Measurements and Instrumentation [SBE206A] (Fall 2018)
Tutorial 9

Dr. Muhammed Rushdi   Asem Alaa

December 17, 2018

Descriptive Statistics for Measurements Errors

Drawing Histogram

1. Count n as the number of data points.
2. Determine the number of histogram bins using Sturges' rule: $k = \lceil \log_2 n \rceil + 1$.
3. Determine the range of data points:
   $R = max\{x_i\} - min\{x_i\}$
4. Determine the width w of each bin: $w = \frac{R}{k}$. You may slightly adjust the width to avoid ambiguity.
5. Pinpoint the bins boundary on the graph. To span the whole range using k bins interspaced by w.
6. Calculate the normalized frequency of each histogram $p_i = \Sigma_i^k \frac{m_i}{n}$; where m_i is the number of data points lying in the interval (i.e bin) i.
7. Complete the histogram after determining the height of each bin to be proportional to its normalized frequency.

Testing the Normality of your Data

Suppose that your data (consists of 100 points,$\mu = 499.5$, $\sigma = 8.389$) the first step (which is given) is to plot the histogram of your data. In this demo, the histogram summary is directly given to you as following:

Interval	m_i
$[479.5, 484.5]$	5
$[484.5, 489.5]$	8
$[489.5, 494.5]$	13
$[494.5, 499.5]$	23
$[499.5, 504.5]$	24
$[504.5, 509.5]$	14
$[509.5, 514.5]$	9
$[514.5, 519.5]$	4

The next steps is to map the histogram boundaries to the standardized normal distribution:

- If your data points $n > 31$, use the standardized normal distribution.
- If your data points $n \leq 31$, use the student distribution.

Boundary	479.5	484.5	489.5	494.5	499.5	504.5	509.5	514.5	519.5
$Z(\frac{x-\mu}{\sigma})$	-2.38	-1.792	-1.195	-0.6	-0.004	0.592	1.188	1.784	2.381
$F(z)$	0.0087	0.037	0.116	0.274	0.498	0.723	0.883	0.963	0.991
Interval area									
expected count $m'_i = np$									

Finally,

1. compute the χ^2 as following:

$$\chi^2 = \Sigma \frac{(m'_i - m_i)^2}{m'_i} \tag{1}$$

2. using $K = n - 2 = 8$ as the degree of freedom, compare the obtained χ^2 with the values at confidence level 90% and 95%.

Uncertainty as an Estimated Variance

Defining the squared uncertainty u_i^2 as an estimate of the variance σ_i^2:

$$u_c^2 \approx u_{x1}^2 \left(\frac{\partial r}{\partial x_1}\right)^2 + u_{x2}^2 \left(\frac{\partial r}{\partial x_2}\right)^2 + 2u_{x1x2}\left(\frac{\partial r}{\partial x_1}\right)\left(\frac{\partial r}{\partial x_2}\right) + \cdots \tag{2}$$

Single-Measurement Measurand Experiment

Design-stage uncertainty is expressed as a function of the zero-order uncertainty of the instrument, u_0 , and the instrument uncertainty, u_I , as:

$$u_d = \sqrt{u_I^2 + u_0^2} \tag{3}$$

The **resolution** of an instrument is the *smallest physically indicated division that the instrument displays or is marked*. The zero-order uncertainty of the instrument, u_0 , **is set arbitrarily to be equal to one-half the resolution**.

The instrument uncertainty, u_I, usually is stated by the manufacturer and results from a number of possible elemental instrument uncertainties, e_i . Examples of e_i are hysteresis, linearity, sensitivity, zero-shift, repeatability, stability, and thermal-drift errors. Thus,

$$u_I = \sqrt{\Sigma_i^N (e_i)^2} \tag{4}$$

Problems

1. Exercise

Problem Statement: Some car rental agencies use an onboard global positioning system (GPS) to track an automobile. Assume that a typical GPS's precision is 2% and its accuracy is 5%. Determine the combined standard uncertainty in position indication that the agency would have if

1. it uses the GPS system as is, and
2. it recalibrates the GPS to within an accuracy of 1%.

1. Solution

2. Exercise

Since you cannot measure the kinetic energy (KE) of a motorcycle directly, you settle for measuring its mass (G) and velocity (H). You determined that the average values are $\bar{m} = 500$ kg and $\bar{v} = 20$ m/s. Knowing that

$$\text{KE} = \frac{1}{2}mv^2 \tag{5}$$

what is the most probable uncertainty in your computations of KE if the uncertainty in your mass measurement is 0.3 kg and the uncertainty in your velocity measurement is 0.008 m/s? (Remember to include the unit in your answer)

2. Solution

3. Exercise

A pressure transducer is connected to a digital panel meter. The panel meter converts the pressure transducer's output in volts back to pressure in psi. The manufacturer provides the following information about the panel meter:

Resolution	0.1 psi
Repeatability	0.1 psi
Linearity	with 0.1% of reading
Drift	less than 0.1 psi /6 months within 32 °F to 90 °F range

The only information given about the pressure transducer is that it has "an accuracy of within 0.5 % of its reading". Estimate the combined standard uncertainty in a measured pressure at a nominal value of 100 psi at 70 ° F. Assume that the transducer's response is linear with an output of 1 V for every psi of input.

3. Solution

4. Exercise

Determine the combined standard uncertainty in the density of air, assuming $\rho = P/RT$. Assume negligible uncertainty in R (R air = 287.04 J/kg · K). Let T = 24 °C = 297 °K and P = 760 mmHg.

4. Solution

5. Exercise

A group of biomedical engineering students wish to determine the density of an elliptical cone to be used in the design of a prosthetic device. They plan to determine the density from measurements of the cone's mass, length h, and diameter d, which have instrument resolutions of 0.1 lbm, 0.05 in., and 0.0005 in., respectively. The balance used to measure the weight has an instrument uncertainty (accuracy) of 1%. Each of the different rulers used to measure the length, and diameter presents an instrument uncertainty (accuracy) of

0.5%. Nominal values of the mass, length, and diameter are 4.5 lbm, 6.00 in., and 4.0000 in., respectively.

1. What are the resolution uncertainties for the measurements of the mass, length, and diameter?
2. Compute the sensitivity coefficients of the density with respect to the mass, length, and diameter at the nominal values.
3. Estimate the zero-order uncertainty in the determination of the density.
4. Which measurement contributes the most to the zero-order uncertainty?
5. What are the absolute instrument uncertainties for the measurements of the mass, length, and diameter at the nominal values?
6. Estimate the instrument uncertainty in the determination of the density.
7. Estimate the design-stage uncertainty in the determination of the density.

5. Solution