Measurements and Instrumentation [SBE206A] (Fall 2018)
Midterm Answers

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Question 1: Instrument Types

1. Exercise

Figure 1:

opposite and below figures show a digital travel luggage scale and its technical specifications. Assume the scale has a quoted inaccuracy of $\pm 1\%$ of the full-scale reading. The scale operates as follows:

1. Pull the nylon strap through the handle of the luggage that you wish to weigh.
2. Hook the buckle of the strap into the triangular loop.
3. Press the power button on the scale. The display will show 8888 and the selected weight measuring unit.
4. After about two second the scale will be ready for weighing (0.0 will be shown on the display).
5. Now carefully lift the scale together with the luggage in such a way that no part of the luggage is touching the ground or the mat that it is on.
6. After measuring the weight of the weighed item, the measuring unit will flash twice and remain on the display for 30 seconds so that you have enough time to record the weight.
7. Turn off the scale by holding down the power button for 3 seconds.

TECHNICAL SPECIFICATIONS

Power supply	3V lithium battery (CR2032)
Measuring range	0 – 50 kg
Resolution	100 g
Switching between weight units	kg --> lb --> st
Visible display size	62 x 29 mm
Dimensions (w x d x h)	149 x 30 x 47 mm
Weight	95 g

Answer the following:

1. Specify with justifications whether this instrument is a passive or active instrument,
2. a deflection-type or null-type instrument, and
3. an analog or digital instrument.
4. Comment on the accuracy and ease of use of this instrument.
5. Can the scale detect a change of 50 g, 95 g, or 149 g? Justify your answers.
6. What is the maximum measurement error expected for this instrument?
7. What is the likely measurement error expressed as a percentage of the output reading if the scale is measuring a weight of 20 kg?

1. Solution

1. Active.
2. Deflection.
3. Digital.
4. (Depends on your previous answers).
5. Can detect 149 g, since it is greater than the instrument resolution.
6. 0.5 Kg.
7. 2.5%.

Question 2: Static Characteristics of Instruments

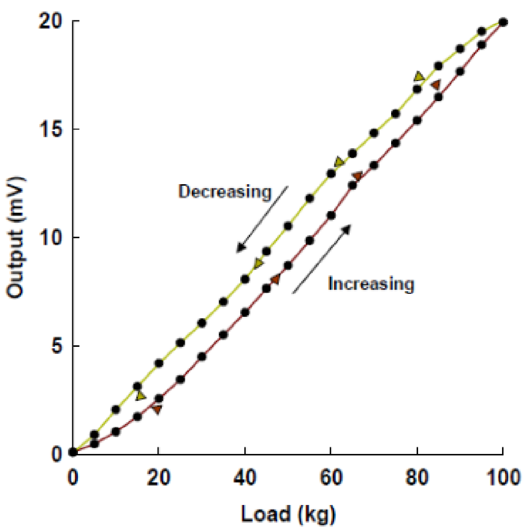
2. Exercise

A load cell is a sensor used to measure weight. A calibration record table is given below. Determine the maximum error (as a percentage of the full-scale output y_{FSO}) for:

1. accuracy $\varepsilon_a = \frac{y_{\text{measured}} - y_{\text{true}}}{y_{\text{FSO}}} \times 100\%$
2. hysteresis $\varepsilon_h = \frac{y_{\text{decreasing}} - y_{\text{increasing}}}{y_{\text{FSO}}} \times 100\%$
3. linearity $\varepsilon_l = \frac{y_{\text{measured}} - y_{\text{L}}}{y_{\text{FSO}}} \times 100\%$

The equation of the best-fit line is $y_L(x) = a_0 + a_1 x$, where $a_1 = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - \Sigma x^2}$, $a_0 = \frac{1}{n}(\Sigma y - a_1 \Sigma x)$, n is the number of data points. Assume that the true or expected output has a linear relationship with the input. In addition, the expected outputs are 0 mV at 0 kg load and 20 mV at 50 kg load.

	Output (mV)	
Load (kg)	Increasing	Decreasing
0	0.08	0.06
5	0.45	0.88
10	1.02	2.04
15	1.71	3.10
20	2.55	4.18
25	3.43	5.13
30	4.48	6.04
35	5.50	7.02
40	6.53	8.06
45	7.64	9.35
50	8.70	10.52
55	9.85	11.80
60	11.01	12.94
65	12.40	13.86
70	13.32	14.82
75	14.35	15.71
80	15.40	16.84
85	16.48	17.92
90	17.66	18.70
95	18.90	19.51
100	19.93	20.02



2. Solution

1. Maximum difference between the measured and true values occurs at point (25,3.43), hence $\varepsilon_a = \frac{3.43-5}{20} = 7.85\%$

2. Maximum difference between the increasing and decreasing pairs occurs at point load 55 Kg, hence $\varepsilon_h = \frac{11.8-9.85}{20} = 9.75\%$
3. Using the calculator (e.g *CASIO fx-570ES PLUS*) on the mode *STAT*: simply issue the *MODE+3*, then select the linear regression model $2: \ \mathrm{A}+\mathrm{BX}$; we obtain the following computed parameters:

n	42
Σx	2100
Σy	409.89
Σx^2	143500
Σxy	28499.45

Accordingly, we compute a_1 and a_0 to get $y_L(x)$ as:
$y_L(x) = -0.6367 + 0.20792x$
The maximum difference between the measured and the fitted values, occurs at load 40 Kg, hence $\varepsilon_l = \frac{7.68-6.53}{20} = 5.75\%$

Question 3: First-Order Instruments

3. Exercise

A thermocouple is immersed in a liquid to monitor its temperature fluctuations. Assume the thermocouple acts as a first-order system.

1. In a planned experiment, the thermocouple is to be exposed to a step change in temperature. The response characteristics of the thermocouple must be such that the thermocouples output reaches 95% of the final temperature within 5 seconds. Assume that the thermocouples bead (its sensing element) is spherical with a density $\rho = 9000\ \mathrm{Kg/m^3}$, a specific heat at constant volume $\mathrm{C_v} = 380\ \mathrm{J/(Kg.K)}$ and a convective heat transfer coefficient $h = 210\ W/(m^2K)$. The time constant of the thermocouple is related to those parameters by $\tau = \frac{\rho \mathrm{d} \mathrm{C_v}}{6h}$, where d is the diameter of the thermocouple's bead. Determine the *maximum* diameter that the thermocouple can have and still meet the desired response characteristics.

2. In another experiment on the same thermocouple, the temperature fluctuations (in $^\circ C$) vary in time as $T(t) = 30 + 25\cos(4t) + 15\sin(2t)$. The output of the thermocouple transducer system $E(t)$ (in mV) is linearly proportional to temperature and has a static sensitivity of $2\ mV/^\circ C$. Find the output $E(t)$ (in mV).

3. Solution

1.

$$\begin{aligned} 0.95 &= 1 - e^{\frac{-5}{\tau}} \\ \tau &= \frac{-5}{\ln(0.05)} \\ &= 1.669s \end{aligned}$$

$$\begin{aligned} 1.669 &= \frac{9000 \times \mathrm{d} \times 380}{6 \times 210} \\ d &= 6.15 \times 10^{-4} \\ &= 0.615mm \end{aligned}$$

2. By solving the system using Laplace transforms:

$$Q_o(s) = \frac{KQ_i(s) + \tau q_o(0)}{\tau s + 1}$$

Subst. $K = 2$, $q_o(0) = 0$, $\tau = 1.67$

$$\begin{aligned}
Q_o(s) &= \frac{2Q_i(s)}{1.67s + 1} \\
\because Q_i(s) &= \frac{30}{s} + \frac{25s}{s^2 + 16} + \frac{30}{s^2 + 4} \\
\therefore Q_o(s) &= \frac{\frac{60}{s} + \frac{50s}{s^2+16} + \frac{60}{s^2+4}}{1.67s + 1} \\
&= \frac{110s^4 + 60s^3 + 1250s^2 + 960s + 3840}{s(1.67s + 1)(s^2 + 16)(s^2 + 4)}
\end{aligned}$$

Using partial fraction decomposition:

$$= \frac{A}{s} + \frac{B}{1.67s + 1} + \frac{Cs + D}{s^2 + 16} + \frac{Es + F}{s^2 + 4} \tag{1}$$

Marking notes:

- Obtaining the Equation (1) secures you 9/12 points of this part.
- Further computation of the constants secures you the full mark.
- Other solutions of DEs definetly apply.

Question 4: Second-Order Instruments

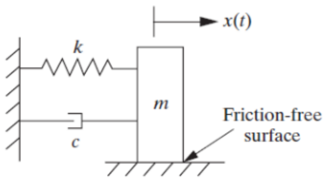


Figure 2:

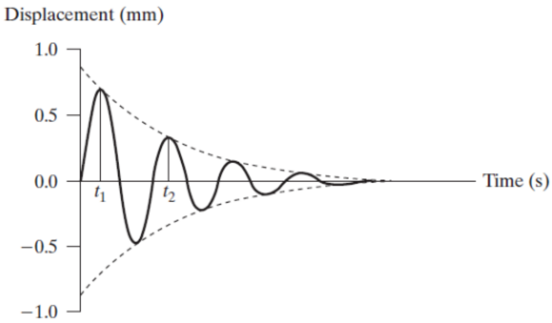


Figure 3:

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4. Exercise

The free (natural) response of a second-order measurement instrument that can be modeled as a mass-spring-damper system in Figure 4 (a) with a mass of 2 kg is recorded to be of the form given in Figure 4 (b). A static deflection test is performed and the stiffness is determined to be $k = 1.5 \times 10^3 \mathrm{N/m}$.

1. Using the displacements at t_1 and t_2, calculate the damping coefficient c.

2. Suggest two ways to make the instrument critically damped.

4. Solution

1.

$$
\begin{aligned}
\omega_n &= \sqrt{\frac{1500}{2}} \\
&= 27.38 \text{rad/s} \\
\frac{D(t_1)}{D(t_2)} &= e^{\alpha \Delta T}
\end{aligned}
$$

from graph:

$$
\frac{0.75}{0.25} = e^{\alpha \Delta T}
$$

applying ln on both sides:

$$
\begin{aligned}
1.1 &= \alpha \Delta T \\
\alpha &= 1.1 f_d \\
&= \frac{1.1\omega_d}{2\pi} \\
\because \omega_n^2 &= \omega_d^2 + \alpha^2 \\
&= \omega_d^2 + (\frac{1.1}{2\pi})^2 w_d^2 \\
\omega_n &= \omega_d \sqrt{1 + (\frac{1.1}{2\pi})^2} \\
\therefore \omega_n &= 1.02\omega_d \\
\therefore \sqrt{1-\zeta^2} &= 1/1.02 = 0.98 \\
\zeta &= 0.2 \\
\because \zeta &= \frac{c}{\sqrt{km}} \\
\therefore c = 11 \text{ N} \cdot \text{Kg/m}
\end{aligned}
$$

2. We manipulate ζ in order to equal 1, by either:
   (a) increasing the damping coeifficient c, or
   (b) decreasing the spring constant.

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5. Exercise

State weather the following statements are true or false and correct the false ones:

1. Measurement sensitivity describes the closeness of output readings when the same input is applied repetitively over a short period of time, with the same measurement conditions.

2. Measurement threshold describes the maximum deviation of a manufactured component from some specified value.

3. A measurement system designer should aim to maximize both of the measurement sensitivity and sensitivity to disturbance.
4. The zero drift of an instrument is a lower limit on the magnitude of the change in the input measured quantity that produces an observable change in the instrument output.
5. The sensitivity drift of an instrument describes the effect where the zero reading of an instrument is modified by a change in ambient conditions.
6. Measurement span is defined as the range of different input values over which there is no change in output value.
7. Null-type and deflection-type instruments require two and three inputs, respectively.
8. Passive instruments have higher resolution than active instruments.
9. The time constant of a first-order instrument is smaller than its half-life time.
10. For an underdamped second-order instrument, the rise time and settling time are equal.

5. Solution

1. False
2. False
3. True
4. False
5. False
6. False
7. False
8. False
9. False
10. False