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LETTER

Symmetry Properties of Ground State Rotational Band for Some Even-Even Superdeformed Light Nuclei

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Abstract - Bohr Theory shows that the energy levels of even-even deformed nuclei are similar to those of an axially symmetric rotator in the lowest rotational band. These energy levels are arranged as 0^+, 2^+, 4^+, …etc., respectively. The sequence of the ratios of energies of the higher excited states to the 2^+ state give rise a number of integers which clarifies symmetry in the formation of the levels in the lowest rotational bands. In this study we have investigated the symmetry properties of energy bands for some even-even superdeformed light nuclei (^{20}Ne, ^{36}Ar, ^{40}Ca, ^{58}Ni, ^{60}Zn) by using of K-quantum number which is projection of the total angular momentum on the axis of symmetry of a nucleus, and also determined energy bands having symmetry.

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Symmetry a fundamental physical quantity in gaining on understanding of the physical laws governing the behavior of nuclei especially in the applied nuclear physics has usually a central role in collective motion of the nucleons which may be described as a vibrational motion about equilibrium position and a rotational motion that maintains the deformed shape of nucleus [1-4]. Thus, as it is well known, superdeformed nuclei, which is provide an opportunity to study many aspects of nuclear structure such as exotic states, single particle motion, collective modes, and pairing of nucleons, and especially in fusion evaporation reactions, have rotational energy spectrum due to their collective motion [5, 6]. These energy spectra generally are defined by the total angular momentum I, parity π and the quantum number K which is the projection of the angular momentum on the intrinsic coordinate axis of the nuclei. Here, K which is the conserved quantum number for the nuclei having axial symmetry could take values $K = I, I\text{-}1, \ldots, -I$. Indeed, K which is defined the angular momentum of the nuclei depending on the attached coordinate system, has a definite and constant value for a specified intrinsic state of any deformed nuclei. Intrinsic energy E_K of slowly rotating nuclei depends on K quantum number and it forms the base of the rotational energy band, and the energy eigenvalues of such a band could be defined as,

$$E(I,K) = E_K + E_{rot}(I) \tag{1}$$

where $E_{rot}(I)$ is the rotational energy.

Many deformed light nuclei, especially with mass ranging from A= 40 to 80, have stable deformation in their ground-states [3]. Such nuclei may rotate due to interactions with an external incident particle or emitting the particle. Rotational energy of an axially symmetric deformed even-even nucleus is given as [6],

$$E_{rot}(I,K) = \frac{\hbar^2}{2}\left[\frac{I(I+1)}{J_0} + \left(\frac{1}{J_3} - \frac{1}{J_0}\right)K^2\right] \tag{2}$$

where I and K are the total angular momentum and its projection on the axis of symmetry, respectively, of a nucleus; J_3 and J_0 are the principal moments of inertia about a symmetry axis and an arbitrary axis perpendicular to the symmetry axis, respectively. Authors of Ref. [6] have used the hydrodynamic moments of inertia restricting the deformed nuclear surface by a quadropole term only and so these nuclei are, on the average, symmetric: that is $J_3 = 0$. Therefore, Eq. (2) will be meaningful only if the value of K is taken identically zero. This yields what is often known as the ground state rotational band, and then we come to the following rotational energy equation:

$$E_{rot} = \frac{\hbar^2}{2J_0} I(I+1), \qquad K=0 \tag{3}$$

The above expression is in good agreement with the observed low-lying energy levels of the even–even deformed nuclei, which is the values of angular momentum I, I = 0, 2, 4, 6,…. As mentioned above the energy level sequence in such a case is called as ground-state rotational band having positive-parity.

More generally, the rotational energy for the ground state bands of even-even nuclei, could be written as an infinite power series,

$$E_{rot}(I) = AI(I+1) + BI^2(I+1)^2 + CI^3(I+1)^3 + ... \tag{4}$$

where A is the well-known rotational constant parameter for sufficiently small values of I, and B, C,…, are the corresponding higher-order constant parameters [7, 8].

In the view of above-mentioned, it seems that the ground state energy bands of superdeformed even-even nucleus have zero quantum number ($K=0$), together with even parity and even nuclear angular momentum. Their energy eigenvalues could be defined by even number starting from zero, as 0^+, 2^+, 4^+, 6^+,…We could obtain an energy ratio, $E(I^+)/E(2^+)$, to define the energy eigenvalues if we use the Eq. (3) given by Ref. 6, such that,

$$R(I) = \frac{E(I^+)}{E(2^+)} = \frac{I(I+1)}{6}. \tag{5}$$

In Eq. (5), $E(2^+)$ represents the energy of the first excited state and could be set as the unit energy for the even-even nuclei. Using Eq. (5) one can be written in the following form,

$$R(2^+):R(4^+):R(6^+):R(8^+):R(10^+):... = 1:3\frac{1}{3}:7:12:18\frac{1}{3}:... \tag{6}$$

However, the real values are a little different from the above ratios in Eq. (6) and these differences are getting smaller and smaller starting from $I=8$. The deviation of the rotational energy from $I(I+1)$ has resulted from the change as a dependence of the moment of inertia, and the dependence of the rotational energy to some other factors [5, 6]. Since we will only concentrate on the symmetry properties of the energy bands, in this study, the natures and other details of the energy bands of interest are not emphasized. On the other hand, we can remark that the energy eigenvalues of the ground states could be given by Eq. (4) more precisely for the various superdeformed light even-even nuclei and their values could be extracted from the experimental data [9]. If we represent $R(4^+)/R(2^+)= r$ in order to find out symmetry properties of the ground state energy bands instead of $R(4^+)/R(2^+)$, in consideration of the measured energy eigenvalues [9] we get an approximated equation,

$$R(2^+):R(4^+):R(6^+):R(8^+):\ldots \approx 1:r:2r:3r:\ldots \tag{7}$$

Instead of Eq. (6), if we consider the experimental data, we should emphasize once more here, that the last approximated equation gives more accurate results than the previous one Eq. (6). The last expression is clearly defines the "equal intervals" and illustrates the "symmetries" in the energy bands. If we rewrite Eq. (7) starting from the ratio $R(4^+)$ then we get the ratio of integers as,

$$R(4^+):R(6^+):R(8^+):\ldots \approx r:2r:3r:\ldots = 1:2:3:\ldots \tag{8}$$

An example of such a symmetry is obtained using the ratio $R(I)=E(I^+)/E(2^+)$ and starting from $I=4$, presented in Figure for the ground state bands in the some superdeformed even-even light nuclei.

We may choose an energy unit $E_1(I_1{}^{\pi})-E_0(I_0{}^{\pi})$ and consider the related ratios to define the mentioned symmetry for the excited bands of the even-even nuclei. Here, $E_0(I_0{}^{\pi})$ is the energy of the lowest state having I_0 *spin* and π parity, and $E_1(I_1{}^{\pi})$ is the energy of the first excited state above the $E_0(I_0{}^{\pi})$ having suitable I_1 and π quantum numbers. In the calculations of the energy ratio for excited bands, instead of Eq. 5, one could use, in general,

$$R_n = \frac{E_n\left(I_n^{\pi}\right) - E_0\left(I_0^{\pi}\right)}{E_1\left(I_1^{\pi}\right) - E_0\left(I_0^{\pi}\right)} \tag{9}$$

where $E_n\left(I_n^{\pi}\right)$ is the energy of the n'th excited state above the $E_0^n\left(I_0^{\pi}\right)$ having suitable I_n and π quantum numbers. In that case using the experimental data [9] we get,

$$R_2 : R_3 : R_4 : \ldots \approx r : 2r : 3r : \ldots : \left(n-1\right) r \tag{10}$$

where,

$$r = \frac{E_2\left(I_2^{\pi}\right) - E_0\left(I_0^{\pi}\right)}{E_1\left(I_1^{\pi}\right) - E_0\left(I_0^{\pi}\right)} \quad . \tag{11}$$

In Eq.(11), the values of energy ratio, r, have changed between 3.1 and 3.3 for different energy bands. In this present work, to illustrate, these energy ratios for ground states of superdeformed even-even light nuclei has been approximately determined as 3.3.

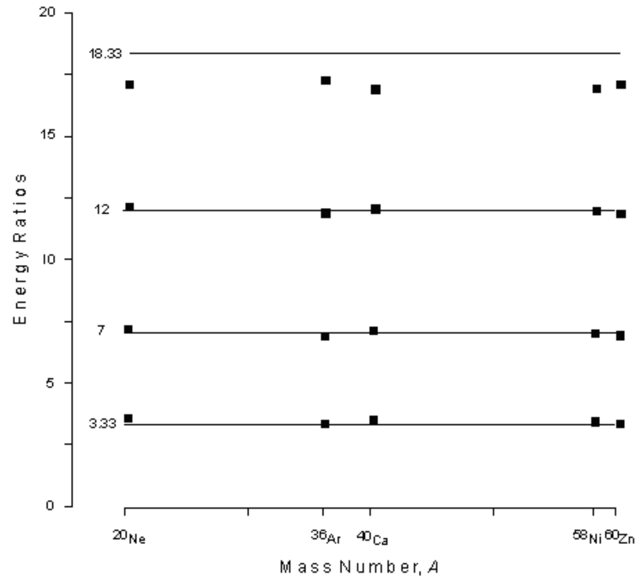

Figure. Mass dependence of energy ratios for the ground state band of superdeformed even-even light nuclei. The horizontal line at the top indicates the values given by the I (I+1) rule.

In the present study, it has seen that mass dependence of energy ratios for the ground state rotational bands in deformed light even-even nuclei approximately satisfies Eq. (11) in Figure. From the figure, it is observed that the obtained energy ratios calculated by Eq. (11) demonstrate a well consistency with Eq. (10) and also show the symmetry of energy levels for the ground state band in superdeformed even-even nuclei. Also, in figure, it has seen that the our calculated energy ratios denote a well accordance with I(I +1) rule, values of calculated Eq. (5). Thus, these results obtained by Eq. (5) almost represent the empirical data values for all energies quite well for the nuclei of interest. This situation illustrates almost equidistant rotation nature of the lower spectra of the nuclei considered. Therefore, the present study we shall consider the rotational excitation modes of almost equidistant form to calculate the symmetry of energy levels.

In conclusion, this property of the energy spectra is concluded very important since the possibility of the use by the collective modes in the identification of the physical characteristic such as nuclear level density parameter, entropy, temperature in the applied nuclear physics.

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