PAGE NO.:
DATE:

Practice Sheet - 1

1] Define algorithm. Explain the characteristics of algorithm.

⇒ Algorithm :-

① An algo can be defined as a finite set of steps, which has to be followed while carrying out a particular problem. It is nothing but a process of executing actions step by step.

② An algorithm can be described by incorporating a natural language such as English, computer language or a hardware language.

characteristics :-

① Input :- It should externally supply zero or more quantities.

② Output :- It results in at least one quantity.

③ Definiteness :- Each instruction should be clear and ambiguous.

④ finiteness :- An algorithm should terminate after executing a finite no. of steps.

⑤ Effectiveness :- Every instruction should be fundamental to be carried out, in principle, by a person using only pen & paper.

⑥ Feasible :- It must be feasible enough to produce each instruction.

⑦ flexibility :- It must be flexible enough to carry out desired changes with no efforts.

(8) Efficient :- The term efficiency is measured in terms of time and space required by the algo to implement.

(9) Independent :- An algo must be language independent, which means that it should mainly focus on the input and the procedure required to derive the output instead of depending upon the language.

Advantage of algo :-

1. Effective communication
2. Easy debugging
3. Easy & efficient coding
4. Independent of programming language.

Disadvantage of algo :-

1. Time consuming
2. Difficult to show branching and looping.

Q.2 Explain various types of algo. design technique.

⇒ following are some of the main algo design technique :-

① Divide and conquer approach
② Greedy technique
③ Dynamic programming
④ Branch and bound
⑤ Randomized algorithms
⑥ Backtracking algo

Q.3 Solve the following recurrence using master's theorem.

$t_n = 0$ if $n=0$
$\quad 1$ if $n=1$
$\quad 3t_{n-1} + 4t_{n-2}$ otherwise

$\Rightarrow$ $t_n = 3t_{n-1} + 4t_{n-2}$
$t_n - 3t_{n-1} - 4t_{n-2} = 0$

put $t_n = t^n$
$t^n - 3t^{n-1} - 4t^{n-2} = 0$

Divide the above eq^n by lowest degree term

$$\frac{t^n}{t^{n-2}} - \frac{3t^{n-1}}{t^{n-2}} - 4\frac{t^{n-2}}{t^{n-2}} = 0$$

$$t^2 - 3t - 4 = 0$$

$$\boxed{r_1 = 4} \quad \boxed{r_2 = -1}$$

Roots are real & different,

Generalized eq^n is :- $a_n = c_1(r_1)^n + c_2(r_2)^n$

$$\therefore \boxed{t_n = c_1(4)^n + c_2(-1)^n} \text{ —— } (1)$$

put $n=0$ in eq^n (1)
$0 = c_1(4)^0 + c_2(-1)^0$
$\therefore c_1 + c_2 = 0$ —— (2)

put $n=1$ in eq^n (1)

$$1 = c_1(4)^1 + c_2(-1)^1$$

$$4c_1 - c_2 = 1 \longrightarrow (3)$$

from eq^n (2) & (3) we get,

$$c_1 = 1/5 = 0.2 \quad , \quad c_2 = -1/5 = -0.2$$

eq^n (1) $\Rightarrow$

$$\boxed{t_n = 0.2(4)^n - 0.2(1)^n}$$

$$\therefore \boxed{t_n = \Theta(4^n)}$$

Q.4 Solve the following recurrence using master's theorem.

① $T(n) = 16T\left(\frac{n}{4}\right) + n$

⟹ compare with master theorem :-

$T(n) = aT\left(\frac{n}{b}\right) + f(n)$

here, $a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b^a} = n^{\log_4^{16}} = n^{\log_4^{4^2}} = n^2$

case ① is applicable :-

$f(n) < n^{\log_b^a}$

$\therefore \quad T(n) = \Theta\left(n^{\log_b^a}\right)$

$\therefore \quad \boxed{T(n) = \Theta(n^2)}$

? ② $T(n) = T(\sqrt{n}) + 1$

⟹ assume $\frac{n}{2^k} = 1$

$n = 2^k$

Taking log on both side.

$\log n = k \log 2$

$\log n = k$

i.e $\boxed{n = 2^k}$

$$T(2^k) = T(2^{k/2}) + 1 \quad \text{——— ①}$$

Put $T(2^k) = S(K)$

eqn ① $\Rightarrow$

$$S(K) = S\left(\frac{K}{2}\right) + 1$$

compair with master theorem,

$$a = 1, \quad b = 2, \quad f(n) = 1$$

$$K^{\log_b^a} = K^{\log_2^1} = K^0 = 1$$

case ② is applicable,

$$\therefore \boxed{f(n) = K^{\log_b^a}}$$

$$\therefore T(K) = \theta\left(K^{\log_b^a} \times \log K\right)$$

$$= \theta(1 \times \log K)$$

$$= \theta(\log K)$$

$$\therefore \boxed{T(n) = \theta(\log(\log n))}$$

③ $T(n) = T\left(\frac{n}{4}\right) + \sqrt{n} + 4$ for $n >= 4$ and $T(1) =$

⇒ compain with master theorem,

$$T(n) = T\left(\frac{n}{4}\right) + \sqrt{n} + 4$$

here, $a = 1$, $b = 4$, $f(n) = \sqrt{n} + 4$

$$\therefore n^{\log_b^a} = n^{\log_4^1} = n^0 = 1$$

$$\therefore f(n) = \sqrt{n} + 4$$

case ③ is applicable :-

$$f(n) > n^{\log_b^a}$$

Regularity condition ⇒

$$a \times f\left(\frac{n}{b}\right) \leq c \cdot f(n)$$

$$1 \times f\left(\frac{n}{4}\right) \leq c \cdot \sqrt{n} + 4$$

$$f\left(\frac{n}{4}\right) \leq c \cdot (\sqrt{n} + 4)$$

$$\sqrt{\frac{n}{4}} + 4 \leq c \cdot (\sqrt{n} + 4)$$

$$\frac{\sqrt{n}}{2} + 4 \leq c \cdot (\sqrt{n} + 4)$$

for $n = 4$

$$\frac{\sqrt{4}}{2} + 4 \le c \cdot (2+4)$$

$$5 \le c \cdot (6)$$

$$\boxed{c = 5/6 < 1}$$

$$\therefore \; T(n) = \Theta(f(n))$$

$$\therefore \; \boxed{T(n) = \Theta(\sqrt{n}+4)}$$

(4) $T(n) = 2T\left(\frac{n}{4}\right) + n$

$\Rightarrow$ compair with the master theorem,

$$a = 2, \quad b = 4, \quad f(n) = n$$

$$n^{\log_b^a} = n^{\log_4^2} = n^{1/2} = \sqrt{n}$$

case (3) is applicable,

$$f(n) > n^{\log_b^a}$$

Regularity condition $\Rightarrow$

$$a \times f\left(\frac{n}{b}\right) \le c \cdot f(n)$$

$$2 \times f\left(\frac{n}{4}\right) \le c \cdot n$$

$$2 \times \frac{n}{4} \le c \cdot n$$

$$\frac{n}{2} \le c \cdot n$$

$$\therefore \boxed{c = 1/2 < 1}$$

$\therefore \; T(n) = \Theta(f(n)) \Rightarrow \boxed{T(n) = \Theta(n)}$

PAGE NO.			
DATE			

⑤ $T(n) = 3T\left(\frac{8n}{4}\right) + n^2$

$\Rightarrow T(n) = 3T\left(\frac{8n/8}{4/8}\right) + n^2$

$T(n) = 3T\left(\frac{n}{0.5}\right) + n^2$

campair with master theorem :-

$a = 3 \ , \ b = 0.5 \ , \ f(n) = n^2$

$n^{\log_b^a} = n^{\log_{0.5}^3} = n^{-1.58}$

$f(n) > n^{\log_b^a}$

case ③ is applicable,

Regularity condition $\Rightarrow$

$a \times f\left(\frac{n}{b}\right) \leq c \cdot f(n)$

$3 \times f\left(\frac{n}{0.5}\right) \leq c \cdot n^2$

$3 \times \left(\frac{n}{0.5}\right)^2 \leq c \cdot n^2$

$3 \times \frac{n^2}{0.25} \leq c \cdot n^2$

$c = 12 \quad \ldots \quad c \nleq 1$

Here the value of c is greater than 1

(6) $T(n) = 6T(n/8) + \log n$

$\Rightarrow$ compair with master theorem :-

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

here, $a = 6$, $b = 8$, $f(n) = \log n$

$$\therefore n^{\log_b^a} = n^{\log_8^6} = n^{0.86} \approx n^0 = 1$$

$$\therefore f(n) \leq n^{\log_b^a}$$

case (1) is applicable :-

$$T(n) = \theta\left(n^{\log_b^a}\right)$$

$$= \theta\left(n^{\log_8^6}\right)$$

$$\therefore \boxed{T(n) = \theta\left(n^{0.86}\right)}$$

(7) $T(n) = 3T(n/4) + n\log n$

$\Rightarrow$ compair with master theorem :-

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

here, $a = 3$, $b = 4$, $f(n) = n\log n$

$$n^{\log_b^a} = n^{\log_4^3} = n^{0.79} \approx n^0 = 1$$

$$\therefore f(n) > n^{\log_b^a}$$

case (3) is applicable :-

checking for regularity condition,

$$a \times f\left(\frac{n}{b}\right) \le c \cdot f(n)$$

$$3 \times f\left(\frac{n}{4}\right) \le c \cdot n\log n$$

$$3 \times \frac{n}{4} \log\left(\frac{n}{4}\right) \le c \cdot n\log n$$

$$\frac{3}{4}\left[\log n - \log 4\right] \le c \cdot \log n$$

$$\frac{3}{4}\left[\log n - 0\right] \le c \cdot \log n$$

$$\frac{3}{4} \log n \le c \cdot \log n$$

$$\boxed{c = \frac{3}{4} < 1}$$

$$T(n) = \theta(f(n))$$

$$\therefore \boxed{T(n) = \theta(n \log n)}$$

(8) $T(n) = 9T\left(\frac{n}{3}\right) + n$

compare with master theorem :-

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

here, $a = 9$, $b = 3$, $f(n) = n$

$$n^{\log_b^a} = n^{\log_3^9} = n^2$$

PAGE NO.:
DATE:

$\therefore f(n) < n^{\log_b^a}$

case (1) is applicable,

$$T(n) = \Theta(n^{\log_b^a})$$

$$\therefore \boxed{T(n) = \Theta(n^2)}$$

(9) $T(n) = 2T\left(\frac{n}{2}\right) + n^3$

$\Rightarrow$ compare with the master theorem:-

$$T(n) = aT\left(\frac{n}{b}\right) + f(n)$$

here, $a = 2$, $b = 2$, $f(n) = n^3$

$$n^{\log_b^a} = n^{\log_2^2} = n$$

$$\therefore f(n) > n^{\log_b^a}$$

case (3) is applicable :-

check for regularity condition :-

$$a \times f\left(\frac{n}{b}\right) \le c \cdot f(n)$$

$$2 \times f\left(\frac{n}{2}\right) \le c \cdot n^3$$

$$2 \times \left(\frac{n^3}{2^3}\right) \le c \cdot n^3$$

$$\frac{n^3}{2^2} \le c \cdot n^3$$

$$\boxed{c = 1/4 < 1}$$

$$\therefore \boxed{T(n) = \Theta(f(n)) = \Theta(n^3)}$$

(10) $T(n) = 9T\left(\frac{3n}{5}\right) + n^3$

$\Rightarrow \quad T(n) = 9T\left(\frac{3n/3}{5/3}\right) + n^3$

$T(n) = 9T\left(\frac{n}{1.7}\right) + n^3$

compais with master theorem :-

$\therefore \quad T(n) = aT\left(\frac{n}{b}\right) + f(n)$

here, $a = 9$, $b = 1.7$, $f(n) = n^3$

$\therefore \quad n^{\log_b^a} = n^{\log_{1.7}^9} = n^{4.14}$

$\therefore \quad f(n) < n^{\log_b^a}$

case (1) is applicable :-

$T(n) = \Theta\left(n^{\log_b^a}\right)$

$\therefore \quad \boxed{T(n) = \Theta\left(n^{4.14}\right)}$

Q.5 solve the following Reccurence.

$$T(n) = \begin{cases} 0 & \text{if } n=0 \\ 5 & \text{if } n=1 \\ 3t_{n-1} + 4t_{n-2} & \text{otherwise} \end{cases}$$

$\Rightarrow$ $T(n) = 3t_{n-1} + 4t_{n-2}$

$t_n - 3t_{n-1} - 4t_{n-2} = 0$

put $t_n = t^n$

$\therefore$ $t^n - 3t^{n-1} - 4t^{n-2} = 0$

Divide the above eqn by lowest degree

$$\frac{t^n}{t^{n-2}} - \frac{3t^{n-1}}{t^{n-2}} - \frac{4t^{n-2}}{t^{n-2}} = 0$$

$\therefore \quad t^2 - 3t - 4 = 0$

$\boxed{r_1 = 4} \qquad \boxed{r_2 = -1}$

Roots are real & different,

Generalized eqn :- $a_n = c_1(r_1)^n + c_2(r_2)^n$

$\therefore \boxed{t_n = c_1(4)^n + c_2(-1)^n}$ —— ①

put $n=0$ in eqn ①

$0 = c_1(4)^0 + c_2(-1)^0$

$\therefore \quad c_1 + c_2 = 0$ —— ②

put $n=1$ in eqn ①

$5 = c_1(4)^1 + c_2(-1)^1$

$\therefore \quad 4c_1 - c_2 = 5$ —— ③

from eqn ② & ③ $\Rightarrow$

$c_1 = 1 \quad , \quad c_2 = -1$

eqn ① becomes,

$\therefore \boxed{t_n = (1)(4)^n - 1(-1)^n}$

$\therefore \boxed{t_n = \theta(4^n)}$

Q.6 Find time complexity for following algorithm

```
sum(a[], n)
{   s = 0.0
    for i 1 to n do
     s = s + a[i]
     return s
}
```

⇒

cost	frequency	Total cost
0	0	0
0	0	0
1	1	1
1	n+1	n+1
1	n	n
1	1	1
0	0	0
		$= 1+n+1+n+1$
		$= 2n+3$
		$= n$

$\boxed{\text{Time complexity} = O(n)}$

Q.7 Explain Asymptotic notations. Find upper bound, lower bound and tight bound range for following.
(i) $21n^2+9n+6$
(ii) $5n+12$

→ Asymptotic notations :-
Asymptotic notation is a way of comparing function that ignores constant factors and small input sizes. Three notations are used to calculate the running time complexity of an algorithm :-
(1) Big-oh notation (O)
(2) Omega (Ω) notation
(3) Theta (Θ) :-

(i) $21n^2+9n+6$
⇒ $f(n) = 21n^2+9n+6$
$g(n) = n^2$

$21n^2+9n+6$

LB	TB	UB
21	21	22

(1) Big-oh :- (upper bound)
$$f(n) \le c \cdot g(n)$$
$$21n^2+9n+6 \le 22 \cdot n^2 \quad \text{—— (1)}$$

Put $n=1$ in eq^n ①

$$21(1)^2+9(1)+6 \le 22(1)^2$$
$$21+9+6 \le 22$$
$$36 \le 22 \quad \text{false}$$

Put $n=3$ in eq^n ①

$$21(3)^2+9(3)+6 \le 22(3)^2$$
$$21\times 9+9\times 3+6 \le 22\times 9$$
$$222 \le 198 \quad \text{false}$$

Put $n=4$ in eq^n ①

$$21(4)^2+9(4)+6 \le 22(4)^2$$
$$21\times 16+9\times 4+6 \le 22\times 16$$
$$378 \le 352 \quad \text{false}$$

for $n=5$, $576 \le 550$ false
for $n=6$, $816 \le 792$ false
for $n=7$, $1098 \le 1078$ false
for $n=8$, $1422 \le 1408$ false
for $n=9$, $1788 \le 1782$ false
for $n=10$, $2196 \le 2200$ True

$$\boxed{\begin{array}{c} f(n) = O(g(n)) \ \forall \ n \ge 10,\ c=22 \\ 21n^2+9n+6 = O(n^2) \ \forall \ n\ge 10,\ c=22 \end{array}}$$

② Omega (Ω) notation :- (lower bound)

$$f(n) \ge c\cdot g(n)$$
$$21n^2+9n+6 \ge 21n^2 \quad \text{— ②}$$

PAGE NO.:
DATE

Put $n=1$ in eq^n 1

$$21(1)^2 + 9(1) + 6 \geq 21(1)^2$$
$$21 + 9 + 6 \geq 21$$
$$36 \geq 21 \quad \text{True}$$

$$\boxed{f(n) = \Omega(n^2) \ \forall \ n \geq 1, \ c = 21}$$

3 Theta (Θ) :-

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$
$$21n^2 \leq 21n^2 + 9n + 6 \leq 22n^2 \quad \text{—— (3)}$$

Put $n=4$ in eq^n 3

$$21(4)^2 \leq 21(4)^2 + 9(4) + 6 \leq 22(4)^2$$
$$336 \leq 378 \leq 352 \quad \text{True}$$

$$\boxed{f(n) = \Theta(n^2) \ \forall \ n \geq 4, \ c_1 = 21, \ c_2 = 22}$$

(ii) $5n+12$

$\Rightarrow$ $f(n) = 5n+12$

$g(n) = n$

$5n+12$

LB	TB	UB
$5n$	$5n$	$6n$

① Big - oh notation :-

$$f(n) \le c \cdot g(n)$$

$$5n+12 \le 6n \quad \text{—— ①}$$

~~to~~ put $n=1$ in eq^n ①

$$5(1)+12 \le 6(1)$$

$$17 \le 6 \quad \text{false}$$

put $n=2$ in eq^n ①

$$5(2)+12 \le 6(2)$$

$$10+12 \le 12$$

$$22 \le 12 \quad \text{false}$$

put $n=4$ in eq^n ①

$$5(4)+12 \le 6(4)$$

$$20+12 \le 24$$

$$32 \le 24 \quad \text{false}$$

put $n=6$ in eq^n ①

$$5(6)+12 \le 6(6)$$

$$30+12 \le 36$$

$$42 \le 36 \quad \text{false}$$

put $n=9$ in eq^n ①

$5(9)+12 \le 6\cdot(9)$

$45+12 \le 54$

$57 \le 54$ false

put $n=12$ in eq^n ①

$5(12)+12 \le 6\cdot(12)$

$60+12 \le 72$

$72 \le 72$ True

$\therefore$ $\boxed{f(n) = \theta(g(n)) = \theta(n) \;\forall\; n \geq 12, c=6}$

② Omega notation :-

$f(n) \geq c \cdot g(n)$

$5n+12 \geq 5 \cdot n$ —②

put $n=1$ in eq^n ②

$5(1)+12 \geq 5\cdot(1)$

$17 \geq 5$ True

$\therefore$ $\boxed{f(n) = \Omega(g(n)) = \Omega(n) \;\forall\; n \geq 1, c=5}$

③ Theta - notation :-

$c_1 g(n) \le f(n) \le c_2 g(n)$

$5n \le 5n+12 \le 6n$ —③

~~put~~ $n=12$ in eq^n ③

$60 \le 72 \le 72$ True

$\therefore$ $\boxed{f(n) = \theta(g(n)) = \theta(n), \forall n \geq 12, c_1=5, c_2=6}$

Q.8 What are the different asymptotic notations? Explain them briefly for the following eqⁿ, find the values of constants using various approaches.

(i) $20n^2 + 8n + 10$

$\Rightarrow$ $f(n) = 20n^2 + 8n + 10$

$g(n) = n^2$

$20n^2 + 8n + 10$

LB	TB	UB
20	20	21

(1) Big - oh notations :-

$$f(n) \le c \cdot g(n)$$

$$20n^2 + 8n + 10 \le 21 \cdot n^2 \quad \text{—— (1)}$$

put $n = 1$ in eqⁿ (1)

$$20(1)^2 + 8(1) + 10 \le 21 \cdot (1)^2$$

$$20 + 8 + 10 \le 21$$

$$38 \le 21 \quad \text{false}$$

put $n = 3$ in eqⁿ (1)

$$20(3)^2 + 8(3) + 10 \le 21(3)^2$$

$$20 \times 9 + 24 + 10 \le 21 \times 9$$

$$214 \le 189 \quad \text{false}$$

DATE

put $n=5$ in eqⁿ ①,

$$20(5)^2 + 8(5) + 10 \le 21(5)^2$$
$$20 \times 25 + 8 \times 5 + 10 \le 21 \times 25$$
$$500 + 50 \le 511$$
$$550 \le 511 \quad \text{false}$$

put $n=8$ in eqⁿ ①,

$$20(8)^2 + 8(8) + 10 \le 21(8)^2$$
$$20 \times 64 + 64 + 10 \le 21 \times 64$$
$$1280 + 74 \le 1344$$
$$1354 \le 1344 \quad \text{false}$$

put $n=10$ in eqⁿ ①,

$$20(10)^2 + 8(10) + 10 \le 21(10)^2$$
$$20 \times 100 + 80 + 10 \le 21 \times 100$$
$$2000 + 90 \le 2100$$
$$2090 \le 2100 \quad \text{True}$$

$$\therefore \boxed{f(n) = O(g(n)) = O(n^2) \ \forall \ n \ge 10, \ c = 21}$$

② Omega (Ω) notation :-

$$f(n) \ge c \times g(n)$$
$$20n^2 + 8n + 10 \ge 20 \times n^2$$
$$20n^2 + 8n + 10 \ge 20n^2 \quad \text{— ②}$$

put $n=1$ in eqⁿ ②,

$$20(1) + 8(1) + 10 \ge 20(1)$$
$$20 + 8 + 10 \ge 20$$
$$38 \ge 20 \quad \text{True}$$

$\therefore$ $\boxed{f(n) = \Omega(g(n)) = \Omega(n^2) \quad \forall\ n \geq 1 \ ,\ c = 20}$

③ Theta (Θ) :-

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$

$$20n^2 \leq 20n^2 + 8n + 10 \leq 21n^2 \quad \text{--- ③}$$

Put $n=1$ in eq^n ③ ,

$$20(1) \leq 20(1) + 8(1) + 10 \leq 21(1)$$

$$20 \leq 20 + 8 + 10 \leq 21$$

$$20 \leq 38 \leq 21 \quad \text{false}$$

Put $n=10$ in eq^n ③ $\rightarrow$

$$20(10)^2 \leq 20(10)^2 + 8(10) + 10 \leq 21(10)^2$$

$$20 \times 100 \leq 20 \times 100 + 80 + 10 \leq 21 \times 100$$

$$2000 \leq 2000 + 90 \leq 2100$$

$$2000 \leq 2090 \leq 2100 \quad \text{True}$$

$\therefore$ $\boxed{f(n) = \Theta(g(n)) = \Theta(n^2) \quad \forall\ n \geq 10 \ ,\ c_1 = 20, c_2 = 21}$

(ii) $2n+5$

$\Rightarrow f(n) = 2n+5$

$g(n) = n$

$2n+5$

LB	TB	UB
2	2	3

① Big-oh notation :-

$$f(n) \le c \cdot g(n)$$

$$2n+5 \le 3n \quad \text{—— } ①$$

put $n=1$ in eqn ①

$$2+5 \le 3$$

$$7 \le 3 \text{ false}$$

put $n=2$ in eqn ①

$$2(2)+5 \le 3 \times 2$$

$$4+5 \le 6$$

$$9 \le 6 \text{ false}$$

Put $n=4$ in eqn ①

$$2(4)+5 \le 3(4)$$
$$8+5 \le 12$$
$$13 \le 12 \quad \text{false}$$

put $n=5$ in eqn ①

$$2(5)+5 \le 3(5)$$
$$10+5 \le 15$$
$$15 \le 15 \quad \text{True}$$

$$\therefore \boxed{f(n) = O(g(n)) = O(n) \ \forall \ n \ge 5, \ c=3}$$

② omega (Ω) notation :-

$$f(n) \ge c \cdot g(n)$$
$$2n+5 \ge 2 \cdot n$$
$$2n+5 \ge 2n \quad \text{——} ②$$

put $n=1$ in eqn ②

$$2(1)+5 \ge 2(1)$$
$$7 \ge 2 \quad \text{True}$$

$$\therefore \boxed{f(n) = \Omega(g(n)) = \Omega(n) \ \forall \ n \ge 1 \text{ and } c=2}$$

③ Theta notation :-

$$c_1 g(n) \le f(n) \le c_2 g(n)$$

$$2n \le 2n+5 \le 3n \quad \text{—— ③}$$

put $n=5$ in eqⁿ ③,

$$2(5) \le 2(5)+5 \le 3(5)$$

$$10 \le 15 \le 15 \quad \text{True}$$

$$\therefore \boxed{f(n) = \theta(g(n)) = \theta(n) \ \forall\ n \ge 5, c_1 = 2, c_2 = 3}$$

(iii) $5n^3 + n^2 + 6n + 2$

→ $f(n) = 5n^3 + n^2 + 6n + 2$

$g(n) = n^3$

$5n^3 + n^2 + 6n + 2$

LB	TB	UB
5	5	6

① Big- oh notations :-

$$f(n) \le c\, g(n)$$

$$5n^3 + n^2 + 6n + 2 \le 6 \times n^3 \quad \text{—— ①}$$

put $n=2$ in eqⁿ ①

$$5(2)^3 + (2)^2 + 6(2) + 2 \le 6(2)^3$$

$$5 \times 8 + 4 + 12 + 2 \le 6 \times 8$$

$$58 \le 48 \quad \text{false.}$$

Put $n=3$ in eqⁿ ①

$$5(3)^3 + (3)^2 + 6(3) + 2 \le 6(3)^3$$
$$5\times 27 + 9 + 18 + 2 \le 6\times 27$$
$$164 \le 162 \quad \text{false}$$

put $n=4$ in eqⁿ ①

$$5(4)^3 + (4)^2 + 6(4) + 2 \le 6\times(4)^3$$
$$362 \le 384 \quad \text{True}$$

$\therefore \boxed{f(n) = O(g(n)) = O(n^3) \quad \forall\, n \geq 4 \;, \; c=6}$

② Omega (Ω) notation :-

$$f(n) \geq c.g(n)$$
$$5n^3 + n^2 + 6n + 2 \geq 5n^3 \quad \text{——} ②$$

put $n=1$ in eqⁿ ①

$$5(1)^3 + (1)^2 + 6(1) + 2 \geq 5(1)^3$$
$$5+9 \geq 5$$
$$14 \geq 5 \quad \text{True}$$

$\therefore \boxed{f(n) = \Omega(n^3) \quad \forall\, n \geq 1 \;, \; c=5}$

③ Theta (θ) notation :-

$$c_1 g(n) \le f(n) \le c_2 g(n)$$
$$5n^3 \le 5n^3 + n^2 + 6n + 2 \le 6n^3 \quad \text{——} ③$$

Put $n=4$ in eqⁿ ③,

$$5(4)^3 \le 5(4)^3 + (4)^2 + 6(4) + 2 \le 6(4)^3$$
$$320 \le 362 \le 384 \quad \text{True}$$

$\therefore \boxed{f(n) = \theta(n^3) \quad \forall\, n \geq 4 \,,\; c_1 = 5 \,,\; c_2 = 6}$

(iv) $4 * 2^n + 3n$

$\Rightarrow f(n) = 4 * 2^n + 3n$

$g(n) = 2^n$

$4 * 2^n + 3n$

LB	TB	UB
4	4	5

① Big - oh :-

$f(n) \leq c \cdot g(n)$

$4 \cdot 2^n + 3n \leq 5 \cdot 2^n$ —— ①

put $n = 4$ in eq^n ①

$4 \cdot 2^4 + 3(4) \leq 5 \cdot 2^4$

$76 \leq 80$ (True)

$$\boxed{f(n) = O(g(n)) = O(2^n) \ \forall \ n \geq 4, \ c = 4}$$

② Omega :-

$f(n) \geq c \cdot g(n)$

$4 \cdot 2^n + 3n \geq 4 \cdot 2^n$ —— ②

put $n = 1$ in eq^n ②

$11 \geq 8$ (True)

$f(n) = \Omega(g(n))$

$$\boxed{f(n) = \Omega(2^n) \ \forall \ n \geq 1, \ c = 4}$$

PAGE NO.:
DATE

(3) Theta - notation :-

$$c_1 g(n) \le f(n) \le c_2 g(n) \quad \text{——— (3)}$$

for $n = 4$ in eq^n (3)

$$4 \cdot 2^n \le 4 \cdot 2^n + 3n \le 5 \, 2^n$$

$$64 \le 76 \le 80 \quad (\text{True}$$

$$\boxed{f(n) = \theta(g(n)) = \theta(2^n) \;\forall\; n \ge 4,\; c_1 = 4,\; c_2 = 5}$$

(V) $10n^2+4n+2$

$\Rightarrow f(n) = 10n^2+4n+2$

$g(n) = n^2$

$10n^2+4n+2$

LB	TB	UB
10	10	11

(1) Big - oh notation :-

$$f(n) \le c \cdot g(n)$$

$$10n^2+4n+2 \le 11 \cdot n^2 \quad \text{--- (1)}$$

Put $n=1$ in eq^n (1),

$$10(1)^2+4(1)+2 \le 11 \cdot (1)^2$$

$$10+4+2 \le 11$$

$$16 \le 11 \quad \text{false}$$

Put $n=3$ in eq^n (1),

$$10(3)^2+4(3)+2 \le 11 (3)^2$$

$$90+12+2 \le 11 \times 9$$

$$104 \le 99 \quad \text{false}$$

Put $n=5$ in eq^n (1),

$$10(5)^2+4(5)+2 \le 11 (5)^2$$

$$250+20+2 \le 25 \times 11$$

$$272 \le 275 \quad \text{True}$$

$$\therefore \boxed{f(n) = O(g(n)) = O(n^2) \quad \forall\, n \ge 5,\ c = 1}$$

② omega (Ω) notations :-

$$f(n) \geq c \cdot g(n)$$
$$10n^2+4n+2 \geq 10n^2 \quad \text{---} ②$$

put $n=1$ in eq^n ②,

$$10(1)^2 + 4(1)+2 \geq 10(1)^2$$
$$10+4+2 \geq 10$$
$$16 \geq 10 \quad \text{True}$$

$\therefore \boxed{f(n) = \Omega(g(n)) = \Omega(n^2) \ \forall \ n \geq 1 \ , \ c=10}$

③ Theta (Θ) notations :-

$$c_1 g(n) \leq f(n) \leq c_2 g(n)$$
$$10n^2 \leq 10n^2+4n+2 \leq 11n^2 \quad \text{---} ③$$

put $n=1$ in eq^n ③,

$$10(1)^2 \leq 10(1)^2+4(1)+2 \leq 11(1)^2$$
$$10 \leq 10+4+2 \leq 11$$
$$10 \leq 16 \leq 11$$

Put $n=5$ in eq^n ③

$$10(5)^2 \leq 10(5)^2+4(5)+2 \leq 11(5)^2$$
$$250 \leq 275 \leq 275 \quad \text{True}$$

$\therefore \boxed{f(n) = \Theta(g(n)) = \Theta(n^2) \ \forall \ n \geq 5 \ , \ c_1=10, c_2=11}$

PAGE NO.:
DATE:

155^{162}

Q.11 Explain the asymptotic notation of analysis of algorithm. Also find θ notation for
(i) $5n^3+n^2+3n+2$
(ii) $27n^2+16n$

(1) $5n^3+n^2+3n+2$

$\Rightarrow$ $f(n) = 5n^3+n^2+3n+2$
$g(n) = n^3$

$5n^3+n^2+3n+2$

LB	TB	UB
5	5	6

Theta (θ) notation :-

$c_1 g(n) \le f(n) \le c_2 g(n)$
$5n^3 \le 5n^3+n^2+3n+2 \le 6n^3$

for $n=1$, $5 \le 11 \le 6$ false
for $n=2$, $40 \le 52 \le 48$ false
for $n=4$, $320 \le 350 \le 384$ True
for $n=3$, $135 \le 155 \le 162$ True

$\therefore$ $\boxed{f(n) = \theta(g(n)) = \theta(n^3)\ \forall\ n \geq 3,\ c_1 = 5,\ c_2 = 6}$

(ii) $27n^2 + 16n$

$\Rightarrow$ $f(n) = 27n^2 + 16n$

$g(n) = n^2$

$27n^2 + 16n$

LB	TB	UB
27	27	28

Theta (Θ) notation :-

$$c_1 g(n) \le f(n) \le c_2 g(n)$$

$$27n^2 \le 27n^2 + 16n \le 28n^2 \quad \text{--- (1)}$$

Put $n=1$ in eq^n (1) $\Rightarrow$

for $n=1$, $27 \le 43 \le 28$ false

for $n=3$, $243 \le 291 \le 252$ false

for $n=4$, $432 \le 496 \le 448$ false

for $n=6$, $972 \le 1068 \le 1008$ false

for $n=8$, $1728 \le 1856 \le 1792$ false

for $n=10$ $2700 \le 2860 \le 2800$ false

for $n=12$ $3888 \le 4080 \le 4032$ false.

Q.13 Find time complexity for following algorithm

Algorithm	cost	frequency	Total cost
Add (A, B, n)	0	0	0
{	0	0	0
for (i = 0 ; i < n ; i++)	1	n+1	n+1
{	0	0	0
for (j = 0 ; j < n ; j++)	1	n(n+1)	n(n+1)
{	0	0	0
c[i][j] = a[i][j] + b[i][j];	1	n^2	n^2
}	0	0	0
}	0	0	0
}	0	0	0
			$= n+1+n^2+n+n^2$
			$= 2n^2+2n+1$
			$= n^2$

$\therefore$ $\boxed{\text{Time complexity} = O(n^2)}$

PAGE NO.
DATE

Q.12 Solve the following reccurrence using substitution method by first guessing the solⁿ and then use induction to find constants and show that the solution work

$$T(n) = \begin{cases} 1 & \text{if } n=1 \\ 2T(n/2)+n & \text{if } n>1 \end{cases}$$

$\rightarrow$ $T(n) = 2T\left(\frac{n}{2}\right) + n$ —— ①

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{n}{2}$ —— ②

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{n}{4}$ —— ③

eqⁿ ① $\Rightarrow$ $T(n) = 2T\left(\frac{n}{2}\right) + n$

$$T(n) = 2\left[2T\left(\frac{n}{4}\right) + \frac{n}{2}\right] + n$$

$$T(n) = 4T\left(\frac{n}{4}\right) + n + n$$

$$T(n) = 4\left[2T\left(\frac{n}{8}\right) + \frac{n}{4}\right] + 2n$$

$$T(n) = 8T\left(\frac{n}{8}\right) + n + 2n$$

$$T(n) = 8T\left(\frac{n}{8}\right) + 3n$$

$$\therefore T(n) = 2^3 T\left(\frac{n}{2^3}\right) + 3n$$

$$\therefore T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn$$

assume $\frac{n}{2^k} = 1 \Rightarrow n = 2^k \rightarrow \log n = k$

So, $T(n) = n\,T(1) + n\log n$

$T(n) = n + n\log n$ $\{T(n) = 1 \text{ if } n = 1\}$

$$\therefore \boxed{T(n) = \theta(n\log n)}$$

PAGE NO.:
DATE:

Q15 solve using substitution method.

$$T(n) = T(n-1) + \log n \quad \text{if } n>1$$
$$T(1) = 1 \quad \text{if } n=1$$

$\Rightarrow$ $T(n) = T(n-1) + \log n$ — (1)

$T(n-1) = T(n-2) + \log(n-1)$ — (2)

$T(n-2) = T(n-3) + \log(n-2)$ — (3)

in eqn (1) $\Rightarrow$

$$T(n) = T(n-1) + \log n$$
$$= T(n-2) + \log(n-1) + \log n$$
$$T(n) = T(n-3) + \log(n-2) + \log(n-1) + \log n$$

$\therefore T(n) = T(n-k) + \log(n-(k-1)) + \log(n-(k-2)) + \log n$

$$n-k = 1, \quad k = n-1$$

$\therefore T(n) = T(1) + \log(n-(n-1-1)) + \log(n-(n-1-2)) + \log n$

$\therefore T(n) = T(1) + \log(2) + \log(3) + \log n$

$\therefore T(n) = 1 + \log 2 + \log 3 + \log n$

$= 1 + \log(2 \cdot 3 \cdot 4 \cdots (n-1)n)$

$= 1 + \log(n!)$

$= 1 + \log(n^n)$

$\therefore T(n) = 1 + n\log n$

$\therefore \boxed{T(n) = \theta(n\log n)}$