12 Confidence Interval Versus Point Estimation

CONFIDENCE INTERVAL VERSUS POINT ESTIMATION

We have discussed inferential statistics as procedures used to estimate the parameters of a population. *Estimate* is a definitive word here. Sample statistics are not expected to predict with absolute certainty the parameters of a population because of sampling error. Statistics vary from one sample to another. If we used a single sample mean to estimate the mean of the population, we would be using what is called a point estimate.

A ***point estimate*** is a single value, based on sample data, that is used to estimate the parameter of a population.

However, the accuracy of our point estimate would remain unknown. Our estimate should be somewhere *close* to the population value, but it would probably not be exact. If we drew another random sample, we would likely get some other (but close) value due to random sampling error. We could be more certain of our estimation by using confidence interval estimation, which takes sampling error into consideration.

A ***confidence interval*** is a range of values that, with a specified degree of confidence, is expected to include a population parameter.

Let us examine these ideas more closely.

POINT ESTIMATES

For point estimates of population parameters, the obtained sample values – with no adjustments for standard error – are reported (i.e., the numerator in our working formula for the t-statistic).

- For one-sample t tests, the point estimate for μ is M.
- For two-sample t tests, independent measures design, the point estimate for $\mu_1 - \mu_2$ is $M_1 - M_2$.
- For two-sample t tests, repeated measures design, the point estimate for μ_D is M_D.

While point estimates are straightforward and readily understood, their values can only be expected to be somewhere in the neighborhood of the population values. Confidence intervals specify the boundaries of that neighborhood.

CONFIDENCE INTERVALS

Because confidence intervals include a range of values, they are not as specific as point estimates. However, they allow us a greater degree of confidence in our estimation. Degree of confidence is expressed as a percentage. The higher the percentage, the greater confidence we can have that the true value of the population parameter will be included in the established interval. Researchers usually want a high degree of assurance, so 95% or 99% confidence levels are frequently used, but other levels can be applied as well.

The specific values to use in the formulas for confidence intervals will be a little different depending on the statistic for which we want to establish confidence, but the general elements are the same. In essence, we will be calculating lower and upper limits for our obtained statistic, using the following basic formulas:

LL = obtained sample statistic − (t)(estimated standard error)
UL = obtained sample statistic + (t)(estimated standard error)

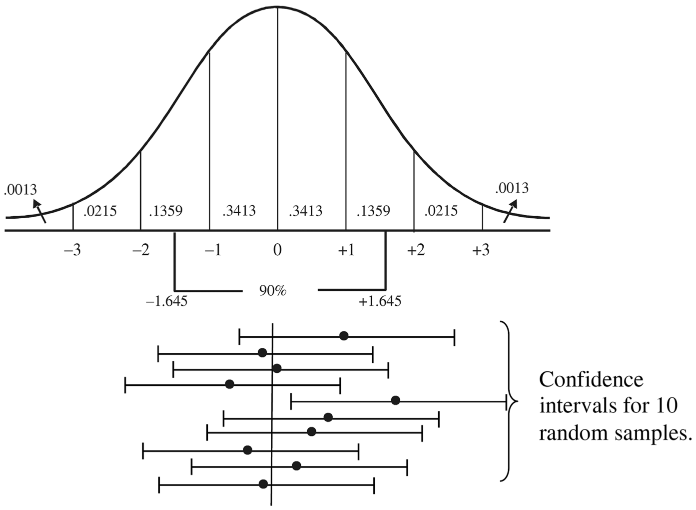
We know that samples do not predict population parameters with perfect accuracy. Sampling error is to be expected. The last two components of the formula provide a measure of how much sampling error to expect for specified degrees of confidence.

Tip!

> The lower and upper limits discussed here are different from the lower and upper real limits of continuous variables that were discussed earlier, which were used in the calculation of the range and the median. Here, we are establishing lower and upper limits as boundaries within which population values are expected to lie.

To get a conceptual understanding of confidence intervals, suppose that μ is the population parameter of interest. Suppose further that we want to include the true value of μ in our interval about 90% of the time. In a sampling distribution with $df = \infty$, about 90% of the scores lie within plus or minus 1.645 standard errors, as shown in the graph below. The horizontal lines below the graph illustrate 90% confidence intervals for 10 random samples. The means are shown as dots located in the center of the intervals. The boundaries of the intervals extend 1.645 standard errors in either direction of the mean.

Sampling distribution of means with $df = \infty$



Notice that only one out of the ten confidence intervals does not include μ, which is what would be expected. In other words, given a 90% confidence level, the true value of the population mean (μ) would be captured in our interval 90% of the time, and 10% of the time it would fail to be captured.

The specific confidence interval formulas for the t tests we have covered are as follows:

- For one-sample t tests:

  $\text{LL} = M - t(s_M)$ and $\text{UL} = M + t(s_M)$

- For two-sample t tests, independent samples design:

  $\text{LL} = (M_1 - M_2) - t(s_{M_1-M_2})$ and $\text{UL} = (M_1 - M_2) + t(s_{M_1-M_2})$

- For two-sample t tests, repeated measures design:

  $\text{LL} = M_D - t(s_{M_D})$ and $\text{UL} = M_D + t(s_{M_D})$

With these formulas, we will be adding to and subtracting from our obtained sample statistic however many standard errors it would take to encompass a given percentage of the distribution. If we were using a 95% confidence level, then we would use the t values that separate the middle 95% of the distribution from the extreme 5% (i.e., $\alpha = .05$, two-tailed). If we were using a 99% confidence level, we would use the t values that separate the middle 99% of the distribution from the extreme 1% (i.e., $\alpha = .01$, two-tailed). The same degrees of freedom should be used that are appropriate for that particular test.

ONE-SAMPLE *t* TEST

Sample Research Question

The average typing speed for the secretaries of a large company is 52 words per minute. A long-time secretary has developed finger dexterity exercises that she reports have improved her speed dramatically. The owner of the company thus hires a researcher to test the effectiveness of the exercise program. After four weeks of training in the finger exercises, the typing speed of 17 secretaries is measured. The mean number of words per minute was $M = 57$ with $SS = 3600$.

A. What would the point estimate of μ be after using the finger exercise program?
B. Establish a 90% confidence interval for the mean and write a sentence of interpretation.

A. The point estimate of μ is $M = 57$.
B. $\text{LL} = M - t(s_M)$ and $\text{UL} = M + t(s_M)$

To use the formulas for establishing confidence intervals, we need values for M, t, and s_M. The value for $M = 57$. For a 90% confidence interval with $df = 16$, the value for $t = \pm 1.746$. We now have to calculate the estimated standard error (s_M).

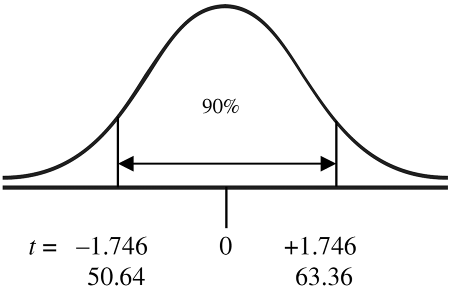
$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{3600}{17-1}} = 15 \qquad s_M = \frac{s}{\sqrt{n}} = \frac{15}{\sqrt{17}} = 3.64$$

$$\begin{aligned}\text{LL} &= 57 - 1.746(3.64)\\ &= 57 - 6.36\\ &= 50.64\end{aligned} \qquad \text{and} \qquad \begin{aligned}\text{UL} &= 57 + 1.746(3.64)\\ &= 57 + 6.36\\ &= 63.36\end{aligned}$$

Interpretation

We can be 90% confident that the population mean (after the finger dexterity program) would be between 50.64 and 63.36.

This is what the t-distribution would look like for this interval:



The t-values of ±1.746 form the boundaries of the middle 90% of the distribution when $df = 16$.

Your Turn!

I. Confidence Interval for a One-Sample *t* Test

Research Question. A history professor was curious about the knowledge of history of graduating seniors in her state. She thus gave a 100-item history test to a random sample of 75 graduating seniors with a resulting $M = 72$ and $SS = 7400$.

A. Make a point estimate of the population mean for that state.

B. Establish a 99% confidence interval for the mean.

C. Write a sentence of interpretation.

TWO-SAMPLE *t* TEST: INDEPENDENT SAMPLES DESIGN

Sample Research Question

A high school counselor has developed a pamphlet of memory-improving techniques designed to help students in preparing for exams. One group of students is given the pamphlets and instructed to practice the techniques for three weeks, after which their memories are tested. The memories of another group of students, who did not receive the pamphlets, are also tested at this time.

Memory-pamphlet group	*No-pamphlet group*
$n_1 = 36$	$n_2 = 40$
$M_1 = 79$	$M_2 = 68$
$SS_1 = 478$	$SS_2 = 346$

A. Make a point estimate of how much improvement in memory results from following the techniques.
B. Establish a 90% confidence interval around the value for the difference between means and write a sentence of interpretation.

A. The point estimate for the population difference between means $(\mu_1 - \mu_2)$ is: $M_1 - M_2 = 79 - 68 = 11$

B. $\text{LL} = (M_1 - M_2) - t(s_{M_1-M_2})$ and $\text{UL} = (M_1 - M_2) + t(s_{M_1-M_2})$

$$
\begin{aligned}
s_{M_1-M_2} &= \sqrt{\left(\frac{SS_1 + SS_2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} \\
&= \sqrt{\left(\frac{478 + 346}{36 + 40 - 2}\right)\left(\frac{1}{36} + \frac{1}{40}\right)} \\
&= \sqrt{(11.14)(.05)} \\
&= .75
\end{aligned}
$$

$$
\begin{aligned}
\text{LL} &= 11 - 1.671(.75) \\
&= 11 - 1.25 \\
&= 9.75
\end{aligned}
$$

and

$$
\begin{aligned}
\text{UL} &= 11 + 1.671(.75) \\
&= 11 + 1.25 \\
&= 12.25
\end{aligned}
$$

Interpretation

We can be 90% confident that the difference between population means would be between 9.75 and 12.25.

Your Turn!

II. Two-Sample *t* Test: Independent Samples Design

Research Question. A psychologist is investigating the effects of suggestion on conceptual problem solving. He administers the Concepts Application Test (CAT) to two groups of subjects. Prior to administering the test to one group, the researcher comments that subjects usually report that they enjoy taking the test and that it gives them

Your Turn!
(continued)

a sense of accomplishment. Another group is simply administered the test with no such remarks. The CAT scores for the two groups are presented below:

CAT with remarks	*CAT alone*
49	26
38	35
36	40
42	32
44	34
37	30
47	

a. Conduct a two-tailed t test using $\alpha = .05$ and determine the size of the effect.
b. Make a point estimate of the difference between means.
c. If the results are significant, establish a 95% confidence around the difference value and write a sentence of interpretation.

A. Step 1: Formulate Hypotheses

Step 2: Indicate the Alpha Level and Determine Critical Values

Step 3: Calculate Relevant Statistics

Step 4: Make a Decision and Report the Results

Effect size

(*Continued*)

Your Turn!
(continued)

B. Point Estimate

C. Confidence Interval

Interpretation

TWO-SAMPLE *t* TEST: REPEATED MEASURES DESIGN

Sample Research Question

A promising math tutoring program was developed by a senior math major who also tutored students. His math professors were impressed and decided to test its effectiveness. A comprehensive math test was given at the end of the semester to $n = 45$ students who were struggling in math in the spring semester. Over the summer, they were tutored in the new program and then retested on an alternate form of the test. The mean for the sample of difference scores was $M_D = 12$ with $SS_D = 1648$.

A. Make a point estimate of μ_D.
B. Establish a 90% confidence interval around the difference value and write a sentence of interpretation.

A. The point estimate for μ_D is: $M_D = 12$

B. $\text{LL} = M_D - t\left(s_{M_D}\right)$ and $\text{UL} = M_D + t\left(s_{M_D}\right)$

$$s_D = \sqrt{\frac{SS_D}{n-1}} = \sqrt{\frac{1648}{45-1}} = 6.12$$

$$s_{M_D} = \frac{s_D}{\sqrt{n}} = \frac{6.12}{\sqrt{45}} = .91$$

$$\begin{aligned} LL &= 12 - 1.684(.91) \\ &= 12 - 1.53 \\ &= 10.47 \end{aligned} \qquad \text{and} \qquad \begin{aligned} UL &= 12 + 1.687(.91) \\ &= 12 + 1.53 \\ &= 13.53 \end{aligned}$$

Interpretation

We can be 90% confident that the increase in scores will be between 10.47 and 13.53.

Your Turn!

III. Two-Sample *t* Test: Repeated Measures Design

Research Question. A study is being conducted at a sleep disorders clinic to determine if a regimen of swimming exercises affects the sleep patterns of individuals with insomnia. The number of hours of nightly sleep of 26 insomniac patients is recorded for a two-week period. The patients are then exposed to two weeks of daily swimming exercises, and their sleep is monitored for two more weeks. For this sample of patients, the amount of nightly sleep increased by $M_D = 1.2$ hours with $SS_D = 112$.

a. On the basis of this data, did the swimming regimen affect the amount of sleep? Use a two-tailed test with $\alpha = .05$ and determine the effect size.
b. Make a point estimate of how much sleep increases on average as a result of the swimming regimen.
c. Establish a 95% confidence around the mean difference and write a sentence of interpretation.
d. Establish a 99% confidence around the mean difference and write a sentence of interpretation.

A. Step 1: Formulate Hypotheses

Step 2: Indicate the Alpha Level and Determine Critical Values

(*Continued*)

Your Turn!
(continued)

Step 3: Calculate Relevant Statistics

Step 4: Make a Decision and Report the Results

Effect size

B. Point Estimate

C. 95% Confidence Interval

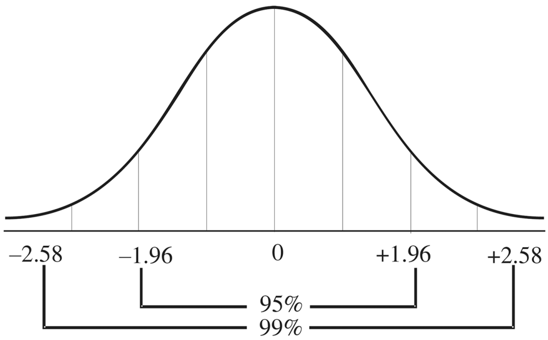
Interpretation

D. 99% Confidence Interval

Interpretation

DEGREE OF CONFIDENCE VERSUS DEGREE OF SPECIFICITY

As was apparent in the last "Your Turn!" practice exercise, when we increase the level of confidence, we lose specificity. The range of values becomes broader and is thus more likely to include the true population parameters. Conversely, when we reduce the level of confidence, we gain specificity but lose confidence in our estimation. This is illustrated in the normal distribution below.



The 95% confidence level establishes a narrower (and more specific) interval of values, while the 99% level provides a wider (and less specific) interval of values.

Answers to "Your Turn!" Problems

I. Confidence Interval for a One-Sample *t* Test

A. $M = 72$
B. LL = 68.94 and UL = 75.06

We can be 99% confident that the population mean would be between 68.94 and 75.06.

II. Two-Sample *t* Test: Independent Samples Design

A. Step 1:	Step 2:	Step 3:
$H_0: \mu_1 = \mu_2$	$\alpha = .05$	$SS_1 = 154.86$
$H_1: \mu_1 \neq \mu_2$	$df = 11$	$SS_2 = 112.83$
	$t_{crit} = \pm 2.201$	$s_{M_1-M_2} = 2.75$
		$t_{obt} = +3.28$

(*Continued*)

Answers to "Your Turn!" Problems

(continued)

Step 4:

Subjects who heard positive remarks about taking the CAT scored significantly higher than the group who heard no such remarks. Reject H_0, $t(11) = +3.28$, $p < .05$.

Effect size: $d = 1.83$, large effect

B. $M_1 - M_2 = 9.03$

C. LL = 2.98 and UL = 15.08

We can be 95% confident that the difference between population means would be between 2.98 and 15.08.

III. Two-Sample *t* Test: Repeated Measures Design

A. Step 1:

$H_0: \mu_D = 0$

$H_1: \mu_D \neq 0$

Step 2:

$\alpha = .05$

$df = 25$

$t_{crit} = \pm 2.060$

Step 3:

$s_D = 2.12$

$S_{M_D} = .42$

$t_{obt} = +2.86$

Step 4:

The swimming regimen had a significant effect on sleep. Reject H_0, $t(25) = +2.86$, $p < .05$.

Effect size: $d = .57$, moderate effect

B. $M_D = 1.2$

C. $LL = 1.2 - (2.060)(.42) = .33$ and $UL = 1.2 + (2.060)(.42) = 2.07$

We can be 95% confident that the increased amount of sleep will be between .33 and 2.07 hours.

D. $LL = 1.2 - (2.787)(.42) = .03$ and $UL = 1.2 + (2.787)(.42) = 2.37$

We can be 99% confident that the increased amount of sleep will be between .03 and 2.37 hours.

Using Microsoft Excel for Data Analysis

If you are using Excel for the first time for statistical analysis, you may need to load the add-in tool that allows these functions. The information for loading the Data Analysis ToolPak as well as general instructions for using Excel for data analysis are at the beginning of the Excel section in Chapter 4.

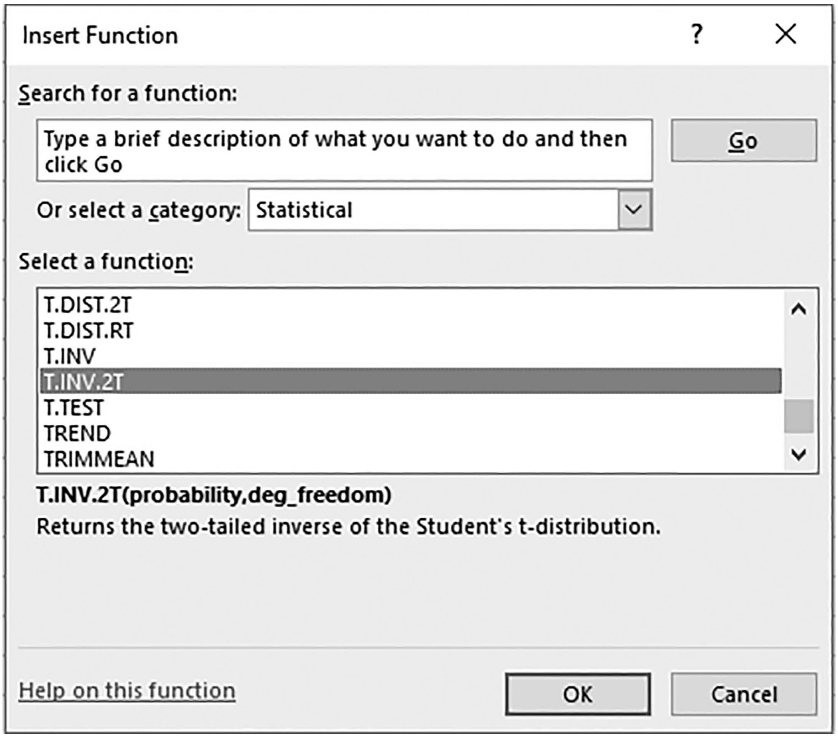
Confidence Interval for a One-Sample *t* Test

This chapter looked at confidence intervals for three different kinds of *t* tests. To calculate these confidence intervals, we will be providing the formulas and instructing Excel to grab the values needed in the formulas from the cells where they are located the spreadsheet. We'll start by using Excel to calculate a confidence interval for the one-sample *t* test using the example from your text about finger dexterity exercises for typing speed. The formula for the lower limit of the confidence interval for a one-sample *t* test is: $LL = M - t(S_M)$. The formula for the upper limit is: $UL = M + t(S_M)$.

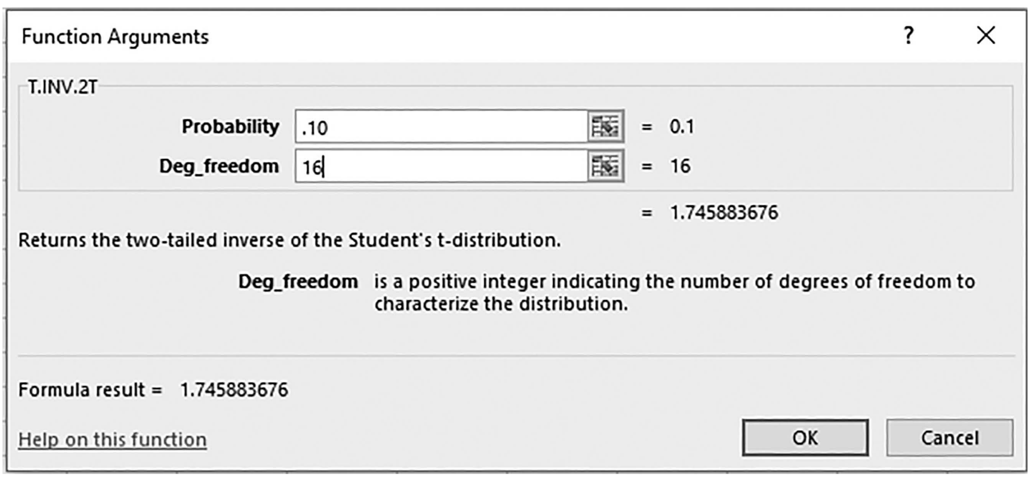
First, type in the information shown below in column A of the spreadsheet that will be needed for the calculations. Also, type in the summary information in column C that was provided for this problem. We will now instruct Excel to perform the remaining operations:

	A	B	C
1	Mean		57
2	SS		3600
3	n		17
4	t crit		
5	Sample SD		
6	Standard error		
7	Lower Limit		
8	Upper Limit		
9			

1. **Critical *t*-values**. Rather than using the table at the back of your text, Excel will determine your t_{crit} values. With your cursor in cell C4, go to the **Formulas** tab and click on the **Insert Function (*fx*)** command from the ribbon. From the **Insert Function** dialog box, scroll down and highlight **T.INV.2T**. This is the function that will provide the t_{crit} values for a two-tailed test. Click **OK**.



Since we are establishing a 90% confidence interval, we will enter .10 in the probability window of the **Function Arguments** box as well as the appropriate *df* (16 in this case). After clicking **OK**, the t_{crit} values needed for the confidence interval formulas (1.745884) will appear in the cell.



2. **Sample Standard Deviation.** To determine the standard error of the mean needed for the confidence interval formulas, you will first have to obtain the sample standard deviation. With your cursor in cell C5, type the following exactly as written: = SQRT(C2/(C3–1)) and hit the enter key. The sample standard deviation (15) will appear in the cell.

	A	B	C	D
1	Mean		57	
2	SS		3600	
3	n		17	
4	t crit		1.745884	
5	Sample SD		=SQRT(C2/(C3-1))	
6	Standard error			
7	Lower Limit			
8	Upper Limit			

3. **Standard Error**. With your cursor in cell C6, type the following exactly as written: = C5/SQRT(C3) and hit the enter key. The standard error (3.638034) will appear in the cell.
4. **Confidence Intervals**. Finally, we can calculate the confidence intervals by instructing Excel to grab the values from the cells needed for the formulas. For the lower limit, place your cursor in cell C7. Type the following exactly as written: = C1-(C4*C6) and hit the enter key. For the upper limit, place your cursor in cell C8. Type the following exactly as written: = C1+(C4*C6) and hit the enter key. Both limits will now appear in the designated cells.

	A	B	C
1	Mean		57
2	SS		3600
3	n		17
4	t crit		1.745884
5	Sample SD		15
6	Standard error		3.638034
7	Lower Limit		50.64842
8	Upper Limit		63.35158

Confidence Interval for a Two-Sample *t* Test: Independent Samples Design

Here again, we will provide Excel with the information it needs to calculate the confidence intervals. The formula for the lower limit of the confidence interval for an independent samples *t* test is: $LL = (M_1 - M_2) - t\left(s_{M_1 - M_2}\right)$. The formula for the upper limit is: $LL = (M_1 - M_2) + t\left(s_{M_1 - M_2}\right)$.

Here, we will use the sample research question pertaining to the pamphlet for memory-improving techniques. Type in the information shown below in column A of the spreadsheet that will be needed for the calculations. Also, type in the summary information in column C that was provided for this problem. Notice that a cell for "df used" is included. Remember that the table for the *t*-distribution at the back of your book is abbreviated and not all *df* values are included. Be sure to enter the *df* that was used from the table rather than the actual *df* so that your comparison with Excel's output will match (aside from rounding differences).

	A	B	C
1	M_1		79
2	M_2		68
3	SS_1		478
4	SS_2		346
5	n_1		36
6	n_2		40
7	df used		60
8	t crit		
9	Standard error		
10	LL		
11	UL		
12			

1. **Critical *t*-values**. Follow the instructions in step one above for the one-sample *t* test except place your cursor in the cell where you want the *t* values to appear (e.g., C8). We are again establishing a 90% confidence interval and so the probability in the **Function Arguments** dialog box will be .10 and *df* will be 60.
2. **Standard Error**. With your cursor in cell C9, type the following exactly as written: =SQRT((C3+C4)/(C5+C6–2)*(1/C5+1/C6)) and hit the enter key. The standard error (.766608) will appear in the designated cell.
3. **Confidence Intervals**. For the lower limit, place your cursor in cell C10. Type the following exactly as written: =(C1-C2)-C8*C9 and hit the enter key. For the upper limit, place your cursor in cell C11. Type the following exactly as written: =(C1-C2)+C8*C9 and hit the enter key. Both limits will appear in the designated cells. There will be some rounding differences but the values are close to what you calculated by hand.

	A	B	C
1	M_1		79
2	M_2		68
3	SS_1		478
4	SS_2		346
5	n_1		36
6	n_2		40
7	df used		60
8	t crit		1.670649
9	Standard error		0.766608
10	LL		9.719267
11	UL		12.28073
12			

Confidence Interval for a Two-Sample *t* Test: Repeated Measures Design

The formula for the lower limit of the confidence interval for a repeated measures *t* test is: $LL = M_D - t\left(s_{M_D}\right)$. The formula for the upper limit is: $LL = M_D + t\left(s_{M_D}\right)$. We will use the sample research question pertaining to the math tutoring program. Type in the information shown below in column A of the spreadsheet that will be needed for the calculations. Also, type in the summary information in column C that was provided for this problem as well as the *df* that was used from the table in the back of your book (rather than the actual *df*).

	A	B	C
1	M_D		12
2	SS_D		1648
3	n		45
4	df used		40
5	t crit		
6	Standard deviation (s_D)		
7	Standard error (s_{M_D})		
8	LL		
9	UL		
10			

1. **Critical *t*-values**. Follow the instructions in step one for the one-sample *t* test except place your cursor in the cell where you want the *t* values to appear (e.g., C5). For a 90% confidence interval, the probability in the **Function Arguments** dialog box will be .10 and *df* will be 40.
2. **Standard Deviation**. Here, you will provide Excel with the formula for the standard deviation as well as the locations where the values needed for the calculation are located. With your cursor in cell C6, type the following exactly as written: =SQRT(C2/(C3–1)) and hit the enter key.
3. **Standard Error**. With your cursor in cell C7, type the following exactly as written: =C6/SQRT(C3) and hit the enter key.
4. **Confidence Intervals**. For the lower limit, place your cursor in cell C8. Type the following exactly as written: =C1-C5*C7 and hit the enter key. For the upper limit, place your cursor in cell C9. Type the following exactly as written: =C1+C5*C7 and hit the enter key. Both limits will appear in the designated cells. There will be some rounding differences but the values are close to what you calculated by hand.

	A	B	C
1	M_D		12
2	SS_D		1648
3	n		45
4	df used		40
5	t crit		1.683851
6	Standard deviation (s_D)		6.120012
7	Standard error (s_{M_D})		0.912318
8	LL		10.46379
9	UL		13.53621
10			

Additional Practice Problems

Answers to odd numbered problems are in Appendix C at the end of the book.

1. Define what is meant by a point estimate and a confidence interval and explain the difference between them.
2. What statistics would be used to estimate point estimates for
   a. a one-sample t test?
   b. an independent measures t test?
   c. a repeated measures t test?
3. What statistics would be used to estimate point estimates for
   a. μ?
   b. $\mu_1 - \mu_2$?
   c. μ_D?
4. Given $M = 28$, $n = 10$, and $s_M = 2.2$, construct a
   a. 99% confidence interval for μ.
   b. 95% confidence interval for μ.
   c. 90% confidence interval for μ.
   d. What can be said about the relationship between the width of the confidence interval and level of confidence?
5. A sample was measured with the following scores on the dependent variable: 6, 8,10, 7, 12, and 8. What is the point estimate for μ?
6. One type of treatment resulted in the following scores: 48, 52, 49, 50, 53, 47, and 45. The scores for a second type of treatment were: 34, 40, 38, 45, 41, 37, and 42. What is the point estimate for $\mu_1 - \mu_2$?
7. Pre-test scores were 68, 75, 79, 69, 80, and 70. Post-test scores were 96, 90, 89, 85, 76, and 87. What is the point estimate for μ_D?
8. Based on the summary data below, establish a 90% confidence interval for μ.
   $M = 30$
   $n = 17$
   $SS = 101$
9. Based on the summary data below, establish a 95% confidence for $\mu_1 - \mu_2$
   $M_1 = 72$ $M_2 = 64$
   $n_1 = 9$ $n_2 = 11$
   $SS_1 = 424$ $SS_2 = 347$
10. Based on the summary data below, establish a 99% confidence interval for μ_D.
   $M_D = 27$
   $n = 15$
   $SS_D = 682$
11. A professor in a substance abuse counselor training program surveyed a sample of $n = 25$ students about the age at which they consumed their first alcoholic beverage. The mean age was 13.8 years with a $s_M = 1.11$. Establish a 95% confidence interval for μ and write a sentence of interpretation.
12. The head of a nursing program is considering the addition of mindfulness training in the curriculum as a way of cultivating cognitive empathy in nursing students. To test the effectiveness of the program, one group of students completes the training program and subsequently takes an empathy assessment. A second group of nursing students takes the assessment without completing the mindfulness training. Summary data for the study are listed below. Higher scores reflect greater cognitive empathy.

Training	*No Training*
$M_1 = 55$	$M_2 = 47$
$n_1 = 9$	$n_2 = 7$
$s_{M_1-M_2} = 3.28$	

a. What is the point estimate for $\mu_1 - \mu_2$?
b. Conduct a two-tailed t test using $\alpha = .05$.
c. Establish a 95% confidence interval for $\mu_1 - \mu_2$ and write a sentence of interpretation.

13. A study was conducted to determine the effectiveness of a new type of exercise designed to help people lose belly fat. The researcher took the waist measurements of a sample of $n = 27$ subjects both before and after the exercise program was implemented. Waist measurement decreased by an average of $M_D = -2.3$ inches with a $s_{M_D} = .81$. Establish a 99% confidence for μ_D and write a sentence of interpretation.