MATLAB HW 6

MATH 3043

a. Write a MATLAB function that solves a linear system $\mathbf{Ax} = \mathbf{b}$ by the gradient method. It should accept as inputs the matrix $\mathbf{A}$, the right handside $\mathbf{b}$, an error tolerance ϵ and an initial guess $\mathbf{x}_0$. It should output an approximation, $\mathbf{x}^*$, and N, the number of iterations it takes to reach the error tolerance ϵ.

b. Use the Poisson matrix code provided to generate a matrix A and a vector b by taking $m = 18$ and a matrix A' and vector b' by taking $m = 98$. Set $\mathbf{A} = -A$, $\mathbf{A}' = -A'$. We do this so that the matrices are positive definite, otherwise they are in fact negative definite.

c. Use your function from (1) to solve the linear system $\mathbf{Ax} = -\mathbf{b}$ and $\mathbf{A}'\mathbf{x} = -\mathbf{b}'$ using an error tolerance of 10^{-8}. This will give two approximations, $\mathbf{x}^*$ and $\mathbf{x}'^*$ and two integers N and N'.

d. Let $M = \max\{N, N'\}$. Modify your function in (1) so that instead of accepting an error tolerance ϵ as input, the function accepts n, the number of iterations to perform.

   Use this modification to plot (using semilogy) the error between approximations generated and $\mathbf{x}^*$ and $\mathbf{x}'^*$ on the grid $(1 : \Delta : M)$, where Δ chosen so that you have around $50 - 100$ grid points.

e. Now repeat $a. - d.$ for the conjugate gradient method. Try plotting everything on a single semilogy plot. Be sure to fully label the plot in detail.

Date: May 2017.