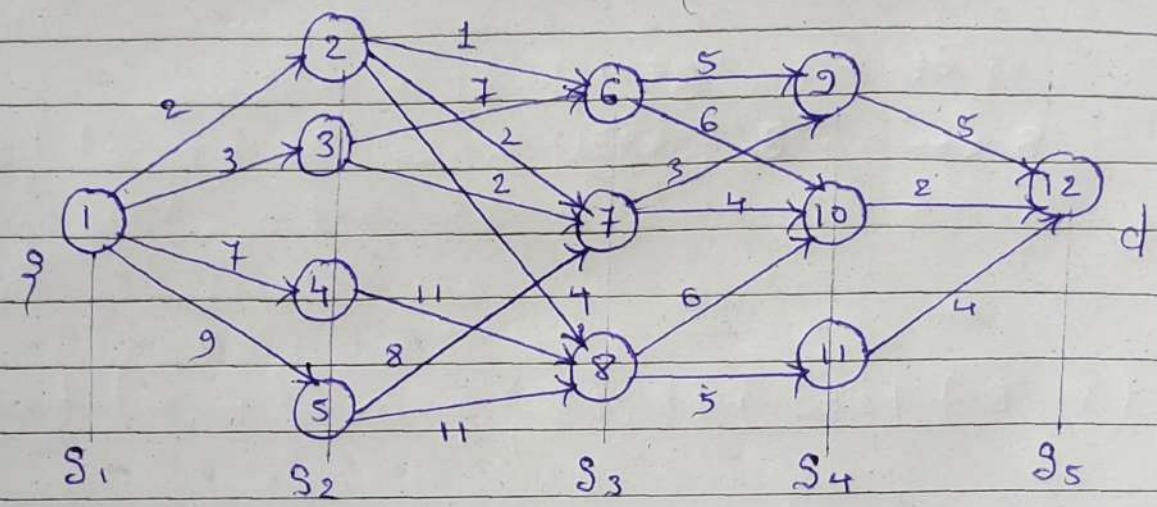
$$d(5,2) = 8$$
$$d(4,8) = 5$$
$$d(3,5) = 3$$
$$d(2,3) = 1$$

* Shortest Path :-

$$= 2 - 8 - 5 - 3 - 1$$

$$= 3 + 2 + 5 + 2 = 12$$

Q.2) What is multistage graph? For the following graph find shortest path from source to destination. * (Forward Approach)



Soln :-

$$cost(i,j) = \min \{ c(j,l) + cost(i+1, l) \}$$

Vertex	1	2	3	4	5	6	7	8	9	10	11	12
Cost	8	8	8	13	14	8	6	8	5	2	4	0
dist	2	6	7	8	7	10	10	10	12	12	12	12

$Cost(5, 12) = 0$

$Cost(4, 9) = 5$

$Cost(4, 10) = 2$

$Cost(4, 11) = 4$

$$Cost(3, 6) = \min \{ c(6, 9) + Cost(4, 9), \; c(6, 10) + Cost(4, 10) \}$$

$$= \min \{ 5 + 5, \; 6 + 2 \} = \min \{ 10, 8 \}$$

$Cost(3, 6) = 8$

$d(3, 6) = 10$

$Cost(3,7) = \min\{ c(7,9) + cost(4,9),$
$c(7,10) + cost(4,10) \}$

$= \min\{ 3+5, 4+2 \}$
$= \min\{ 8, 6 \}$
$Cost(3,7) = 6$
$d(3,7) = 10$

$Cost(3,8) = \min\{ c(8,10) + Cost(4,10),$
$c(8,11) + Cost(4,11) \}$

$= \min\{ 6+2, 5+4 \}$
$= \min\{ 8, 9 \}$
$Cost(3,8) = 8$
$d(3,8) = 10$

$Cost(2,2) = \min\{ c(2,6) + Cost(3,6),$
$c(2,7) + cost(3,7),$
$c(2,8) + cost(3,8) \}$

$= \min\{ 1+18, 2+16, 4+18 \}$
$Cost(2,2) = \min\{ 19, 18, 22 \} = 18$
$d(2,2) = 7$

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Cost (2,3) = min { c(3,6) + Cost(3,6),
c(3,7) + Cost(3,7), }

= min { 7 + 8, 2 + 6 }

Cost(2,3) = min { 15, 8 } = 8

d(2,3) = 7

Cost (2,4) = min { c(4,8) + Cost(8,8) }

= min { 11 + 8 }

= 19

d(2,4) = 8

Cost (2,5) = min { c(5,7) + Cost(3,7) },
c(5,8) + cost(3,8) }

= min { 8 + 6, 11 + 8 }

Cost(2,5) = min { 14, 18 } = 14

d(2,5) = 7

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$Cost(1,1) = \min\{c(1,2) + cost(2,2),$

$c(1,3) + cost(2,3),$

$c(1,4) + cost(2,4),$

$c(1,5) + cost(2,5)\}$

$= \min\{2+8, 3+8, 7+8, 9+14\}$

$= \min\{8, 11, 15, 23\}$

$Cost(1,1) = 10 = 10$

$d(1,1) = 2$

→ shortest path.

$d(1,1) = 2$

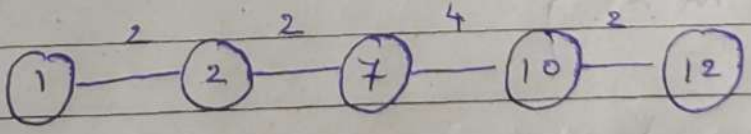
$d(2,2) = 7$

$d(3,7) = 10$

$d(4,10) = 12$

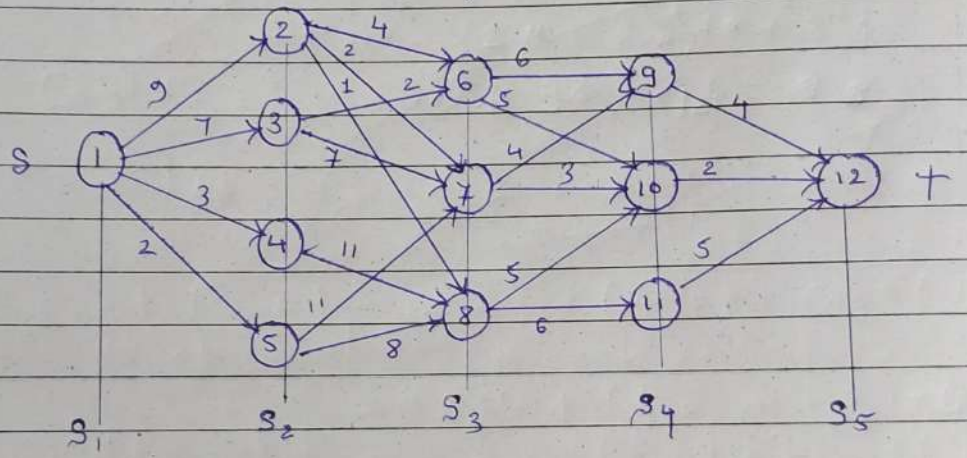
$d(5,12) = 16$

path



Shortest dist^n from source to destination => 2 + 2 + 4 + 2 = 10

5) Find the minimum cost path from "s" to "t" in multistage graph shown. Using forward approach.



n:- $Cost(i, j) = \min \{ c(j, l) + Cost(i+1, l) \}$

Vertex	1	2	3	4	5	6	7	8	9	10	11	12
Cost	16	7	9	18	15	7	5	7	4	2	5	0
dist	2	7	6	8	8	10	10	10	12	12	12	12

$Cost(5, 12) = 0$
$Cost(4, 9) = 4$
$Cost(4, 10) = 2$
$Cost(4, 11) = 5$

$Cost(3, 6) = \min \{ c(6, 9) + cost(4, 9),$
$c(6, 10) + Cost(4, 10) \}$
$= \min \{ 6 + 4, 5 + 2 \}$
$Cost(3, 6) = \min \{ 10, 7 \} = 7$
$d(3, 6) = 10$

Cost (3, 7) = min { c(7, 5) + Cost (4, 5),
c(7, 10) + Cost (4, 10) }
= min { 4 + 4, 3 + 2 }
= min { 8, 5 }
Cost(3, 7) = 5
d (3, 7) = 10

Cost (3, 8) = min { c(8, 10) + Cost (4, 10),
c(8, 11) + Cost (4, 11) }
= { 5 + 2, 6 + 5 }
Cost(3, 8) = { 7, 11 }
Cost(3, 8) = 7
d (3, 8) = 10

Cost (2, 2) = min { c(2, 6) + cost (3, 6),
c(2, 7) + cost (3, 7),
c(2, 8) + cost (3, 8) }
= min { 4 + 7, 2 + 5, 1 + 7 }
= min { 11, 7, 8 }
Cost (2, 2) = 7
d (2, 2) = 7

$$Cost(2,3) = \min\{ c(3,6) + Cost(3,6),\ c(3,7) + Cost(3,7) \}$$

$$= \min\{ 2 + 7, 7 + 5 \}$$

$$Cost(2,3) = \min\{ 9, 12 \} = 9$$

$$d(2,3) = 6$$

$$Cost(2,4) = \min\{ c(4,8) + Cost(3,8) \}$$

$$= \min\{ 11 + 7 \} = \min\{ 18 \} = 18$$

$$d(3,4) = 8$$

$$Cost(2,5) = \min\{ c(5,7) + Cost(3,7),\ c(5,8) + Cost(3,8) \}$$

$$= \min\{ 11 + 5, 8 + 7 \}$$

$$Cost(2,5) = \min\{ 16, 15 \} = 15$$

$$d(2,5) = 8$$

$$Cost(1,1) = \min\{ c(1,2) + Cost(2,2),\ c(1,3) + Cost(2,3),\ c(1,4) + Cost(2,4),\ c(1,5) + Cost(2,5) \}$$

$= \min\{9+7, 7+9, 3+18, 2+15\}$

$= \min\{16, 16, 21, 17\}$

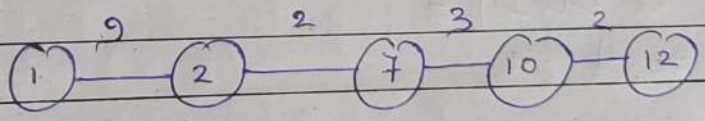
Cost(1,,) = 16

d(1,1) = 2

Shortest path

d(1,1) = 2
d(2,2) = 7
d(3,7) = 10
d(4,10) = 2

path



Shortest dist^n from 1 to 12 = 9 + 2 + 3 + 2 = 16

Q.4) Differentiate bet'n

1) Dynamic Programming	Greedy Strategy
1) Dynamic Programming is used to obtain the optimal solution.	1.) Greedy Method is also used to get the optimal solution.
2) Breaks down the problem into subproblems.	2.) Makes choices without considering the future.
3) Less efficient as compared to a greedy approach.	3) More efficient as compare to a greedy approach.
4) It is guaranteed that Dynamic Programming will generate an optimal sol'n using Principle of optimality.	4) In Greedy Method, there is no such guarantee of getting optimal solution.
5) Memorization is required	5) No memorization is required.
6) Slow results Comparatively	6) Fast results.
7) Every problem can be solved by dynamic algorithm.	7) Every problem can't be solved by a greedy algorithm.

8)	LCS, knapsack problem	8) Shortest path problem, Huffman Coding

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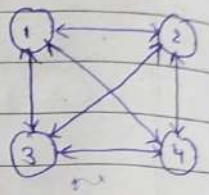
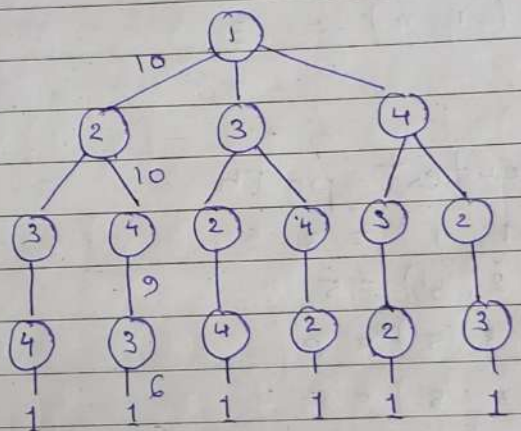
Q.] Travelling Salesman Problem ? Implement TSP for following matrix multiplication

	1	2	3	4
1	0	10	15	20
2	5	0	9	10
3	6	13	0	12
4	8	8	9	0



Soln :-



$$g(i, s) = \min_{j \in s} \{ c_{ij} + g(j, s - \{j\}) \}$$

Let the starting vertex 1

* If $|s| = \phi$. → Consider null set elemen

$g(2, \phi) = C_{21} = 5$

$g(3, \phi) = C_{31} = 6$

$g(4, \phi) = C_{41} = 8$

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* If |s| = 1 → Consider 1 set element

$g(2, \{3,4\}) = C_{23} + g(3, \phi) = 9 + 6 = 15$

$g(2, \{4\}) = C_{24} + g(4, \phi) = 10 + 8 = 18$

,

$g(3, \{2\}) = C_{32} + g(2, \phi) = 13 + 5 = 18$

$g(3, \{4\}) = C_{34} + g(4, \phi) = 12 + 8 = 20$

$g(4, \{3\}) = C_{43} + g(3, \phi) = 9 + 6 = 15$

$g(4, \{2\}) = C_{42} + g(2, \phi) = 8 + 5 = 13$

* If |s| = 2 → Consider two set element

$g(2, \{3,4\}) = \min \{ C_{23} + g(3, \{4\}),$
$C_{24} + g(4, \{3\}) \}$

$= \min \{ 9 + 20, 10 + 15 \}$

$= \min \{ 29, 25 \}$

$g(2, \{3,4\}) = \underline{25}$

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$g(4, \phi) = C_{41} =$

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$g(2, \{4,3\}) = \min \{ c(2,4) + g(4, \{3\}),$
$c(2,3) + g(3, \{4\}) \}$

$= \min \{ 10 + 15, 9 + 20 \}$
$= \min \{ 25, 29 \}$
$g(2, \{4,3\}) = 25$

$g(3, \{2,4\}) = \min \{ c(3,2) + g(2, \{4\}),$
$c(3,4) + g(4, \{2\}) \}$

$= \min \{ 13 + 18, 12 + 13 \}$
$= \min \{ 31, 25 \}$
$g(3, \{2,4\}) = 25$

$g(3, \{4,2\}) = \min \{ c(3,4) + g(4, \{2\}),$
$c(3,2) + g(2, \{4\}) \}$

$= \min \{ 12 + 13, 18 + 18 \}$
$= \min \{ 25, 31 \}$

$g(3, \{4,2\}) = 25$

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$g(4, \{2,3\}) = \min \{ c_{42} + g(2, \{3\}),$
$c_{43} + g(3, \{4\}) \}$
$= \min \{ 8 + 15, 9 + 20 \}$
$= \min \{ 23, 29 \}$
$g(4, \{2,3\}) = \underline{23}$

$g(4, \{3,2\}) = \min \{ c_{43} + g(3, \{2\}),$
$c_{42} + g(2, \{3\}) \}$
$= \min \{ 9 + 18, 8 + 15 \}$
$= \min \{ 27, 23 \}$
$g(4, \{3,2\}) = 23$

* If $|s| = 3$.

$g(1, \{2,3,4\}) = \min \{ c_{12} + g(2, \{3,4\}),$
$c_{13} + g(3, \{2,4\}),$
$c_{14} + g(4, \{2,3\}) \}$

$= \min \{ 10 + 25, 15 + 25, 20 + 23 \}$
$= \min \{ 35, 40, 43 \}$
$g(1, \{2,3,4\}) = 35$
path is $1 \xrightarrow{10} 2 \xrightarrow{10} 4 \xrightarrow{15} 3 \xrightarrow{6} 1$
And cost is 35

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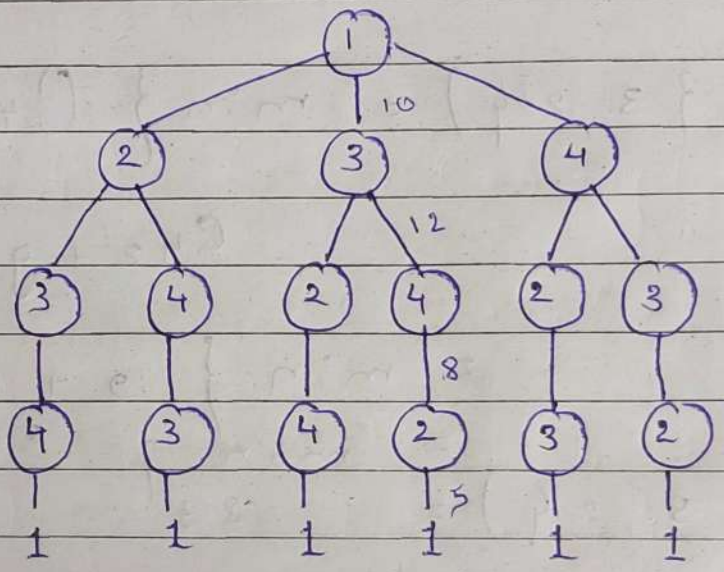
Q.10) Implement T. Salesman problem for following matrix.

	1	2	3	4
1	0	15	10	20
2	5	0	10	9
3	6	13	0	12
4	8	8	9	0

Soln :-



$$g(i, s) = \min \{ c_{ij} + g(j, s - \{j\}, 4) \}$$

* If $|s| = \phi$

$$g(2, \phi) = C_{21} = 5$$
$$g(3, \phi) = C_{31} = 6$$
$$g(4, \phi) = C_{41} = 8$$

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* If $|q| = 1$.

$g(2, \{3\}) = \min\{c_{23} + g(3, \phi)\} = 10 + 6 = 16$

$g(2, \{4\}) = \min\{c_{24} + g(4, \phi)\} = 9 + 8 = 17$

$g(3, \{2\}) = \min\{c_{32} + g(2, \phi)\} = 13 + 5 = 18$

$g(3, \{4\}) = \min\{c_{34} + g(4, \phi)\} = 12 + 8 = 20$

$g(4, \{2\}) = \min\{c_{42} + g(2, \phi)\} = 8 + 5 = 13$

$g(4, \{3\}) = \min\{c_{43} + g(3, \phi)\} = 9 + 6 = 15$

* If $|q| = 2$

$g(2, \{3,4\}) = \min\{c_{23} + g(3, \{4\}),$
$c_{24} + g(4, \{3\})\}$

$= \min\{10 + 20, 9 + 15\}$
$= \min\{30, 24\}$

$g(2, \{3,4\}) = 24$

$g(2, \{4,3\}) = \min\{c_{24} + g(4, \{3\}),$
$c_{23} + g(3, \{4\})\}$

$= \min\{9 + 15, 10 + 20\}$
$= \min\{24, 30\}$

$g(2, \{4,3\}) = 24$

$g(4, \phi) = c_{41} = 2$

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$g(3, \{2,4\}) = \min\{c_{32} + g(2, \{4\}),$
$c_{34} + g(4, \{2\})\}$

$= \min\{13+17, 12+13\}$

$= \min\{30, 25\}$

$g(3, \{2,4\}) = 25$

$g(3, \{4,2\}) = \min\{c_{34} + g(4, \{2\}),$
$c_{32} + g(2, \{4\})\}$

$= \min\{12+13, 13+17\}$

$g(3, \{4,2\}) = \min\{25, 30\}$

$g(4, \{2,3\}) = \min\{c_{42} + g(2, \{3\}),$
$c_{43} + g(3, \{2\})\}$

$= \min\{8+16, 9+18\}$

$= \min\{24, 27\}$

$g(4, \{2,3\}) = 24$

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$g(4, \{3,2\}) = \min\{c_{43} + g(3, \{2\}),$
$c_{42} + g(2, \{3\})\}$

$= \min\{9 + 18, 8 + 16\}$
$= \min\{27, 24\}$
$g(4, \{3,2\}) = 24$

$g(4, \{2,3\}) = \min\{c_{42} + g(2, \{3\}),$
$c_{43} + g(3, \{2\})\}$

$= \min\{8 + 16, 9 + 18\}$
$= \min\{24, 27\}$
$g(4, \{2,3\}) = 24$

* If $|s| = 3$

$g(1, \{2,3,4\}) = \min\{c_{12} + g[2, \{3,4\}),$
$c_{13} + g(3, \{2,4\}),$
$c_{14} + g(4, \{2,3\})$

$= \min\{15 + 24, 10 + 25, 20 + 24\}$
$= \min\{39, 35, 44\}$
$g(1, \{2,3,4\}) = 35$

path is $1 \xrightarrow{10} 3 \xrightarrow{12} 4 \xrightarrow{8} 2 \xrightarrow{5} 1$

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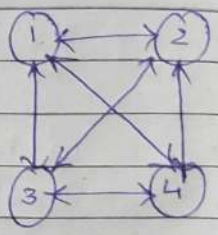
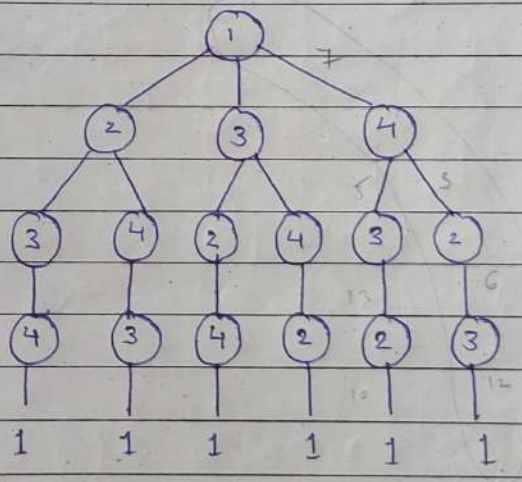
$\therefore$ path cost is 35

Q.11) Travelling Saleman Problem ?

	1	2	3	4
1	0	9	8	7
2	10	0	6	5
3	12	13	0	8
4	2	3	5	0



Soln :-



$$g(i, s) = \min_{j \in s} \{ c_{ij} + g(j, s - \{j\}) \}$$

Let the starting vertex 1

If $|s| = \phi$ → Consider no set element

$g(2, \phi) = C_{21} = 10$

$g(3, \phi) = C_{31} = 12$

$g(4, \phi) = C_{41} = 2$

If $|s| = 1$

$g(2, \{3\}) = \min\{C_{23} + g(3, \phi)\} = \min$

$= 6 + 12 = 18$

$g(2, \{4\}) = \min\{C_{24} + g(4, \phi)\}$

$= 5 + 2 = 7$

$g(3, \{2\}) = \min\{C_{32} + g(2, \phi)\}$

$= 13 + 10 = 23$

$g(3, \{4\}) = \min\{C_{34} + g(4, \phi)\}$

$= 8 + 2 = 10$

$g(4, \{3\}) = \min\{C_{43} + g(3, \phi)\}$

$= 5 + 12 = 17$

$g(4, \{2\}) = \min\{C_{42} + g(2, \phi)\}$

$= 3 + 10 = 13$

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If $|s| = 2$

$g(2, \{3,4\}) = \min\{ C_{23} + g(3, \{4\}),$
$C_{24} + g(4, \{3\}) \}$

$= \min\{ 6 + 10, 5 + 17 \}$

$= \min\{ 16, 22 \} = 16$

$g(2, \{4,3\}) = \min\{ C_{24} + g(4, \{3\}),$
$C_{23} + g(3, \{4\}) \}$

$= \min\{ 5 + 17, 6 + 10 \}$
$= \min\{ 22, 16 \} = 16$

$g(3, \{2,4\}) = \min\{ C_{32} + g(2, \{4\}),$
$C_{34} + g(4, \{2\}) \}$

$= \min\{ 13 + 7, 8 + 13 \}$
$= \min\{ 20, 21 \} = 20$

$g(3, \{4,2\}) = \min\{ C_{34} + g(4, \{2\}),$
$C_{32} + g(2, \{4\}) \}$

$= \min\{ 8 + 13, 13 + 7 \}$
$= \min\{ 21, 20 \} = 20$

$g(4, \{2,3\}) = \min\{ c_{42} + g(2, \{3\}),$
$c_{43} + g(3, \{2\}) \}$

$= \min\{ 3+18, 5+23 \}$

$= \min\{ 21, 28 \} = 21$

$g(4, \{3,2\}) = \min\{ c_{43} + g(3, \{2\}),$
$c_{42} + g(2, \{3\}) \}$

$= \min\{ 5+23, 3+18 \}$

$= \min\{ 28, 21 \} = 21$

* If $|s| = 3$

$g(1, \{2,3,4\}) = \min\{ c_{12} + g(2, \{3,4\}),$
$c_{13} + g(3, \{2,4\}),$
$c_{14} + g(4, \{2,3\}) \}$

$= \min\{ 9+16, 8+20, 7+21 \}$

$= \min\{ 25, 28, 28 \}$

$= 25$

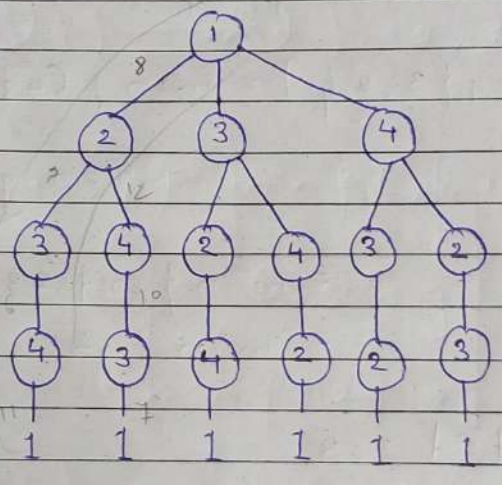
path is $1 \xrightarrow{7} 4 \xrightarrow{3} 2 \xrightarrow{6} 3 \xrightarrow{12} 1$

Cost $\Rightarrow$ $7 + 3 + 6 + 12 = 28$

12)

	1	2	3	4
1	0	8	16	15
2	14	0	9	12
3	7	10	0	6
4	11	13	10	0

Soln :-



Set

$$g(i, S) = \min_{j \in S} \{ c_{ij} + g(j, S - \{j\}) \}$$

* If $|S| = \phi$

$$g(2, \phi) = c_{21} = 14$$
$$g(3, \phi) = c_{31} = 7$$
$$g(4, \phi) = c_{41} = 11$$

* If $|S| = 1$

$$g(2, \{3\}) = \min \{ c_{23} + g(3, \phi) \}$$

$$= \min \; 9$$

$g(2,\{4\}) = \min\{c_{24} + g(4,\phi)\} = 12$

$g(3,\{2\}) = \min\{c_{32} + g(2,\phi)\} = 10$

$g(3,\{4\}) = \min\{c_{34} + g(4,\phi)\} = 6$

$g(4,\{2\}) = \min\{c_{42} + g(2,\phi) = 13$

$g(4,\{3\}) = \min\{c_{43} + g(3,\phi) = 10$

* If $|q| = 2$

$g(2,\{3,4\}) = \min\{c_{23} + g(3,\{4\}),$
$c_{24} + g(4,\{3\})\}$

$= \min\{9+6, 12+10\}$

$g(2,\{3,4\}) = \min\{15, 22\} = 15$

$g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}),$
$c_{34} + g(4,\{2\})\}$

$= \min\{10+12, 6+13\}$

$= \min\{22, 19\} = 19$

$$g(3, \{4,2\}) = \min\{ C_{34} + g(4, \{2\}),$$
$$C_{32} + g(2, \{4\}) \}$$
$$= \min\{ 6+13, 10+12 \}$$
$$g(3, \{4,2\}) = \min\{ 19, 22 \} = 19$$

$$g(2, \{4,3\}) = \min\{ C_{24} + g(4, \{3\}),$$
$$C_{23} + g(3, \{4\}) \}$$
$$= \min\{ 12+10, 9+6 \}$$
$$g(2, \{4,3\}) = \min\{22, 15\} = 15$$
$$g(4, \{2,3\}) = \min\{ C_{42} + g(2, \{3\}),$$
$$C_{43} + g(3, \{2\})$$
$$= \min\{ 13+9, 10+10 \}$$
$$g(4, \{2,3\}) = \min\{22, 20\} = 20$$
$$g(4, \{3,2\}) = \min\{ C_{43} + g(3, \{2\}),$$
$$C_{42} + g(2, \{3\})$$

$= \min \{ 10 + 10, \quad 13 + 9 \}$

$(4, \{3, 2\}) = \min \{ 20, 22 \} \quad = 20.$

* If $|s| = 3$

$g(1, \{2, 3, 4\}) = \min \{ C_{12} + g(2, \{3, 4\}),$

$C_{13} + g(3, \{2, 4\}),$

$C_{14} + g(4, \{2, 3\}) \}$

$= \min \{ 8 + 15, \quad 16 + 19, \quad 15 + 20 \}$

$= \min \{ 23, \quad 35, \quad 35 \}$

$g(1, \{2, 3, 4\}) = \quad 23$

path is.

1 - 2 - 3 - 4 - 1

path cost is $= 8 + 9 + 6 + 11$

$= \quad 34$

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Q.16) Determine LCS : X = { A G G T A B Z },
Y = { G X T X A Y B }.

X \ Y		G	X	T	X	A	Y	B
	0	0	0	0	0	0	0	0
A	0	←0	←0	←0	←0	↑1	←1	←1
G	0	↑1	←1	←1	←1	←1	←1	←1
G	0	↑1	←1	←1	←1	←1	←1	←1
T	0	↑1	←1	↑2	←2	←2	←2	←2
A	0	↑1	←1	↑2	←2	↑3	←3	←3
B	0	↑1	←1	↑2	←2	↑3	←3	↑4
Z	0	↑1	←1	↑2	←2	↑3	←3	↑4

We get the maximum length of common subsequence as 4.

So, Final string of LCS ⇒ G T A B

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Q.20) Longest Common subsequence.
Find optimal solution for given sequence.
X = (A, B, C, D, B, A, B) & Y = (B, C, D, A, B, A).

X = (A, B, C, D, B, A, B)
Y = (B, C, D, A, B, A)

X \ Y		B	C	D	A	B	A
	0	0	0	0	0	0	0
A	0	←0	←0	←0	↑1	←1	↑1
B	0	↑1	←1	←1	←1	↑2	←2
C	0	↑1	↑2	←2	←2	←2	←2
D	0	↑1	↑2	↑3	←3	←3	←3
B	0	↑1	↑2	↑3	←3	↑4	←4
A	0	↑1	↑2	↑3	↑4	←4	↑5
B	0	↑1	↑2	↑3	↑4	↑5	←5

We get the maximum length of common subsequence as 5.

So, Final string of longest common subsequence ⇒ B C D A B

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21) Longest Common Subsequence.

X = a a b a a b a b a a
Y = b a b a a b a b

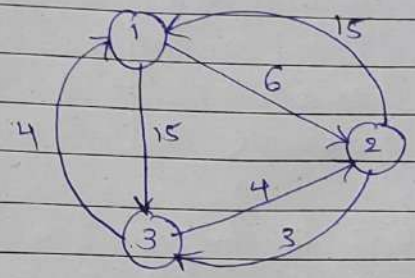
X \ Y		b	a	b	a	a	b	a	b
	0	0	0	0	0	0	0	0	0
a	0	←0	↑1	←1	↑1	↑1	←1	↑1	←1
a	0	←0	↑1	←1	↑2	↑2	←2	↑2	←2
b	0	↑1	←1	↑2	←2	←2	↑3	←3	↑3
a	0	↑1	↑2	←2	↑3	↑3	←3	↑4	←4
a	0	↑1	↑2	←2	↑3	↑4	←4	↑4	←4
b	0	↑1	↑2	↑3	←3	↑4	↑5	←5	↑5
a	0	↑1	↑2	↑3	↑4	↑4	↑5	↑6	←6
b	0	↑1	↑2	↑3	↑4	←4	↑5	↑6	↑7
a	0	↑1	↑2	↑3	↑4	↑5	←5	↑6	↑7
a	0	↑1	↑2	↑3	↑4	↑5	←5	↑6	↑7

We get the maximum length of common subsequence as 7

So, Final string of LCS ⇒ a b a a b a b

Shrikrupa

Q. 23



Soln :-

$$D^0 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0 & 6 & 15 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

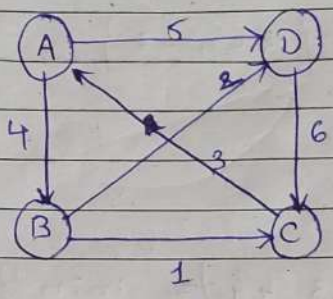
$$D^1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0 & 6 & 15 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

$$D^2 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0 & 6 & 9 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

$$D^3 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & 0 & 6 & 9 \\ 2 & 7 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

D^3 gives the shortest path betn each pair of vertices.

Q.25) Floyd Warshall.



Soln :-

$$D^0 = \begin{array}{c|cccc} & 1\,A & 2\,B & 3\,C & 4\,D \\ \hline 1\,A & 0 & 4 & \infty & 5 \\ 2\,B & \infty & 0 & 1 & 2 \\ 3\,C & 3 & \infty & 0 & \infty \\ 4\,D & \infty & \infty & 6 & 0 \end{array}$$

$$D^1 = \begin{array}{c|cccc} & A\,1 & B\,2 & C\,3 & D\,4 \\ \hline A\,1 & 0 & 4 & \infty & 5 \\ B\,2 & \infty & 0 & 1 & 2 \\ C\,3 & 3 & 7 & 0 & 8 \\ D\,4 & \infty & \infty & 6 & 0 \end{array}$$

$$D^2 = \begin{array}{c|cccc} & A\,1 & B\,2 & C\,3 & D\,4 \\ \hline 1 & 0 & 4 & 5 & 5 \\ 2 & \infty & 0 & 1 & 2 \\ 3 & 3 & 7 & 0 & 8 \\ 4 & \infty & \infty & 6 & 0 \end{array}$$

$$D^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 4 & 5 & 5 \\ 4 & 0 & 1 & 2 \\ 3 & 7 & 0 & 8 \\ 9 & 13 & 6 & 0 \end{bmatrix} \end{array}$$

$$D^4 = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 4 & 3 & 5 \\ 4 & 0 & 1 & 2 \\ 3 & 7 & 0 & 8 \\ 9 & 13 & 6 & 0 \end{bmatrix} \end{array}$$

→ D^4 gives the shortest path between each pair of vertices.

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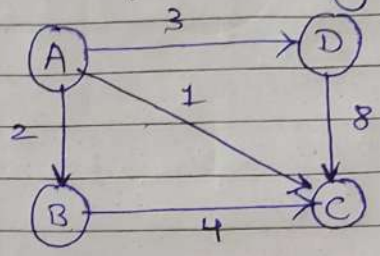
Q.26 Find all shortest paths using Floyd Warshall algorithm



Soln :-

D^0 =

		1 A	2 B	3 C	4 D
1	A	0	2	1	3
2	B	∞	0	4	∞
3	C	∞	∞	0	∞
4	D	∞	∞	8	0

D^1 =

	A	B	C	D
A	0	2	1	3
B	∞	0	4	∞
C	∞	∞	0	∞
D	∞	∞	8	0

D^2 =

	A	B	C	D
A	0	2	1	3
B	∞	0	4	∞
C	∞	∞	0	∞
D	∞	∞	6	0

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$D^3 =$

	A	B	C	D
A	0	2	1	3
B	∞	0	4	∞
C	∞	∞	0	∞
D	∞	∞	6	0

$D^4 =$

	A	B	C	D
A	0	2	1	3
B	∞	0	4	∞
C	∞	∞	0	∞
D	∞	∞	6	0

→ D^4 gives the shortest path betn each pair of vertices.