Intro to Modern Algebra
Recitation 7

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Step 3:

Let $f(x) = 3x^3 + x^2 + 2x$ and $g(x) = 2x^2 + x + 4$

b) If f and g are elements of $\mathbb{Z}_5[x]$, use long division to find $q(x)$ and $r(x)$ so that $f(x) = g(x)q(x) + r(x)$.

Long division:

$$\begin{array}{r|llll}
 & & & 4x & +1 \\
\cline{2-5}
2x^2 + x + 4 & 3x^3 & +x^2 & +2x & +2 \\
 & -3x^3 & -4x^2 & -x & \\
\cline{2-5}
 & 0 & +2x^2 & +x & +2 \\
 & & -2x^2 & -x & -4 \\
\cline{3-5}
 & & 0 & +0 & +3
\end{array}$$

Therefore, $q(x) = 4x + 1$ and $r(x) = 3$ in the form $f(x) = g(x)q(x) + r(x)$.

c) What happens if you try to do the same thing viewing f and f as elements of $\mathbb{Z}_6$?

When trying to do the long division, one immediately runs into a problem when trying to find a solution to $2x = 3$. This occurs in the first step trying to cancel out the dominant term in the polynomial $f(x) = 3x^3 + x^2 + 2x$. This is (probably) because $\mathbb{Z}_6$ is a ring, while $\mathbb{Z}_5$ is a field.