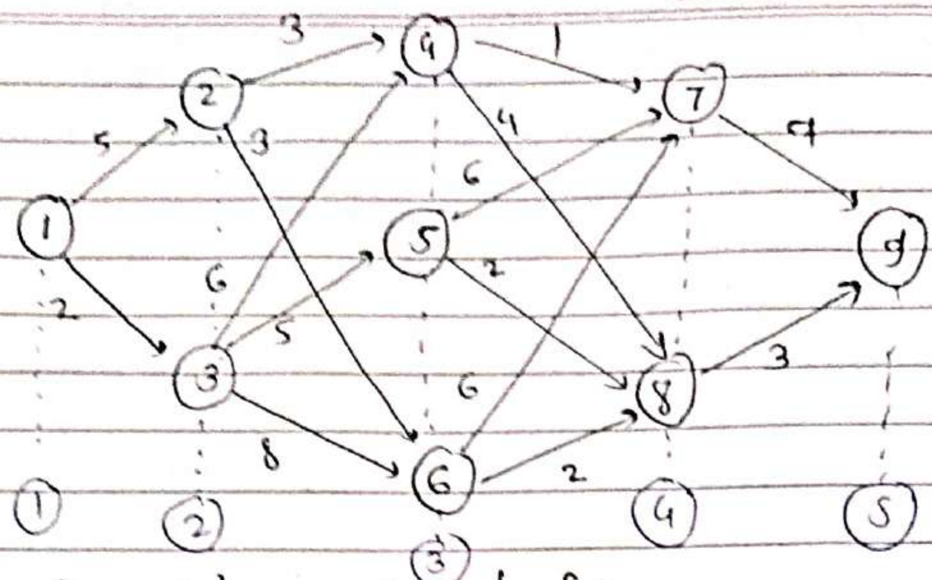
Q.1



Backward approach :-

Sol^n: vertex 1 2 3 4 5 6 7 8 9

cost

d

computation at level ① :-

cost (1, 1) = 0 ⟹ d(1,1) = 1

level vertex

computation at level ② :-

cost (2, 2) = 5 ⟹ d(2, 3) = 1

cost (2, 3) = 2 ⟹ d(2, 3) = 1

computation at level ③ :-

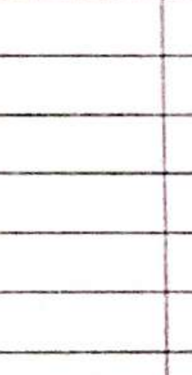
$$cost(i, j) = \min\{ c(l, j) + cost(i-1, l)\}$$

$$cost(3,4) = \min\{ c(2,4) + cost(2,2),\ c(3,4) + cost(2,3)\}$$

i j

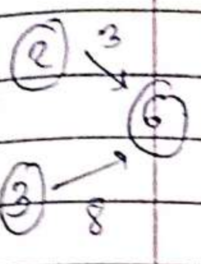
$$= \min\{3+5,\ 6+2\}$$

$\boxed{cost(3,4) = 8}$ d(3,4) = 2 or 3



$$\text{cost}(3,5) = \min\{c(3,5) + \text{cost}(2,3)\}$$
$$= \min\{5+2\}$$

$$\text{cost}(3,5) = \underline{\underline{7}}$$

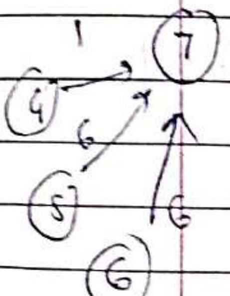


$$\text{cost}(3,6) = \min\{c(2,6) + \text{cost}(2,2),\; c(3,6) + \text{cost}(2,3)\}$$

$$= \min\{3+5,\; 8+2\}$$

$$\therefore \text{cost}(3,6) = \underline{\underline{8}} \Rightarrow d(3,6) = 2$$

computation at level (4) :-

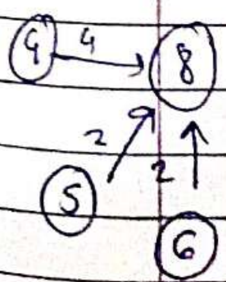


$$\text{cost}(4,7) = \min\{c(4,7) + \text{cost}(3,4),\; c(5,7) + \text{cost}(3,5),\; c(6,7) + \text{cost}(3,6)\}$$

$$= \min\{1+8,\; 6+7,\; 6+8\}$$

$$\text{cost}(4,7) = \underline{\underline{9}} \Rightarrow d(4,7) = 4$$

~~com~~:

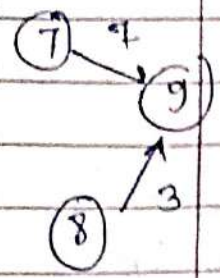


$$\text{cost}(4,8) = \min\{c(4,8) + \text{cost}(3,4),\; c(5,8) + \text{cost}(3,5),\; c(6,8) + \text{cost}(3,6)\}$$

$$= \min\{4+8,\; 2+7,\; 2+8\}$$

$$\text{cost}(4,8) = \underline{\underline{9}} \Rightarrow d(4,8) = 5$$

computation at level (5):



$$\text{cost}(5,9) = \min\{c(7,9)+\text{cost}(4,7),\ c(8,9)+\text{cost}(4,8)\}$$

$$= \min\{7+9,\ 3+9\}$$

$$\text{cost}(5,9) = \underline{12} \Rightarrow d(5,9) = 8$$

Now, $d(5,9) = 8$

$d(4,8) = 5$

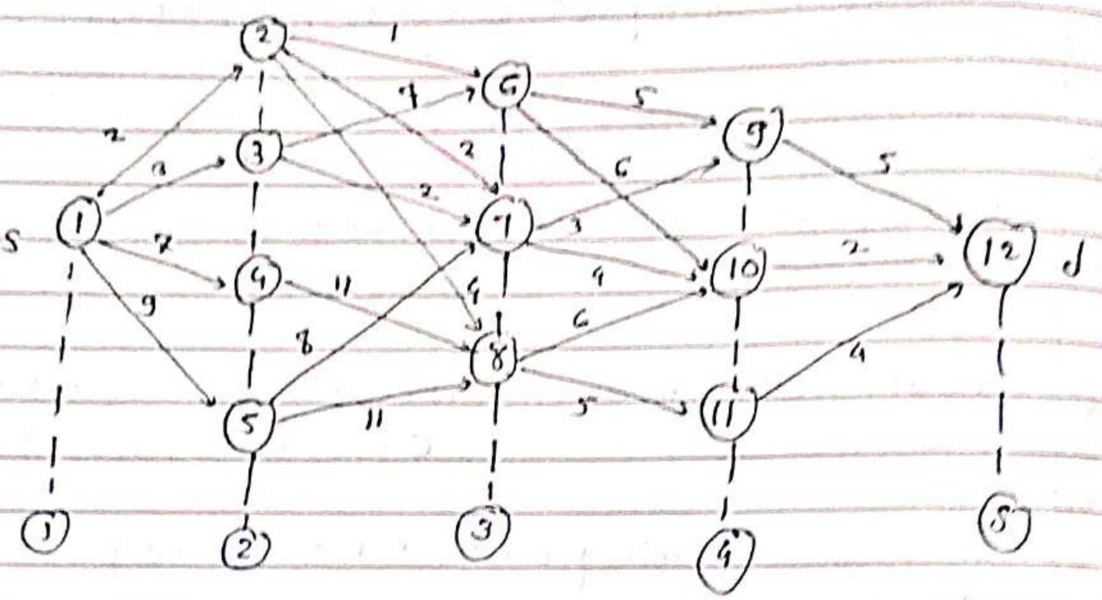
$d(3,5) = 3$

$d(2,3) = 1$

shortest path is $9-8-5-3-1$

$= \underline{12}$

(2) what is multistage graph? For the following graph, find shortest path from source to destination.



Sol^n:-

v	1	2	3	4	5	6	7	8	9	10	11	12
cost	10	8	8	19	14	8	6	8	5	2	4	0
d	2	7	7	8	7	10	10	10	12	12	12	12

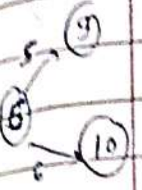
computation at level 5 :-

cost (5, 12) = 0 ⟹ d (5, 12) = 12

computation at level 4 :-

cost (4, 9) = 5 ⟹ d (4, 9) = 12
cost (4, 10) = 2 ⟹ d (4, 10) = 12
cost (4, 11) = 4 ⟹ d (4, 11) = 12

computation at level 3 :-



$cost(3,6) = \min\{c(6,9) + cost(4,9),\ c(6,10) + cost(4,10)\}$

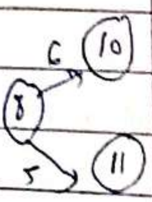
$= \min\{5+5,\ 6+2\}$

$cost(3,6) = 8 \Rightarrow d(3,6) = 10$

$cost(3,7) = \min\{c(7,9) + cost(4,9),\ c(7,10) + cost(4,10)\}$

$= \min\{3+5,\ 4+2\}$

$cost(3,7) = 6 \Rightarrow d(3,7) = 10$

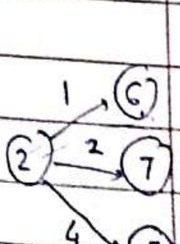


$cost(3,8) = \min\{c(8,10) + cost(4,10),\ c(8,11) + cost(4,11)\}$

$= \min\{6+2,\ 5+4\}$

$cost(3,8) = 8 \Rightarrow d(3,8) = 10$

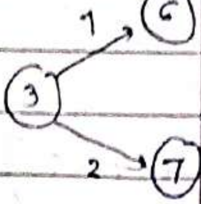
computation at level 2 :-



$cost(\underset{i}{\underset{\uparrow}{2}}, \underset{j}{\underset{\uparrow}{2}}) = \min\{c(2,6) + cost(3,6),\ c(2,7) + cost(3,7),$
$c(2,8) + cost(3,8)\}$

$= \min\{1+8,\ 2+6,\ 4+8\}$

$cost(2,2) = 8 \Rightarrow d(2,2) = 7$

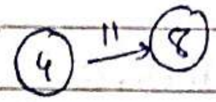


$cost(2,3) = \min\{c(3,6)+cost(3,6),\ c(3,7)+cost(3,7)\}$

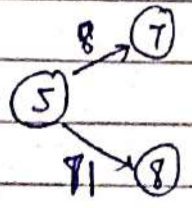
$= \min\{7+8,\ 2+6\}$

$cost(2,3) = 8 \Rightarrow d(2,3) = 7$

$cost(2,4) = 18 \Rightarrow d(2,4) = 8$



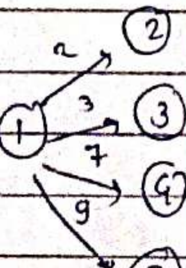
$cost(2,5) = \min\{c(5,7)+cost(3,7),\ c(5,8)+cost(3,8)\}$



$= \min\{8+6,\ 11+8\}$

$cost(2,5) = 14 \Rightarrow d(2,5) = \cancel{8}\ 7$

computation at level 1 :-



$cost(1,1) = \min\{c(1,2)+cost(2,2),\ c(1,3)+cost(2,3),$
$c(1,4)+cost(2,4),\ c(1,5)+cost(2,5)\}$

$= \min\{2+8,\ 3+8,\ 7+19,\ 9+14\}$

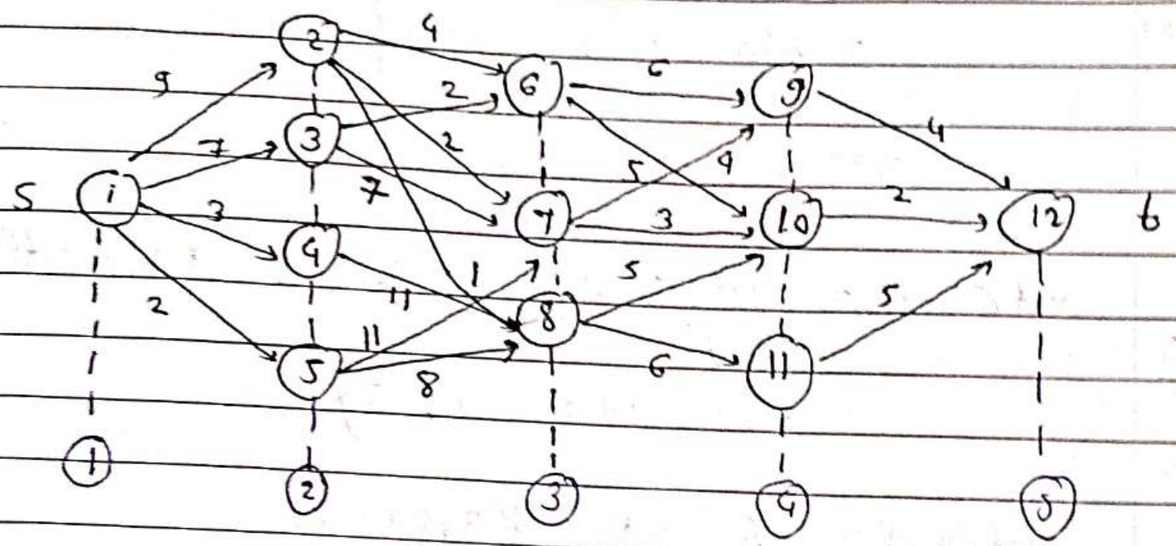
$cost(1,1) = 10 \Rightarrow d(1,1) = 2$

$d(1,1) = 2$
$d(2,2) = 7$
$d(3,7) = 10$
$d(4,10) = 12$
$d(5,12) = 12$

$1 \xrightarrow{2} 2 \xrightarrow{2} 7 \xrightarrow{4} 10 \xrightarrow{2} 12$

$cost(1,1) = 10$

Q.3 Find the minimum cost path from "s" to "t" in multistage graph shown. using forward approach.



Solⁿ :- computation at level 5 :-

$cost(5, 12) = 0 \Rightarrow d(5,12) = 12$

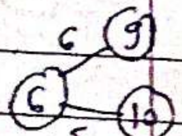
computation at level 4 :-

$cost(4, 9) = 4 \Rightarrow d(4,9) = 12$

$cost(4, 10) = 2 \Rightarrow d(4,10) = 12$

$cost(4, 11) = 5 \Rightarrow d(4,11) = 12$

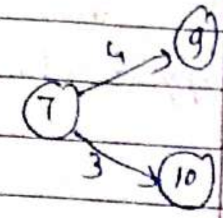
computation at level 3 :-



$cost(3,6) = \min\{c(6,9) + cost(4,9),\ c(6,10) + cost(4,10)\}$

$= \min\{6+4,\ 5+2\}$

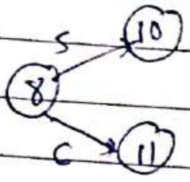
$cost(3,6) = 7 \Rightarrow d(3,6) = 10$



$cost(3,7) = \min\{c(7,9) + cost(4,9),\ c(7,10) + cost(4,10)\}$

$= \min\{4+4,\ 3+2\}$

$cost(3,7) = 5 \Rightarrow d(3,7) = 10$

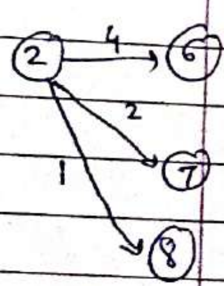


$cost(3,8) = \min\{c(8,10) + cost(4,10),\ c(8,11) + cost(4,11)\}$

$= \min\{5+2,\ 6+5\}$

$cost(3,8) = 7 \Rightarrow d(3,8) = 10$

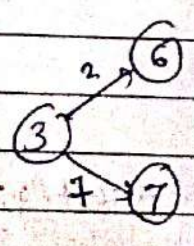
computation at level ② :-



$cost(2,2) = \min\{c(2,6) + cost(3,6),\ c(2,7) + cost(3,7),$
$c(2,8) + cost(3,8)\}$

$= \min\{4+7,\ 2+5,\ 1+7\}$

$cost(2,2) = 7 \Rightarrow d(2,2) = 7$



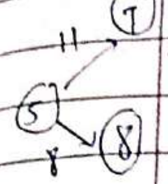
$cost(2,3) = \min\{c(3,6) + cost(3,6),\ c(3,7) + cost(3,7)\}$

$= \min\{2+7,\ 7+5\}$

$cost(2,3) = \underline{\underline{9}} \Rightarrow \cancel{cost}\ d(2,3) = 6$

0)}

cost(2, 4) = min{ 18 19 ⟹ d(2, 4) = 8

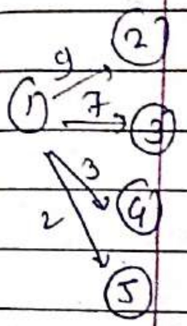


cost (2, 5) = min{ c(5, 7) + cost(3, 7) , c(5, 8) + cost(3, 8) }

= min{ 11 + 5 , 8 + 7 }

cost(2, 5) = 15 ⟹ d(2, 5) = 8

computation at level 1 :-



cost (1, 1) = min{ c(1, 2) + cost(2, 2) , c(1, 3) + cost(2, 3) , c(1, 4) + cost(2, 4) , c(1, 5) + cost(2, 5) }

= min{ 9+7 , 7+9 , 3+18 , 2+15 }

cost(1, 1) = 16 ⟹ d(1, 1) = 2 / 3

d(1, 1) = 2
d(2, 2) = ~~8~~ 7
d(3, 7) = 10
d(4, 10) = 12
d(5, 12) = 12

$\{S\}\ (1) \xrightarrow{9} (2) \xrightarrow{2} (7) \xrightarrow{3} (10) \xrightarrow{2} (12)\ \{t\}$

∴ cost (1, 1) = 16

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Q.4 Differentiate between :- Dynamic & greedy strategy.

→

Dynamic programming	Greedy strategy
① A single sequence of the decision is generated.	① various no. of sequence of the decision are generated.
② Not reliable	② very reliable
③ Follows serial forward approach.	③ Follows bottom-up or top-down approach.
④ An optimal solⁿ may not be achieved.	④ The optimal solⁿ is achieved every time
⑤ memory efficient	⑤ memory consumed more
⑥ Quicker results.	⑥ Slower result comparatively.
⑦ ex. fractional knapsack problem.	⑦ ex. 0/1 knapsack problem
⑧ Time complexity :- $O(E\log V + V\log V)$	⑧ $O(V \times E)$
⑨ Makes locally optimal choices at each step without considering the global picture.	⑨ Breaks down problem into subproblem & solve each subproblem only once.

Q.5 Find the minimum no. of multiplication required to multiply the matrix of given dimension using chained matrix multiplication. dimension of matrices are :-

$A = 13 \times 5$, $B = 5 \times 89$, $C = 89 \times 3$, $D = 3 \times 34$

Sol^n :- The given product factor is $P = \{\overset{P_0}{13}, \overset{P_1}{5}, \overset{P_2}{89}, \overset{P_3}{3}, \overset{P_4}{34}\}$

$A = 13 \times 5$

$B = 5 \times 89$

$C = 89 \times 3$

$D = 3 \times 34$

The structure of matrix m & s are follow :-

$j \longrightarrow$

1	2	3	4	$i \downarrow$
0	5785	1530	2856	1
	0	1335	1845	2
		0	9078	3
			0	4

$m(i, j)$

$j \longrightarrow$

2	3	4	$i \downarrow$
1	1	3	1
	2	3	2
		3	3

$S(i, j)$ → s for K value

formula :-

$$m(i,j) = \begin{cases} 0 & \text{if } i = 0 \\ \min\{m(i,k) + m(k+1,j) + P_{i-1} \cdot P_k \cdot P_j\} & \text{if } i \le k < j \end{cases}$$

* computation of diagonal 1 :-

$m(1,1) = 0$, $m(2,2)$, $m(3,3)$, $m(4,4) = 0$

* computation of diagonal 2 :-

$m(\overset{i}{1}, \overset{j}{2})$

so, $i \le k < j$ ..... $1 \le k < 2$

$\therefore \boxed{k = 1}$

$m(1,2) = m(1,1) + m(2,2) + P_0 \cdot P_1 \cdot P_2$

$= 0 + 0 + 13 \cdot 5 \cdot 89$

$\boxed{m(1,2) = 5785}$

~~$m(1,3) = m(i,j)$~~

~~if $i \le k < j$~~

~~$\boxed{k=1}$, $\boxed{k=2}$~~

for $k=1 \Rightarrow \therefore m(1,3) = m(1,1) + m(2,3) + P_0 \cdot P_1 \cdot P_3$

$= 0 +$

for $m(2,3)$:

if $i \le k < j$

i.e $2 \le k < 3$

$\therefore \boxed{k = 2}$

$m(2,3) = m(2,2) + m(3,3) + P_1 \cdot P_2 \cdot P_3$

$= 0 + 0 + 5 \cdot 89 \cdot 3$

$\boxed{m(2,3) = 1335}$

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for $m(3,4)$
if $i \le k < j$ i.e $3 \le k < 4$
$\therefore \boxed{k = 3}$

$m(3,4) = m(3,3) + m(4,4) + P_2 \cdot P_3 \cdot P_4$
$= 0 + 0 + 89 \times 3 \times 34$
$\boxed{m(3,4) = 9078}$

* computation of diagonal 3 :-

for $m(1,3)$
if $i \le k < j$ i.e $1 \le k < 3$
$\therefore \boxed{k = 1, 2}$

for $k = 1$
$\therefore m(1,3) = m(1,1) + m(2,3) + P_0 \cdot P_1 \cdot P_3$
$= 0 + 1335 + 13 \cdot 5 \cdot 3$
$\boxed{m(1,3) = 1530}$ $\therefore$ 1530 is optimal no. of multipl-

for $k = 2$
$\therefore m(1,3) = m(1,2) + m(3,3) + P_0 \cdot P_2 \cdot P_3$
$= 5785 + 0 + 13 \cdot 89 \cdot 3$
$\therefore m(1,3) = 9256$

for $m(2,4)$
if $i \le k < j$ i.e $2 \le k < 4$
$\therefore \boxed{k = 2, 3}$

for $k=2$

$m(2,4) = m(2,2) + m(3,4) + P_1 \cdot P_2 \cdot P_4$

$= 0 + 9078 + 5 \times 89 \times 34$

$m(2,4) = 24,208$

for $k=3$

$m(2,4) = m(2,3) + m(4,4) + P_1 \cdot P_3 \cdot P_4$

$= 1335 + 0 + 5 \times 3 \times 34$

$\boxed{m(2,4) = 1845}$ ... 1845 is optimal no. of multiplication

for $m(1,4)$

if $i \le k < j$ i.e $1 \le k < 4$

$\therefore \boxed{k = 1, 2, 3}$

for $k=1$

$m(1,4) = m(1,1) + m(2,4) + P_0 \cdot P_1 \cdot P_4$

$= 0 + 1845 + 13 \cdot 5 \cdot 34$

$m(1,4) = 4055$

for $k=2$

$m(1,4) = m(1,2) + m(3,4) + P_0 \cdot P_2 \cdot P_4$

$= 5785 + 9078 + 13 \cdot 89 \cdot 34$

$m(1,4) = 54201$

for $k=3$

$m(1,4) = m(1,3) + m(4,4) + P_0 \cdot P_3 \cdot P_4$

$= 1530 + 0 + 13 \cdot 3 \cdot 34$

$\boxed{m(1,4) = 2856}$ .... 2856 is optimal sol^n.

?

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construction of optimal parantization :-

[A B C D].

$S[1,4] = 3$

$$\left(A \cdot B \cdot C \,\Big|_{13 \times \cancel{5} \times 3} \quad D_{3 \times 34} \right) \Rightarrow 13 \times 3 \times 34 = 1326$$

$+$

$S[1,3] = 1$

$$\left((A)_{13 \times 5} \; (B \times C)_{5 \times 3} \;\Big|\; \cancel{D_{3 \times 39}} \right) \Rightarrow 13 \times 5 \times 3 = 195$$

$+$

$$(B \times C)_{5 \times 3} \Rightarrow 5 \times 89 \times 3 = 1335$$

$$\boxed{2856}$$

Q.6 using chained matrix multiplication method find out minimum no. of operation required to multiply. Following matrices and also find the best sequence.

$A = 6 \times 10$

$B = 10 \times 12$

$C = 12 \times 5$

$D = 5 \times 8$

Sol^n:- The given product factor is $P = \{\overset{P_0}{6} \times \overset{P_1}{10} \times \overset{P_2}{12} \times \overset{P_3}{5} \times \overset{P_4}{8}\}$

$A = 6 \times 10$

$B = 10 \times 12$

$C = 12 \times 5$

$D = 5 \times 8$

The structure of matrix m & S are as follow:-

$j \longrightarrow$

1	2	3	4	$i \downarrow$
0	720	900	1140	1
	0	600	100	2
		0	480	3
			0	4

m(i, j)

$j \longrightarrow$

2	3	4	i
1	1	3	1
	2	3	2
		3	3

s(i, j)

Computation of diagonal ① :-

$m(1,1) = 0$, $m(2,2) = 0$, $m(3,3) = 0$, $m(4,4) = 0$

Computation of diagonal ② :-

$m(1,2)$

if $i \le k < j$ i.e $1 \le k < 2$

$\therefore \boxed{k = 1}$

$\therefore m(1,2) = m(1,1) + m(2,2) + P_0 \cdot P_1 \cdot P_2$

$= 0 + 0 + 6 \times 10 \times 12$

$\therefore \boxed{m(1,2) = 720}$

for $m(2,3)$

if $i \le k < j$ i.e $2 \le k < 3$

$\therefore \boxed{k = 2}$

$\therefore m(2,3) = m(2,2) + m(3,3) + P_1 \cdot P_2 \cdot P_3$

$= 0 + 0 + 10 \times 12 \times 5$

$\boxed{m(2,3) = 600}$

for $m(3,4)$

if $i \le k < j$ i.e $3 \le k < 4$

$\therefore \boxed{k = 3}$

$\therefore m(3,4) = m(3,3) + m(4,4) + P_2 \cdot P_3 \cdot P_4$

$= 0 + 0 + 12 \times 5 \times 8$

$\boxed{m(3,4) = 480}$

computation of diagonal 3 :-

$m(1,3)$

if $i \le k \le j$ i.e $1 \le k < 3$

$\therefore \boxed{k = 1, 2}$

$$m(1,3) = \min \begin{cases} m(1,1) + m(2,3) + P_0 \cdot P_1 \cdot P_3 , \\ m(1,2) + m(3,3) + P_0 \cdot P_2 \cdot P_3 \end{cases}$$

$$= \min \{ 0 + 600 + 6 \times 10 \times 5 ,\ 720 + 0 + 6 \times 12 \times 5 \}$$

$$= \min \{ 900 , 1080 \}$$

$\boxed{m(1,3) = 900}$

for $m(2,4)$

if $i \le k \le j$ i.e $2 \le k \le 4$

$\boxed{k = 2, 3}$

$$m(2,4) = \min \begin{cases} m(2,2) + m(3,4) + P_1 \cdot P_2 \cdot P_4 , \\ m(2,3) + m(4,4) + P_1 \cdot P_3 \cdot P_4 \end{cases}$$

$$= \min \{ 0 + 480 + 10 \times 12 \times 8 ,\ 600 + 10 \times 5 \times 8 \}$$

$$= \min \{ 1440 , 1000 \}$$

900
480
1440

$\boxed{m(2,4) = 1000}$

computation of diagonal 4 :-

$m(1,4)$

if $i \le k < j$ i.e $1 \le k < 4$

$\therefore \boxed{k = 1, 2, 3}$

$$m(1,4) = \min \begin{cases} m(1,1) + m(2,4) + P_0 \cdot P_1 \cdot P_4 , \\ m(1,2) + m(3,4) + P_0 \cdot P_2 \cdot P_4 , \\ m(1,3) + m(4,4) + P_0 \cdot P_3 \cdot P_4 \end{cases}$$

$$= \min \{ 0 + 1000 + 6 \times 10 \times 8 ,\ 720 + 480 + 6 \times 12 \times 8 ,\ 900 + 0 + 6 \times 5 \times 8 \}$$

$$= \min \{ 1480 , 1776 , 1140 \}$$

$\boxed{m(1,4) = 1140}$

optimal no. of multiplication is 1140

$S[1,4] = 3$

$((A\ B\ C)_{6 \times 12 \times 5} (D)_{5 \times 8})$ 6

$(((A)_{6 \times 10} (B C)_{10 \times 5}) (D)_{5 \times 8})$ $S[1,3]$ 1

$((A (BD))_{6 \times 5} (D)_{5 \times 8})$

?

$((A (BD)) D)_{6 \times 8}$

Q.7 Calculate the minimum no. of scalar multiplication for following set of matrix using matrix chain multiplication?
$A1 = 10 \times 20$, $A2 = 20 \times 13$, $A3 = 13 \times 15$, $A4 = 15 \times 12$
Also write correct parenthesization?

Sol^n: The given product factors $P = \{10, 20, 13, 15, 12\}$ (P_0, P_1, P_2, P_3, P_4)

$A = 10 \times 20$
$A_2 = 20 \times 13$
$A_3 = 13 \times 15$
$A_4 = 15 \times 12$

The structure of matrix m & s are as follows:

$m(i, j)$ ($j \rightarrow$ across, $i \downarrow$ down)

i \ j	1	2	3	4
1	0	2600	4550	6350
2		0	3900	5460
3			0	2340
4				0

$s(i, j)$

i \ j	2	3	4
1	1	2	3
2		2	2
3			3

Computation of diagonal 1 :-

$m(1,1) = 0$, $m(2,2) = 0$, $m(3,3) = 0$, $m(4,4) = 0$

Computation of diagonal 2 :-

for $m(1,2)$
if $i \le k < j$ i.e $1 \le k < 2$
$\therefore \boxed{k = 1}$

$m(1,2) = m(1,1) + m(2,2) + P_0 \cdot P_1 \cdot P_2$
$= 0 + 0 + 10 \cdot 20 \cdot 13$
$\boxed{m(1,2) = 2600}$

for $m(2,3)$
if $i \le k < j$ i.e $2 \le k < 3$
$\therefore \boxed{k = 2}$

$m(2,3) = m(2,2) + m(3,3) + P_1 \cdot P_2 \cdot P_3$
$= 0 + 0 + 20 \times 13 \times 15$
$\boxed{m(2,3) = 3900}$

for $m(3,4)$
if $i \le k < j$ i.e $3 \le k < 4$
$\boxed{k = 3}$

$m(3,4) = m(3,3) + m(4,4) + P_2 \cdot P_3 \cdot P_4$
$= 0 + 0 + 13 \cdot 15 \cdot 12$
$\boxed{m(3,4) = 2340}$

Computation of diagonal 3 :-

for $m(1,3)$
if $i \le k < j$ i.e. $1 \le k < 3$
$\boxed{k = 1, 2}$

$\boxed{K = 1, 2}$

$m(1,3) = \min\{ m(1,1) + m(2,3) + P_0 \cdot P_1 \cdot P_3,\; m(1,2) + m(3,3) + P_0 \cdot P_2 \cdot P_3 \}$

$= \min\{ 0 + 3900 + 10 \times 20 \times 15,\; 2600 + 0 + 10 \times 13 \times 15 \}$

$= \min\{ 6900, 4550 \}$

$\boxed{m(1,3) = 4550}$

$i \leq K < j$ i.e $\boxed{K = 2, 3}$

$m(2,4) = \min\{ m(2,2) + m(3,4) + P_1 \cdot P_2 \cdot P_4,\; m(2,3) + m(4,4) + P_1 \cdot P_3 \cdot P_4 \}$

$= \min\{ 0 + 2340 + 20 \times 13 \times 12,\; 3900 + 0 + 20 \times 15 \times 12 \}$

$= \min\{ 5460, 7500 \}$

$\boxed{m(2,4) = 5460}$

computation of diagonal (4) :-

$m(1,4)$

if $i \leq K < j$ ie $1 \leq K < 4$

$\boxed{K = 1, 2, 3}$

$m(1,4) = \min\{ m(1,1) + m(2,4) + P_0 \cdot P_1 \cdot P_4,\; m(1,2) + m(3,4) + P_0 \cdot P_2 \cdot P_4,\; m(1,3) + m(4,4) + P_0 \cdot P_3 \cdot P_4 \}$

$= \min\{ 0 + 5460 + 10 \times 20 \times 12,\; 2600 + 2340 + 10 \times 13 \times 12,\; 4550 + 0 + 10 \times 15 \times 12 \}$

$= \min\{ 7860, 6500, 6350 \}$

$\boxed{m(1,4) = 6350}$

optimal parenthesization :-

A1 A2 A3 A4

$s[1,4] = 3$

$((A_1 A_2 A_3)_{10 \times 15} A_4{}_{15 \times 12}) \longrightarrow 10 \times 15 \times 12 = 1800$

$s[1,3] = 2$ +

$(A_1 A_2)_{10 \times 13} (A_3)_{13 \times 15} \longrightarrow 10 \times 13 \times 15 = 1950$

+

$(A_1 A_2)_{10 \times 13} \longrightarrow 10 \times 20 \times 13 = 2600$

6350

Q.8 calculate the minimum no. of scalar multiplication for following set of matrix using matrix chain multiplication.

$A = 4 \times 5$
$B = 5 \times 3$
$C = 3 \times 2$
$D = 2 \times 7$

Sol^n:- The given product factor is $P = \{ \overset{P_0}{4}, \overset{P_1}{5}, \overset{P_2}{3}, \overset{P_3}{2}, \overset{P_4}{7} \}$

$A = 4 \times 5$
$B = 5 \times 3$
$C = 3 \times 2$
$D = 2 \times 7$

j →

1	2	3	4	i ↓
0	60	70	126	1
	0	30	100	2
		0	42	3
			0	4

m(i, j)

j →

2	3	4	i ↓
1	1	3	1
	2	3	2
		3	3

S(i, j)

computation at diagonal (1) :-

$m(1,1) = m(2,2) = m(3,3) = m(4,4) = 0$

computation at diagonal (2) :-

$m(1,2) = m(i,j)$
if $i \le k \le j$ i.e $1 \le k < 2$
$\therefore \boxed{k = 1}$

$m(1,2) = m(1,1) + m(2,2) + P_0 \cdot P_1 \cdot P_2$
$= 0 + 0 + 4 \times 5 \times 3$
$\boxed{m(1,2) = 60}$

$m(2,3)$
if $i \le k < j$ i.e $2 \le k < 3$
$\boxed{k = 2}$

$m(2,3) = m(2,2) + m(3,3) + P_1 \cdot P_2 \cdot P_3$
$= 0 + 0 + 5 \times 3 \times 2$
$\boxed{m(2,3) = 30}$

for $m(3,4)$
if $i \le k < j$ i.e $3 \le k < 4$
$\boxed{k = 3}$

$m(3,4) = m(3,3) + m(4,4) + P_2 \cdot P_3 \cdot P_4$
$= 0 + 0 + 3 \times 2 \times 7$
$m(3,4) = \cancel{56}\ 42$

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computation at level 3 :-

$m(1,3)$
if $i \le k < j$ i.e $1 \le k < 3$

$\boxed{k = 1, 2}$

$m(1,3) = \min \{ m(1,1) + m(2,3) + P_0 \cdot P_1 \cdot P_3,$
$m(1,2) + m(3,3) + P_0 \cdot P_2 \cdot P_3 \}$

$= \min \{ 0 + 30 + 4 \times 5 \times 2,$
$60 + 0 + 4 \times 3 \times 2 \}$

$= \min \{ 70, 84 \}$

$\boxed{m(1,3) = 70}$

$m(2,4)$

$\boxed{k = 2, 3}$

$m(2,4) = \min \{ m(2,2) + m(3,4) + P_1 \cdot P_2 \cdot P_4,$
$m(2,3) + m(4,4) + P_1 \cdot P_3 \cdot P_4$
$\}$

$= \min \{ 0 + 42 + 5 \times 3 \times 7,$
$30 + 0 + 5 \times 2 \times 7 \}$

$= \min \{ 147, 100 \}$

$\boxed{m(2,4) = 100}$

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computation at diagonal 4 :-

$m(1,4) =$

$\boxed{k = 1, 2, 3}$

$m(1,4) = \min \{ m(1,1) + m(2,4) + P_0 \cdot P_1 \cdot P_4,$
$m(1,2) + m(3,4) + P_0 \cdot P_2 \cdot P_4,$
$m(1,3) + m(4,4) + P_0 \cdot P_3 \cdot P_4 \}$

$= \min \{ 0 + 100 + 4 \times 5 \times 7,$
$60 + 42 + 4 \times 3 \times 7,$
$70 + 0 + 4 \times 2 \times 7 \}$

$= \min \{ 240, 242, 126 \}$

$\boxed{m(1,4) = 126}$

A B C D
$s[1,4] = 3$

$((A\ B\ C)_{4\times2}\ D_{2\times7}) \Rightarrow 4 \times 2 \times 7 = 56$

+

$s[1,3] = 1$

$(A)_{4\times5}\ (B \times C)_{5\times2} \Rightarrow 4 \times 5 \times 2 = 40$

+

$(B \times C)_{5\times2} \rightarrow 5 \times 3 \times 2 = 30$

126

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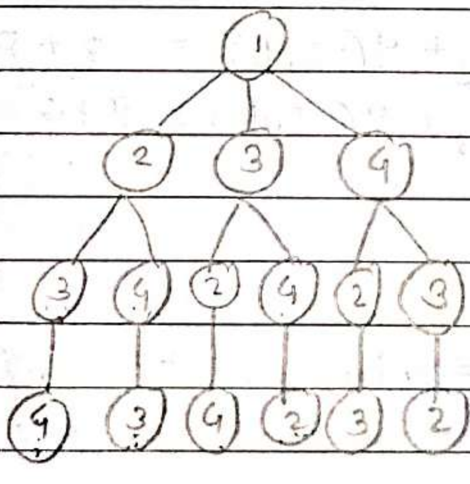
Q.90 What is travelling salesman problem? Implement travelling salesman problem for the following matrix representation of complete graph.

$$\begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

Sol^n :- Let label the matrix by 1, 2, 3, 4

i.e

$$c_{ij} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 10 & 15 & 20 \\ 5 & 0 & 9 & 10 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix} \end{array}$$



10
+ 96
9
6

formula :-

$$g(i, S) = \min \{ c_{ij} + g(j, S - \{j\}) \} \quad j \in S$$

let the starting vertex is node 1.

if $|S| = \phi$

$g(2, \phi) = c_{21} = 5$

$g(3, \phi) = c_{31} = 6$

$g(4, \phi) = c_{41} = 8$

if $|S| = 1$

$g(2, \{3\}) = c_{23} + g(3, \phi) = 9 + 6 = 15$

$g(2, \{4\}) = c_{24} + g(4, \phi) = 10 + 8 = 18$

$g(3, \{2\}) = c_{32} + g(2, \phi) = 13 + 5 = 18$

$g(3, \{4\}) = c_{34} + g(4, \phi) = 12 + 8 = 20$

$g(4, \{2\}) = c_{42} + g(2, \phi) = 8 + 5 = 13$

$g(4, \{3\}) = c_{43} + g(3, \phi) = 9 + 6 = 15$

if $|S| = 2$

$g(2, \{3,4\}) = \min\{c_{23} + g(3, \{4\}), c_{24} + g(4, \{3\})\}$

$= \min\{9 + 20, 10 + 15\}$

$g(2, \{3,4\}) = 25$

$g(3, \{2,4\}) = \min\{c_{32} + g(2, \{4\}), c_{34} + g(4, \{2\})\}$

$= \min\{13 + 18, 12 + 13\}$

$g(3, \{2,4\}) = 25$

$g(4, \{2,3\}) = \min\{c_{42} + g(2, \{3\}), c_{43} + g(3, \{2\})\}$

$= \min\{8 + 15, 9 + 18\}$

$= \min\{23, 27\}$

$= \underline{\underline{23}}$

if $|S| = 3$

$g(1, \{2,3,4\}) = \min\{c_{12} + g(2, \{3,4\}), c_{13} + g(3, \{2,4\}), c_{14} + g(4, \{2,3\})\}$

$= \min\{\underline{10 + 25}, 15 + 25, 20 + 23\}$

$g(1, \{2,3,4\}) = 35$

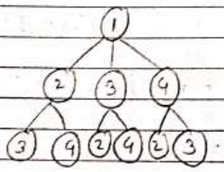
path 1 - 2 - 4 - 3 - 1 & cost is $\underline{\underline{35}}$

Q.10 Implement travelling salesman problem for following matrix.

$$\begin{bmatrix} 0 & 15 & 10 & 20 \\ 5 & 0 & 10 & 9 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix}$$

Sol^n:- consider,

$$C_{ij} = \begin{array}{c|c} & 1 \quad\; 2 \quad\; 3 \quad\; 4 \\ \begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix} 0 & 15 & 10 & 20 \\ 5 & 0 & 10 & 9 \\ 6 & 13 & 0 & 12 \\ 8 & 8 & 9 & 0 \end{bmatrix} \end{array}$$



let the starting vertex is 1.

if $|S| = \phi$

$g(2,\phi) = c_{21} = 5$
$g(3,\phi) = c_{31} = 6$
$g(4,\phi) = c_{41} = 8$

if $|S| = 1$

$g(2,\{3\}) = c_{23} + g(3,\phi) = 10 + 6 = 16$
$g(2,\{4\}) = c_{24} + g(4,\phi) = 9 + 8 = 17$

$g(3,\{2\}) = c_{32} + g(2,\phi) = 13 + 5 = 18$
$g(3,\{4\}) = c_{34} + g(4,\phi) = 12 + 8 = 20$

$g(4,\{2\}) = c_{42} + g(2,\phi) = 8 + 5 = 13$
$g(4,\{3\}) = c_{43} + g(3,\phi) = 9 + 6 = 15$

if $|S| = 2$

$g(2,\{3,4\}) = \min\{c_{23} + g(3,\{4\}),\ c_{24} + g(4,\{3\})\}$

$= \min\{10 + 20,\ 9 + 15\}$

$g(2,\{3,4\}) = 24$

$g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}),\ c_{34} + g(4,\{2\})\}$

$= \min\{13 + 17,\ 12 + 13\}$

$g(3,\{2,4\}) = 25$

$g(4, \{2,3\}) = \min\{c_{42} + g(2,\{3\}), c_{43} + g(3,\{2\})\}$

$= \min\{8 + 16, 9 + 18\}$

$g(4, \{2,3\}) = 24$

if $|S| = 1$

$g(1, \{2,3,4\}) = \min\{c_{12} + g(2,\{3,4\}),$

$c_{13} + g(3,\{2,4\}),$

$c_{14} + g(4,\{2,3\})\}$

$= \min\{15+24, 10+25, 20+24\}$

$g(1, \{2,3,4\}) = 35$

Path is 1 - 3 - 4 - 2 - 1 & cost is $\underline{35}$.

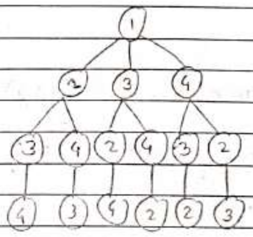
Q.11 what is TSP ? calculate the TSP tour for following matrix.

~~0~~	~~15~~	~~10~~	~~20~~
~~5~~	~~0~~	~~10~~	~~9~~
~~6~~	~~13~~	~~0~~	~~12~~
~~8~~	~~8~~	~~9~~	~~0~~

0	9	8	7
10	0	6	5
12	13	0	8
2	3	5	0

Soln:- consider,

$$c_{ij} = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 9 & 8 & 7 \\ 2 & 10 & 0 & 6 & 5 \\ 3 & 12 & 13 & 0 & 8 \\ 4 & 2 & 3 & 5 & 0 \end{array}$$



let the starting vertex is 1

if $|S| = \phi$

$g(2, \phi) = c_{21} = 10$
$g(3, \phi) = c_{31} = 12$
$g(4, \phi) = c_{41} = 2$

if $|S| = 1$

$g(2, \{3\}) = c_{23} + g(3, \phi) = 6 + 12 = 18$
$g(2, \{4\}) = c_{24} + g(4, \phi) = 5 + 2 = 7$

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$g(3,\{2\}) = c_{32} + g(2,\phi) = 13+10 = 23$
$g(3,\{4\}) = c_{34} + g(4,\phi) = 8+2 = 10$

$g(4,\{2\}) = c_{42} + g(2,\phi) = 3+10 = 13$
$g(4,\{3\}) = c_{43} + g(3,\phi) = 5+12 = 17$

if $|S| = 2$

$g(2,\{3,4\}) = \min\{c_{23} + g(3,\{4\}), c_{24} + g(4,\{3\})\}$
$= \min\{6+10, 5+17\}$

$g(2,\{3,4\}) = 16$

$g(3,\{2,4\}) = \min\{c_{32} + g(2,\{4\}), c_{34} + g(4,\{3\})\}$
$= \min\{13+7, 8+17\}$

$g(3,\{2,4\}) = 20$

$g(4,\{2,3\}) = \min\{c_{42} + g(2,\{3\}), c_{43} + g(3,\{2\})\}$
$= \min\{3+18, 5+23\}$

$g(4,\{2,3\}) = 21$

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if $|S| = 3$

$g(1,\{2,3,4\}) = \min\{c_{12} + g(2,\{3,4\}),$
$c_{13} + g(3,\{2,4\}),$
$c_{14} + g(4,\{2,3\})\}$
$= \min\{9+16, 8+20, 7+21\}$

$g(1,\{2,3,4\}) = 25$

Path is 1-2-3-4-1 & cost is 25.

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Q.16 Determine LCS : X = {AGGTABZ}, Y = {GXTXAYB} write the recurrence eqn for LCS. Also state the longest common subsequence.

⇒

Y ⟶ (j)

X ↓ (i)

	•	G	X	T	X	A	Y	B
•	0	0	0	0	0	0	0	0
A	0	←0	←0	←0	←0	↖1	←1	←1
G	0	↖1	←1	←1	←1	←1	←1	←1
G	0	↖1	←1	←1	←1	←1	←1	←1
T	0	↑1	←1	↖2	←2	←2	←2	←2
A	0	↑1	←1	↑2	←2	↖3	←3	←3
B	0	↑1	←1	↑2	←2	↑3	←3	↖4
Z	0	↑1	←1	↑2	←2	↑3	←3	↑4

GTAB

We get the longest common subsequence as GTAB of length = 4.

OR

length of LCS = 4

& longest common subsequence = GTAB

	•	B
A	←	↖1

Defn :- LCS (longest common subsequence) is defined as the longest subsequence which is common in all given input sequences.
ex. str1 = AGGTAB str2 = GXTXAYB
LCS = GTAB

?

ponial

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Q.17 Determine LCS of
X = (e, x, p, o, n, e, n, t, i, a, l)
Y = (p, o, l, y, n, o, m, i, a, l)
by using dynamic programming algorithm.

Solⁿ:-

	•	e	x	p	o	n	e	n	t	i	a	l
•	0	0	0	0	0	0	0	0	0	0	0	0
p	0	←0	←0	↖1	←1	←1	←1	←1	←1	←1	←1	←1
o	0	←0	←0	↑1	↖2	←2	←2	←2	←2	←2	←2	←2
l	0	←0	←0	↑1	↑2	←2	←2	←2	←2	←2	←2	↖3
y	0	←0	←0	↑1	↑2	←2	←2	←2	←2	←2	←2	↑3
n	0	←0	←0	↑1	↑2	↖3	←3	↖3	←3	←3	←3	←3
o	0	←0	←0	↑1	↖2	↑3	←3	←3	←3	←3	←3	←3
m	0	←0	←0	↑1	↑2	↑3	←3	←3	←3	←3	←3	←3
i	0	↑0	←0	↑1	↑2	↑3	←3	←3	←3	↖4	←4	←4
a	0	←0	←0	↑1	↑2	↑3	←3	←3	←3	↑4	↖5	←5
l	0	←0	←0	↑1	↑2	↑3	↑3	↑3	↑3	↑4	↑5	↖6

length of LCS = 6 lnoial

& longest common subsequence = lnoial

X (wrong)

(solve again)

← ↑ a b a a b   a b a b

Q.18 Determine LCS of $x = \{a, b, a, b, a, a, b, a\}$ & $Y = \{a, b, a, a, b, a, b\}$

Sol^n:-

$Y \longrightarrow (j)$, $x \downarrow (i)$

	•	a	b	a	a	b	a	b
•	0	0	0	0	0	0	0	0
a	0	↖1	←1	↖1	↖1	←1	↖1	←1
b	0	↑1	↖2	←2	←3	↖2	←2	↖2
a	0	↖1	↑2	↖3	↖3	←3	↖3	←3
b.	0	↑1	↖2	↑3	←3	↖4	←4	↖4
a.	0	↖1	↑2	↖3	↖4	←4	↖5	←5
a	0	↖1	↑2	↖3	↖4	←4	↖5	←5
b.	0	↑1	↖2	↑3	↑4	↖5	←5	↖6
a.	0	↖1	↑2	↖3	↖4	↑5	↖6	←6

a b a a b a

6

a b a b a a b   a a b a a b a

a b a a b a b

~~b a b~~ a b

'b

Q.19 A = CONDITION

B = RECURSION

write an algo to find LCS & print LCS.

Sol^n:-

	•	C	O	N	D	I	T	I	O	N
•	0	0	0	0	0	0	0	0	0	0
R	0	←0	←0	←0	←0	←0	←0	←0	←0	←0
E	0	←0	←0	←0	←0	←0	←0	←0	←0	←0
C	0	↖1	←1	←1	←1	←1	←1	←1	←1	←1
U	0	↑1	←1	←1	←1	←1	←1	←1	←1	←1
R	0	↑1	←1	←1	←1	←1	←1	←1	←1	←1
S	0	↑1	←1	←1	←1	←1	←1	←1	←1	←1
I	0	↑1	←1	←1	←1	↖2	←2	↖2	←2	←2
O	0	↑1	↖2	←2	←2	←2	←2	←2	↖3	←3
N	0	↑1	↑2	↖3	←3	←3	←3	←3	←3	↖4

length of LCS = 4

& longest common subsequence = CION

Q.1) write an algorithm to find longest common subsequence. Also find its complexity. Find optimal solution for given sequence.
X = (A, B, C, B, D, A, B) & Y = (B, C, D, A, B, A)

→

		•	B	C	D	A	B	A
	•	0	0	0	0	0	0	0
X	A	0	←0	←0	←0	↖1	←1	↖1
	B	0	↖1	←1	←1	←1	↖2	←2
	C	0	↑1	↖2	←2	←2	←2	←2
BCDAB	D	0	↑1	↑2	↖3	←3	←3	←3
	B	0	↖1	↑2	↑3	←3	↖4	←4
	A	0	↑1	↑2	↑3	$\boxed{↖4}$	←4	↖5
	B	0	↖1	↑2	↑3	↑4	$\boxed{↖5}$	$\boxed{←5}$

length of LCS = 5
& longest common subsequence is BCDAB

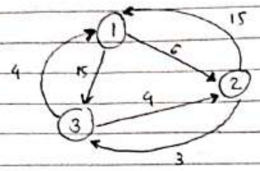
Q.2) write an algo to generate longest common subsequence (LCS). Apply the algo for the following string & generate LCS with the help of LCS matrix.
X = a, a, b, a, a, b, a, b, a, a
Y = b, a, b, a, a, b, a, b

Soln:

		•	a	a	b	a	a	b	a	b	a	a
	•	0	0	0	0	0	0	0	0	0	0	0
	b	0	←0	←0	↖1	←1	←1	↖1	←1	↖1	←1	←1
abaabab	a	0	↖1	↖1	←1	↖2	↖2	←2	↖2	←2	↖2	↖2
	b	0	↑1	←1	↖2	←2	←2	↖3	←3	↖3	←3	←3
	a	0	↖1	↖2	←2	↖3	↖3	←3	↖4	←4	↖4	↖4
	a	0	↖1	↖2	←2	↖3	↖4	←4	↖4	←4	↖5	↖5
	b	0	↑1	↑2	↖3	←3	↑4	↖5	←5	↖5	←5	←5
	a	0	↖1	↖2	↑3	↖4	↖4	↑5	↖6	←6	↖6	↖6
	b	0	↑1	↑2	↖3	↑4	←4	↖5	↑6	$\boxed{↖7}$	←7	$\boxed{←7}$

length of LCS = 7
& longest common subsequence = abaabab

Q.23 write an algo for floyd warshall & calculate distance and path matrix of following.



Soln:-

$$D^0 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 6 & 15 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

Now,

$$D^1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 6 & 15 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

$D^0[2,3] \quad D^0[2,1] + D^0[1,3]$

$3 < 15 + 15$

$D^0[3,2] \quad D^0[3,1] + D^0[1,2]$

$4 < 4 + 6$

$$D^2 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 6 & 9 \\ 2 & 15 & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

$D^1[1,3] \quad D^1(1,2) + D^1(2,3)$

$15 \quad 6 + 3$

$15 > 9$

$D^1[3,1] \quad D^1(3,2) + D^1(2,1)$

$4 \not> 4 + 15$

$4 \not> 19$

$$D^3 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 6 & 9 \\ 2 & \cancel{17} & 0 & 3 \\ 3 & 4 & 4 & 0 \end{array}$$

$D^2[1,2] \quad D^2(1,3) + D^2(3,2)$

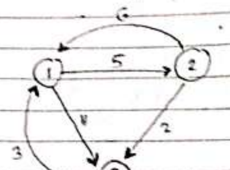
$6 < 9 + 4$

$D^2[2,1] \quad D^2(2,3) + D^2(3,1)$

$15 > 3 + 4$

$15 > 7$

Q.24 Find out shortest distance between all pair of vertices & write the floyd warshall all pair shortest path algorithm.



soln:-

$$D^0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} 0 & 5 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{array}\right] \end{array}$$

$$D^1 = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \left[\begin{array}{ccc} 0 & 5 & 11 \\ 6 & 0 & 2 \\ 3 & 8 & 0 \end{array}\right]$$

$D^0[2,3] \quad D^0[2,1] + D^0[1,3]$

$2 < 6 + 11$

$D^0[3,2] \quad D^0[3,1] + D^0[1,2]$

$\infty > 3 + 5$

$\infty > 8$

$$D^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} 0 & 5 & 7 \\ 6 & 0 & 2 \\ 3 & 8 & 0 \end{array}\right] \end{array}$$

$D^1[1,3] \quad D^1[1,2] + D^1[2,3]$

$11 > 5 + 2$

$11 > 7$

$D^1[3,1] \quad D^1[3,2] + D^1[2,1]$

$3 < 8 + 6$

$$D^3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc} 0 & 5 & 7 \\ 5 & 0 & 2 \\ 3 & 8 & 0 \end{array}\right] \end{array}$$

$D^2[1,2] \quad D^2[1,3] + D^2[3,2]$

$5 < 7 + 8$

$D^2[2,1] \quad D^2[2,3] + D^2[3,1]$

$6 \quad 2 + 3$

$6 > 5$