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Physical Significances Of Fifth-Order Nonlinearity For Pulse Dynamics In Monomode Optical Fibres

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Abstract - We discuss, with illustrations, some physical significances of fifth-order nonlinear susceptibility for pulse dynamics in monomode optical fibres. The amplitude dynamic governing equation is the cubic-quintic nonlinear Schrödinger equation, (CQNLSE), which has soliton properties similar to the cubic nonlinear Schrödinger equation, (CNLSE), based on solutions by a variational method. Some differences, with regards to pulse durations in the range from 10 picoseconds to a few femtoseconds that make CQNLSE experimentally more viable are explained.

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1. Introduction

The technology of optical telecommunications [1] and many other modern optical devices, [1,2] where optical fibres are the media for pulse transmission from one point to the other, is rapidly advancing. For the physics of the propagating pulses in those optical devices, the governing amplitude propagation equation has always been described with the cubic nonlinear Schrödinger equation, (CNLSE), which implies an intensity dependent nonlinear refractive index of the form $n(\omega,|E|^2) \sim n_0 + n_2|E|^2$ where n_0 denotes linear refractive index, E is the electric field and $n_2 \equiv 3\chi^{(3)}_{xxxx}/(8n_0)$, is the nonlinear refractive index corresponding to the third-order nonlinear susceptibility tensor $\chi^{(3)}_{xxxx}$. There are, however, a few main physical reasons justifying the necessity of using the cubic-quintic nonlinear Schrödinger equation, (CQNLSE), of which refractive index takes the form $n(\omega,|E|) \sim n_0 + n_2|E|^2 + n_4|E|^4$ where $n_4 \equiv 5\chi^{(5)}_{xxxxxx}/(32n_0)$, the nonlinear refractive index that corresponds to the leading fifth-order susceptibility tensor. One reason for using the latter expression of the nonlinear refractive index that has been well illustrated [3] is to do with very high input intensity. A far-reaching reason, however, is that at most of the input intensities wherein CNLSE is applied, the CQNLSE gives correct dynamics and does reveal some physical significances that are useful for device modeling applications [1].

In most of the existing discussions [2 – 6], one would find tendencies to prefer either one of the two governing equations of which the trends are in favour of the CNLSE against the CQNLSE, [1, 7]. In a few reported considerations of the effects or consequences of the fifth-order nonlinearity, [8], inherent in the CQNLSE, two significant phenomena that were not realized are the saturation effect and the two-state solution. The present piece of work gives explicit descriptions, with illustrations, of the two phenomena in manners unreported to date. (See section 4).

2. Dynamic Governing Equation

From the first principles, i.e., from the Maxwell's equations, the dynamic governing amplitude propagation equation can be obtained following the method of ref. [4] extended to include n_4 signifying effects of $\chi^{(5)}_{xxxxxx}$. The equation would reveal two perturbation terms: the third-order dispersion and self-steepening terms which have distortion effects on the propagating pulses. If these are neglected, the dimensionless form of the CQNLSE takes the form

$$i\frac{\partial U}{\partial \xi} - \frac{\delta_0}{2}\frac{\partial^2 U}{\partial T^2} - \alpha_0 U + \delta_1 |U|^2 U + \delta_2 \nu |U|^4 U = 0 \tag{1}$$

where $i \equiv \sqrt{-1}$, $U(\xi, T) \equiv A(\xi, T)/A_0$ is the dimensionless complex amplitude with $A(\xi, T)$ and A_0 respectively denoting actual pulse amplitude and initial or input amplitude, $\xi (\equiv z/L_D)$ is the dimensionless propagation distance with z and L_D respectively denoting actual propagation distance and dispersion length; $T \equiv (t - z/v_g)/\tau_0$ denotes the shifted dimensionless time where t is the actual time, v_g is the group velocity and τ_0, the real pulse duration.

Other parameters are defined as following. For normal dispersion $\delta_0 = +1$, $\delta_1 = +1$ and $\delta_2 = \pm 1$; for the anomalous dispersion to be considered in this note, $\delta_0 = -1$, $\delta_1 = +1$ and $\delta_2 = \pm 1$; ν is the most crucial parameter [1] that has an expression of the form

$$\nu = \frac{2}{3}\frac{|n_4|}{n_2^2}\frac{|\beta_2|}{\pi}\frac{\lambda}{\tau_0^2} \tag{2}$$

where λ is the optical wavelength of the propagating pulse in the monomode optical fibre, and $|\beta_2|$ is the magnitude of dispersion parameter in its second-order term such that $\lambda > 1{\cdot}3\mu m$ for β_3 (i.e., the third-order dispersion term) to be insignificant numerically and physically. The parameter, $\alpha_0 \equiv L_D \lambda \varpi^2 / 4\pi$, where ϖ^2 has the meaning of the separation constant; when α_0 is determined from appropriate initial conditions for equation (1), ϖ^2 is simply evaluated.

In addition to previously reported variational model of equation (1), [1], another model is presented in this contribution. Through the soliton theory and saturation effects, in the present model, differences between CQNLSE and CNLSE are illustrated with explanation.

3. Description By The Variational Method

The CQNLSE (1) has a Lagrangian of the form

$$L = \frac{i}{2}\left(U \frac{\partial U^*}{\partial \xi} - U^* \frac{\partial U}{\partial \xi} \right) - \frac{\delta_0}{2}\left| \frac{\partial U}{\partial T} \right|^2 + \alpha_0 |U|^2 - \frac{\delta_1}{2}|U|^4 - \frac{\delta_2}{3}\nu |U|^6 \tag{3}$$

where asterisks denote complex conjugates. In the Ritz variational procedures [5] the Gaussian trial functions for both initial and subsequent profiles have been proved to be close approximations of the analytical profiles via the criterion of integral contents [1,5]. With the trial function defined [1] for the subsequent pulses, and then used in equation (3) a Lagrangian density with respect to the dimensionless time is obtained. According to the variational principle $\delta\int\langle L\rangle d\xi = 0$, the function $\int_{-\infty}^{\infty} L_G \, d\tau$ is the reduced Lagrangian where L_G defines the Lagrangian density. The reduced Lagrangian is the dependent variable for the Euler-Lagrange equation given by

$$\frac{\delta\langle L\rangle}{\delta(i)} = \frac{\partial}{\partial \xi}\left\{ \frac{\partial \langle L\rangle}{\partial[\partial(i)/\partial \xi]} \right\} + \frac{\partial}{\partial T}\left\{ \frac{\partial \langle L\rangle}{\partial[\partial(i)/\partial T]} \right\} - \frac{\partial\langle L\rangle}{\partial(i)} = 0 \tag{4}$$

where (*i*) denotes anyone of the Gaussian parameters [1] for the propagating pulse; these are the complex amplitude, the pulsewidth and chirp function of the propagation distance. If equation (4) is worked for the parameters, variational equations, in differential forms, are obtained. The details so far briefly given here are available in ref. [1].

Solutions of the variational equations contain all results to completely describe pulse dynamics. Harmonic oscillator equation is a principal result. In turn, potential function is obtained from the harmonic oscillator equation typifying the pulse as a particle in a potential well [1]. Since the details of the procedures are elsewhere [1], the potential function is stated explicitly thus

$$\Phi(y) = \frac{1}{y^2} + \frac{\xi_r}{y} - (1 + \xi_r) \tag{5}$$

where $y(\xi) \equiv g(\xi)/g_0$ is the normalized pulsewidth in which $g(\xi)$ is the dimensionless pulsewidth and g_0 defines initial pulsewidth; ξ_r is the crucial parameter that defines a factor of pulse compression/decompression of which expression is

$$\xi_r = \frac{9\sqrt{2}\delta_1 E_0 g_0}{9\delta_0 + 4\sqrt{3}\delta_2 \nu E_0^2} \tag{6}$$

where $E_0 \equiv g_0|G_0|^2$ such that $|G_0|$ is the input amplitude of the pulse.

In the bright solitary wave configuration, $\xi_r = -2$ as deducible from the set of allowed intervals of values for pulse compression/decompression factor [1,5]. Equation (6) can then be shown to yield

$$\frac{1}{g_0} = \frac{|G_0|}{3\sqrt{2}}\left\{ 9\sqrt{2} + 8\sqrt{3}\delta_2 \nu |G_0|^2 \right\}^{1/2}. \tag{7}$$

by putting $\nu = 0$ to correspond to CNLSE [5] one obtains

$$\frac{1}{g_0} = \frac{|G_0|}{2^{1/4}}. \tag{8}$$

that is, by applying all of the detailed procedures described here to CNLSE, equation (8) would be obtained for the bright soliton pulse.

4. Discussion Of The Variational Model

In the anomalous dispersion regime of pulse propagation for which $\beta_2 < 0$, equation (2) may be used to experimentally observe variation of $|\beta_2|$ with optical wavelength, λ, assuming fixed magnitudes of other parameters. One significant implication of (2) is that $\nu \propto 1/(\tau_0)^2$ in complete analogy to coefficients of perturbation terms which have been advanced as necessary for CNLSE to be valid for pulse durations in femtoseconds [1,6]. Observe that this means that as τ_0 decreases, both ν and input or incident power increase significantly so that the CQNLSE (1) would describe distortionless propagation [7]. As noted previously, [1], the parameter ν of the order 0.044 can correspond to τ_0 = 10·0ps for given values of $|\beta_2|$ and λ. Thus, durations from 10.0ps necessarily require inclusion of n_4 if a numerical significance of order 10^{-3} is set for ν.

Fig. 1 simulates variation of input dimensionless pulsewidth with respect to input dimensionless pulseheight, giving another variational model comparable to the previous one [1,7]. The essence of the present model is rooted in clearer illustration of difference between the CNLSE and CQNLSE through the phenomena of saturation and two-state solution which are the main physical significances of $\chi^{(5)}$. Saturation implies that at certain values of the pulsewidth, value of pulseheight does not change significantly. Correspondingly, except at the minimum value of the pulsewidth, every other value of the pulsewidth corresponds to two values of pulseheight thus defining two-state solution.

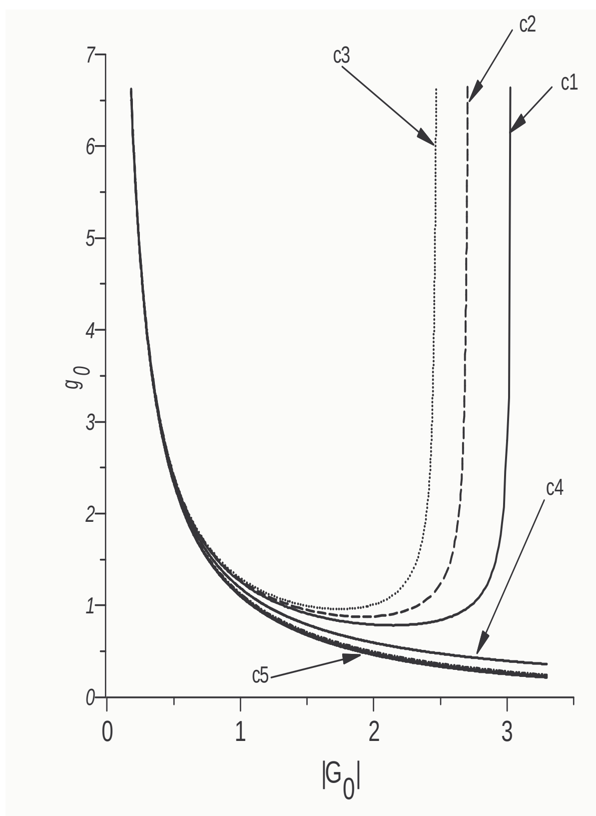

Fig. 1. Two-state solution and saturation effect of $\chi^{(5)}_{xxxxxx}$ through the nonlinearity coefficient ν: (a) Curves c1, c2 and c3 obtained from equation (7) for $\delta_2 = -1$, corresponds to ν = 0·1 , 0·125, and 0·15, curve c4 obtained from equation (8) for CNLSE corresponds to $\nu = 0$ in equation (7), and curve c5 depicts three superimposed curves for $\delta_2 = +1$ with ν = 0·1, 0·125, and 0·15 respectively.

For the monomode optical fibre, $\delta_2 = -1$, curves c1, c2 and c3 of Fig. 1 respectively simulate two-state solutions of the CQNLSE for $\nu = 0{\cdot}1$, $0{\cdot}125$, and $0{\cdot}15$. It could be seen that one value of g_0 gives two values of $|G_0|$. The three values have respective minimum values $g_{0(min)} \sim 0.8324$, 0.931 and 1.0194 corresponding to $|G_0|_{min} \approx 2.475$, 2.213 and 2.021. The unique values may be considered as one form of saturation effect. As the curves depict, at $|G_0| \approx 3.03$, 2.711 and 2.475 further increases of g_0 does not produce significant increases in $|G_0|$, i.e., the respective maximum amplitudes are saturated. Due to a reason that most optical media have $n_4 < 0$, CNLSE precludes two-state phenomena. Curves c4 and c5 of Fig. 1 simulate saturation effects of ν respectively for CNLSE and CQNLSE (1) whenever $n_4 > 0$ for $\delta_2 = +1$.

Curve c4 corresponds to $\nu = 0$ in equation (8). Actually, curve c5 corresponds to three superimposed curves drawn from equation (7) for $\delta_2 = +1$ with $\nu = 0.1$, 0.125 and 0.15. That is, the curves seem to overlap. In nonlinear theory, the differences must be considered significant. However, at one value of $|G_0|$, the input amplitude as from $|G_0| \sim 0.75$, it could be seen that the magnitude of differences between pulsewidths get larger against the CNLSE as this would imply errors in the latter. Moreover, curve c4 reaches saturation at higher amplitude compared to anyone of curves c5.

Fig. 2 depicts the same model. It is a generalized form of relations between the pulsewidth parameter and input intensity parameter for CQNLSE. Curve (1) corresponds to $\delta_2 = -1$and curve (2) is for $\delta_2 = +1$.

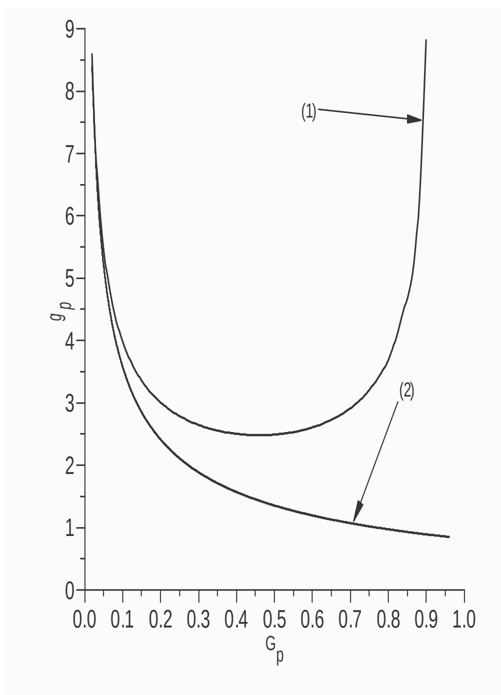

Fig. 2. Generalized relation obtained for CQNLSE from equation (7) where curve (1) corresponds to $\delta_2 = -1$ and curve (2) for $\delta_2 = +1$. Pulsewidth parameter, $g_p \equiv g_0/(\nu)^{1/2}$ and input intensity parameter $G_p \equiv \nu|G_0|^2$. Note that the input amplitude parameter is $G_A \equiv \sqrt{G_p} = \nu^{1/2}|G_0|$.

5. Conclusion

From the results of variational solution of CQNLSE [1], physical significances of fifth-order nonlinear susceptibility have been illustrated using another variational model that describes bright solitary wave. This model enables us to clearly explain differences between the CNLSE and CQNLSE. In its dimensionless form as usually applicable in soliton theory, the latter sustains saturation of the input amplitude and two-state solution. The two-state solution, though has formerly been demonstrated to be inherent in most optical media [1, 7], due to their negative valued fifth-order nonlinear refractive index, n_4, has now been shown to be absent for $n_4 > 0$. It could, thus, be seen from Fig. 1 that with a given set of values of the propagation parameters, for pulse durations as from 10.0 ps to a few femtoseconds, the CNLSE is not adequate for the dynamics of bright solitary waves.

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