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LETTER

Transformation equations for the kinetic energy of tardyon and photon via the Bertozzi's experiment

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Abstract - Transformation equations for the kinetic energy of an electron and of a photon are derived starting with the Bertozzi's experiment considered from the rest frame of the experimental device and from a reference frame relative to which the device moves with constant speed. The electrons involved in this experiment move in the positive direction of the overlapped axes OX(OX') of the two frames. This derivation is based on the transformation equation for parallel speeds. The formula accounting for the transformation of the tardyon kinetic energy has at limit with particle rest energy approaching zero and with particle speed approaching c a particular form which reminds the transformation equation for the photon (kinetic) energy.

1. Introduction

Bertozzi's experiment [1, 2] is a landmark in understanding the fact that the Newtonian concept of absolute mass is unable to account for the dynamic behavior of the electrons traveling at high-speed approaching the light speed in empty space. In such an experiment the electrons are accelerated to a given measurable high speed, after which they enter a space where their speed is measured and their number is counted. Finally they collide with a target giving up their kinetic energy. The increase of the target temperature is a measure of the kinetic energy of the incident electrons. Let I' be the rest inertial reference frame of the Bertozzi's experimental device. Herein the electrons move in the positive direction of the O'X' axis with speed u'. Using the work-kinetic energy theorem we find out the relationship between u', the particle rest energy $E_0 = m_0c^2$ and the kinetic energy W' of the particle measured from I' [3] expressed as:

$$W' = E_0\left[(1-\frac{u'^2}{c^2})^{-1/2} - 1\right] \tag{1}$$

Let I be another inertial reference frame relative to which I' moves with speed V and relative to which the electrons move with speed u. The kinetic energy of the electrons in this reference frame is W related to u by:

$$W = E_0\left[(1-\frac{u^2}{c^2})^{-1/2}-1\right]. \tag{2}$$

The purpose of our paper is to find out a transformation equation that relates W and W'. Textbooks devoted to the subject do not present it - probably because the kinetic energy is not a component of a four vector. The speed u and u' are related by the relativistic addition law of parallel velocities [4]

$$u = \frac{u'+V}{1+\frac{u'V}{c^2}}. \tag{3}$$

For the clarity and consolidation of the mathematical interference developed below we rewrite this equation in the equivalent form:

$$\beta_u = \frac{\beta_{u'}+\beta_V}{1+\beta_{u'}\beta_V}, \tag{4}$$

where we have used the shorthand notations $\beta_V = V/c$, $\beta_u = u/c$ and $\beta_{u'} = u'/c$. With a little bit of algebraic work we obtain another equivalent but useful form for the addition law of parallel velocities written as:

$$\gamma_u = \gamma_{u'}\gamma_V\left(1+\beta_{u'}\beta_V\right). \tag{5}$$

Here we have further used the classic relations $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \sqrt{1-\frac{1}{\gamma^2}}$. Expressing the right side of (2) as a function of u' via (3) the result is

$$W = E_0\left(\frac{1+\frac{Vu'}{c^2}}{\sqrt{1-\frac{V^2}{c^2}}\sqrt{1-\frac{u'^2}{c^2}}}-1\right). \tag{6}$$

Equation (1) leads to

$$\frac{u'}{c} = \frac{\sqrt{\frac{W'}{E_0}\left(2+\frac{W'}{E_0}\right)}}{1+\frac{W'}{E_0}}, \tag{7}$$

and to

$$\sqrt{1-\frac{u'^2}{c^2}}=\frac{1}{1+\frac{W'}{E_0}} \tag{8}$$

with which (6) becomes

$$\frac{W}{E_0}=\gamma_V\left[\left(1+\frac{W'}{E_0}\right)+\beta_V\sqrt{\frac{W'}{E_0}\left(2+\frac{W'}{E_0}\right)}\right]-1. \tag{9}$$

This equation represents the transformation equation for the kinetic energy of the tardyon. By rearranging it in the equivalent form

$$1+\frac{W}{E_0}=\gamma_V\left[1+\frac{W'}{E_0}+\beta_V\sqrt{\left(1+\frac{W'}{E_0}\right)^2-1}\right], \tag{10}$$

and introducing $E = W+E_0$ as the total energy of the particle we obtain the transformation equation for the total energy in the form:

$$\frac{E}{E_0}=\gamma_V\left[\frac{E'}{E_0}+\beta_V\sqrt{\left(\frac{E'}{E_0}\right)^2-1}\right] \tag{11}$$

Furthermore, using the notion of relativistic mass m as introduced by Einstein $E = m\cdot c^2$, we obtain the equation for the transformation of the relativistic mass in the form

$$\frac{m}{m_0}=\gamma_V\left[\frac{m'}{m_0}+\beta_V\sqrt{\left(\frac{m'}{m_0}\right)^2-1}\right]. \tag{12}$$

If we replace the term $\frac{E}{E_0}=\frac{1}{\sqrt{1-\beta^2}}$ and respectively $\frac{m}{m_0}=\frac{1}{\sqrt{1-\beta^2}}$ in the last two equations above then we return to the particular expression illustrated by equation (5) which basically represents the relativistic addition law for parallel velocities.

In order to illustrate the dependence of the relative kinetic energy W/E_0 as a function of W'/E_0 we present in figure 1 the variation of W/E_0 with β_V for different values of W'/E_0 as a parameter.

As expected, when the electron is at rest in I' but moving with speed V relative to I we have $W'/E_0 = 0$ and equation (10) leads to

$$W=E_0\left(\gamma_V-1\right). \tag{13}$$

Consider now a new experiment similar to the Bertozzi's historical one where we replace the source of electrons with a source of non-corpuscular radiation such as e.g. a monochromatic light source. Additionally we replace the target by a blackbody that absorbs the energy of the incident photons being able to count the number of these photons [5] and measure their total energy. Therefore we are able to calculate the energy carried by a single photon. We start in this case with (10) presented as

$$W = \frac{E_0 + W' + \beta_V \sqrt{W'(W' + 2E_0)}}{\sqrt{1 - \frac{V^2}{c^2}}} - E_0. \tag{14}$$

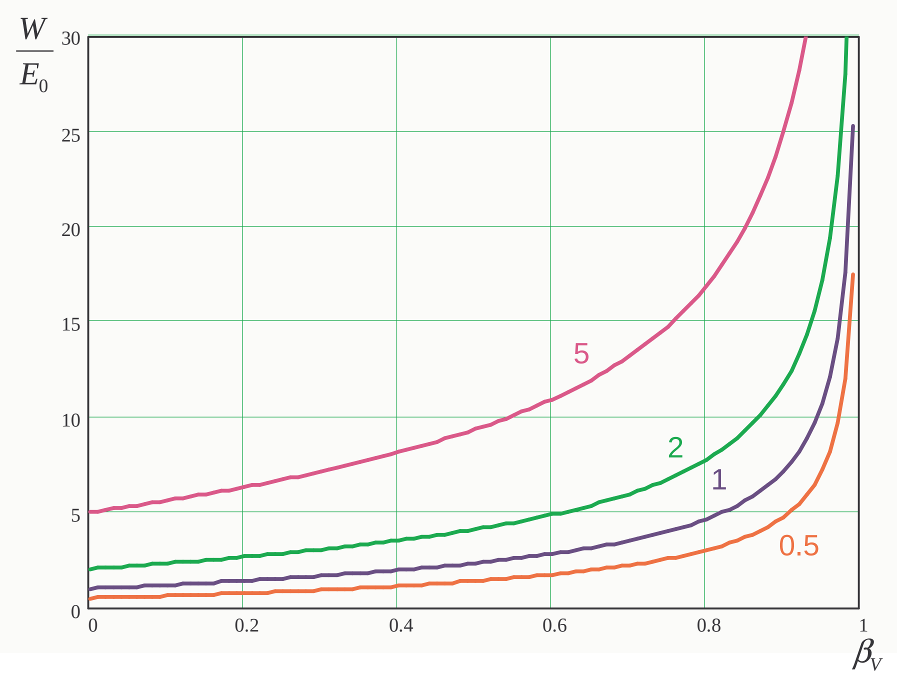

Fig. 1. The variation of W/E_0 with β_V=V/c for different values of W'/E_0 as a parameter

To be applicable for the photon energy we replace E_0=0 (the rest energy of the photon is equal to zero) and u'=c (the photon speed is invariant) with the end result being that

$$W_{photon} = \frac{1+\beta_V}{\sqrt{1-\beta_V^2}} W'_{photon}, \tag{15}$$

or

$$h\nu = h\nu' \sqrt{\frac{1+\beta_V}{1-\beta_V}}, \tag{16}$$

where h represents the Planck's constant, with ν and ν' representing the frequency of the same photon as detected from I and respectively I'. We present in Figure 2 the variation of W/W' with β_V ($0<\beta_V<1$).

The fact that the energy of the photon is in essence kinetic energy is confirmed by the formula which relates the total energy, the kinetic energy and the rest energy of a tardyon

$$E = W + E_0 \tag{17}$$

This becomes in the case of photon (E_0=0)

$$E_{photon} = W_{photon} . \qquad (18)$$

The results obtained above could be another evidence for the famous Feynman's statement: "So what we learn about the properties of electrons will apply also to all particles including photons of light".[6]

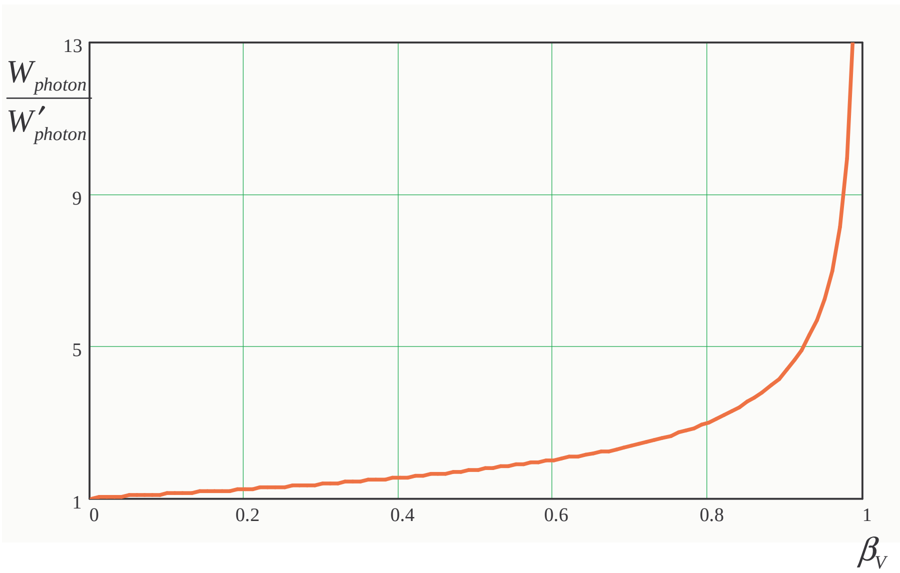

Fig. 2. The variation of W_{photon}/W'_{photon} with β_V

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