Radiometric Detection of Spread-Spectrum Signals in Noise of Uncertain Power

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The standard analysis of the radiometric detectability of a spread-spectrum signal assumes a background of stationary, white Gaussian noise whose power spectral density can be measured very accurately. This assumption yields a fairly high probability of interception, even for signals of short duration. By explicitly considering the effect of uncertain knowledge of the noise power density, it is demonstrated that detection of these signals by a wideband radiometer can be considerably more difficult in practice than is indicated by the standard result. Worst-case performance bounds are provided as a function of input signal-to-noise ratio (SNR), time-bandwidth (TW) product and peak-to-peak noise uncertainty. The results are illustrated graphically for a number of situations of interest. It is also shown that asymptotically, as the TW product becomes large, the SNR required for detection becomes a function of noise uncertainty only and is independent of the detection parameters and the observation interval.

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I. INTRODUCTION

The theory of signal detection in noise provides a number of methods for ascertaining the presence of a signal in noise. The applicability of a given method principally depends upon the extent to which the signal structure is known. Generally, the type of signal to be detected may range from the one extreme of a signal with features that are precisely known to the other extreme of a signal which resides in a certain, possibly very wide frequency band, with features that are totally unknown. Unifying treatments of a range of possible detector structures are provided in [1 and 2].

When the signal is known, the optimal detector in stationary Gaussian noise consists of a matched filter (or the equivalent correlator) followed by a threshold comparator. When little or no knowledge of the signal structure is available to the interceptor, one is driven to using a general-purpose detector, the radiometer. Indeed, even when the signal structure is partially known, a radiometer may be chosen for the simplicity of its hardware and the robustness of its performance in the face of changing signal characteristics.

This work focuses on radiometric detection of spread-spectrum signals in additive white Gaussian noise for the realistic situation of uncertain knowledge of the noise power. The performance prediction of the standard (i.e., noise power known exactly) theory of radiometry [3, 4, 8] is shown to grossly overestimate the actual detectability of such signals due to the limited accuracy with which the detection threshold can be set. Sensitivity to the threshold setting is, of course, a characteristic that the radiometer shares with many detectors. However, in the case of the wideband radiometer, this sensitivity is so extreme that the standard performance equations must be modified considerably.

To properly set the stage for the problem, we begin in Section II with a brief review of the standard formulation of radiometer performance for the situation of perfect knowledge of noise power. These results are extended to the case of uncertain knowledge of noise power in Section III.

II. RADIOMETRIC DETECTION WITH PERFECT KNOWLEDGE OF NOISE POWER

Consider a transmitted signal having a duration T and a spectrum spread over a bandwidth W. The signal power during the time interval T is P_s and the noise is assumed to be stationary, white, and Gaussian, with a one-sided power-spectral-density, N_0. The value of N_0 is assumed known. In the following section, the consequences of relaxing this assumption are considered.

As shown in Fig. 1, a wideband radiometer consists of a filter of bandwidth W, followed by a square-law device and an integrator. The output X

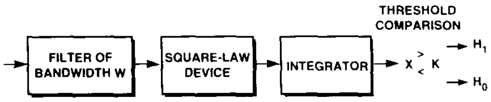

Fig. 1. Wideband radiometer.

of the integrator is compared with a threshold K and a decision is made: hypothesis H_1 (signal present) is chosen if $X > K$, hypothesis H_0 (signal absent) is chosen if $X < K$. For a spread-spectrum signal, the time-bandwidth product (TW) is typically large enough (even for relatively small values of T) to allow the use of a Gaussian approximation for the distribution of X. Thus, under hypothesis H_i $(i = 0, 1)$, X has a normal distribution with mean μ_i and variance σ_i^2. As shown in [3], the means and variances of the test statistic are

$$\mu_0 = N_0 TW, \qquad \sigma_0^2 = N_0^2 TW \tag{1}$$

$$\mu_1 = N_0 TW(\mathrm{SNR}+1), \qquad \sigma_1^2 = N_0^2 TW(2\mathrm{SNR}+1) \tag{2}$$

where we have defined the input signal-to-noise-ratio (SNR) as[1]

$$\mathrm{SNR} \triangleq \frac{P_s}{N_0 W}.$$

The probability of false alarm, P_{FA}, and the probability of detection P_D, are then given by

$$P_{\mathrm{FA}} = Q\left(\frac{K-\mu_0}{\sigma_0}\right) \tag{3}$$

and

$$P_{\mathrm{D}} = Q\left(\frac{K-\mu_1}{\sigma_1}\right) \tag{4}$$

where

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy.$$

Setting the threshold K_0 for a desired probability of false alarm, $P_{\mathrm{FA,des}}$, we obtain

$$K_0 = \mu_0 + \sigma_0 Q^{-1}(P_{\mathrm{FA,des}}) = GN_0 \tag{5}$$

where

$$G = [TW + \sqrt{TW} Q^{-1}(P_{\mathrm{FA,des}})].$$

Using (2), (4) and (5) yields

$$P_{\mathrm{D}} = Q\left[\frac{Q^{-1}(P_{\mathrm{FA,des}}) - \mathrm{SNR}\sqrt{TW}}{\sqrt{1+2\mathrm{SNR}}}\right]. \tag{6}$$

Solving for the SNR gives

$$\mathrm{SNR} = \frac{B}{\sqrt{TW}} + \frac{A}{TW}[A - \sqrt{A^2 + TW + 2\sqrt{TW}B}] \tag{7}$$

[1]The symbol $\triangleq$ denotes mathematical definition.

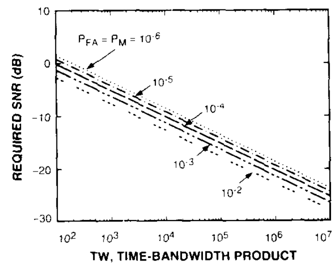

Fig. 2. Required SNR: known noise level.

where $A \triangleq Q^{-1}(P_{\mathrm{D,des}})$ and $B \triangleq Q^{-1}(P_{\mathrm{FA,des}})$. Graphs of this equation for several values of false alarm probability (P_{FA}) and missed-detection probability

$$P_{\mathrm{M}} \triangleq 1 - P_{\mathrm{D}}$$

are plotted in Fig. 2. Fig. 2 and subsequent figures should be interpreted cautiously since the normal approximation assumed in (3) and (4) becomes inaccurate in situations of small TW products (e.g., < 100) and small P_{FA} and/or P_{D} values.

Since $|A|$ and $|B|$ are both fairly small (< 10) for reasonable detection probabilities, the spread-spectrum condition $TW \gg 1$ allows (7) to be approximated as

$$\mathrm{SNR} \approx \frac{B-A}{\sqrt{TW}} = \frac{Q^{-1}(P_{\mathrm{FA,des}}) - Q^{-1}(P_{\mathrm{D,des}})}{\sqrt{TW}}. \tag{8}$$

For a fixed bandwidth, the SNR required[2] to achieve the desired detection probabilities is proportional to $T^{-1/2}$. Note that signals can be detected at as low an SNR as desired, provided the detection interval is long enough.

Implicit in this, the standard formulation for the detection of large time-bandwidth signals, is the assumption that the detector has perfect knowledge of the noise level. However, as shown below, realistic limitations on the detector's knowledge of the noise level produce serious degradation in the detector's performance.

III. RADIOMETRIC DETECTION WITH UNCERTAIN KNOWLEDGE OF NOISE POWER

Consider (5) which shows the threshold K_0 is proportional to N_0. In almost all practical situations, N_0 (and, consequently, K_0) would need to be estimated by the interceptor. Denote these estimates by $\hat{N}_0$ and $\hat{K}_0$, and assume that the error in estimating N_0 is

[2]Our definition of SNR as $P_S/(N_0 W)$, chosen to simplify some of the formulations in this paper, should be kept in mind in interpreting (8). When considering the more typical parameter P_s/N_0, the equation indicates a proportionality to $\sqrt{W}/\sqrt{T}$, not to $1/\sqrt{TW}$.

bounded by

$$(1-\epsilon_1)N_0 \le \hat{N}_0 \le (1+\epsilon_2)N_0 \tag{9}$$

with $0 \le \epsilon_1 < 1$ and $\epsilon_2 \ge 0$.

From (3) and (5) it is clear that an *underestimate* of the true noise level causes $P_{FA} > P_{FA,des}$. Therefore, to guarantee $P_{FA} \le P_{FA,des}$ over the entire range of estimator values given by (9), the purposely biased threshold estimate

$$\hat{K}_0 = \frac{\hat{N}_0}{(1-\epsilon_1)} \cdot G \tag{10}$$

should be used in (3).

We now consider the effect of using the estimate (10) on the probability of signal detection. Substituting $\hat{K}_0$ into (4) and using (2) gives

$$P_D = Q\left[\frac{\hat{K}_0 - N_0 TW(\mathrm{SNR}+1)}{N_0\sqrt{TW(2\mathrm{SNR}+1)}}\right]. \tag{11}$$

Since $Q(\cdot)$ is a monotonically decreasing function of its argument, P_D is minimized (i.e., worst case P_D) when $\hat{K}_0$ takes its *maximum* value. This occurs when

$$\hat{N}_0 = (1+\epsilon_2)N_0$$

in which case,

$$\hat{K}_0 = G\left(\frac{\hat{N}_0}{1-\epsilon_1}\right) = G\left(\frac{1+\epsilon_2}{1-\epsilon_1}\right)N_0 = UK_0 \tag{12}$$

where we have defined the peak-to-peak uncertainty U as

$$U \triangleq \frac{1+\epsilon_2}{1-\epsilon_1} \ge 1. \tag{13}$$

Thus,

$$P_{D,\text{worst case}} = Q\left[\frac{UK_0 - N_0 TW(\mathrm{SNR}+1)}{N_0\sqrt{TW(2\mathrm{SNR}+1)}}\right].$$

We can guarantee that $P_D \ge P_{D,des}$ over the entire range of noise uncertainty by requiring that $P_{D,\text{worst case}} = P_{D,des}$. Making this substitution and using (1) and (5) in the last expression gives

$$P_{D,des} = Q\left[\frac{(U-1)\sqrt{TW} + UQ^{-1}(P_{FA,des}) - \mathrm{SNR}\sqrt{TW}}{\sqrt{1+2\mathrm{SNR}}}\right].$$

The SNR needed to achieve $P_D \ge P_{D,des}$ and $P_{FA} \le P_{FA,des}$ over the entire range of noise uncertainty (9) is obtained by solving the last equation for SNR:

$$\mathrm{SNR} = (U-1) + U\left(\frac{B}{\sqrt{TW}}\right) + \frac{A}{TW}\left[A - \sqrt{A^2 + (2U-1)TW + 2\sqrt{TW}B}\right]. \tag{14}$$

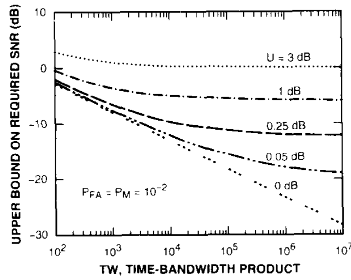

Fig. 3. Upper bound on required SNR: 0 dB $\le U \le$ 3.0 dB; $P_{FA} = P_M = 10^{-2}$.

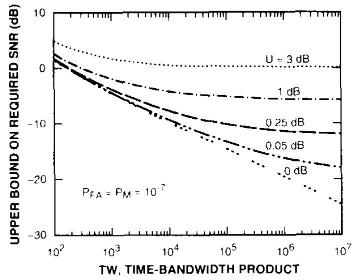

Fig. 4. Upper bound on required SNR: 0 dB $\le U \le$ 3.0 dB; $P_{FA} = P_M = 10^{-7}$.

In the case of absolutely certain noise, $U = 1$, and (14) degenerates to the previous result, (7). For $TW \gg 1$, (14) becomes

$$\mathrm{SNR} \approx (U-1) + \frac{UB - A\sqrt{2U-1}}{\sqrt{TW}} = (U-1) + O\left(\frac{1}{\sqrt{TW}}\right) \ge (\mathrm{SNR})_{\min} \tag{15}$$

where the term $(\mathrm{SNR})_{\min}$, defined as

$$(\mathrm{SNR})_{\min} \triangleq (U-1) \tag{16}$$

is the minimum SNR needed to overcome the noise-level uncertainty, *regardless of the detector parameters $P_{D,des}$ and $P_{FA,des}$ and the length of the detection interval.* This situation contrasts with the case of certainly known noise level, as described by (8), where there is an unlimited tradeoff of T versus $(\mathrm{SNR})_{\text{required}}$.

The broad validity of (15) is obvious from the comparison of Figs. 3 and 4, which graph (14) for several values of U and for two vastly different sets of detection probabilities: $P_{FA} = P_M = 10^{-2}$ and $P_{FA} = P_M = 10^{-7}$.

To facilitate the use of these performance bounds, Figs. 5–8 present parametric graphs of the required SNR. These figures show that radiometric interception of spread-spectrum signals is significantly affected by noise-level uncertainties, even those as small as a fraction of a dB.

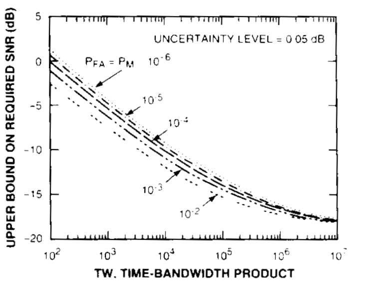

Fig. 5. Upper bound on required SNR: uncertain noise level; $U = 0.05$ dB.

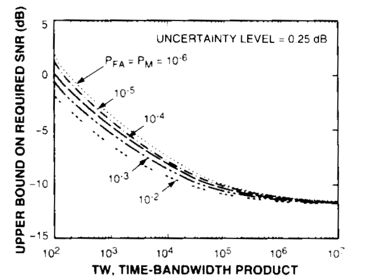

Fig. 6. Upper bound on required SNR: uncertain noise level; $U = 0.25$ dB.

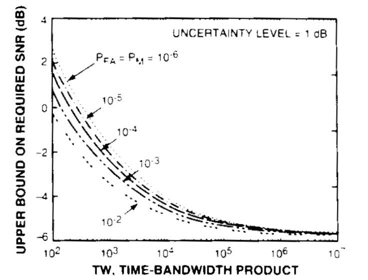

Fig. 7. Upper bound on required SNR: uncertain noise level; $U = 1.0$ dB.

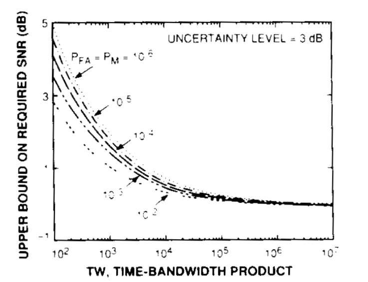

Fig. 8. Upper bound on required SNR: uncertain noise level; $U = 3.0$ dB.

A useful asymptotic expression for the relative increase in SNR engendered by noise-level uncertainty can be obtained by using the spread-spectrum condition ($TW \gg 1$) on the ratio of (14) and (7):

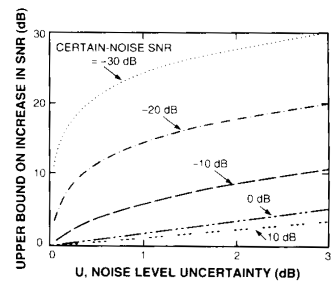

Fig. 9. Upper bound on SNR increase required: $0 \text{ dB} \leq U \leq 0.3$ dB.

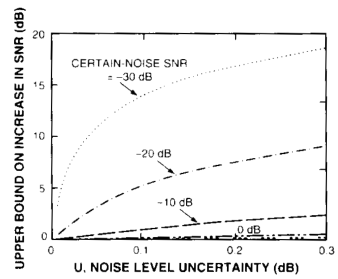

Fig. 10. Upper bound on SNR increase required: $0 \text{ dB} \leq U \leq 3.0$ dB.

$$\begin{aligned}\Delta\,\mathrm{SNR} &\triangleq \frac{(\mathrm{SNR})_{U=U}}{(\mathrm{SNR})_{U=1}} \\ &\approx \frac{UB - A\sqrt{2U-1}}{B-A} + \frac{U-1}{(\mathrm{SNR})_{U=1}} \\ &\approx U + \frac{U-1}{(\mathrm{SNR})_{U=1}}.\end{aligned} \tag{17}$$

This equation is plotted in Figs. 9 and 10. As an example of the use of these figures, if the certain noise $(\mathrm{SNR})_{\mathrm{required}}$ is -20 dB, a peak-to-peak uncertainty of 0.1 dB causes $(\mathrm{SNR})_{\mathrm{required}}$ to be increased by 5.3 dB.

Note that (15) indicates that to detect a spread-spectrum signal ($TW \gg 1$) when there is a substantial uncertainty in noise level, the SNR required is approximately $(\mathrm{SNR})_{\mathrm{min}}$.[3]

A. Bounding of Noise Uncertainty Parameter U

It is important to note that because of the worst case assumptions used to obtain (14), if the radiometer has an input SNR as given by (14) (and by the curves in Figs. 4–8), then the actual P_{FA} and P_{M} will be *no greater* than the values indicated. That is, the

[3]This basic phenomenon has been investigated previously in [5], but only to the extent of numerically calculating the performance of a few particular cases, without obtaining a general analytic result. In passing, it should be noted that the heuristic formula $(\mathrm{SNR})_{\mathrm{min}} = (U-1)/U$, given in [5] in terms of system noise temperatures, is incorrect, as examination of (15) and (16) above shows.

radiometer performance will be *no worse* than is indicated by the curves. In practice, the usefulness of these worst case bounds is dependent on the judicious determination of the noise uncertainty parameter U. For instance, if $\hat{N}_0/N_0$ is log-normally distributed, then, strictly speaking, $\epsilon_1 = 1$, $\epsilon_2 = \infty$, and $U \doteq \infty$ and the resulting bound (14) is useless in that it requires an infinite SNR. In the following, we show that even if the range of the random variable $\hat{N}_0$ is unbounded, it is possible to determine effective values of ϵ_1, ϵ_2, and U in terms of the underlying distribution of $\hat{N}_0$.

Heuristically, from the derivation of (14), it is clear that if the distribution of $\hat{N}_0$ is known, the quantities ϵ_1 and ϵ_2 in the definition of U (9) should be chosen to be no larger than are required to satisfy

$$P[\hat{N}_0 \leq (1-\epsilon_1)N_0] \approx P_{\mathrm{FA,des}}$$

$$P[\hat{N}_0 \geq (1+\epsilon_2)N_0] \approx P_{\mathrm{M,des}}$$

i.e., we need not be concerned with extreme underestimates of N_0 that occur so rarely that their probabilities are less than $P_{\mathrm{FA,des}}$, nor with extreme overestimates that occur with probabilities less than $P_{M,des}$. This heuristic argument is quantified below.

Determination of ϵ_1: Let the distribution of $\hat{N}_0/N_0$ be known and let ϵ_1^* be defined by

$$P[\hat{N}_0/N_0 \leq 1-\epsilon_1^*] = \alpha_1 \cdot P_{\mathrm{FA,des}} \tag{18}$$

where α_1 is a constant to be chosen (in practice, $\alpha_1 = 1/2$ will suffice).

Let F denote the event of false alarm. Then, if ϵ_1 is set to ϵ_1^*, we have

$$\begin{aligned} P_{\mathrm{FA}} &= P(F \mid \hat{K}_0 < K_0)P(\hat{K}_0 < K_0) \\ &\quad + P(F \mid \hat{K}_0 \geq K_0)P(\hat{K}_0 \geq K_0) \\ &\leq P(\hat{K}_0 < K_0) + P(F \mid \hat{K}_0 \geq K_0) \\ &\leq P\left(\frac{\hat{N}_0}{1-\epsilon_1^*} < N_0\right) + P(F \mid \hat{K}_0 = K_0) \\ &= \alpha_1 P_{\mathrm{FA,des}} + P_{\mathrm{FA,des}} = (1+\alpha_1)P_{\mathrm{FA,des}}. \end{aligned}$$

From the last result it is seen that the upper-bound on P_{FA} can be made arbitrarily close to $P_{\mathrm{FA,des}}$ provided the α_1 defining ϵ_1^* in (18) is chosen small enough. In typical situations a false alarm probability of 1.5 $P_{\mathrm{FA,des}}$ is not practically distinguishable from $P_{\mathrm{FA,des}}$ and thus a choice of $\alpha_1 = 1/2$ is reasonable.

Determination of ϵ_2: Let M denote the event of a miss. Let ϵ_2^* be defined by

$$P[\hat{N}_0/N_0 \geq 1+\epsilon_2^*] = \alpha_2 P_{\mathrm{M,des}} \tag{19}$$

where α_2 is a constant to be chosen.

Define $U^* = (1+\epsilon_2^*)/(1-\epsilon_1^*)$ and SNR* by (14) with $U = U^*$. Then, if ϵ_2 is set to ϵ_2^*, we have

$$\begin{aligned} &P(M \mid \mathrm{SNR} = \mathrm{SNR}^*) \\ &\quad = P(M \mid \hat{K}_0 < U^* K_0; \mathrm{SNR} = \mathrm{SNR}^*) \\ &\qquad \cdot P(\hat{K}_0 < U^* K_0) + P(M \mid \hat{K}_0 \geq U^* K_0; \mathrm{SNR} = \mathrm{SNR}^*) \\ &\qquad \cdot P(\hat{K}_0 \geq U^* K_0) \\ &\quad \leq P(M \mid \hat{K}_0 < U^* K_0; \mathrm{SNR} = \mathrm{SNR}^*) + P(\hat{K}_0 \geq U^* K_0) \\ &\quad \leq P_{\mathrm{M,des}} + P\left(\frac{\hat{N}_0}{1-\epsilon_1^*} \geq \frac{1+\epsilon_2^*}{1-\epsilon_1^*} N_0\right) \\ &\quad = P_{\mathrm{M,des}} + P(\hat{N}_0 \geq (1+\epsilon_2^*)N_0) \\ &\quad = P_{\mathrm{M,des}} + \alpha_2 \cdot P_{\mathrm{M,des}} = (1+\alpha_2)P_{\mathrm{M,des}}. \end{aligned}$$

It is seen that the upper bound on P_{M} can be made arbitrarily close to $P_{\mathrm{M,des}}$ by judicious choice of α_2 in (19). Again, $\alpha_2 = 1/2$ is often a reasonable choice.

Illustrative Example: We illustrate the above method for determining U for the case where the fractional error $\hat{N}_0/N_0$ is log-normally distributed. That is, on a dB scale, the difference between $\hat{N}_0$ and N_0 is normally distributed with a mean of 0 dB and a standard deviation of σ dB. For simplicity, we choose $\alpha_1 = \alpha_2 = 1$ in (18) and (19).

From (18) ϵ_1^* is determined by

$$P[\hat{N}_0/N_0 \leq 1-\epsilon_1^*] = P_{\mathrm{FA,des}}$$

or on a dB scale:

$$P[\hat{N}_0(\mathrm{dB}) - N_0(\mathrm{dB}) \leq 10\log_{10}(1-\epsilon_1^*)] = P_{\mathrm{FA,des}}.$$

Since $\hat{N}_0(\mathrm{dB}) - N_0(\mathrm{dB})$ is normally distributed as $N(0,\sigma)$, we may solve for ϵ_1^* as

$$\epsilon_1^* = 1 - 10^{[\sigma \cdot Q^{-1}(1-P_{\mathrm{FA,des}})]/10}. \tag{20}$$

Similarly, ϵ_2^* may be obtained from (19) as

$$\epsilon_2^* = 10^{\sigma Q^{-1}(P_{\mathrm{M,des}})/10} - 1. \tag{21}$$

Consider, for example, the case where $\sigma = 1$ dB and $P_{\mathrm{FA,des}} = P_{\mathrm{M,des}} = 10^{-3}$. Then (20) and (21) give

$$\epsilon_1^* = 0.5; \qquad \epsilon_2^* = 1$$

and

$$U^* = \frac{1+1}{1-0.5} = 4 = 6 \text{ dB}$$

i.e., we allow for a peak-to-peak uncertainty of ± 3 dB which corresponds to the upper and lower 10^{-3} tails of the noise uncertainty distribution.

IV. DISCUSSION

The extreme sensitivity of the wideband radiometer to uncertainty in N_0 is due to the fact that the radiometer simply relies on detection of changes in total received energy. The large dimensionality (equal to $2TW$) of the spread-spectrum signals it is attempting to detect implies that the detection interval contains much more noise energy than it does signal energy for signals with typical SNRs. Since the signal causes a very small fractional increase in the total energy in the interval, uncertainty in the measurement of the noise energy seriously degrades the detection of such an increase by the radiometer. More precisely, as is evident from (1) and (2), the critical deficiency of the radiometer is that the noise contributes to the *mean* of the test statistic under both hypotheses and, for the low SNRs of interest, dominates the contribution of the signal. Moreover, as is evident from (15) and the related text, averaging (i.e., increasing T) does not appreciably help performance since it only reduces the variance. This deficiency of the radiometer has been frequently noted in a heuristic manner (see for instance [1, 6, and 8]). The results presented here are believed to be the first analytical treatment of this effect.

Troublesome levels of noise-level uncertainty can arise from a number of sources. First, antenna noise temperature is known to vary as a function of weather, pointing angle, and frequency [5]. Second, the accuracy of the noise-level estimator is inherently limited by finite averaging and by implementation errors. Finally, the channel may contain nonstationary noise and extraneous signals.

Whatever the sources of the noise-level uncertainty, their conglomerate effect on the performance of the radiometer can be approximated using the bounds presented here. If that performance is deemed unacceptable and if the characteristics of the target signal are at least partially known, more complex detectors may be considered. For instance, feature detectors such as chip-rate or carrier frequency detectors may be implemented [1, 7]. These detectors have the advantage that, in contrast to the radiometer, the noise does not contribute to the mean of the test statistic under either hypothesis. As a consequence, it can be shown that these detectors are relatively robust in the presence of noise uncertainty. However, they may be defeated by suitable filtering of the transmitted wideband signal [7]. Another detector which is robust in the presence of noise uncertainty is the cross-correlation radiometer involving the correlation of inputs from two spatially separated antennas. This detector, which under ideal conditions has performance nearly identical to the radiometer [8], is, however, considerably more complicated to implement and requires that the noises at the two antenna outputs be uncorrelated [10].

Another way in which knowledge of some of the signal characteristics can be utilized is in the improvement of the noise estimate. In particular, if a measurement orthogonal to the signal can be used to estimate the actual noise present during the detection interval, the noise can be tracked through all its fluctuations. The performance of detectors, whether or not they are of the radiometric type, would be improved by this scheme since it would effectively remove the noise-level uncertainty.

V. PERFORMANCE OF OPTIMAL DETECTORS OF DIRECT-SEQUENCE PSEUDONOISE WAVEFORMS

Polydoros and Weber [9] have considered optimal detectors of a direct-sequence pseudonoise (DSPN) signal with a structure that is completely known except for the random pseudonoise (PN) pattern. In this situation the maximum likelihood detector consists of an expression involving the matched filter outputs for all the possible PN sequences over the detection interval. For the case of large time-bandwidth products, the detection procedure consists of testing a Gaussian statistic against a threshold, K. As before, the detection probabilities are given by (3) and (4). The hypothesis-dependent means and variances of the test statistic are [9][4]

$$\mu_0 = N_0 N \xi; \qquad \sigma_0^2 = N_0^2 N \xi \tag{22}$$

$$\mu_1 = (\mathrm{SNR} + \xi) N_0 N; \qquad \sigma_1^2 = (2\,\mathrm{SNR} + \xi) N_0^2 N \tag{23}$$

where N is the number of chips in the observed PN signal, and the constant ξ equals 1/2 for the coherent case and 1 for the noncoherent case.

Upon equating N with the time-bandwidth product TW, (1) and (2) are seen to be identical to (22) and (23) for the noncoherent detector, and nearly identical for the coherent detector. Therefore, the equations of Section III, or simple variants of them, can be used to approximate the effectiveness of these optimal synchronous detectors in the presence of noise-level uncertainty.

VI. CONCLUSION

It has been demonstrated that presence of even small levels of noise power uncertainty make the standard radiometer calculations far too optimistic in their indication of the detectability of signals spread over a wide bandwidth. The deterioration in the performance of the detector was analytically upper-bounded in terms of an expression involving the peak-to-peak range of noise uncertainty, U. An illustrative example was provided to show how the parameter U should be calculated in practice.

The method was also used to consider the effectiveness of optimal detection of low SNR DSPN waveforms with known structure. Lastly, we also note that the results can also be used as a rough

[4]For convenience we have normalized to 1 the chip time T_c appearing in the formulas of [9].

approximation of the performance in nonstationary noise of a wideband radiometer.

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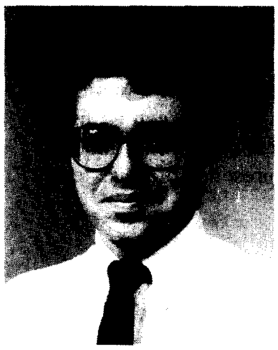

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