Hypothesis testing

1. μ is known $\Longrightarrow$ Normal distribution

Null hypothesis

Alternative hypothesis

Type I $\Rightarrow$ H_0 , $\mu = \mu_0$
$\Rightarrow$ H_1 , $\mu \neq \mu_0$

Type II $\Rightarrow$ H_0 , $\mu \leq \mu_0$
$\Rightarrow$ H_1 , $\mu > \mu_0$

Type III $\Rightarrow$ H_0 , $\mu \geq \mu_0$
$\Rightarrow$ H_1 , $\mu < \mu_0$

Type 1 hypothesis testing

$$\left.\begin{aligned} H_0 &: \mu = \mu_0 \\ H_1 &: \mu \neq \mu_0 \end{aligned}\right\} \quad x_1, x_2, \ldots x_n \rightarrow N(\mu, \sigma^2)$$

step 1. σ^2 is known

$$x_n \sim N(\mu, \sigma^2)$$

$$\overline{x}_n \sim N(\mu, \sigma^2/n)$$

$$Z = \frac{\overline{x}_n - \mu}{\sigma/\sqrt{n}} \longrightarrow N(0,1)$$

step 2 : since σ^2 & n are known

lets assume null hypothesis i.e $\mu = \mu_0$

$$Z = \frac{\overline{x}_n - \mu_0}{}$$

$\overline{}$
$\sigma/\sqrt{n}$

now plug the values and find Z

from from problems wkt 95% chance of Z to lie in $+2$ to -2.

(Null hypothesis) → try to obstruct change as much as possible using confidence interval even after that if you are seeing change → (Alternative hypothesis) then alternative hypothesis must be true.

We will accept H_0 and reject H_1 or accept H_1 and reject H_0.

<u>Type II : hypothesis testing</u>

$x_1, x_2, \dots, x_n \rightarrow N(\mu, \sigma^2)$, σ^2 is known

$H_0 : \mu \le 25$ → existing knowledge car milage can never be more than 25

$H_1 : \mu > 25$ → the car company is claiming that milage is greater than 25.

<u>step1</u>

$$\overline{X}_n \rightarrow N(\mu, \sigma^2/n)$$

$$\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \longrightarrow N(0,1)$$

consider $\mu \le 25 \Rightarrow -\mu \ge -25$

$$Z = \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \ge \frac{\overline{X}_n - 25}{\sigma/\sqrt{n}}$$

for standard normal

$\therefore$ If confidence Interval is 95% Then $Z < 1.64$

$$\frac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}} \leq Z < 1.64$$

$\Rightarrow$ If this is true Then null hypothesis is correct. The claim by alternative hypothesis That car milage is greater Than 25 is false

H_0 accepted , H_1 rejected

$\Rightarrow$ If this is false then null hypothesis is wrong. The claim made by alternative hypothesis that car milage is greater than 25 is true.

H_0 rejected , H_1 accepted

Type 3

$H_0 : \mu \geq 25$, $x_n \longrightarrow N(\mu, \sigma^2)$

$H_1 : \mu < 25$, $\bar{x}_n \longrightarrow N(\mu, \sigma/\sqrt{n})$

$\therefore$ If confidence Interval is 95% Then $Z > 1.64$

$$\boxed{\frac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}} \geq Z > -1.64}$$

$\longrightarrow$ If this is true null hypothesis is correct. H_0 is accepted and H_1 is rejected

$\longrightarrow$ If this is false alternative hypothesis is correct. H_0 is rejected & H_0 is accepted.

Type 1

$H_0 : \mu = \mu_0$

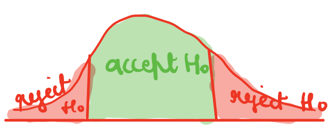
$H_1 : \mu \neq \mu_0$

TS: $\dfrac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}}$

$-2 \le TS \le +2$ — true → accept H_0

false → reject H_0

95%



Type 2

$H_0 : \mu \le \mu_0$

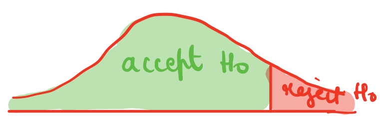
$H_1 : \mu > \mu_0$

TS: $\dfrac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}}$

$TS < 1.64$

true → accept H_0

false → reject H_0



Type 3

$H_0 : \mu \ge \mu_0$

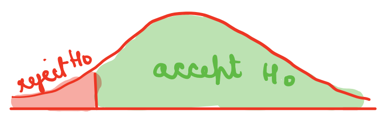
$H_1 : \mu < \mu_0$

TS: $\dfrac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}}$

$TS > -1.64$

true ⇒ accept H_0

false ⇒ reject H_0



Hypothesis testing

Student t - distribution

change the 'σ^2' the population's variance to 'S^2' variance of sample's variance

Type 1

$H_0 : \mu = \mu_0$

$H_1 : \mu \neq \mu_0$

TS: $\bar{x}_n - \mu_0 \rightarrow t_{n-1}$

Type 2

$H_0 : \mu \le \mu_0$

$H_1 : \mu > \mu_0$

TS: $\bar{x}_n - \mu_0 \rightarrow t_{n-1}$

Type 3

$H_0 : \mu \ge \mu_0$

$H_1 : \mu < \mu_0$

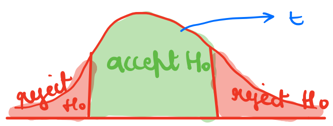
TS: $\bar{x}_n - \mu_0 \rightarrow t_{n-1}$

$S/\sqrt{n}$

$-1.98 \leq TS \leq +1.98$ } 95%

accept H_0 — true

reject H_0 — false

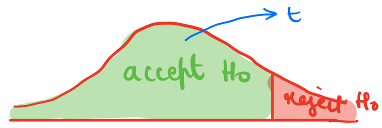


$S/\sqrt{n}$

$TS < 1.68$

true → accept H_0

false → reject H_0

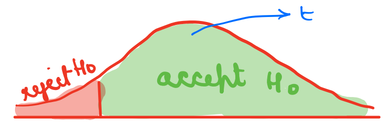


$S/\sqrt{n}$

$TS > -1.68$

true ⇒ accept H_0

false ⇒ reject H_0



P value

→ P value is the probability the null hypothesis is true.

→ We don't accept or reject anything

Calculate $TS = \dfrac{\bar{x}_n - \mu}{\sigma/\sqrt{n}}$ = 'some value'

In null hypothesis significance testing, the p-value is the probability of obtaining test results at least as extreme as the results actually observed, under the assumption that null hypothesis is correct.

Type 2

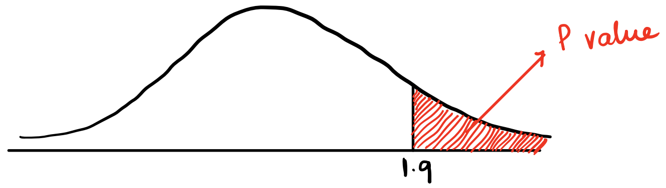
$H_0 : \mu \leq \mu_0$

$H_1 : \mu \geq \mu_0$

$TS = \dfrac{\bar{x}_n - \mu_0}{}$

$\sigma/\sqrt{n}$

ex ⇒ value 1.9, The extreme value would be more than 1.9



Programming grocery shopping

1) $\mu_0 < \$120$

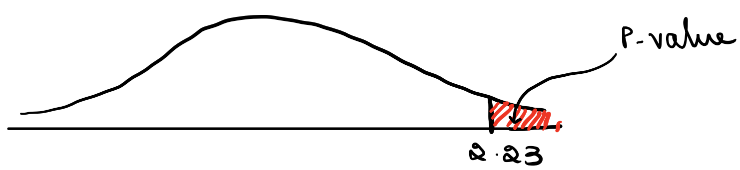
$n = 80 \begin{cases} \bar{x}_n = \$130 \\ \sigma = \$40 \end{cases}$ $\because n \geq 30$, normal distribution

$H_0 \rightarrow \mu_0 \leq 120$

$H_1 \rightarrow \mu_0 > 120$

soln : Therefore this is type 2 hypothesis

$$\frac{\bar{x}_n - \mu_0}{\sigma/\sqrt{n}} = \frac{130-120}{40/\sqrt{80}} = \frac{10}{40/4\sqrt{20}} \cdot \frac{\sqrt{20}}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5} = 2.23$$



p-value is — right hand side area from 2.23

1 - stats.t.cdf (2.23, df = 79)

$= \underline{0.014} \Rightarrow 1.4\%$

This means there is only 1.4% chance that null hypothesis is true. So we reject null hypothesis and accept alternative hypothesis.

<u>problem 2</u> <u>Call centre</u>

18 months data $\mu = 4$ min, $\sigma = 3$ min → this is normal

mean = 4.0

std = 3

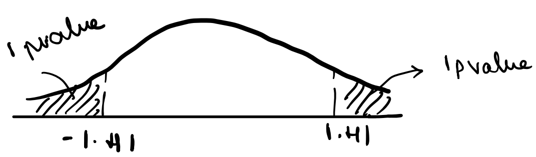
Sample mean = 50 (n)

sample mean = 4.6

<u>solu^n</u> : $H_0 : \mu = 4$

$H_1 : \mu \neq 4$

$$TS = \frac{\bar{x} - 4}{\sigma/\sqrt{n}} = \frac{4.6 - 4}{3/\sqrt{50}} = \frac{0.6}{3/\sqrt{50}} = \frac{0.2}{\sqrt{50}} = 1.41$$



given sample size

$\therefore$ p-value = 2* stats.t.cdf $(-TS, \underbrace{df = 49}_{\text{degrees of freedom} = n-1 = 50-1})$

0.16 ≃ 16%

= 49

→ There is greater chance of null hypothesis being true.

One tail test

Problem 3 : $n = 9$

samples :

Two sample t-Tests

Hypothesis is Type I

If you have multiple datas then

a) two items

$\Rightarrow C_1 \quad C_2 \Rightarrow$ 1 test

b) 3 items

$\Rightarrow C_1 \quad C_2 \quad C_3 \Rightarrow$ 3 test

c) 4 items

$C_1 \quad C_2 \quad C_3 \quad C_4 \Rightarrow {}^4C_2$

${}^nC_2$

$\boxed{{}^nC_2}$ → This is not feasible for higher values of 'n' so we do

ANOVA