Intro to Modern Algebra: Homework #2

Due on November 2 at 9:30am

Professor Lorenz 9:30-11:00

Sam Cook

Problem 6

Which of the following subsets of $\mathbb{R}[x]$ are subrings of $\mathbb{R}[x]$.

a All polynomials with constant term of 0_R

b All polynomials of degree 2.

Solution

a) This is a subring. We will show that it is a nonempty subset closed under subtraction and multiplication. First, zero is a polynomial with the constant term 0_R, so it is a nonempty subset. Next, consider polynomials $a, b \epsilon \mathbb{R}[x]$. Then if we subtract them and 0_R, we get $(a + 0_R) - (b + 0_R) = (a - b) + (0_R - 0_R) = a - b$, and since $a, b \epsilon R$, it is closed under subtraction. Similarly for multiplication, we get $(a + 0_R) * (b + 0_R) = (ab) + (0_R * b) + (0_R * a) + 0_R = ab$, so it is similarly closed under multiplication. Therefore, it is a subring of $\mathbb{R}[x]$.

b) This is not a subring because it is not closed under multiplication. Consider the polynomials x^2 and $x^2 + 1$.When we multiply them, $x^2 * (x^2 + 1) = x^4 + x^2$, which is not a degree 2 polynomial, and therefore it is not closed under multiplication and cannot be a ring. Furthermore, 0_R is not of degree 2 and therefore is not in the subset.

Problem 20

Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map defined by

$$D(a_0 + a_1x + a_2x^2 + ... + a_nx^n) = a_1 + 2a_x + 3a_3x^2 + ... + na_nx^{n-1}$$

Is D a homormorphism of rings? An isomorphism?

Solution

This is neither a homomorphism of rings nor an isomorphism. We will prove this by showing D does not preserve multiplication. Consider $x^2, x^3 \epsilon \mathbb{R}[x]$. When we multiply them, $x^2 * x^3 = x^5$. Now, we will apply D and multiply them. $D(x^2) * D(x^3) = 2x * 3x^2 = 6x^3 \neq x^5$. Therefore, D does not preserve multiplication and cannot be a homomorphic or isomorphic.