Spectral Correlation of Modulated Signals: Part I—Analog Modulation

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Abstract—**The importance of the concept of cyclostationarity in design and analysis of signal detectors, synchronizers, and extractors in communication systems is briefly discussed, and the central role of spectral correlation, in the characterization of random processes that are cyclostationary in the wide sense, is explained. A spectral correlation function that is a generalization of the power spectral density function is described, and a corresponding generalization of the Wiener-Khinchine relation and several other fundamental spectral correlation relations also are described. Explicit formulas for the spectral correlation function for various types of analog-modulated signals are derived. This includes pulse and carrier amplitude modulation, quadrature amplitude carrier modulation, and phase and frequency carrier modulation. To illustrate the differing spectral correlation characteristics of different modulation types, the magnitudes of the spectral correlation functions are graphed or described in graphical terms as the heights of surfaces above a bifrequency plane.**

I. INTRODUCTION

A cyclostationary process is a random process with probabilistic parameters, such as the autocorrelation function, that vary periodically with time. Cyclostationary processes are appropriate probabilistic models for signals that have undergone periodic transformations, such as sampling, scanning, modulating, multiplexing, and coding operations, provided that the signal is appropriately modeled as a stationary process before undergoing the periodic transformation. In response to the increasing demands on communication system performance, systems analysts are beginning to discover the importance of recognizing the cyclostationary character of most communications signals, and to abandon the simpler more approximate stationary models that have been used in the past. The growing awareness of the relevance of the concept of cyclostationarity is illustrated by recent work in synchronization [1]–[10], crosstalk interference and modulation transfer noise [11], [12], transmitter and receiver filter design [13]–[16], adaptive filtering and system identification [17]–[19], coding [20], [21], queueing [22], [23], and detection [19], [24], [25]. In addition, the growing role of cyclostationarity in other signal processing areas is illustrated by recent work in biomedical engineering [26], climatology [27], and hydrology [28], and by recent developments in basic theory for prediction [29], extraction [13], [19], detection [19], [25], and signal modeling and representation [30]–[33].

It is shown in this paper that spectral correlation is a characteristic property of wide-sense cyclostationarity (cyclostationarity of the autocorrelation), and a spectral correlation function that is a generalization of the power spectral density function is described. The spectral correlation concept has associated with it four fundamental properties of processes that are of significant practical value: 1) different types of modulated signals (such as BPSK, QPSK, and SQPSK) that have identical power spectral density functions can have highly distinct spectral correlation functions; 2) stationary noise and interference exhibit no spectral correlation (the spectral correlation function is identically zero); 3) the spectral correlation function contains phase and frequency information related to timing parameters in modulated signals; and 4) the existence of spectral correlation in a signal means that some spectral components can be estimated using other spectral components of the signal. Furthermore, all optimum signal processors (such as estimators and detectors) that are specified in terms of the cyclostationary autocorrelation function can be interpreted and implemented in terms of the spectral correlation function. For example, the low-SNR likelihood ratio for detection and the low-SNR likelihood function for parameter estimation for random signals in white Gaussian noise are, in general, completely specified by the signal autocorrelation function (cf. [10], [19], [34]). Similarly, the optimum periodically time-variant filter for a cyclostationary signal in stationary noise is completely specified by the signal and noise autocorrelation functions [13], [19], [35]. Also, the optimum quadratic transformation for generation of maximum SNR spectral lines from a cyclostationary process for purposes of detection and synchronization is completely specified by the spectral correlation function for the signal and the power spectral density for the noise [10], [19]. Consequently, properties 1)–4) can be exploited for detection, classification, parameter estimation, and extraction of signals buried in noise and further masked by interference. Although the spectral correlation function is a second-order (quadratic) statistic or probabilistic parameter like the power spectral density function, these four properties enable it to be used to accomplish tasks that are impossible to accomplish with the power spectral density function [19]. This includes synchronization [10] and noise and interference rejection for signal extraction and detection [19]. For example, modulated signals that are severely masked by other interfering signals as well as noise can, in some applications, be more effectively detected by detection of spectral correlation rather than detection of energy. This is so, for instance, when the energy level of the background noise or interference fluctuates unpredictably [25]. Also, there are situations in which cyclostationarity is problematic, rather than being a property to be exploited. For example, nonlinearities in transmission systems (e.g., traveling-wave tube amplifiers and noise limiters) can inadvertently generate spectral lines from cyclostationary signals, and these spectral lines can cause severe interference effects (cf. [12]). As explained in [10], the characteristics of spectral lines (their frequencies and strengths) that can be generated with quadratic nonlinearities are determined by the spectral correlation function.

Since constraints on the length of this paper do not allow for explicit discussion of the various applications of spectral correlation, only one particular application is described here and the description is brief. This application concerns the problem of extracting a signal of interest from a background of both broad-band noise and band-limited interference. If

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interference in some portions of the signal band is so strong that it swamps the signal in those partial bands, then the optimum time-invariant filter (Wiener filter) simply rejects those bands. However, if the spectral components of the signal in those bands are correlated with spectral components of the signal in other bands, then the optimum periodically time-variant filter will use those correlated components to estimate and replace the rejected components [19]. If the correlation is high and the noise and interference in the bands with correlated components are low, the replaced components can be of high fidelity. If an adaptive periodically time-variant filter is used, the appropriate weighting of the frequency-shifted components is learned through the process of adaptation [19]. In order to gain an understanding of the potential for this approach to signal extraction, the spectral correlation characteristics of signals of interest must be determined.

The primary objective of this paper is, therefore, to introduce a general approach and several general formulas for calculating the spectral correlation function, and to obtain explicit formulas for the spectral correlation functions for many of the most commonly used types of modulation. As a visual aid, the magnitudes of these calculated functions are graphed (or described in graphical terms) as the heights of surfaces above a bifrequency plane. In this Part I, the modulation types considered are all analog and include pulse and carrier amplitude modulation, quadrature amplitude carrier modulation, and phase and frequency carrier modulation. In Part II [43], various types of digital pulse and carrier modulation are considered.

Although the probabilistic approach based on expected values, which is conventional for power spectral density calculations, could be adopted here, as in [35], the nonprobabilistic approach based on limiting time-average values is used instead because of its closer conceptual ties with empirical methods for measurement of spectral correlation [41]. The link between these two approaches, which is briefly discussed herein, is explained in more detail in [42].

II. Spectral Correlation

This section is a very brief review of the fundamental concepts, definitions, and a few basic properties of spectral correlation. An in-depth introductory treatment is given in the tutorial paper [42] which is prerequisite for a full understanding of the notion of spectral correlation.

In order to measure the local spectral content of a waveform $x(t)$ over the time interval $[t - T/2, t + T/2]$, we use the finite-time Fourier transform

$$X_T(t, \nu) \triangleq \int_{t-T/2}^{t+T/2} x(u)e^{-i2\pi\nu u}\, du, \tag{1}$$

evaluated at frequency ν. The correlation between two such local spectral components at frequencies $\nu = f + \alpha/2$ and $\nu = f - \alpha/2$ (where f is the midpoint and α is the separation of the two frequencies) normalized by $\sqrt{T}$, measured over an interval of length Δt, is given by

$$S_{x_T}^{\alpha}(f)_{\Delta t} \triangleq \frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2} \frac{1}{\sqrt{T}} X_T(t, f+\alpha/2) \cdot \frac{1}{\sqrt{T}} X_T^*(t, f-\alpha/2)\, dt \tag{2}$$

(where the superscript asterisk denotes complex conjugation.) An idealized measure of spectral correlation for a persistent waveform is then given by the double limit

$$\hat{S}_x^{\alpha}(f) \triangleq \lim_{T\to\infty}\lim_{\Delta t\to\infty} S_{x_T}^{\alpha}(f)_{\Delta t}. \tag{3}$$

As $\Delta t \to \infty$, the measure of correlation becomes ideal (all random effects are averaged away), and as $T \to \infty$, the bandwidth of the spectral components being correlated becomes infinitesimal. The order of the two limits in (3) is crucial since for any truly random waveform (one whose spectral density function contains continuous components, not just Dirac deltas), the limit as $T \to \infty$ does not exist for finite Δt [19]. It is shown in [42] that the idealized spectral correlation function (3) can be characterized as the Fourier transform

$$\hat{S}_x^{\alpha}(f) = \int_{-\infty}^{\infty} \hat{R}_x^{\alpha}(\tau)e^{-i2\pi f\tau}\, d\tau \tag{4}$$

of the limit function[1]

$$\hat{R}_x^{\alpha}(\tau) \triangleq \lim_{\Delta t\to\infty} \frac{1}{\Delta t}\int_{-\Delta t/2}^{\Delta t/2} x(t+\tau/2)x(t-\tau/2)e^{-i2\pi\alpha t}\, dt. \tag{5}$$

To show this, one first shows that the product of normalized finite-time Fourier transforms in (2) is itself characterized by a Fourier transform

$$\frac{1}{T}X_T(t, f+\alpha/2)X_T^*(t, f-\alpha/2) = \int_{-T}^{T}\left[\frac{1}{T}\int_{-(T-|\tau|)/2}^{(T-|\tau|)/2} x(t+u+\tau/2)x(t+u-\tau/2) e^{-i2\pi\alpha(t+u)}\, du\right] e^{-i2\pi f\tau}\, d\tau \tag{6}$$

and then the limit as $\Delta t \to \infty$ in (3) and the integral with respect to t in (2) are interchanged with the integrals with respect to τ and u in (6) to obtain

$$\hat{S}_x^{\alpha}(f) = \lim_{T\to\infty}\int_{-T}^{T}\frac{1}{T}\int_{-(T-|\tau|)/2}^{(T-|\tau|)/2} \hat{R}_x^{\alpha}(\tau)\, du e^{-i2\pi f\tau}\, d\tau \tag{7}$$

which can be shown to yield (4).

If α is set equal to zero, then (2) becomes the temporal mean-square value (average power) of the local spectral component, and therefore (3) becomes the average power spectral density function $\hat{S}_x^0(f) \equiv \hat{S}_x(f)$. Also, in this case, (5) becomes the autocorrelation function $\hat{R}_x^0(\tau) \equiv \hat{R}_x(\tau)$, and (4) therefore becomes the Wiener relation [36] (which is commonly called the *Wiener–Khinchine relation* in the probabilistic setting). For $\alpha \neq 0$, $\hat{R}_x^{\alpha}(\tau)$ in (5) is the strength of any finite additive sine wave component with frequency α that might be contained in the lag product waveform $y_\tau(t) \triangleq x(t + \tau/2)x(t - \tau/2)$. Relation (4) reveals that there is a one-to-one correspondence between such periodicity in lag products, which is called *cyclostationarity*, and spectral correlation. This correspondence, (4), is called the *cyclic Wiener relation* [42]. The function $\hat{R}_x^{\alpha}(\tau)$ is called the *cyclic autocorrelation*, and the spectral correlation function $\hat{S}_x^{\alpha}(f)$ is also called the *cyclic spectral density* [42]. If $\hat{R}_x^{\alpha}(\tau) \equiv 0$ for all $\alpha \neq 0$ and $\hat{R}_x(\tau) \not\equiv 0$, then $x(t)$ is said to be *purely stationary*. If $\hat{R}_x^{\alpha}(\tau) \not\equiv 0$ only for α = integer$/T_0$ for some period T_0, then $x(t)$ is said to be *purely cyclostationary* with period T_0. If $\hat{R}_x^{\alpha}(\tau) \not\equiv 0$ for values of α that are not all integer multiples of some fundamental frequency $1/T_0$, then $x(t)$ is said to *exhibit cyclostationarity*. In modulated signals, the periods of cyclostationarity correspond to carrier frequencies, pulse rates, spreading code repetition rates, time-division multiplexing rates, and so on.

If $x(t)$ itself contains no finite additive sine wave component, then the temporal mean of $X_T(t, f)$ is zero for $f \neq 0$. Consequently, $\hat{S}_x^{\alpha}(f)$ is actually a spectral covariance function

[1] The relationship between the limit function (5) and the radar ambiguity function is discussed in [42].

in this case, and the power spectral density $\hat{S}_x(f)$ is actually a variance. Therefore, the normalized function

$$\hat{C}_x^\alpha(f) \triangleq \frac{\hat{S}_x^\alpha(f)}{[\hat{S}_x(f+\alpha/2)\hat{S}_x(f-\alpha/2)]^{1/2}} \tag{8}$$

is a complex correlation coefficient which satisfies

$$|\hat{C}_x^\alpha(f)| \leq 1. \tag{9}$$

This function $\hat{C}_x^\alpha(f)$ is called the *spectral autocoherence function* [42]. The waveform $x(t)$ is said to be *completely coherent* at f and α if $|\hat{C}_x^\alpha(f)| = 1$, and it is said to be *completely incoherent* at f and α if $\hat{C}_x^\alpha(f) = 0$. The parameter α is called the *cycle frequency*, and the parameter f is called the *spectrum frequency* [42].

In the probabilistic approach, a stochastic process $x(t)$ is defined to be *cyclostationary* (in the wide sense) with period T_0 if the probabilistic autocorrelation

$$R_x(t, \tau) \triangleq E\{x(t+\tau/2)x(t-\tau/2)\} \tag{10}$$

(in which $E\{\cdot\}$ denotes expected value) is periodic with period T_0, $R_x(t+T_0, \tau) \equiv R_x(t, \tau)$ [35]. The coefficients of the sine wave components of this autocorrelation

$$R_x^\alpha(\tau) \triangleq \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} R_x(t, \tau)e^{-i2\pi\alpha t}\,dt \tag{11}$$

(where α = integer/T_0) are called the *cyclic autocorrelations*. For a stochastic process that exhibits cyclostationarity at more than one fundamental frequency, $R_x(t, \tau)$ consists of a denumerable sum of finite additive periodic components and has the Fourier series representation

$$R_x(t, \tau) = \sum_\alpha R_x^\alpha(\tau)e^{i2\pi\alpha t} \tag{12}$$

where

$$R_x^\alpha(\tau) \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} R_x(t, \tau)e^{-i2\pi\alpha t}\,dt, \tag{13}$$

and the sum in (12) ranges over all values of α for which (13) is not identically zero. If only one periodic component is present in $R_x(t, \tau)$ (i.e., if $R_x(t, \tau)$ is periodic), then (13) reduces to (11). The nonprobabilistic counterpart of $R_x(t, \tau)$ in either (10) or (12) is given by

$$\hat{R}_x(t, \tau) = \sum_\alpha \hat{R}_x^\alpha(\tau)e^{i2\pi\alpha t} \tag{14}$$

where $\hat{R}_x^\alpha(\tau)$ is defined by (5). For a purely cyclostationary $x(t)$, $\hat{R}_x(t, \tau)$ is also given by [42]

$$\hat{R}_x(t, \tau) = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(t+\tau/2+nT_0)x(t-\tau/2+nT_0), \tag{15}$$

whereas for an $x(t)$ that exhibits cyclostationarity with the periods $T = T_1, T = T_2, T = T_3, \cdots, T = T_M$ for some integer M (or $M = \infty$), we have [19], [42]

$$\hat{R}_x(t, \tau) = \hat{R}_x^0(\tau) + \sum_{i=1}^{M} [\hat{R}_x(t, \tau; T_i) - \hat{R}_x^0(\tau)] \tag{16}$$

where $\hat{R}_x(t, \tau; T_i)$ is given by (15) with $T_0 = T_i$. Thus, there is a duality between the more common probabilistic theory of stochastic processes that exhibit cyclostationarity [35] and the nonprobabilistic theory [19], [42] adopted here. If the stochastic process is cycloergodic [19], [31], [35], then $\hat{R}_x^\alpha(\tau) \equiv R_x^\alpha(\tau)$ and these two dual theories merge into one.

A particularly convenient method for calculating the spectral correlation function for many types of modulated signals is to model the signal as a purely stationary waveform transformed by a linear periodically time-variant transformation. Specifically, consider a vector-input, scalar-output, *linear periodically time-variant* (LPTV) system with period T_0, and with impulse-response function $\boldsymbol{h}$, expanded in a Fourier series:

$$y(t) = \int_{-\infty}^{\infty} \boldsymbol{h}(t, u)\boldsymbol{x}(u)\,du \tag{17}$$

$$\boldsymbol{h}(t, u) = \sum_{n=-\infty}^{\infty} \boldsymbol{g}_n(t-u)e^{i2\pi nu/T_0} \tag{18}$$

where $\boldsymbol{h}(t, u)$ is a row vector and $\boldsymbol{x}(u)$ is a column vector. The system function, which is defined by the row vector

$$\boldsymbol{G}(t, f) \triangleq \int_{-\infty}^{\infty} \boldsymbol{h}(t, t-\tau)e^{-i2\pi f\tau}\,d\tau, \tag{19}$$

is then given by the Fourier series

$$\boldsymbol{G}(t, f) = \sum_{n=-\infty}^{\infty} \boldsymbol{G}_n(f+n/T_0)e^{i2\pi nt/T_0} \tag{20}$$

for which $\boldsymbol{G}_n$ is the Fourier transform of $\boldsymbol{g}_n$. It can be shown by straightforward (but tedious) calculation that the cyclic autocorrelations and cyclic spectra of the input $\boldsymbol{x}(t)$ and output $y(t)$ of the LPTV system are related by the formulas[2] [42]

$$\hat{R}_y^\alpha(\tau) = \sum_{n,m=-\infty}^{\infty} \int_{-\infty}^{\infty} \mathrm{tr}\,\{[\hat{\boldsymbol{R}}_x^{\alpha-(n-m)/T_0}(t) \cdot e^{-i\pi(n+m)t/T_0}]\boldsymbol{r}_{nm}^\alpha(t-\tau)\}\,dt \tag{21}$$

$$\hat{S}_y^\alpha(f) = \sum_{n,m=-\infty}^{\infty} \boldsymbol{G}_n(f+\alpha/2)\hat{\boldsymbol{S}}_x^{\alpha-(n-m)/T_0}(f-[n+m]/2T_0) \cdot \boldsymbol{G}_m'(f-\alpha/2)^* \tag{22}$$

in which $\hat{\boldsymbol{R}}_x^\alpha$ is the matrix of cyclic cross correlations for the vector-valued waveform $\boldsymbol{x}(t)$, $\hat{\boldsymbol{S}}_x^\alpha$ is the corresponding matrix of Fourier transforms, and $\boldsymbol{r}_{nm}^\alpha$ is the matrix of *finite cyclic cross correlations* of the Fourier-coefficient functions $\boldsymbol{g}_n$, which is defined by

$$\boldsymbol{r}_{nm}^\alpha(\tau) \triangleq \int_{-\infty}^{\infty} \boldsymbol{g}_n'(t+\tau/2)\boldsymbol{g}_m(t-\tau/2)e^{-i2\pi\alpha t}\,dt. \tag{23}$$

Formula (22) is used in the following sections for calculating the spectral correlation for various types of modulated signals. This formula can be generalized for linear time-variant systems with more than one periodicity, in which case the sum in (18) is expanded to include all harmonics of each of the fundamental frequencies corresponding to each periodicity, by simply expanding the double sum in (22) similarly [19].

A special case of interest is a time-invariant filter, for which all $n \neq 0$ terms in (18) and (20) are zero. In this case, (22) reduces to

$$\hat{S}_y^\alpha(f) = \boldsymbol{G}_0(f+\alpha/2)\hat{\boldsymbol{S}}_x^\alpha(f)\boldsymbol{G}_0'(f-\alpha/2)^*, \tag{24}$$

and for $\alpha = 0$, this is recognized as the conventional formula for the power spectral density at the output of a multiinput filter.

[2] In (21) and (22), the prime denotes matrix transposition and tr$\{\cdot\}$ denotes the matrix trace operation.

For a discrete time-series $x(nT_*)$, the counterparts of (4) and (5) are [42] the Fourier-series transform

$$\tilde{S}_x^\alpha(f) \triangleq \sum_{k=-\infty}^{\infty} \tilde{R}_x^\alpha(kT_*)e^{-i2\pi kT_* f} \tag{25a}$$

and the limit sequence

$$\tilde{R}_x^\alpha(kT_*) \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x(nT_* + kT_*) \cdot x(nT_*)e^{-i2\pi\alpha(n+k/2)T_*}. \tag{26}$$

The inverse Fourier-series transform is given by

$$\tilde{R}_x^\alpha(kT_*) = T_* \int_{-1/2T_*}^{1/2T_*} \tilde{S}_x^\alpha(f)e^{i2\pi fkT_*}\, df. \tag{25b}$$

For the case in which the time-series $x(nT_*)$ is obtained by time sampling a waveform $x(t)$, it can be shown [42] that (4) and (25a) are related by the *spectral aliasing formula*

$$\tilde{S}_x^\alpha(f) = \frac{1}{T_*} \sum_{n,m=-\infty}^{\infty} \hat{S}_x^{\alpha+m/T_*}(f - m/2T_* - n/T_*). \tag{27}$$

As an example, for the special case of $\alpha = 0$, the formula (27) becomes

$$\tilde{S}_x(f) = \frac{1}{T_*} \sum_{n,m=-\infty}^{\infty} \hat{S}_x^{m/T_*}(f - m/2T_* - n/T_*) \tag{28}$$

for the power spectral density of the time-sampled waveform. The terms corresponding to $m \neq 0$ account for the fact that the powers of superposed (aliased) spectral components do not add directly when the components are correlated [33]. If the waveform $x(t)$ is purely stationary, then it has no spectral correlation and the $m \neq 0$ terms vanish.

Another useful formula for calculating spectral correlation is the *spectral-correlation convolution* relation

$$\hat{S}_y^\alpha(f) = \sum_\beta \int_{-\infty}^{\infty} \hat{S}_x^{\alpha-\beta}(f - v)\hat{S}_w^\beta(v)\, dv \tag{29}$$

which holds for the product

$$y(t) = x(t)\,w(t) \tag{30}$$

of two statistically independent[3] waveforms [42]. The sum in (29) extends over all values of β for which neither factor in (29) is identically zero. As an example, for the special case $\alpha = 0$, the formula (29) becomes

$$\hat{S}_y(f) = \sum_\beta \int_{-\infty}^{\infty} \hat{S}_x^{-\beta}(f - v)\hat{S}_w^\beta(v)\, dv \tag{31}$$

for the power spectral density of the product of independent waveforms. If either factor $x(t)$ or $w(t)$ is purely stationary, then all terms except $\beta = 0$ are zero, and this is recognized as the conventional convolution formula for the power spectral density of the product of two purely stationary waveforms.

In viewing the graphs of spectral correlation presented in the following sections, it should be kept in mind that the surface above the f axis where $\alpha = 0$ is the conventional PSD. This provides a benchmark against which the strength of spectral correlation can be measured. Also, because of the symmetries $\hat{S}_x^\alpha(-f) = \hat{S}_x^\alpha(f)$ and $\hat{S}_x^{-\alpha}(f) = \hat{S}_x^\alpha(f)^*$, only a quarter of the bifrequency (f, α) plane is shown in some cases. Impulses (Dirac deltas) in the spectral correlation function are shown as narrow pulses added to the continuous part. The relative heights of these pulses shown in the graphs correctly reflect the relative values of the areas of the impulses, but the absolute heights are scaled for graphical convenience. The scale factors used can be deduced from the data in the captions.

[3] It is shown in [19] that in the fraction-of-time probabilistic sense, every periodic waveform (or sum of periodic waveforms) is statistically independent of every other waveform.

III. Pulse and Carrier Amplitude Modulation

In this section, we study the spectral correlation properties of conventional amplitude-modulated sine wave carriers with and without carrier phase fluctuation, conventional amplitude-modulated pulse trains with and without pulse-timing jitter, and other more specialized amplitude modulation types such as stacked-carrier spread spectrum and pulsed-noise signals. All these signals can be derived from product modulation and filtering operations on unmodulated signals.

Consider the generalized *amplitude modulation* (AM) waveform

$$x(t) = a(t)p(t) \tag{32a}$$

for which $p(t)$ is a periodic (or almost periodic) carrier with Fourier series

$$p(t) = \sum_\beta P_\beta e^{i2\pi\beta t}. \tag{32b}$$

Examples of this type of modulation are conventional AM signals, stacked carrier spread-spectrum signals, and pulsed-noise signals. Equation (32) is an LPTV transformation of $a(t)$ for which the Fourier coefficient functions $g_\beta(\tau)$ of the impulse-response function are given by

$$g_\beta(\tau) = P_\beta \delta(\tau) \tag{33a}$$

where

$$P_\beta \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t)e^{-i2\pi\beta t}\, dt. \tag{33b}$$

Therefore, formula (21) (with $x(t)$ and $y(t)$ there replaced by $a(t)$ and $x(t)$, respectively) can be applied to obtain the cyclic autocorrelation

$$\hat{R}_x^\alpha(\tau) = \sum_{\beta,\nu} P_\nu P_{\beta-\nu} \hat{R}_a^{\alpha-\beta}(\tau)e^{-i\pi(\beta-2\nu)\tau}. \tag{34}$$

Fourier transformation of (34) [or direct use of (22)] yields the cyclic spectral density

$$\hat{S}_x^\alpha(f) = \sum_{\beta,\nu} P_\nu P_{\beta-\nu} \hat{S}_a^{\alpha-\beta}(f - \nu + \beta/2). \tag{35}$$

For the special case in which $a(t)$ is purely stationary, (35) reduces to

$$\hat{S}_x^\alpha(f) = \sum_\nu P_\nu P_{\alpha-\nu} \hat{S}_a(f - \nu + \alpha/2). \tag{36}$$

If $a(t)$ is white, then it follows from (36) that the autocoherence magnitude is given by

$$|\hat{C}_x^\alpha(f)| = \frac{\left|\sum_\nu P_\nu P_{\alpha-\nu}\right|}{\sum_\nu |P_\nu|^2} \tag{37}$$

where $P_{-\nu} = P_\nu^*$. Equation (37) is a kind of correlation-coefficient sequence for the sequence of Fourier coefficients $\{P_\nu\}$. As an example of (37), if $x(t)$ is a pulsed noise signal, then $\{P_\beta\}$ are the Fourier coefficients of an on–off square wave, with period T_0 and duty cycle of, say, 100 η percent:

$$P_\beta=\frac{1}{T_0}\int_0^{\eta T_0} e^{-i2\pi\beta t}\,dt=e^{-i\pi\eta\beta T_0}\frac{\sin(\pi\eta\beta T_0)}{\pi\beta T_0}.$$

As an example of (36), if $x(t)$ is a stacked carrier AM signal, then P_β is a constant for a finite set of values of β, such as $\beta = f_0 + n\Delta f$ for $n = 0, 1, 2, 3, \cdots, N-1$, and P_β is zero for all other values of β. Let us consider some other examples in more detail.

Example (Amplitude Modulation): If $p(t)$ is given by

$$p(t)=\cos(2\pi f_0 t+\phi_0) \tag{38}$$

then we have conventional AM and (34) and (35) reduce to

$$\hat{R}_x^\alpha(\tau)=\frac{1}{2}\hat{R}_a^\alpha(\tau)\cos(2\pi f_0\tau)+\frac{1}{4}\hat{R}_a^{\alpha+2f_0}(\tau)e^{-i2\phi_0}+\frac{1}{4}\hat{R}_a^{\alpha-2f_0}(\tau)e^{+i2\phi_0} \tag{39}$$

and

$$\hat{S}_x^\alpha(f)=\frac{1}{4}[\hat{S}_a^\alpha(f+f_0)+\hat{S}_a^\alpha(f-f_0)+\hat{S}_a^{\alpha+2f_0}(f)e^{-i2\phi_0}+\hat{S}_a^{\alpha-2f_0}(f)e^{+i2\phi_0}]. \tag{40}$$

This result can be used to obtain the cyclic spectra for other types of modulation that involve an amplitude-modulated carrier. Examples for which $a(t)$ is cyclostationary are binary phase-shift keying and amplitude-shift keying, which are treated in [43]. For the special case in which the amplitude $a(t)$ is purely stationary, $x(t)$ is purely cyclostationary with period $T_0 = 1/2f_0$, and (40) reduces to

$$\hat{S}_x^\alpha(f)=\begin{cases}\frac{1}{4}\hat{S}_a(f+f_0)+\frac{1}{4}\hat{S}_a(f-f_0), & \alpha=0\\ \frac{1}{4}\hat{S}_a(f)e^{\pm i2\phi_0}, & \alpha=\pm 2f_0\\ 0, & \text{otherwise.}\end{cases} \tag{41}$$

A typical graph of the spectral correlation magnitude surface $|\hat{S}_x^\alpha(f)|$ is shown in Fig. 1. It follows from (41) that the magnitude of the spectral autocoherence of $x(t)$ (8) is given by

$$|\hat{C}_x^\alpha(f)|=\frac{\hat{S}_a(f)}{[\hat{S}_a(f)^2+\hat{S}_a(f+\alpha)\hat{S}_a(f)+\hat{S}_a(f-\alpha)\hat{S}_a(f)+\hat{S}_a(f+\alpha)\hat{S}_a(f-\alpha)]^{1/2}} \tag{42}$$

for $\alpha = \pm 2f_0$. Consequently, if $a(t)$ is band limited such that $\hat{S}_a(f) = 0$ for $|f| \geqslant B$ and $\hat{S}_a(f) \neq 0$ for $|f| < B$, with $B < f_0$, then

$$|\hat{C}_x^\alpha(f)|=\begin{cases}1, & |f|<B \text{ and } |\alpha|=2f_0,\ 0\\ 0, & |f|\geqslant B \text{ or } |\alpha|\neq 2f_0,\ 0\end{cases} \tag{43}$$

and $x(t)$ is completely coherent for $\alpha = \pm 2f_0$ and all frequencies f for which it is not completely incoherent because there is no power density. But if $B > f_0$, then $|\hat{C}_x^\alpha(f)| < 1$ for $|f| > 2f_0 - B$. For example, if $a(t)$ is white ($\hat{S}_a(f) = A_0$ for $-\infty < f < \infty$), then $|\hat{C}_x^\alpha(f)| = 1/2$ for $-\infty < f < \infty$ and $|\alpha| = 2f_0$.

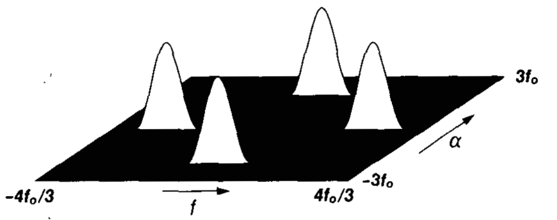

Fig. 1. Spectral correlation magnitude surface for an amplitude-modulated sine wave with carrier frequency f_0.

Example (Pulse-Amplitude Modulation): If $p(t)$ is given by

$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT_0), \tag{44}$$

then $P_\beta = 1/T_0$ for $\beta = m/T_0$ for all integers m, and therefore (35) yields

$$\hat{S}_x^\alpha(f)=\frac{1}{T_0^2}\sum_{n,m=-\infty}^{\infty}\hat{S}_a^{\alpha-m/T_0}(f-n/T_0+m/2T_0). \tag{45}$$

If the product time series (32) and (44) is filtered using an impulse-response function $q(t)$, then $y(t) = x(t) \otimes q(t)$ is the *pulse-amplitude modulation* (PAM) signal

$$y(t)=\sum_{n=-\infty}^{\infty}a(nT_0)q(t-nT_0). \tag{46}$$

Application of the input–output spectral correlation relation for filters (24) (with G_0 there replaced by Q) to (45) yields

$$\hat{S}_y^\alpha(f)=\frac{1}{T_0^2}Q(f+\alpha/2)Q^*(f-\alpha/2)\cdot\sum_{n,m=-\infty}^{\infty}\hat{S}_a^{\alpha-m/T_0}(f-n/T_0+m/2T_0) \tag{47}$$

for this PAM signal. The spectral correlation aliasing formula (27) [with $x(t)$ there replaced by $a(t)$] applied to (47) yields the alternative formula for PAM:

$$\hat{S}_y^\alpha(f)=\frac{1}{T_0}Q(f+\alpha/2)Q^*(f-\alpha/2)\tilde{S}_a^\alpha(f). \tag{48}$$

If $a(t)$ is purely stationary, then (48) reduces to

$$\hat{S}_y^\alpha(f)=\begin{cases}\frac{1}{T_0}Q(f+\alpha/2)Q^*(f-\alpha/2)\tilde{S}_a(f+\alpha/2), & \alpha=k/T_0\\ 0; & \alpha\neq k/T_0\end{cases} \tag{49}$$

(for all integers k) by using

$$\tilde{S}_a^{\alpha+k/T_0}(f)=\tilde{S}_a^\alpha(f+k/2T_0).$$

It follows from (49) that the spectral autocoherence magnitude (8) is given by

$$|\hat{C}_y^\alpha(f)| = 1, \qquad \alpha = k/T_0 \tag{50}$$

for all f for which $\hat{S}_y(f \pm \alpha/2) \neq 0$. Therefore, the PAM signal is completely coherent at $\alpha = k/T_0$ for all integers k and all f for which the signal is not completely incoherent because there is no power density. A graph of the spectral correlation magnitude surface $|\hat{S}_y^\alpha(f)|$ for a white amplitude sequence $\{a(nT_0)\}$ and a rectangle pulse $q(t)$ of width T_0 is shown in Fig. 2.

Example (Jittered PAM): The more realistic model of PAM that incorporates pulse-timing jitter

$$y(t) = \sum_{n=-\infty}^{\infty} a_n q(t - nT_0 - \epsilon_n) \tag{51}$$

can be reexpressed as

$$y(t) = [a(t)\, w(t)] \otimes q(t) \tag{52}$$

where $w(t)$ is the jittered impulse train

$$w(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0 - \epsilon_n) \tag{53}$$

and $a_n = a(nT_0 - \epsilon_n)$. This model is appropriate for either jitter that occurs in the process of forming the PAM signal or jitter that occurs after the PAM signal has been formed, provided in this latter case that $a(t)$ is sufficiently narrow band (low pass) or broad band and that the statistics of a_n are independent of the jitter. The general formulas (24) and (29) applied to (52) (assuming $\{\epsilon_n\}$ and $a(t)$ are statistically independent) yield

$$\hat{S}_y^\alpha(f) = Q(f+\alpha/2)Q^*(f-\alpha/2) \cdot \sum_\beta \int_{-\infty}^{\infty} \hat{S}_a^{\alpha-\beta}(f-v)\hat{S}_w^\beta(v)\, dv, \tag{54}$$

and for a purely stationary amplitude $a(t)$, this reduces to

$$\hat{S}_y^\alpha(f) = Q(f+\alpha/2)Q^*(f-\alpha/2)[\hat{S}_a(f) \otimes \hat{S}_w^\alpha(f)]. \tag{55}$$

If the jitter sequence $\{\epsilon_n\}$ is purely stationary, then $w(t)$ is purely cyclostationary with period T_0, and $y(t)$ is therefore also purely cyclostationary with period T_0. However, the strength of spectral correlation at $\alpha = k/T_0$ will be attenuated by the convolution in (55). On the other hand, if $\{\epsilon_n\}$ is an independent-increment sequence [35] (in the fraction-of-time sense [19]), then $w(t)$ will be purely stationary and so too will $y(t)$. Nevertheless, $y(t)$ can still exhibit reliably measurable spectral correlation locally in time. The former (purely stationary $\{\epsilon_n\}$) model is appropriate for jitter relative to a tracking clock synchronized (imperfectly) to $y(t)$, whereas the latter (independent-increment $\{\epsilon_n\}$) model is appropriate for absolute jitter and drift. It can be shown [43] that for an independent sequence of jitter variables $\{\epsilon_n\}$, the spectral correlation function is given by

$$\begin{aligned}\hat{S}_w^\alpha(f) = \frac{1}{T_0^2} \sum_{n=-\infty}^{\infty} &\Psi_\epsilon(2\pi n/T_0)\Psi_\epsilon^*(2\pi[n+k]/T_0) \\ &\cdot \delta(f+n/T_0+k/2T_0) + \frac{1}{T_0}\{\Psi_\epsilon^*(2\pi k/T_0) \\ &- \Psi_\epsilon^*(2\pi[f+k/2T_0])\Psi_\epsilon(2\pi[f-k/2T_0])\}, \\ &\alpha = k/T_0\end{aligned} \tag{56}$$

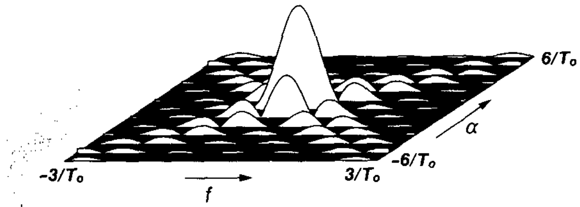

Fig. 2. Spectral correlation magnitude surface for a pulse-amplitude modulated pulse train with pulse rate $1/T_0$.

for all integers k and is zero for all other values of α. The function Ψ_ϵ is the characteristic function for the sequence $\{\epsilon_n\}$, defined by

$$\Psi_\epsilon(\omega) \triangleq \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \exp\{i\omega\epsilon_n\}. \tag{57}$$

In general, the larger the variance of $\{\epsilon_n\}$ is, the narrower Ψ_ϵ will be and the more the spectral correlation in $y(t)$ will be attenuated. As an example, if ϵ_n has Gaussian fraction-of-time density with zero mean and variance σ_ϵ^2, then [19]

$$\Psi_\epsilon(\omega) = \exp\left\{-\frac{1}{2}\sigma_\epsilon^2\omega^2\right\}. \tag{58}$$

Example (Phase-Deviated AM): The more realistic model for AM that incorporates carrier phase deviation

$$x(t) = a(t)\cos[2\pi f_0 t + \phi(t)] \tag{59}$$

is still an example of product modulation, and therefore formula (29) applies (assuming that $\phi(t)$ and $a(t)$ are statistically independent) to yield

$$\hat{S}_x^\alpha(f) = \sum_\beta \int_{-\infty}^{\infty} \hat{S}_a^{\alpha-\beta}(f-v)\hat{S}_w^\beta(v)\, dv \tag{60}$$

where

$$w(t) = \cos[2\pi f_0 t + \phi(t)]. \tag{61}$$

As explained in Section V, the spectral correlation function for the phase-modulated sine wave (61) is given by the Fourier transform of the cyclic autocorrelation function

$$\hat{R}_w^\alpha(\tau) = \begin{cases} \frac{1}{2}\mathrm{Re}\{\Psi_\tau(1,-1)e^{i2\pi f_0\tau}\}, & \alpha = 0 \\ \frac{1}{4}\Psi_\tau(1,1), & \alpha = 2f_0 \\ \frac{1}{4}\Psi_\tau(1,1)^*, & \alpha = -2f_0 \\ 0, & \alpha \neq \pm 2f_0, 0 \end{cases} \tag{62}$$

where $\mathrm{Re}\{\cdot\}$ denotes the real part and the function Ψ_τ is the joint characteristic function for $\phi(t+\tau/2)$ and $\phi(t-\tau/2)$:

$$\Psi_\tau(\omega_1, \omega_2) \triangleq \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \exp\{i[\phi(t+\tau/2)\omega_1 + \phi(t-\tau/2)\omega_2]\, dt. \tag{63}$$

In general, the larger the variance of $\phi(t)$ is, the smaller $\Psi_\tau(1, 1)$ will be in magnitude, and the more the spectral correlation in $x(t)$ will be attenuated. For example, if $\phi(t)$ is a purely stationary Gaussian waveform with zero mean, then [19]

$$\Psi_\tau(1, 1) = \exp\{-[\hat{R}_\phi(0) + \hat{R}_\phi(\tau)]\}. \tag{64}$$

IV. Quadrature-Carrier Amplitude Modulation

Consider the quadrature-carrier amplitude modulation (QAM) waveform

$$x(t) = c(t)\cos(2\pi f_0 t) - s(t)\sin(2\pi f_0 t)$$
$$= a(t)\cos[2\pi f_0 t + \phi(t)] \tag{65a}$$

for which

$$a(t) = [c(t)^2 + s(t)^2]^{1/2}$$
$$\phi(t) = \tan^{-1}[s(t)/c(t)]$$
$$c(t) = a(t)\cos[\phi(t)]$$
$$s(t) = a(t)\sin[\phi(t)]. \tag{65}$$

This is a particular LPTV transformation of the two-dimensional vector of waveforms $[c(t), s(t)]'$ for which the vector of impulse-response functions is

$$h(t, u) = [\cos(2\pi f_0 t)\delta(t-u), \ -\sin(2\pi f_0 t)\delta(t-u)]$$

and the vector of corresponding system functions is

$$G(t, f) = [\cos(2\pi f_0 t), \ -\sin(2\pi f_0 t)].$$

Application of formula (21) yields

$$\hat{R}_x^\alpha(\tau) = \frac{1}{2}[\hat{R}_c^\alpha(\tau) + \hat{R}_s^\alpha(\tau)]\cos(2\pi f_0 \tau)$$
$$-\frac{1}{2}[\hat{R}_{sc}^\alpha(\tau) - \hat{R}_{cs}^\alpha(\tau)]\sin(2\pi f_0 \tau)$$
$$+\frac{1}{4}\sum_{n\in\{-1,1\}} \{[\hat{R}_c^{\alpha+2nf_0}(\tau) - \hat{R}_s^{\alpha+2nf_0}(\tau)]$$
$$- ni[\hat{R}_{sc}^{\alpha+2nf_0}(\tau) + \hat{R}_{cs}^{\alpha+2nf_0}(\tau)]\} \tag{66}$$

and application of formula (22) yields the cyclic spectral density

$$\hat{S}_x^\alpha(f) = \frac{1}{4}\sum_{n\in\{-1,1\}} \{[\hat{S}_c^\alpha(f+nf_0) + \hat{S}_s^\alpha(f+nf_0)]$$
$$- ni[\hat{S}_{sc}^\alpha(f+nf_0) - \hat{S}_{cs}^\alpha(f+nf_0)]\}$$
$$+\frac{1}{4}\sum_{n\in\{-1,1\}} \{[\hat{S}_c^{\alpha+2nf_0}(f) - \hat{S}_s^{\alpha+2nf_0}(f)]$$
$$- ni[\hat{S}_{sc}^{\alpha+2nf_0}(f) + \hat{S}_{cs}^{\alpha+2nf_0}(f)]\}. \tag{67}$$

Examples for which $c(t)$ and $s(t)$ are cyclostationary are quaternary-phase-shift keying and some amplitude-phase-shift keying, which are treated in [43].

For the special case in which the in-phase and quadrature components $c(t)$ and $s(t)$ are jointly purely stationary, $x(t)$ is purely cyclostationary with period $T_0 = 1/2f_0$, and we have

$$\hat{S}_x(f) = \frac{1}{4}[\hat{S}_c(f+f_0) + \hat{S}_c(f-f_0) + \hat{S}_s(f+f_0) + \hat{S}_s(f-f_0)]$$
$$-\frac{1}{2}[\hat{S}_{cs}(f+f_0)_i - \hat{S}_{cs}(f-f_0)_i] \tag{68}$$

and

$$\hat{S}_x^\alpha(f) = \frac{1}{4}[\hat{S}_c(f) - \hat{S}_s(f)] \pm \frac{1}{2} i\hat{S}_{cs}(f)_r, \qquad \alpha = \pm 2f_0 \tag{69}$$

with $\hat{S}_x^\alpha = 0$ for $\alpha \neq 0$ and $\alpha \neq \pm 2f_0$. (In these formulas, the subscripts r and i denote the real and imaginary parts, respectively.) Thus, the cyclic spectral density of QAM is of the same general form as that shown in Fig. 1 for AM, except that the full symmetry shown there is typically not exhibited by QAM. Only symmetry about the f axis and symmetry about the α axis is always exhibited by QAM. Moreover, it follows from the autocoherence inequality (8)–(9) that the heights of the surfaces centered at $(f, \alpha) = (0, \pm 2f_0)$ are lower than or equal to the heights at $(f, \alpha) = (\pm f_0, 0)$. If $c(t)$ and $s(t)$ are band limited such that $\hat{S}_c(f) = \hat{S}_s(f) = 0$ for $|f| \geqslant f_0$, then (68)–(69) yield

$$|\hat{C}_x^\alpha(f)|^2 = \frac{[\hat{S}_c(f) - \hat{S}_s(f)]^2 + 4[\hat{S}_{cs}(f)_r]^2}{[\hat{S}_c(f) + \hat{S}_s(f)]^2 - 4[\hat{S}_{cs}(f)_i]^2}, \qquad \alpha = \pm 2f_0 \tag{70a}$$

for the spectral autocoherence magnitude of $x(t)$. Moreover, it is shown in [35] that *any* process $x(t)$ that is purely cyclostationary with period $1/2f_0$ and is band limited to $f \in (-2f_0, 2f_0)$ can be represented in the form of (65) (Rice's representation, see [32]) for which $c(t)$ and $s(t)$ are purely stationary and band limited to $f \in (-f_0, f_0)$. This includes many band-limited analog-modulated sine wave carriers, with purely stationary modulating signals, used in conventional communications systems and essentially all modulated periodic pulse trains with *excess bandwidth* (beyond the *Nyquist bandwidth*) of 100 percent or less and purely stationary modulating signals. Thus, for all such signals, the spectral autocoherence magnitude is given by (70a). Furthermore, (70a) can be reexpressed as

$$|\hat{C}_x^\alpha(f)|^2 = 1 - \frac{4(1 - |\hat{C}_{cs}(f)|^2)\hat{S}_c(f)\hat{S}_s(f)}{[\hat{S}_c(f) + \hat{S}_s(f)]^2 - 4[\hat{S}_{cs}(f)_i]^2}, \qquad \alpha = \pm 2f_0 \tag{70b}$$

where $\hat{C}_{cs}(f)$ is the cross coherence between $c(t)$ and $s(t)$ [19], [35], and the denominator can be reexpressed as

$$[\hat{S}_c(f) + \hat{S}_s(f)]^2 - 4[\hat{S}_{cs}(f)_i]^2 = 16[\hat{S}_x(f-f_0)_e^2 - \hat{S}_x(f-f_0)_o^2], \qquad |f| < f_0 \tag{71}$$

where the subscripts e and o denote the even and odd parts, respectively, of $\hat{S}_x(f)$ about the point $f = f_0$ for $f > 0$. Consequently, for given spectrum $\hat{S}_x$ and spectral product $\hat{S}_c\hat{S}_s$, the autocoherence magnitude of $x(t)$ increases as the magnitude of the cross coherence between $c(t)$ and $s(t)$ increases. Furthermore, for given cross coherence $|\hat{C}_{cs}|$ and spectral product $\hat{S}_c\hat{S}_s$, the autocoherence magnitude of $x(t)$ increases as the dominance of the even part (about f_0) of the spectrum $\hat{S}_x$ over the odd part increases (assuming $\hat{S}_c(f)\hat{S}_s(f) \neq 0$).

It follows from (70a) that $x(t)$ is completely incoherent ($\hat{C}_x^\alpha(f) = 0$) at $\alpha = \pm 2f_0$ and at any f if and only if $c(t)$ and $s(t)$ are *balanced at f* in the sense that

1) $\hat{S}_c(f) = \hat{S}_s(f)$

and

2) $\hat{S}_{cs}(f)_r = 0$.

It also follows from (70b) that $x(t)$ is completely coherent ($|\hat{C}_x^\alpha(f)| = 1$) at $\alpha = \pm 2f_0$ and at any $|f| < f_0$ if and only if either

3) the supports of $\hat{S}_c$ and $\hat{S}_s$ are disjoint such that $\hat{S}_c(f)\hat{S}_s(f) = 0$ or

4) the waveforms $c(t)$ and $s(t)$ are completely cross coherent at f such that

$$|\hat{C}_{cs}(f)| = 1 \qquad \text{for } |f| < f_0$$

and either 1) or 2) (or both) is violated.

It follows from (70a) and (71) that for a time series $x(t)$ that is completely coherent at $|f| < f_0$, the cyclic spectrum magnitude is characterized by the symmetry of the conventional spectrum:

$$|\hat{S}_x^\alpha(f)|^2 = \hat{S}_x(f-\alpha/2)_e^2 - \hat{S}_x(f-\alpha/2)_o^2, \qquad |f| < |\alpha|/2 \quad \text{for } |\alpha| = 2f_0. \quad (72)$$

When 4) holds, $c(t)$ and $s(t)$ are related (at least in the temporal mean square sense) by an LTI transformation $s(t) = h(t) \otimes c(t)$ (cf. [19], [35]). But there do exist LTI transformations for which neither 1) nor 2) is violated, namely, those for which the transfer functions are unity in magnitude and purely imaginary with arbitrary signs at arbitrary frequencies f, that is, $H(f) = \pm i$.

Example (SSB, DSB, VSB): The Hilbert transform (for which $H(f) = +i$ for $f < 0$ and $H(f) = -i$ for $f > 0$), which yields a *single-sideband* (SSB) signal $x(t)$,

$$\hat{S}_x(f) = 0, \qquad |f| < f_0,$$

results in a waveform that is completely incoherent for all f. In contrast, a transfer function that is a real constant yields a *double-sideband* (DSB) signal $x(t)$ that is completely coherent for all $|f| < f_0$, as established in Section III. Similarly a *vestigial-sideband* (VSB) signal, which is obtained by subjecting a DSB signal to a low-pass filtering operation with bandwidth, say, $B = f_0 + b$, is completely coherent for $|f| < b$ and completely incoherent (for an ideal low-pass filter) for $|f| > b$. For the DSB signal, there is no phase modulation, $\phi(t) =$ constant. However, if the DSB signal is filtered and the filter transfer function is asymmetric about the point $f = f_0$ (for $f > 0$) and nonzero, then $\phi(t)$ in (65) is no longer constant, but $x(t)$ is still completely coherent for all $|f| < f_0$ (since the autocoherence magnitude is invariant to linear time-invariant transformations [19], [35], provided that the transfer function does not equal zero).

In order to determine what type of phase modulation annihilates coherence, one can use the fact that the necessary and sufficient conditions 1) and 2) for complete incoherence for all f are equivalent to the pair of conditions

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} a(t+\tau/2)a(t-\tau/2) \cdot \cos[\phi(t+\tau/2)+\phi(t-\tau/2)]\, dt \equiv 0 \quad (73a)$$

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} a(t+\tau/2)a(t-\tau/2) \cdot \sin[\phi(t+\tau/2)+\phi(t-\tau/2)]\, dt \equiv 0. \quad (73b)$$

It follows from (73) that an equivalent condition is

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} a(t+\tau/2)a(t-\tau/2) \cdot \exp\{i[\phi(t+\tau/2)+\phi(t-\tau/2)]\}\, dt \equiv 0. \quad (74)$$

For example, if $a(t)$ is statistically independent of $\phi(t)$ (e.g., $a(t) =$ constant), then (74) reduces to

$$\Psi_\tau(1, 1) = 0, \qquad -\infty < \tau < \infty \quad (75)$$

for which Ψ_τ is the joint characteristic function for the variables $\phi(t + \tau/2)$ and $\phi(t - \tau/2)$ defined by (63). But (75) is simply the condition under which the phase-modulated waveform

$$x(t) = \cos[2\pi f_0 t + \phi(t)]$$

is completely incoherent. This is discussed further in the next section.

V. Phase and Frequency Carrier Modulation

Consider the phase-modulated (PM) sine wave

$$x(t) = \cos[2\pi f_0 t + \phi(t)] \quad (76)$$

for which $\phi(t)$ contains no periodicity (of any order). It can be shown by direct calculation that the cyclic autocorrelation for $x(t)$ is given by

$$\hat{R}_x^\alpha(\tau) = \begin{cases} (1/2)\,\mathrm{Re}\{\Psi_\tau(1, -1)e^{i2\pi f_0 \tau}\}, & \alpha = 0 \\ (1/4)\Psi_\tau(1, 1), & \alpha = 2f_0 \\ (1/4)\Psi_\tau(1, 1)^*, & \alpha = -2f_0 \\ 0, & |\alpha| \neq 2f_0,\ \alpha \neq 0 \end{cases} \quad (77a)$$

and the cyclic mean of $x(t)$

$$\hat{M}_x^\alpha \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-i2\pi\alpha t}\, dt$$

is given by

$$\hat{M}_x^\alpha = \begin{cases} (1/2)\Psi(1), & \alpha = f_0 \\ (1/2)\Psi(1)^*, & \alpha = -f_0 \\ 0, & |\alpha| \neq f_0 \end{cases} \quad (78a)$$

where Ψ_τ is the joint characteristic function for $\phi(t + \tau/2)$ and $\phi(t - \tau/2)$

$$\Psi_\tau(\omega_1, \omega_2) \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \exp\{i[\phi(t+\tau/2)\omega_1 + \phi(t-\tau/2)\omega_2]\}\, dt \quad (77b)$$

and Ψ is the characteristic function for $\phi(t)$

$$\Psi(\omega) \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \exp\{i\phi(t)\omega\}\, dt. \quad (78b)$$

Thus, $x(t)$ is purely cyclostationary with period $1/2f_0$ except when $\Psi_\tau(1, 1) = 0$ for all τ, and $x(t)$ exhibits spectral lines at $\pm f_0$ except when $\Psi(1) = 0$. Neither $\Psi_\tau(1, 1) \equiv 0$ nor $\Psi(1) = 0$ can hold for a purely stationary Gaussian waveform $\phi(t)$ [cf. (58) and (64)]. However, there are *some* waveforms $\phi(t)$ that do satisfy $\Psi_\tau(1, 1) \equiv 0$ and $\Psi(1) = 0$. An example is the balanced quaternary-valued waveform

$$\phi(t) = \tan^{-1}[s(t)/c(t)]$$

for which $c(t)$ and $s(t)$ are statistically identical, uncorrelated, binary-valued (± 1) waveforms with stationary transition times (e.g., a Poisson point processes [35]). Another example is the balanced quaternary-valued PAM waveform $\phi(t)$, which yields the QPSK signal discussed in [43]. However, since the transition times in this $\phi(t)$ are periodic with period, say T_0, then $x(t)$ is purely cyclostationary with period T_0, even though it is not cyclostationary with period $1/2f_0$. Furthermore, both

$\Psi_\tau(1, 1) \equiv 0$ and $\Psi(1) = 0$ are satisfied by some nonstationary waveforms, such as those that arise from *frequency modulation*. Specifically, let $\phi(t)$ be given by[4]

$$\phi(t) = \int_0^t z(u)\, du \tag{79a}$$

for which $z(t)$ is a purely stationary waveform. Then

$$z(t) = d\phi(t)/dt, \qquad t > 0 \tag{79b}$$

and $z(t)$ is the *instantaneous frequency deviation* (in radians per unit of time) of the modulated sine wave (76). If the spectrum $\hat{S}_z(f)$ is not high pass (or bandpass) in the sense that it does not approach zero faster than linearly in f as $f \to 0$, then $\phi(t)$ can satisfy $\Psi_\tau(1, 1) \equiv 0$ and $\Psi(1) = 0$. For example, if $z(t)$ is white ($\hat{S}_z(f) = k$), then $\phi(t)$ is an *independent-increment waveform* (a *diffusion*), for which it is well known that $x(t)$ is stationary and contains no spectral lines [37]. Thus, frequency-modulated (FM) sine waves with low-pass (or all-pass) modulation are purely stationary, whereas those with high-pass (or bandpass) modulation can be purely cyclostationary. For example, if $z(t)$ is defined by (79b) for a purely stationary $\phi(t)$, then $x(t)$ can be purely cyclostationary. Thus, we see that some FM sine waves are purely stationary, but most PM sine waves of practical interest are purely cyclostationary. Hence, there is a fundamental distinction to be made between the statistical properties of FM sine waves and PM sine waves. This is often not recognized, since it is common practice to use probabilistic models and to introduce a random time-invariant phase variable to render all modulated sine waves stochastically stationary (cf. [33] and [38]), and to adopt relatively arbitrary conventions (e.g., [39]–[40]) for distinguishing between phase modulation and frequency modulation. It should be emphasized, however, that even frequency-modulated sine waves that are stationary in the long run can exhibit reliably measurable properties of cyclostationarity locally in time.

It is also worth clarifying that the transformation (79a) from $z(t)$ to $\phi(t)$ is only marginally stable and can therefore be an inappropriate model. For example, it can be shown that even though $z(t)$ might have a zero temporal mean, the temporal mean of $\phi(t)$ need not be zero. This is easily demonstrated for the simple case $z(t) = \cos(\omega t + \theta)$. More importantly, although the temporal covariance $\hat{R}_z(\tau) - (\hat{M}_z)^2$ of $z(t)$ may approach zero as $\tau \to \infty$, the temporal covariance of $\phi(t)$ can approach a positive constant, which can attenuate the spectral correlation in the model of the FM signal $x(t)$ at $\alpha = \pm 2f_0$. For example, $\hat{R}_\phi(\tau)$ in (64), which must be replaced by the covariance $\hat{R}_\phi(\tau) - [\hat{M}_\phi]^2$ when $\hat{M}_\phi \neq 0$, can be expressed as the sum of its constant asymptote

$$c \triangleq \hat{R}_\phi(\infty) - [\hat{M}_\phi]^2 \tag{80}$$

and its residual,

$$R(\tau) \triangleq \hat{R}_\phi(\tau) - \hat{R}_\phi(\infty). \tag{81}$$

The exponential in (64) can then be factored to obtain

$$\Psi_\tau(1, 1) = \exp\{-c\} \exp\{-[R(0) + R(\tau)]\} \tag{82}$$

for a purely stationary Gaussian waveform $\phi(t)$. Thus, the asymptote c attenuates the cyclic autocorrelation $\hat{R}_x^\alpha(\tau)$ at $\alpha = \pm 2f_0$ (77a). The fact that $c \neq 0$ represents anomalous behavior can be seen from the fact that the strength of the spectral line at $f = 0$ for such a time series is not $[\hat{M}_\phi]^2$, but rather it is $c + [\hat{M}_\phi]^2$. This nonphysical behavior can be avoided by realizing that the integration operation in any physical frequency modulator must be lossy and therefore should be modeled as

$$\phi(t) = \int_0^t e^{-\gamma(t-u)} z(u)\, du \tag{83}$$

for some appropriately small positive value of γ. Unlike (79a), (83) is indeed a stable transformation, and therefore $\hat{R}_\phi(\infty) = [\hat{M}_\phi]^2$ if $\hat{R}_z(\infty) = [\hat{M}_z]^2$, and $\hat{M}_\phi = 0$ if $\hat{M}_z = 0$.

If (83) is used as the model for FM, then the preceding statements about the distinction between PM and FM need to be modified. Specifically, FM modeled with (83) will, in general, be purely cyclostationary regardless of the low-pass spectral content of $z(t)$. However, the strength of the spectral correlation will be weak when the spectrum of $z(t)$ does not approach zero faster than linearly in f, and the magnitude of the spectral correlation will, in fact, approach zero as $\gamma \to 0$ in (83). This can be seen by using (64) and showing that $\hat{R}_\phi(0) \to \infty$ as $\gamma \to 0$.

Before concluding this discussion, let us consider the relationship between the spectral density and the spectral correlation for PM and FM. From (77a), we have

$$\hat{R}_x(\tau) = \frac{1}{2} \operatorname{Re}\{\psi_\tau(1, -1) e^{i2\pi f_0 \tau}\} \tag{84}$$

where Ψ_τ is given by (77b). For a zero-mean Gaussian waveform, Ψ_τ is given explicitly by [19]

$$\Psi_\tau(1, -1) = \exp\{-[\hat{R}_\phi(0) - \hat{R}_\phi(\tau)]\}. \tag{85}$$

Thus, it follows from (77a), (64), (84), and (85) that $\hat{R}_x^{2f_0}(\tau)$ and $\hat{R}_x(\tau)$ differ by only a cosine factor, a factor of 1/2, and a sign. However, this sign difference is very significant.[5] For example, it follows that although $\hat{R}_x(\tau)$ reaches its maximum value of 1/2 at $\tau = 0$, $\hat{R}_x^{2f_0}(\tau)$ reaches its maximum value of

$$\max_\tau \hat{R}_x^{2f_0}(\tau) = \frac{1}{4} \exp\{-\hat{R}_\phi(0)\} \tag{86a}$$

at $\tau \to \infty$ for $\hat{R}_\phi(\tau) \geqslant 0$ and $\hat{R}_\phi(\infty) = 0$. Furthermore, if $\hat{R}_\phi(\tau)$ can be negative, then

$$\max_\tau \hat{R}_x^{2f_0}(\tau) = \frac{1}{4} \exp\{-[\hat{R}_\phi(0) + \min_\tau \hat{R}_\phi(\tau)]\}. \tag{86b}$$

Thus, for a nonoscillatory phase time series ($\hat{R}_\phi(\tau) \geqslant 0$), we have

$$\max_\tau \hat{R}_x^{2f_0}(\tau) << \max_\tau \hat{R}_x(\tau) \tag{87}$$

for $\hat{R}_\phi(0) >> 1$, but for an oscillatory or narrow-band time series for which

$$\hat{R}_\phi(0) + \min_\tau \hat{R}_\phi(\tau) << \hat{R}_\phi(0)$$

(87) need not hold, even if $\hat{R}_\phi(0) >> 1$. In fact, $\hat{R}_x^{2f_0}(\tau)$ will itself be highly oscillatory in this case, and this can result in a highly oscillatory transform $\hat{S}_x^{2f_0}(f)$. Consequently, even though the area of $\hat{S}_x^{2f_0}(f)$, which is given by

$$\hat{R}_x^{2f_0}(0) = \frac{1}{4} \exp\{-2\hat{R}_\phi(0)\}$$

will be very small for $\hat{R}_\phi(0) >> 1$ compared to the area of $\hat{S}_x(f)$, its peaks can be comparable to the peaks of $\hat{S}_x(f)$.

As an example to illustrate the difference between PM and

[4] For the one-sided ($t \geqslant 0$) model, all limit averages over $|t| < \infty$ must be multiplied by 2 to produce the appropriate result.

[5] Because of this sign difference, the technique used to develop Woodward's theorem (cf. [35]) for approximating the spectrum of FM for the case of high modulation index cannot be used to approximate the spectral correlation function.

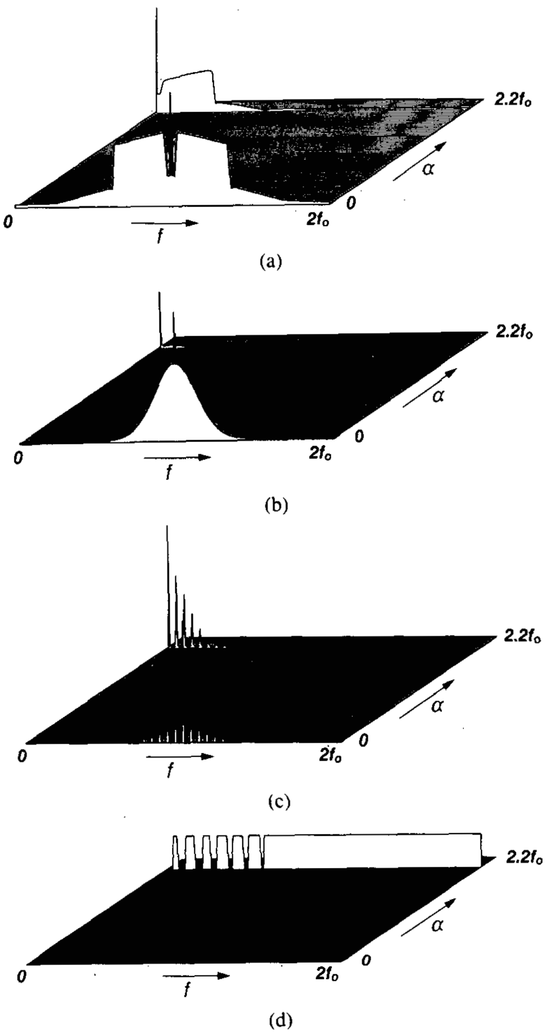

Fig. 3. Spectral correlation surfaces for a unity-power phase-modulated sine wave. (a) Magnitude surface for $f_0 = 9000$, modulating-phase passband = [300, 3300], rms phase = 1. (The spectral-line-pair power = 0.37.) (b) Magnitude surface for $f_0 = 36\,000$, modulating-phase passband = [300, 3300], rms phase = 2.5. (The spectral-line-pair power = 0.082.) (c) Magnitude surface for $f_0 = 636\,000$, modulating-phase passband = [30 300, 33 300], rms phase = 2.5. (The spectral-line-pair power = 0.082.) (d) Phase surface for $f_0 = 636\,000$, modulating-phase passband = [30 300, 33 300], rms phase = 2.5 (range: $0-\pi$).

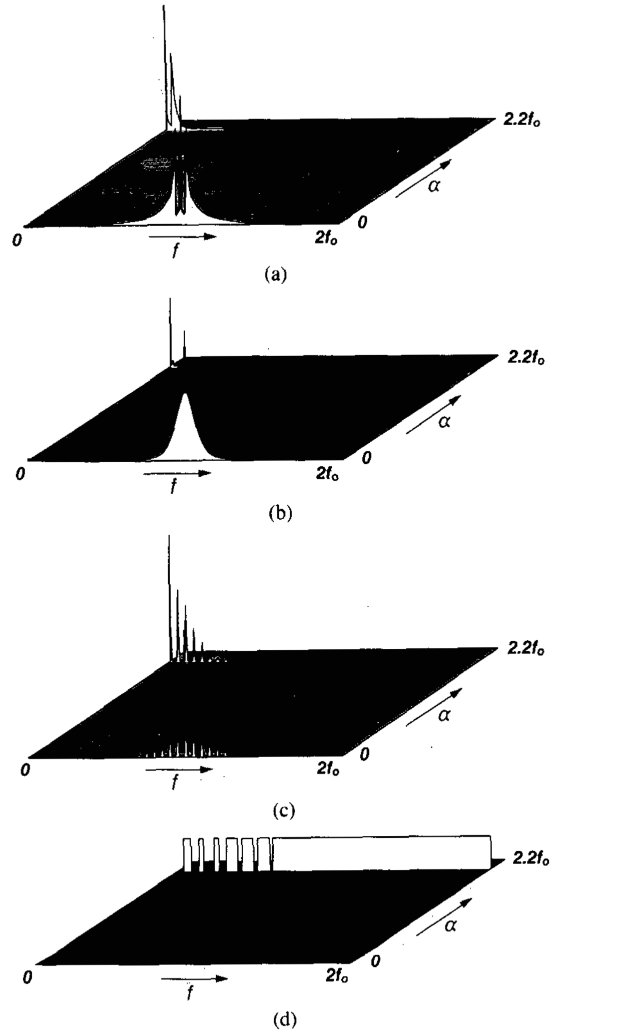

Fig. 4. Spectral correlation surfaces for a unity-power frequency-modulated sine wave. (a) Magnitude surface for $f_0 = 9000$, modulating-frequency passband = [300, 3300], rms phase = 1. (The spectral-line-pair power = 0.37.) (b) Magnitude surface for $f_0 = 36\,000$, modulating-frequency passband = [300, 3300], rms phase = 2.5. (The spectral-line-pair power = 0.082.) (c) Magnitude surface for $f_0 = 636\,000$, modulating-frequency passband = [30 300, 33 300], rms phase = 2.5. (The spectral-line-pair power = 0.082.) (d) Phase surface for $f_0 = 636\,000$, modulating-frequency passband = [30 300, 33 300], rms phase = 2.5 (range: $0-\pi$).

FM and the dependence on mean-square phase deviation and phase bandwidth, we begin by considering a model for the modulating signal $y(t)$ that is typical of speech, namely, $y(t)$ is a zero-mean purely stationary Gaussian waveform with spectral density

$$\hat{S}_y(f) = \begin{cases} S_0, & 300 \leqslant |f| \leqslant 3300 \\ 0, & \text{otherwise} \end{cases}$$

and we consider a typical value of mean-square phase deviation $\hat{R}_\phi(0) = (2.5)^2$. We then consider a smaller value $\hat{R}_\phi(0) = 1$ and a smaller relative bandwidth $30\,300 \leqslant |f| \leqslant 33\,300$. For PM, $\phi(t) = y(t)$, and for FM, $\phi(t)$ is given by (83) with $z(t) = y(t)$ and $\gamma << 300$. The spectral correlation magnitude surfaces for these various cases are shown in Figs. 3 and 4.

VIII. Conclusions

A new characteristic of modulated signals, the spectral correlation function, is calculated for a variety of analog modulation types. The results clarify the ways in which cyclostationarity is exhibited by different types of modulated signals. These differing spectral correlation characteristics can be used to empirically classify modulated signals, even when the signals are buried in noise, and they also can be exploited for signal detection, synchronization, and extraction (cf. [10], [19], [25], [35]). A signal can be synchronized to using a quadratic synchronizer if and only if it exhibits spectral correlation. Similarly, the existence of spectral correlation in a signal of interest makes it possible to reject noise and interference for purposes of signal detection and extraction in ways that would be impossible for signals without spectral correlation. The results in this paper are essential for the design and analysis of signal processing systems, such as detectors, classifiers, synchronizers, and extractors, that exploit spectral correlation since one of the first steps in such design or analysis is to determine the spectral correlation characteristics of the signals of interest.

Since the spectral correlation function reduces to the conventional power spectral density function for a cycle frequency of zero, $\alpha = 0$, the spectral correlation formulas derived here yield the well-known formulas for power spectral density as a special case. In Part II [43], the spectral correlation function is calculated for a variety of digital modulation types.

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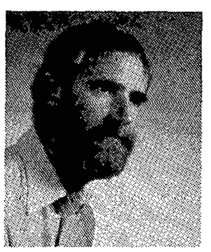

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