Biostatistics: Sheet 3 –
Continuous Random
Variables and Probability
Distributions

1) Suppose X has the pdf $f(x)=3x^2$ for $0<x<1$, $f(x)=0$ otherwise, and the conditional pdf of Y given X=x is:

$$f(y|x) = \begin{cases} \frac{3y^2}{x^3} & for \quad 0 < y < x \\ 0 & otherwise \end{cases}$$

Determine:
(a) The marginal pdf of Y
(b) The conditional pdf of X given Y=y

2) If P, the price of a certain commodity in dollars, and S, total sales(in 10,000) units, are random variables whose joint distribution function can be approximated with the joint probability density

$$f(p,s) = \begin{cases} 5pe^{-ps} & for \ 0.20 < p < 0.40, s > 0 \\ 0 & otherwise \end{cases}$$

Find the probabilities that:

(a) The price will be less 30 cents and sales will exceed 20,000 units
(b) The conditional probability function of X given P=p
(c) The probability that the sales will be less than 30,000 units given P=25 cents
(d) The probability that the sales will be less than 30,000 units given P≤25 cents
(e) The expected sales when P=0.25

3) Suppose X and Y have the joint pdf

$$f(x,y) = \begin{cases} \frac{3}{16}(4-2x-y) & for \ 0 < x, \ 0 < y \ and \ 2x+y < 4 \\ 0 & otherwise \end{cases}$$

a) Find the conditional pdf of Y given X=x
b) $P(Y \geq 2 \mid x = 0.5)$

4) If X and Y are the amount of the overtime and the bonus in thousands of pounds given to a specific worker:

$$f_{XY}(x,y) = e^{-(x+y)} \quad for \ x > 0, y > 0$$

Calculate the probability that this worker will get more than one thousand pounds.

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5) let Z~ N(0,1) (i.e Z is a standard normal random variable)
Find the following probabilities using the tables:
   a) $P(Z \leq 0.8)$
   b) $P(Z \geq -1.43)$
   c) $P(-0.09 \leq Z \leq 1.91)$
   d) $P(-2.2 \leq Z \leq -1.4)$

6) Suppose the cumulative distribution function of the random variable X is:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.25x + 0.5 & -2 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

Determine the following:
   a) $P(X < 1.8)$
   b) $P(X > -1.5)$
   c) $P(X < -2)$
   d) $P(-1 < X < 1)$

7) Suppose that X is a binomial random variable with n=100 and p=0.1.

   (a) Compute the exact probability that X is less than 4.
   (b) Approximate the probability that X is less than 4 and compare to the result in part (a).
   (c) Approximate the probability that 8 <X<12

8) The life of automobile voltage regulators has an exponential distribution with a mean life of six years. You purchase an automobile that is six years old, with a working voltage regulator, and plan to own it for six years.

   (a) What is the probability that the voltage regulator fails during your ownership?
   (b) If your regulator fails after you own the automobile three years and it is replaced, what is the mean time until the next failure?

9) Calls to the help line of a large computer distributor follow a Possion distribution with a mean of 20 calls per minute.
   (a) What is the mean time until the one-hundredth call?
   (b) What is the mean time between call numbers 50 and 80?
   (c) What is the probability that three or more calls occur within 15 seconds?

10) The time between arrivals of customers at an automatic teller machine is an exponential random variable with a mean of 5 minutes.
   (a) What is the probability that more than three customers arrive in 10 minutes?
   (b) What is the probability that the time until the fifth customer arrives is less than 15 minutes?