

2nd Order RC Circuit

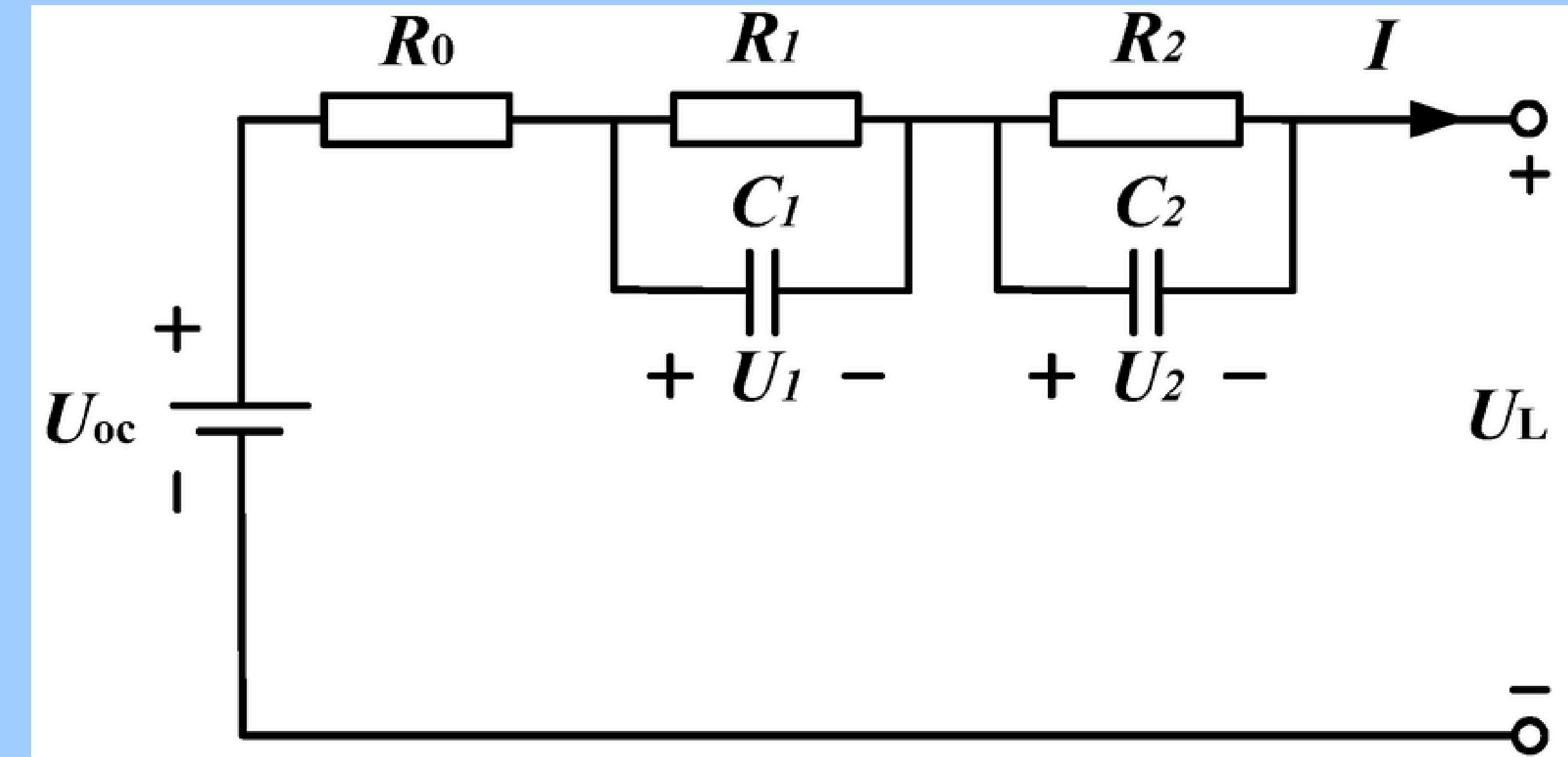


Second Order RC Circuit Using Laplace Transform

- A second-order circuit contains two independent energy storage elements
- In RC circuits, energy storage elements are capacitors
- Two capacitors \Rightarrow second-order system

Laplace Transform:

- Converts differential equations to algebra
- Simplifies analysis of higher-order circuits



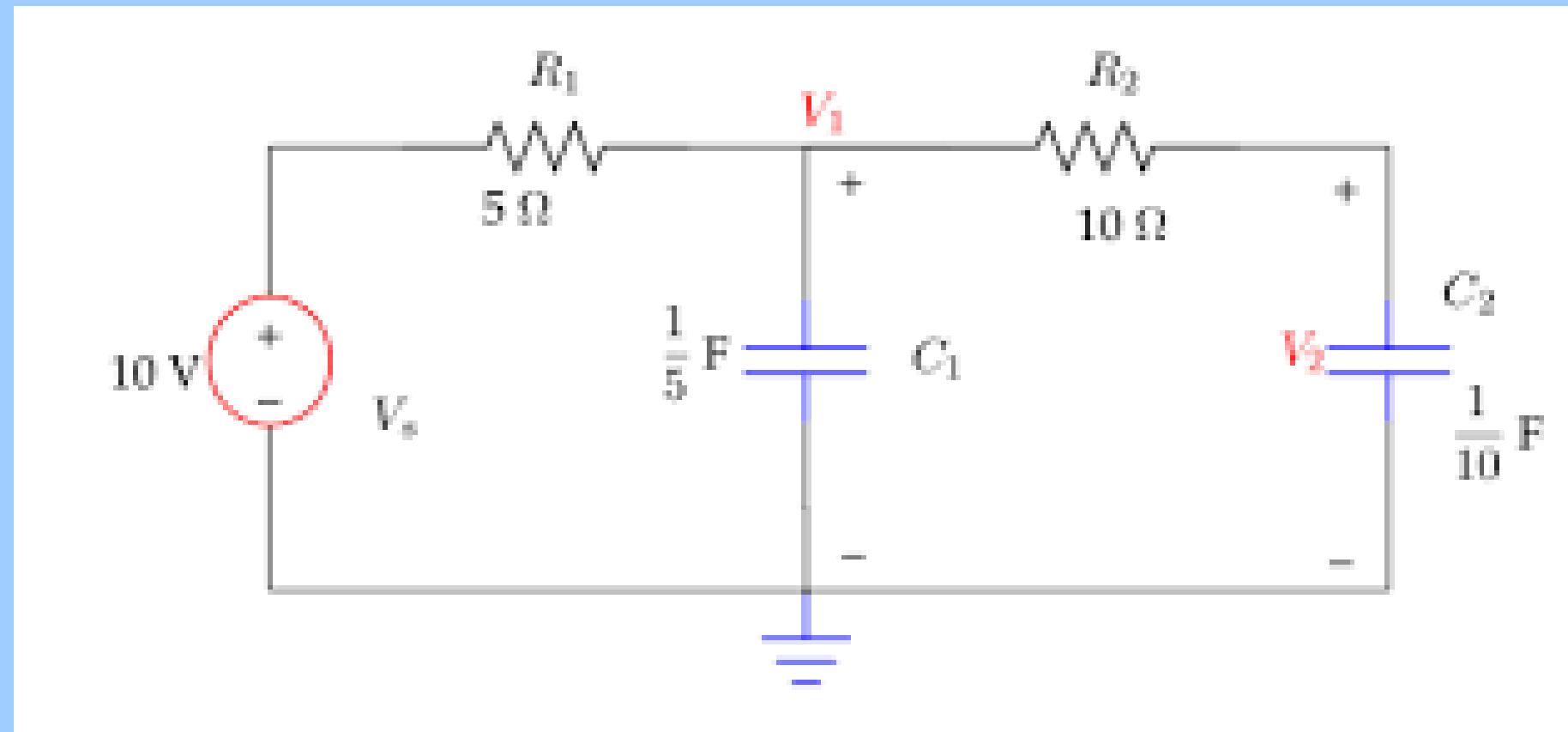
Basic Second-Order RC Circuit Structure

- Contains:

Resistors: R_1, R_2, R_1, R_2

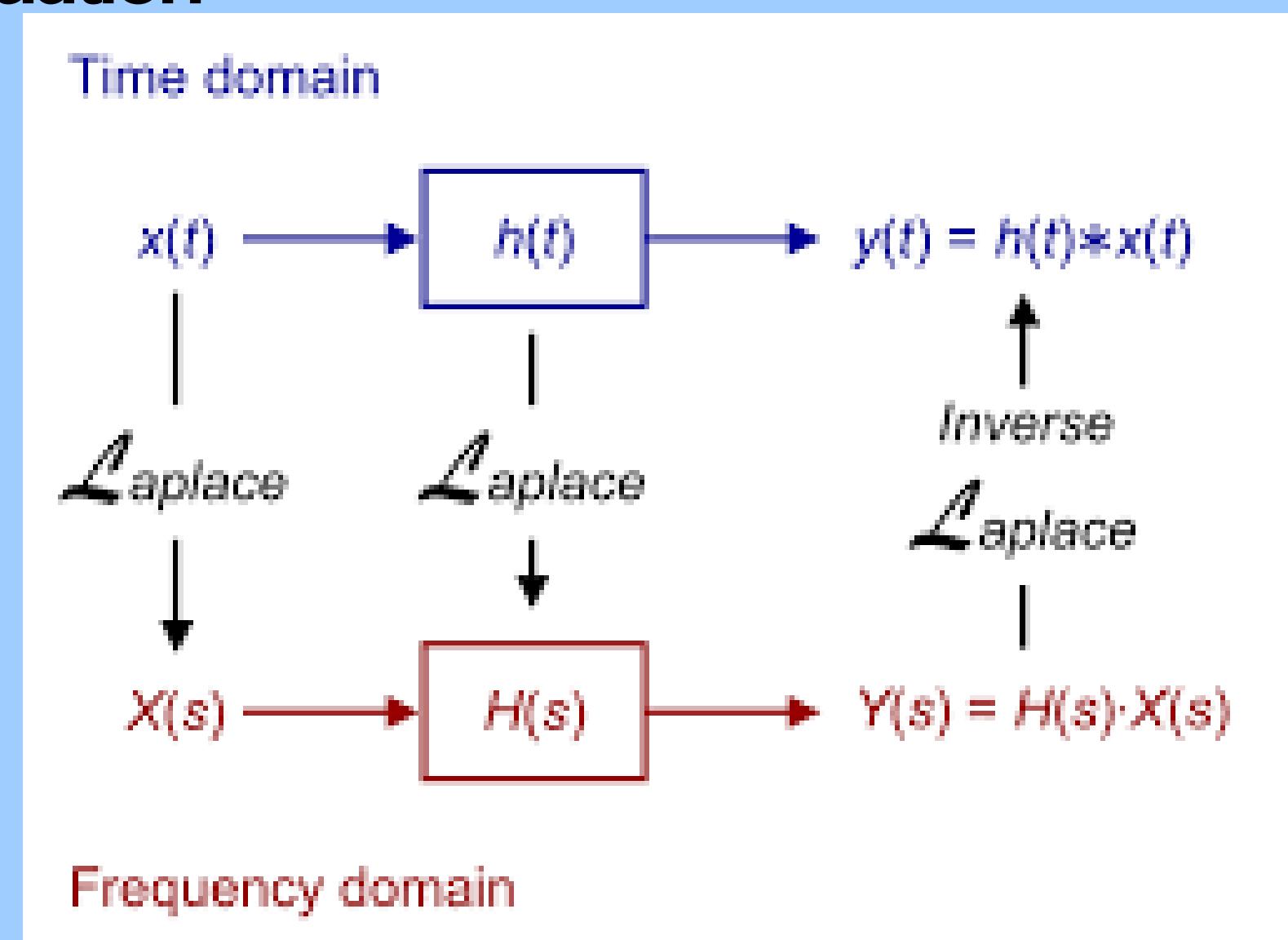
Capacitors: C_1, C_2, C_1, C_2

- Input: $V_i(t)$, Output: $V_o(t)$
- Two capacitors \Rightarrow two energy storage states
- Leads to a second-order differential equation



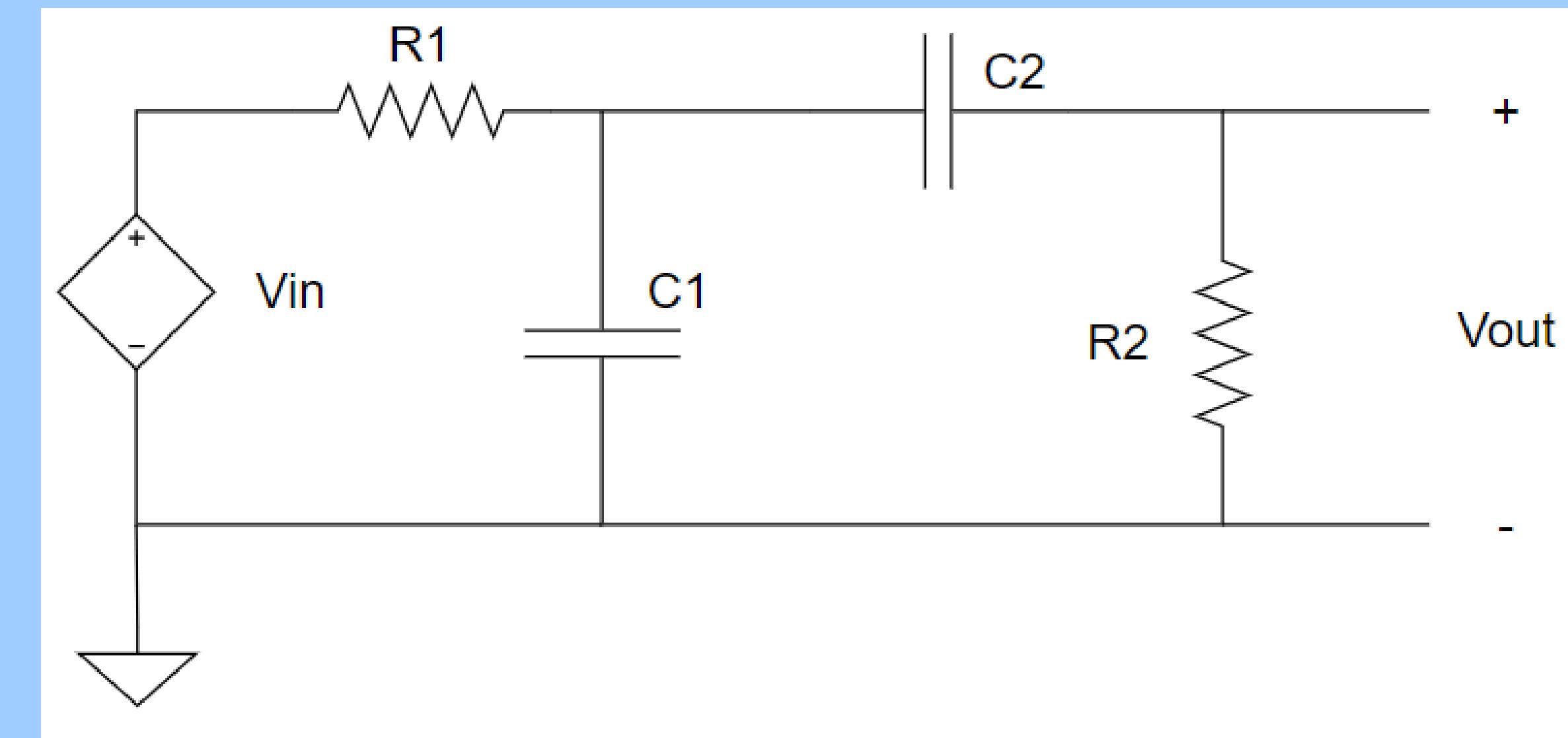
Why Use Laplace Transform?

- Time domain equations include derivatives:
- $i(t) = Cdv(t)/dt$, $d[sqrt{v(t)}]/(dt^2)i(t) = C$
- Two capacitors → second-order differential equation
- Laplace Transform:
- Converts differentiation to multiplication by s
- Handles initial conditions
- Makes algebraic circuit equations



Laplace Domain Model of Second-Order RC

- Impedance in s-domain:
 $Z_R = R$, $Z_C = 1/sC$
- Solve using voltage division or mesh equations
- Final transfer function:
 $H(s) = V_o(s)/V_i(s) = K/s^2 + as + b$



Where:

- $a = \text{sum of resistive \& capacitive terms}$
- $b = \text{product of resistive and capacitive terms}$

Conclusion:

Highest power of s is 2 \Rightarrow second-order circuit

- Denominator:

$$s^2 + as + b = 0 \quad s^2 + a s + b = 0 \quad s^2 + as + b = 0$$

- Roots determine behavior:

Real & distinct \Rightarrow Overdamped

Real & equal \Rightarrow Critically damped

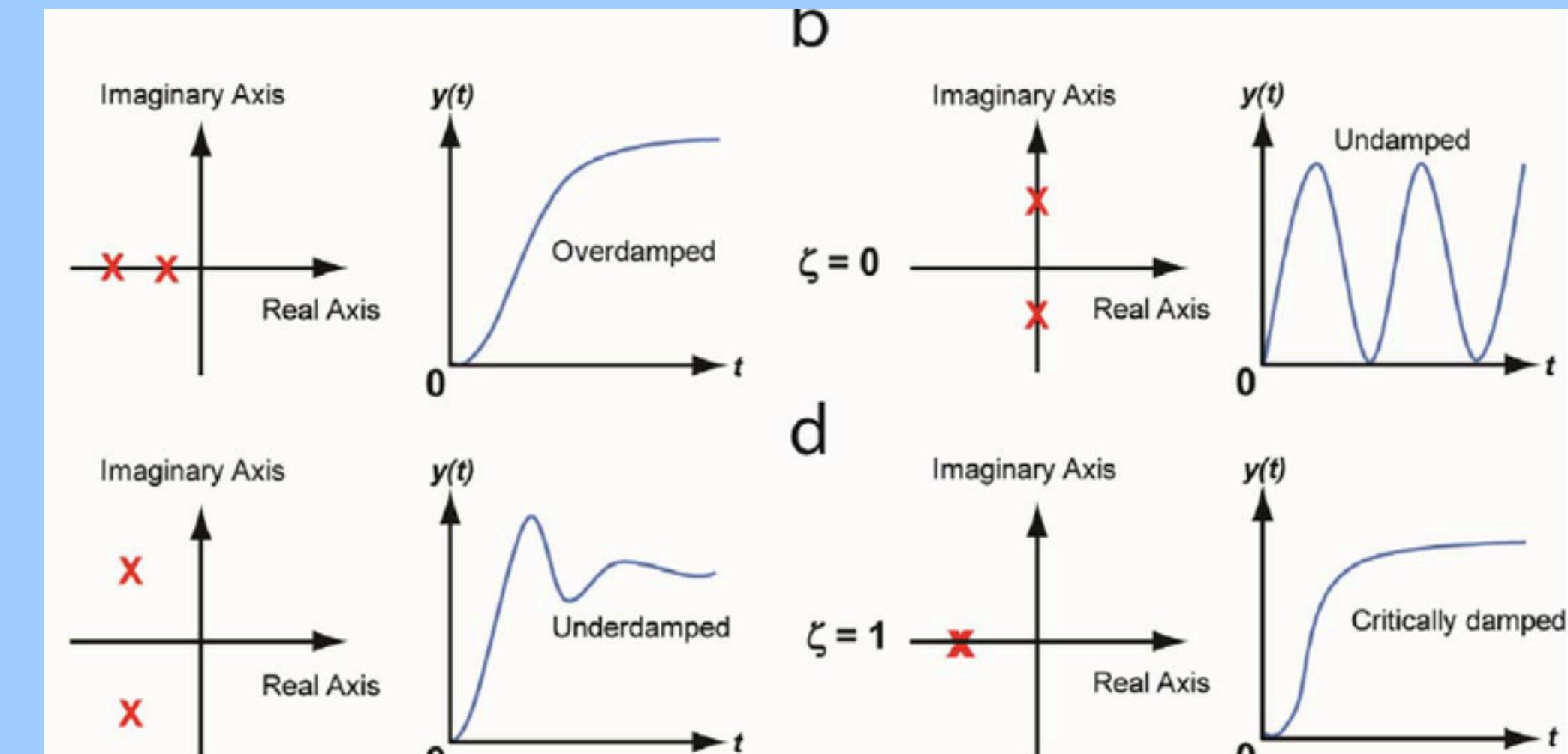
- Complex conjugate \Rightarrow Underdamped

- RC networks usually show

overdamped/critically damped behavior (no oscillation)

- Key Insight:

- Response shaped by two poles



Frequency Response & Practical Uses

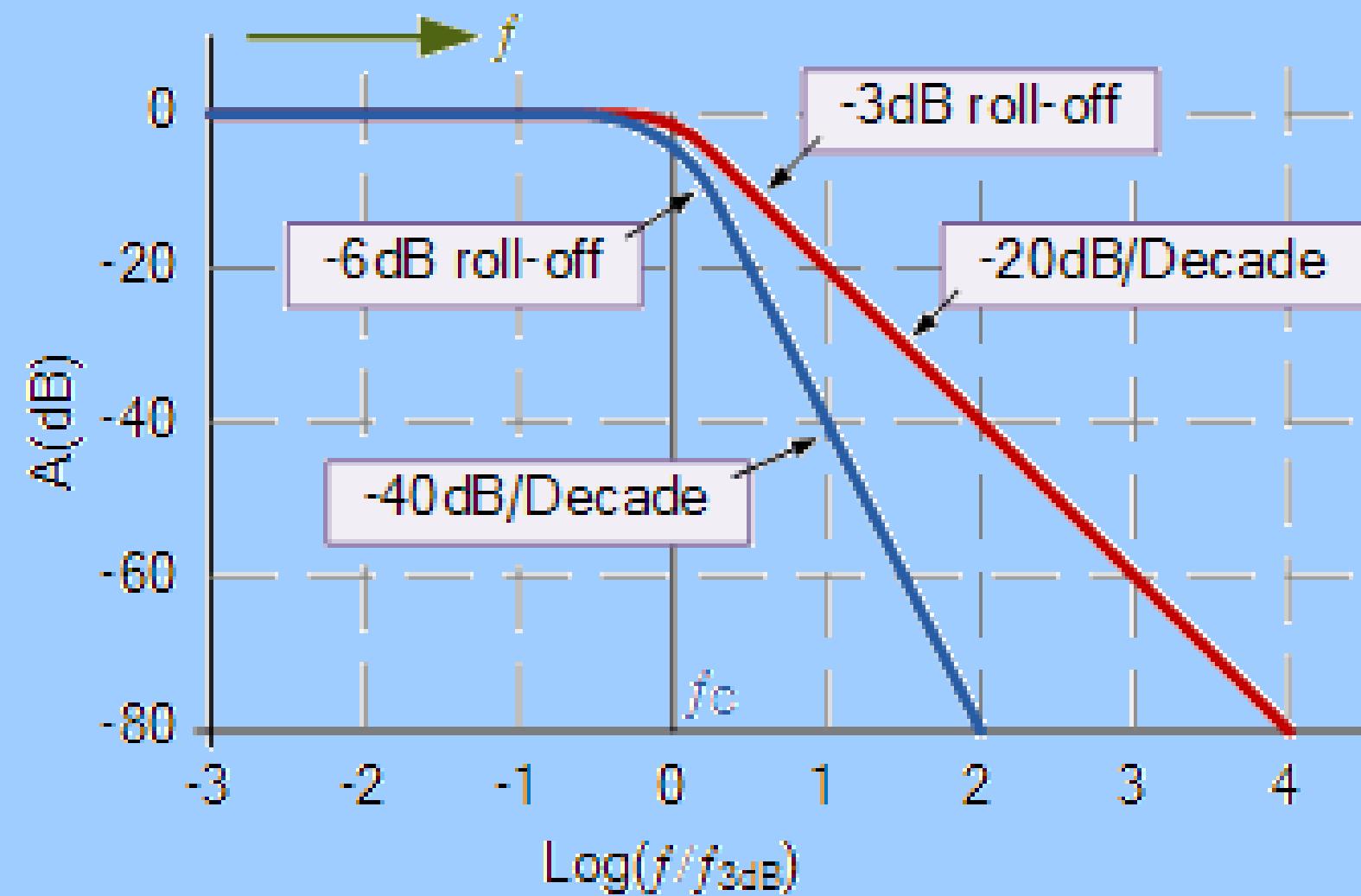
- Second-order RC can be: Low-pass, High-pass
- Band-pass depending on configuration
- Compared to first order: Steeper attenuation

Better filtering

- Roll-off: -40 dB/decade

Applications

- Power supply noise filtering
- Audio signal conditioning
- Communication circuits



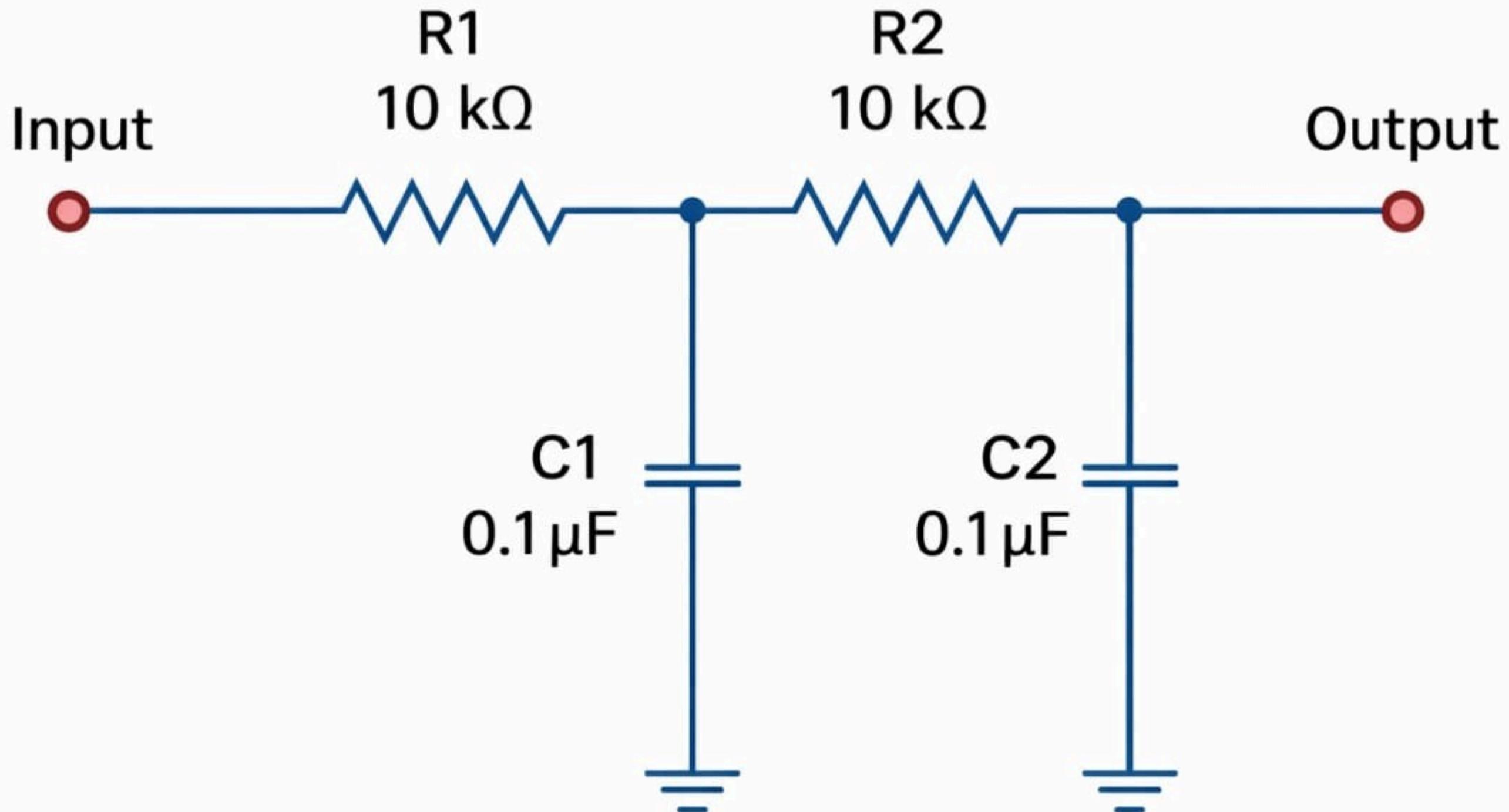
Application :Passive Noise Reduction Using Second Order RC Circuit

- **Noise is an unwanted high-frequency signal present in electrical systems**
- **Passive noise reduction uses R and C elements only**
- **Second order RC circuits provide:**
- **Better noise attenuation than first order circuits**
- **Sharper frequency roll-off**
- **Laplace Transform is used to:**
- **Model noise filtering behavior mathematically**
- **Analyze system response efficiently**

Objective

- **To demonstrate how a second-order RC circuit reduces noise using Laplace Transform**

Second-Order RC Low-Pass Filter Circuit



Second Order RC Low-Pass Filter Configuration

Circuit consists of:

- Two resistors: $R_1, R_2, R_{_1}, R_{_2}$
- Two capacitors: $C_1, C_2, C_{_1}, C_{_2}$
- Two cascaded RC sections

Each RC section contributes:

- One pole

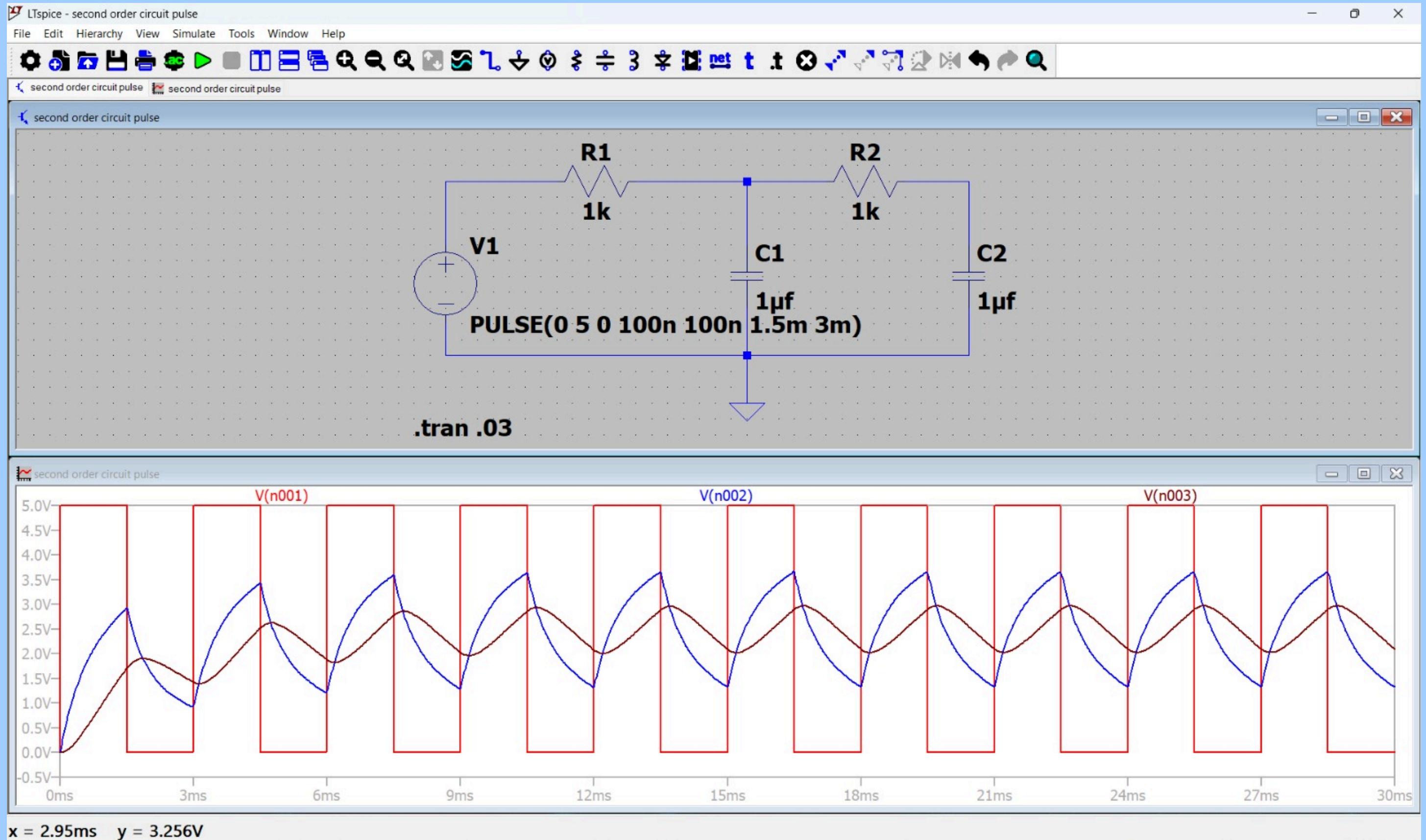
Total system order:

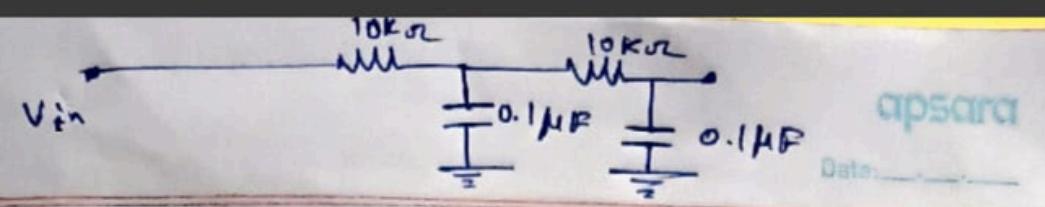
- Two poles \rightarrow second-order system

Filtering Action

- Low-frequency signals pass with minimal attenuation
- High-frequency noise is significantly reduced

Simulation and theoretical approach





Given in

parameter value

Input voltage 1.5 V_{pp}

Frequency 1.5 kHz

Output voltage 40mV_{pp}

R

10 k_Ω

C

0.1 μF

Circuit

2nd Order RC

Theoretical Calculation.

Cutoff frequency.

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times (10k) (0.1\mu F)} = 159 \text{ Hz}$$

Frequency ratio

$$\frac{f}{f_c} = \frac{1500}{159} \approx 9.48$$

Gain of 2nd order RC filter.

$$= |H(j\omega)| = \frac{1}{1 + (\omega RC)^2}$$

$$= \frac{1}{1 + (9.48)^2}$$

$$= \frac{1}{1 + 88.9} \Rightarrow 0.011$$

Expected Output voltage.

$$V_{out} = 1.5 \times 0.011$$

$$V_{out} = 0.0267 \text{ V} \approx 27 \text{ mV.}$$

Practical value (we got)

$$V_{out} = 40 \text{ mV}$$

The variation is due to Capacitor tolerance
of $\pm 20\%$.