Measurements and Instrumentation [SBE206A] (Fall 2018)
Tutorial 6 (Revision Problems)

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Problems on first-order systems

1. Exercise

Output variable y responds like a first-order dynamic system when exposed to a sudden change of input variable x. The half-life of the system is 2.00 s. When x is suddenly increased, y grows from an initial value of 100 to a final value of 132. Calculate the time (in seconds) required for y to reach a value of 124.

1. Solution

Typical 2nd-order systems

Second-order spring-mass-damper system

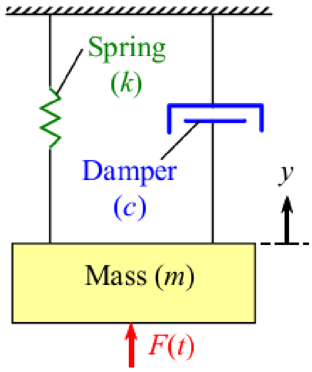

Figure 1: Second-order spring-mass-damper system.

Derivation of the 2nd-order equation

By applying Newton's second law, on the free body diagram in **??**:

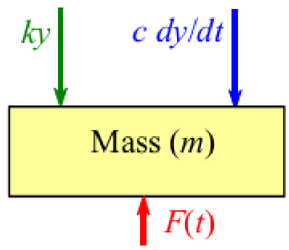

Figure 2: Free body diagram.

$$\sum F = ma \tag{1}$$

$$F_{\text{spring}} + F_{\text{damper}} + F_{\text{input}}(t) = m\frac{d^2y}{dt^2} \tag{2}$$

$$-ky - c\frac{dy}{dt} + F_{\text{input}}(t) = m\frac{d^2y}{dt^2} \tag{3}$$

Applying Laplace transform on Eqn **??**:

$$
\begin{aligned}
-kY(s) - c\left[sY(s) - y(0)\right] + F(s) &= m\left[s^2Y(s) - y'(0) - sy(0)\right] \\
F(s) + y(0) + my'(0) + msy(0) &= \left[ms^2 + cs + k\right]Y(s) \\
Y(s) &= \frac{F(s) + y(0) + my'(0) + msy(0)}{ms^2 + cs + k} \\
&= \frac{F(s)/m + y(0)/m + y'(0) + sy(0)}{s^2 + \frac{c}{m}s + \frac{k}{m}}
\end{aligned}
$$

By equating the denumerator to the characteristic polynomial of the 2$^{\text{nd}}$-order system $N(s) = (s^2 + 2\eta\omega_n s + {\omega_n}^2)$, we realize that:

- $\omega_n = \sqrt{\frac{k}{m}}$
- $\eta = \frac{c}{2\sqrt{km}}$

Problems on 2$^{\text{nd}}$-order systems

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2. Exercise

A spring-mass-damper system is set up with the following properties: mass $m = 22.8$g, spring constant $k = 51.6$ N/cm, and damping coefficient $c = 3.49$ N·s/m. The forcing function is a step function (sudden jump).

1. Calculate the damping ratio of this system. Will it oscillate?
2. If the system will oscillate, calculate the oscillation frequency in hertz. [Note: Calculate the physical frequency, not the radian frequency.] Compare the actual oscillating frequency to the undamped natural frequency of the system.

2. Solution

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3. Exercise

The following second-order ODE:

$$5\frac{d^2y}{dt^2} + \frac{dy}{dt} + 1000y = x(t)$$

The forcing function is a step function (sudden jump):

- $x(t) = 0$ for $t < 0$
- $x(t) = 25$ for $t > 0$

1. Calculate the natural frequency and damping ratio of this system.
2. Calculate the equilibrium response (as $t \to \infty$, what is y?)

3. Solution

4. Exercise

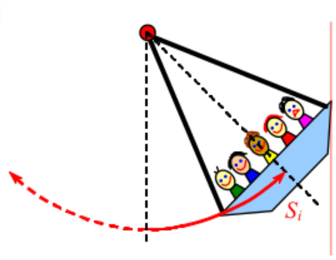

Figure 3: A pendulum-type system.

A pendulum-type amusement park ride behaves as a 2nd-order dynamic system with damping ratio $\eta = 0.1$ and $f_n = 0.125$ Hz. For an initial displacement $S_i = 10.0$ m, calculate the damped natural frequency, the undamped natural frequency, and how long it takes for the oscillations to damp out to within 5% of S_i (the 95% response time).

4. Solution