CIS 3223 Miniquiz 7

Dr Anthony Hughes

Name:
Temple ID (last 4 digits:
Section (circle): morning afternoon

Instructions

You have 90 minutes.

You may prepare a single sheet (both sides) of notes.

You may use a simple non-graphing calculator.

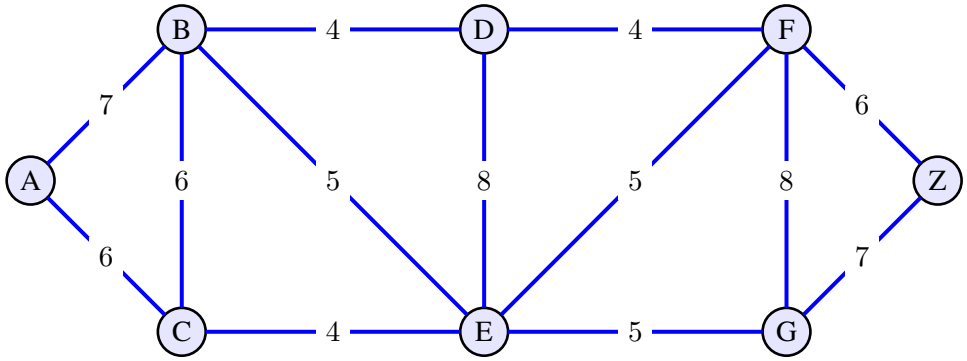
You should take the quiz in a single session.

Do not discuss the quiz.

This quiz cannot be dropped.

I certify that I have complied with the written instructions _____

1 (8 pts) For the following graph use Kruskal's algorithm to find a minimum-cost spanning tree, and then determine the minimum cost. Use alphabetical ordering.



Construct a hash table using the lengths of the edges to store the edges.

Sort the edges in each bucket using alphabetical ordering.

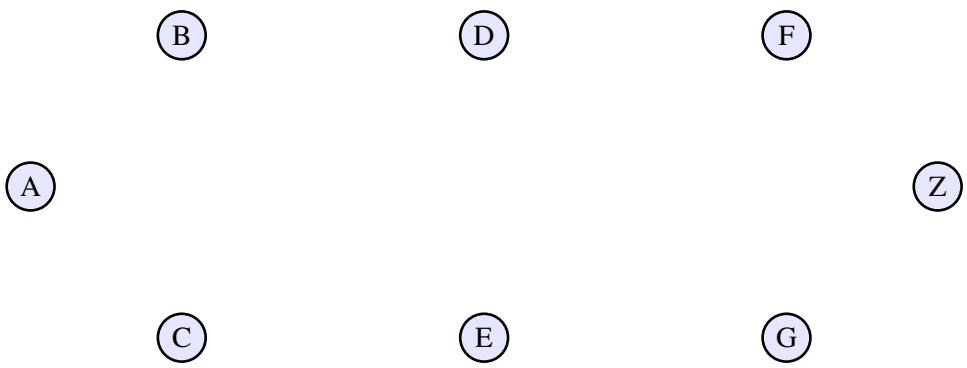
Edge List (Hash Table)

4	CD	BD	DF

Sorted Edge List (Hash Table)

4	BD	CE	DF

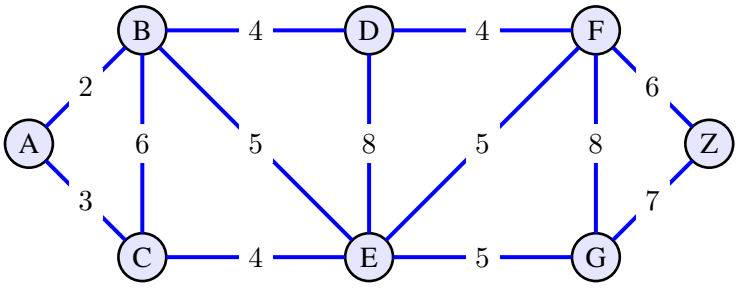
Construct a minimum cost spanning tree by using the edges in each bucket moving from left to right starting with bucket with the lowest value:



Minimum Cost ____

Is the graph bipartite? yes no

2 (4 pts) Consider the following weighted graph:



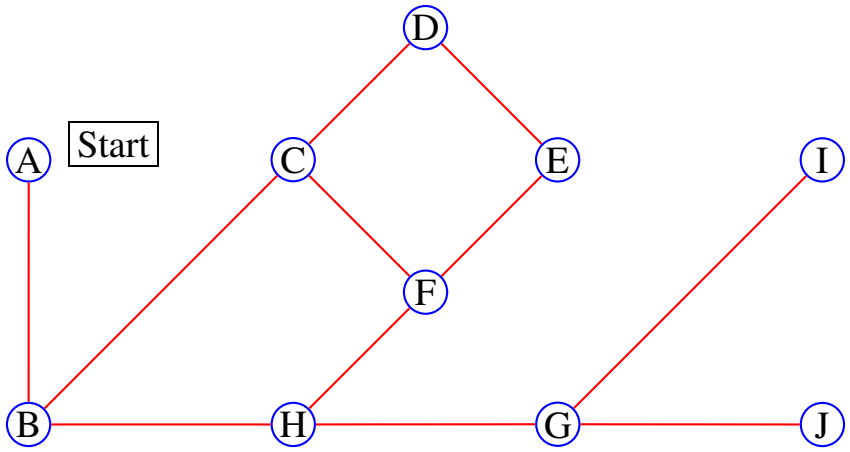
(a) Find the length of the shortest cycle containing the vertex E. (e.g. lengh of E-F-Z-G is 23)

(b) Describe an algorithm for the following task.

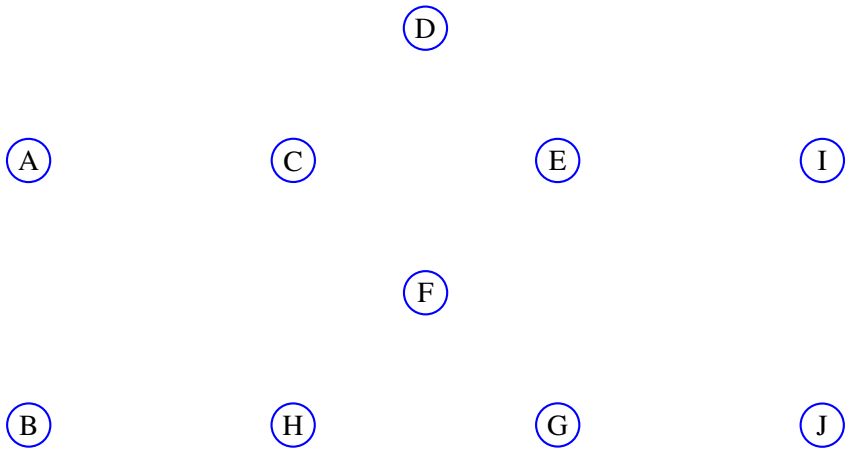
Input: An undirected graph $G = (V, E)$; edge lengths $w_e > 0$, a vertex $u \in V$

Output: The length of the shortest cycle containing the vertex u

3 (4 pts) Perform a **bfs** on the following graph to construct a spanning tree; whenever there is a choice of vertices, pick the one that is alphabetically first (so start at vertex A). Show all work.



VERTEX QUEUE: A _____



Is the graph bipartite? yes no

4 (4 pts) A binary tree T is *full* if each node is either a leaf or possesses exactly two child nodes.

(a) Draw a full binary T tree with 11 nodes.

(b) How many leaves does T have?

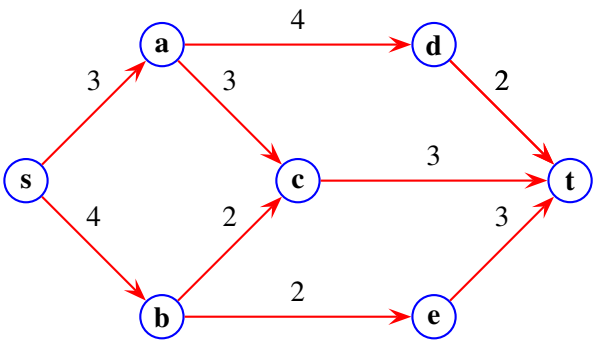
(c) Let G be a full binary tree with k internal nodes. Use strong induction to prove that G has $k + 1$ leaves.

Base case: G is a single node. In this case G has 0 internal nodes and 1 leaf. Result holds.

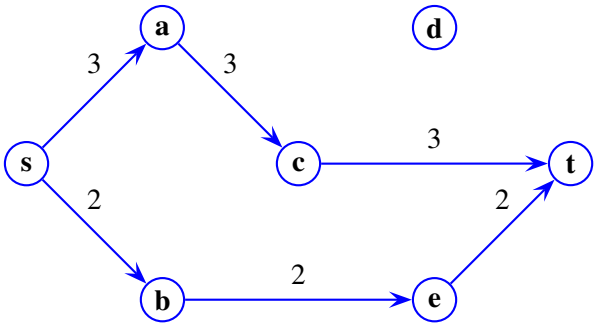
Inductive case: Assume true for full binary trees that have at most k internal nodes.

Let G be a full binary tree with $k + 1$ internal nodes.

5 (8 pts) Given the flow network G and a flow f below

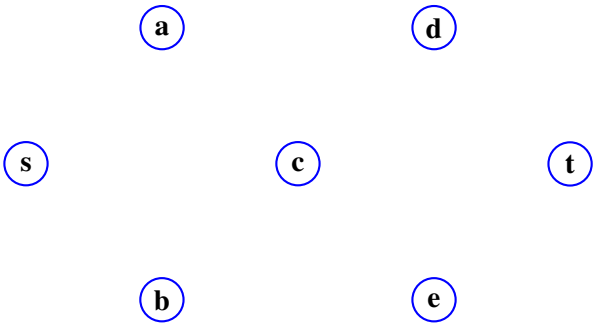


Flow network G

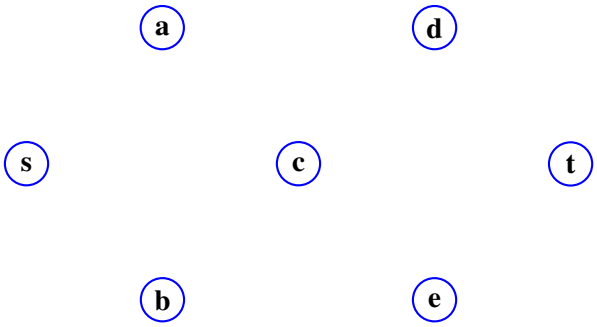


Flow f

complete the next step of the Edmunds-Karp algorithm.

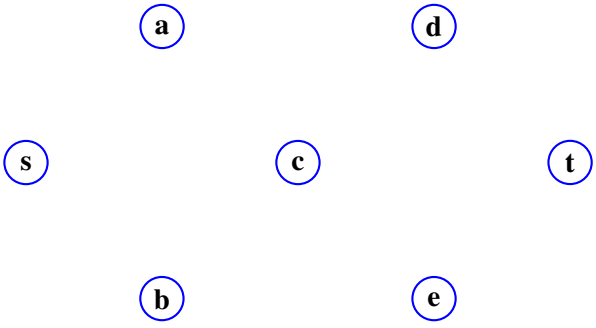


Residual network G^f

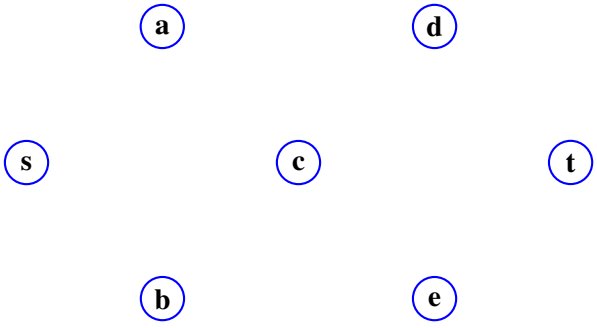


bfs on G starting at s

Determine **bfs** using a queue:



Augmenting flow f



Flow f

Is the resulting flow a max flow? yes no

6 (2 pts) Consider a node at position j, $j \geq 2$, in a complete 5-arry tree. Find the positions of its parent and its five children. Note that the position of the root of the tree is 1.

position of parent: _____

positions of children: _____ _____ _____ _____ _____

7 ([Google] extra credit, 2 pts) Four people need to cross a rickety bridge at night. Unfortunately, they have only one torch and the bridge is too dangerous to cross without one. The bridge is only strong enough to support two people at a time. Not all people take the same time to cross the bridge. Times for each person: 1 min, 3 mins, 7 mins and 10 mins. What is the shortest time needed for all four of them to cross the bridge? Justify your answer.