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Special relativity with non-standard synchronized clocks: "Everyday" clock synchronization simplified

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Abstract - We show that the Lorentz transformations for the space-time coordinates of the same event, solve problems in which the synchronization of the clocks of a given inertial reference frame are synchronized following a clock synchronization procedure different from that proposed by Einstein. The case of the so called "everyday clock synchronization" procedure is considered recovering all the results obtained without using them in a relatively complicated way, for those who just start learning special relativity theory.

Keywords: special relativity, clock synchronization

1. Introduction

The concept of *observer* is of fundamental importance in the foundation of special relativity theory. An observer is a physicist who knows the laws of physics and is able to handle measuring devices. Blatter and Greber [1] make a net distinction between two concepts of relativistic observers:

1. Observers of Einstein type
2. Observers synchronized by and working with light signals

The first concept of observers was introduced by Einstein. This is an observer who collects and transmits data on events in space with a whole set of identical wristwatches evenly distributed in space. A meter stick is used to find out the space coordinates of any point. Identical wristwatches are used to attribute time coordinates to events which take place in front of each of them. The different watches need to be synchronized by a clock synchronization procedure. We will consider the procedure proposed by Einstein [2]. Consider the clocks C_1 and C_2 located at the points $M_1(x_1,y_1)$ and $M(x_2,y_2)$ in a two space dimensions approach. A light signal is sent from M_1 when C_0 measures time t_1 and is received at M_2 when C_2 measures time t_2. We say that they are *Einstein-synchronized* if

$$t_2 = t_1 + \frac{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}{c} \tag{1}$$

where c represents the “two way” speed of light in empty space, the second term in the right side of (1) representing the time during which the light signal propagates from point M_0 to point M. The *two-way* speed of light is measured by sending a light signal from M_0 to M and reflected back to M_0 and calculated as the ratio of the total distance traveled ($2M_0M$) divided by the duration of the journey measured by C_0. This two-way speed is experimentally known to be the same between all pairs of points for all observers.

A second type of the concept of relativistic observer is located at one point in space and collects information by the light arriving at his or her location. At each point of the *OX* axis (in a one space dimension approach) we find an observer $R(x)$ equipped with a clock $C(x)$ synchronized with clock $C_0(0)$ and with radio and television receivers. An approach to special relativity theory based on a feasible clock synchronization procedure [2, 3] (described in the next section), different from that proposed by Einstein, offers the opportunity to compare the way in which two observers, of the two types mentioned above, respectively, located at the same point in space, act in order to solve problems associated with relativistic telemetry.

The point the authors are making is that there are many different coordinate systems to choose from apart from the standard Einstein coordinates. Einstein coordinates are arguably the best coordinates but they are not the only coordinates, and it is educational to consider some of the other possibilities (radar and photographic detection of the space-time coordinates of a distant event). The “everyday” coordinates the authors define are one possibility. The purpose of our paper is to compare the results they obtain in the two approaches respectively and to convince the student that there are lots of different coordinate systems to choose apart from the standard “Einstein-synchronized” coordinates. Einstein coordinates are arguably the best coordinates but they are not the only coordinates and it is instructive to consider some different possibilities. The “everyday” approach is one such possibility.

2. “Everyday” clock synchronization.

The approach proposed by Leubner *et al* [3, 4] goes as follows. Consider a radio station located on Earth at the origin *O* of the *I* frame that emits the following information: “At the sound of the last tone, it will be 12 o’clock”. The radio signal propagates in the positive direction of the *OX* axis and arrives after a given time propagation to the location of an observer $R(x)$ with a wristwatch $C(x)$ which reads $t_E(x)$ being synchronized with the other clocks of that frame following the procedure proposed by Einstein. Neglecting the time during which the information propagates, the observer fixes the clock to display 12 o’clock and we say that an *everyday clock synchronization* took place. In order to make the approach more transparent we replace the radio station with a television studio which continuously transmits the image of the clock $C_0(0)$ displaying the time $t_E(0)$ when the television signal was emitted. Under such conditions $R(x)$ has a wrist watch $C(x)$ reading $t_E(x)$ and a clock image on the television screen reading $t_T(x)$. The readings of the two clocks are related by

$$t_E(x) = t_T(x) + \frac{|x|}{c} \tag{2}$$

an equation that accounts for the fact that clocks $C(x)$ and $C_0(0)$ are Einstein -synchronized. To simplify the analysis, we shall consider only the case where $x \geq 0$, so that in Eq. (2) we can replace $|x|$ by x.

Knowing that the Lorentz transformations hold correctly only in the case of Einstein-synchronized clocks they should also work when we express in them t_E as a function of combinations of other physical quantities i.e. like (2). Working as Einstein observers, they know that it is considered that the two-way speed of light and the one-way speeds are equal to each other having an isotropic character in space. Under such conditions the Lorentz transform, usually expressed as

$$x' = \frac{x - Vt_E}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{3}$$

and

$$t'_E = \frac{t_E - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{4}$$

in a one space dimensions approach, become after expressing t_E as a function of the time t_T displayed by the television clock image. Eqs. (3) and (4) relate the space-time coordinates of events $F(x, t_E)$ and $F'(x', t_E')$ detected by observers from I and I' respectively. The two events take place at the same point in space, when the Einstein-synchronized clocks $C(x)$, and $C'(x')$ located at that point read t and t' respectively. Relativists say that they are the same event. The index E stands for the fact that the clocks are Einstein synchronized. Expressing t_E in Eqs. (3) and (4) as a function of the time displayed by the television clock image Eq. (2) they become

$$x' = \frac{x - V(t_T + \frac{x}{c})}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{x(1 - \frac{V}{c}) - Vt_T}{\sqrt{1 - \frac{V^2}{c^2}}} \tag{5}$$

$$t'_E = \frac{t_T + \frac{x}{c} - \frac{V}{c^2}x}{\sqrt{1 - \frac{V^2}{c^2}}} = \frac{t_T + \frac{x}{c}(1 - \frac{V}{c})}{\sqrt{1 - \frac{V^2}{c^2}}}\,. \tag{6}$$

In order to express the left side of Eq. (6) as a function of a time displayed by a television clock image we express $t'_E = t'_T + \frac{x'}{c}$ and combine Eqs. (5) and (6) in order to obtain

$$t'_T = t_T\sqrt{\frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}}\,. \tag{7}$$

Knowing that at the origin of time the origins O and O' are located at the same point in space, it is not surprising at all that Eq. (7) accounts for the longitudinal Doppler effect, $(t_T - 0)$ representing a proper time interval measured by observer R_0, whereas $(t'_T - 0)$ representing the same time interval measured by observer R'_0. Knowing that V is measured using Einstein-synchronized clocks, it is of interest to find out its equivalent V_T measured using television image clocks. Consider that observers from I intend to measure the speed of clock $R'_0(0)$ located at the origin O' of I' and moving together with it at the Einstein speed V relative to I. A measuring rod is located along the OX axis with its left end **L** located at O. When the moving clock arrives at the end **L** the clock $C_0(0)$ located there reads $t = 0$. Arriving at a point $M(x)$, the Einstein clock located there reads t_E and so the Einstein speed V is by definition

$$V = \frac{x}{t_E} = \frac{x}{t_T + \frac{x}{c}} = \frac{V_T}{1 + \frac{V_T}{c}} \tag{8}$$

where by definition

$$V_T = \frac{x}{t_T}. \tag{9}$$

is the relative speed of I and I' measured using television image clocks. With Eq. (8) we obtain

$$1 - \frac{V}{c} = \frac{1}{1 + \frac{V_T}{c}} \tag{10}$$

$$\sqrt{1 - \frac{V^2}{c^2}} = \frac{\sqrt{1 + 2\frac{V_T}{c}}}{1 + \frac{V_T}{c}} \tag{11}$$

$$\sqrt{\frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}} = \sqrt{1 + 2\frac{V_T}{c}} \tag{12}$$

The transform we are looking for is

$$x' = \frac{x - V_T t_T}{\sqrt{1 + 2\frac{V_T}{c}}} \tag{13}$$

and

$$t'_T = t_T \sqrt{1 + 2\frac{V_T}{c}}. \tag{14}$$

If a particle moves at a television speed u_T relative to I and at a television speed u'_T relative to I' (both in the positive direction of the overlapped axes) they transform as

$$u'_T = \frac{x'}{t'_T} = \frac{u_T - V_T}{1 + 2\frac{V_T}{c}}. \qquad (15)$$

With Eq. (15) we find out that the fundamental postulate in special relativity theory, claiming the invariability of light speed, fails short in this case. For a light signal propagating with light speed c in I, its speed as measured in frame I' will be $c_{I'} = c\frac{c - V_T}{c + 2V_T} < c$. For a light signal propagating with light speed c in I' its speed as measured in frame I will be $c_I = V_T + c\left(1 + 2\frac{V_T}{c}\right) = c + 3V_T > c$.

We have recovered all the results obtained by Leubner *et al* [3] without using the concept of base vector [4] which is not in an easy grasp even for experienced physicists.

3. "Everyday" clock synchronization in 3D

We propose the scenario presented in Figure 1 as detected from the inertial reference frame I (XOY). It involves a first clock $C_0(0,0)$ and a source of light $S(0,0)$ both located at the origin O. At any point $M(x=r\cos\theta, y=r\sin\theta)$ in plane we find a pair of clocks, the first clock $C_E(r\cos\theta, r\sin\theta)$ being stopped and set to read $t_E = 0$ and the second clock $C_T(r\cos\theta, r\sin\theta)$ stopped and expressly set to read $t_T = r/c$. The source of light S emits a short signal in all directions, this signal being further used to synchronize any other clock in plane. Arriving at the point M the light signal that propagates along a direction that makes an angle θ with the positive direction of the OX axis starts the two clocks located there. By definition the clocks C_E and C_0 are synchronized à la Einstein whereas the clocks C_T and C_0 are synchronized following the everyday clock synchronization procedure. From then on, we will have in any point in plane the same relationship $t_E=t_T+r/c$ between the readings of any paired clocks.

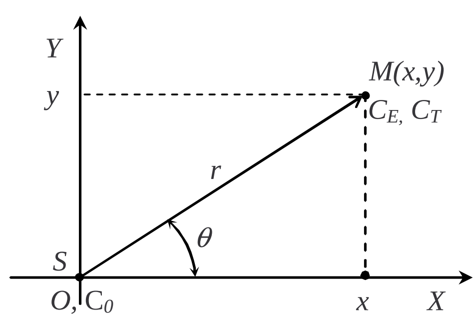

Fig. 1. Scenario for the 3D approach to the "everyday" clock synchronization

In frame I the event associated with the fact that the light signal starts the local pair of clocks located at point M is characterized by the space-time coordinates:

$$x = r\cos\theta \;\; (16) \;\; ; \;\; y = r\sin\theta \;\; (17) \; ; \; t_E = t_T + \frac{r}{c} \qquad (18)$$

Detected from another inertial reference frame I' (in the standard arrangement with I) the same event is characterized by the space-time coordinates x', y', r' and θ'. Using furthermore the notation $\beta_T = \frac{V_T}{c}$ we obtain the following auxiliary relations:

$$x' = \frac{r[\cos\theta(1+\beta_T) - \beta_T] - ct_T\beta_T}{\sqrt{1+2\beta_T}} \tag{19}$$

$$y' = y = r\sin\theta = r'\sin\theta' \tag{20}$$

Eq. (19) reveals the fact that all points in the frame I situated on the line with slope θ=const and given by the relation $\cos\theta = \frac{\beta_T}{1+\beta_T}$ remap in I' onto a vertical line $x' = -\frac{ct_T\beta_T}{\sqrt{1+2\beta_T}}$ moving with local time in I'. As we see under such conditions the shape of the vertical line is not distorted. Furthermore we have:

$$t'_E = \frac{t_E - \frac{V}{c^2}x}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{t_T + \frac{r}{c} - \frac{V}{c^2}r\cos\theta}{\sqrt{1-\frac{V^2}{c^2}}} = \frac{t_T + \frac{r}{c}(1-\frac{V}{c}\cos\theta)}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$t'_E = \frac{t_T(1+\beta_T) + \frac{r}{c}[1+\beta_T(1-\cos\theta)]}{\sqrt{1+2\beta_T}} \tag{21}$$

Obviously with θ=0 Eq. (19) reduces to Eqs. (13) and (21) reduces to Eq. (6).

The slope of the position vector (polar angle coordinate) θ in I transforms in I' into a new value θ' related by

$$\tan\theta' = \frac{y'}{x'} = \frac{\sqrt{1+2\beta_T}\sin\theta}{(1+\beta_T)\cos\theta - \beta_T - \frac{ct_T}{r}\beta_T} \tag{22}$$

whilst the lengths of the position vectors r and r' transform as

$$r' = r\sqrt{\frac{\left[(1+\beta_T)\cos\theta - \beta_T - \frac{ct_T}{r}\beta_T\right]^2}{1+2\beta_T} + \sin^2\theta} \tag{23}$$

with θ=0 we notice that obviously Eq. (23) reduces to Eq. (13).

4. Transforming the radial velocity and the spherical wave front

Consider now a source of discrete particles (tardyons) situated at the origin of I that emits at the origin of time particles moving with constant velocity in any space direction. Using their television clocks, the observers in I measure a constant radial particle velocity $u = r/t_T$ for any given direction. The particle moving along a line of slope θ in frame I will appear in frame I' under the instant polar angle θ' given by Eq. (22):

$$\tan\theta' = \frac{\sqrt{1+2\beta_T}\sin\theta}{(1+\beta_T)\cos\theta - \beta_T - \frac{c}{u}\beta_T} \tag{24}$$

As Eq. (24) is time invariant we conclude that the observers in I' detect this particle as moving along a line as well. Therefore we found that with the “everyday” clock synchronization approach the transformed world line of the tardyon is also a straight line having a slope given by the equation above. We calculate now the corresponding radial velocity of the particle, as measured by the observers in I', by using their television clocks with:

$$u' = \frac{r'}{t'_T} = \frac{1}{\frac{t'_E}{r'} - \frac{1}{c}} \tag{25}$$

Using Eqs. (21) and (23) with $r = ut_T$ we obtain the radial velocity in I' as:

$$u' = \frac{u}{\frac{1+\beta_T + u/c\left[1+\beta_T(1-\cos\theta)\right]}{\sqrt{\left[(1+\beta_T)\cos\theta - \beta_T(1+u/c)\right]^2 + (1+2\beta_T)\sin^2\theta}} - 1} \tag{26}$$

As Eq. (26) is also time invariant we conclude that in I' the particles appears to propagate along a line with constant radial velocity given by the equation above. However the radial velocity in I' is inhomogeneous having different magnitudes along different directions.

We replace now the particle source by a signal source emitting signals, which propagate in all directions in frame I with constant velocity *c* as measured by observers using their television clocks. In this case the relative radial signal velocity as measured by observers in frame I' by using their television clocks will be from Eq. (26):

$$\frac{c'}{c} = \frac{1}{\frac{2(1+\beta_T) - \beta_T\cos\theta}{\sqrt{\left[(1+\beta_T)\cos\theta - 2\beta_T\right]^2 + (1+2\beta_T)\sin^2\theta}} - 1} \tag{27}$$

With θ =0 we notice that this equation reduces obviously to the already known longitudinal value $c' = c\frac{1-\beta_T}{1+2\beta_T}$. We represent in the figure below the relative radial velocity in frame I' as function of the polar angle for different values of relative frame velocity. As we see c' has different magnitudes along different directions, a common feature with other approaches of non-standard clock synchronization.

In what concerns Eq. (23) let consider that *r* = *constant* describes a circular wave front in I where the signal propagation is isotropic. In this case r' represents the distorted wave front as detected by observers from I'. The result is illustrated in Figure 3.

We observe that for any relative frame velocity we may identify two particular directions that are free of relativistic effects i.e. the radial signal velocity is the same in both frames. These directions are defined by the points where the circular wave front in I intersects the distorted one in I'.

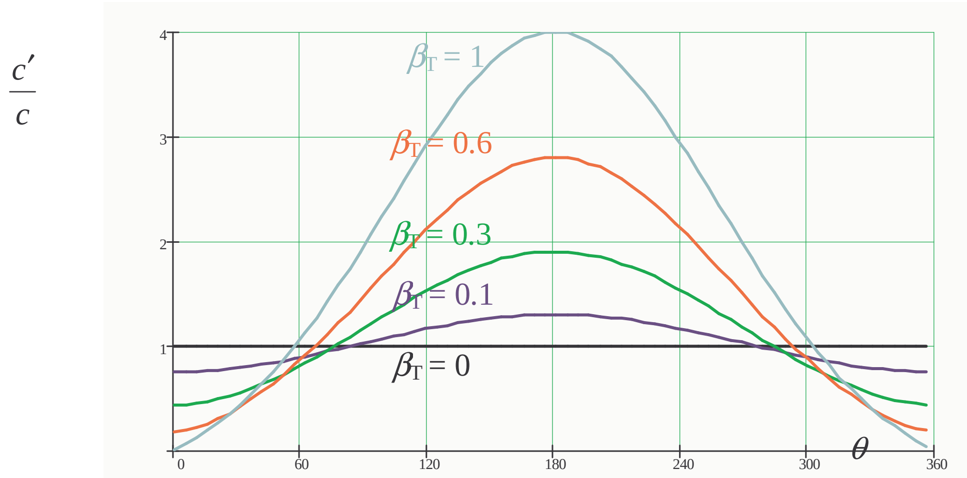

Fig. 2. The non-isotropic light speed c' detected by observers from I' as a function of the angle θ.

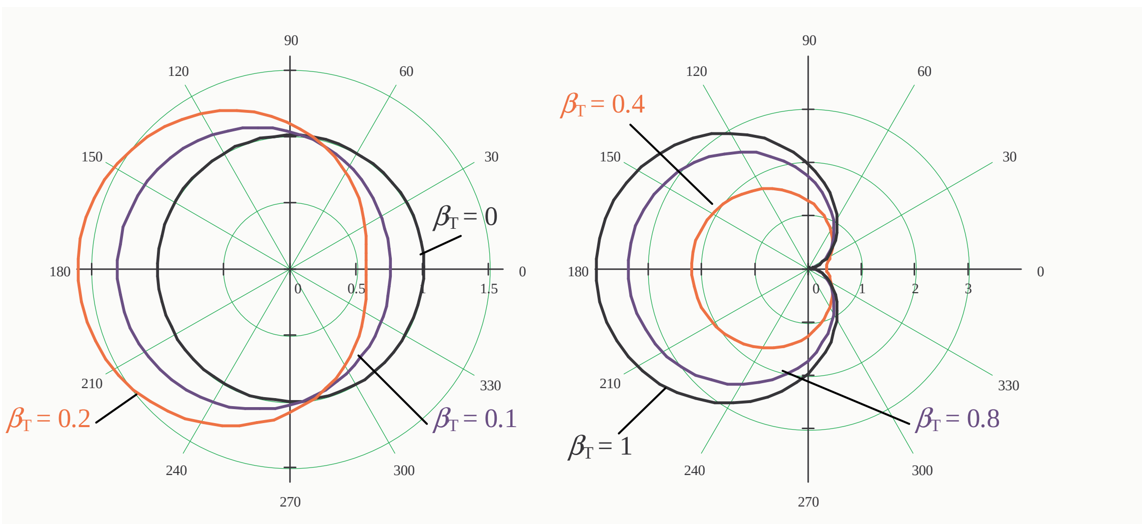

Fig. 3. Apparent wave fronts for various relative frame velocities

We have derived so far the transformation equations for the polar coordinates in the case of the everyday synchronization procedure. What we still have to do is to find the transformation equation that relates t_T and t'_T. Starting with $t'_T = t'_E - \frac{r'}{c}$ we obtain by using Eqs. (21) and (23):

$$t'_T = \frac{t_T(1+\beta_T) + \frac{r}{c}\left[1+\beta_T(1-\cos\theta)\right]}{\sqrt{1+2\beta_T}} - \frac{r}{c}\sqrt{\frac{\left[(1+\beta_T)\cos\theta - \beta_T - \frac{ct_T}{r}\beta_T\right]^2}{1+2\beta_T} + \sin^2\theta} \tag{28}$$

With θ =0 we notice that obviously Eq. (28) reduces to Eq. (14). Reconsider now our source emitting signals from the common origin at the origin of time. The signals propagate with constant radial velocity u_E in all directions and we have $r = u_E t_E$. When using their television clocks to measure the signal velocity, the observers in frame I will detect the wave front as propagating with the equivalent velocity:

$$u_T = \frac{r}{t_T} = \frac{r}{t_E - \frac{r}{c}} = \frac{u_E}{1 - \frac{u_E}{c}} \tag{29}$$

Note that the light signals will appear to have infinite radial velocity in this case. For any other signal propagating with subluminal velocity in frame I we have $r = u_T t_T$. The wave front in I at any time t_T will be a circle. For the particular propagation line with slope θ we have from Eq. (28):

$$t'_T = t_T \left(\frac{(1+\beta_T) + \frac{u_T}{c}\left[1 + \beta_T(1-\cos\theta)\right]}{\sqrt{1+2\beta_T}} - \frac{u_T}{c}\sqrt{\frac{\left[(1+\beta_T)\cos\theta - \beta_T - \frac{c}{u_T}\beta_T\right]^2}{1+2\beta_T} + \sin^2\theta} \right) \tag{30}$$

The equation above reveals that all simultaneous events associated with the propagation of wave front in frame I will be also detected as simultaneous in frame I'. Moreover the geometric locus of all simultaneous events in frame I that remain simultaneous in frame I' is a circle in I. In frame I' this locus is depicted by the Figure 3 above.

5. Conclusions

The Lorentz-Einstein transformations with clocks synchronized following the procedure proposed by Einstein could be used in order to find out the shape of the equations which account for the relativistic effects in the case of non-standard synchronization of the involved clocks.

The anisotropy of the speed of light as revealed above is a characteristic for all particular approaches in relativity theory that replace the clock synchronization proposed by Einstein by other procedures more or less compatible with it [5-7]. The authors quoted hereby consider that there is an inertial reference frame in which the light propagates isotropically, with all other inertial reference frames, which move relative to it, presenting anisotropy in the propagation of light and additionally showing an absolute character for the simultaneity. We consider that these properties are merely a consequence of changing the synchronization procedure.

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