Section 6

Scale Invariance, MOPS, and SIFT

Presentation by *Asem Alaa*

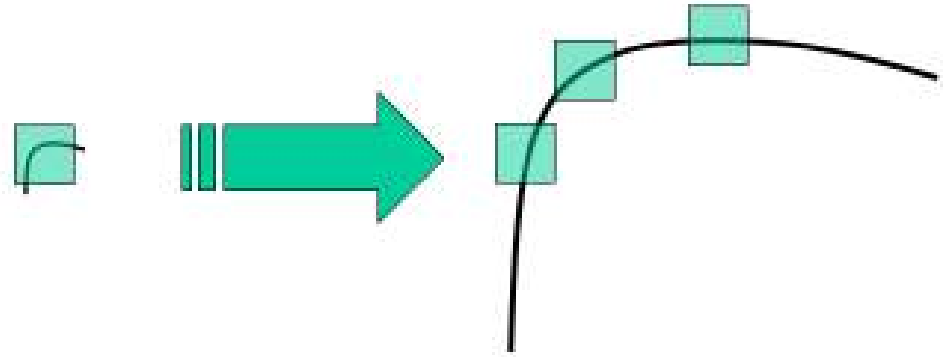
Problem definition

- Features or **key-points** are corners: unique in the image.
- Harris is a corner detector[1]
- Harris operator is invariant for translation, illumination and rotation.
- Harris operator is variant for scaling.

Example

- In smaller image there is a corner.
- Difficult to detect in large scale.
- Not recognized for all scales.
- Size of the window effect detectability.
- Large corners needs large windows, vice versa.

Two different scales



1. {Section 4: Hough Transform and Harris Operator}

How to achieve Scale Invariance

Scale Invariance 1: Multi-Scale Feature Representation

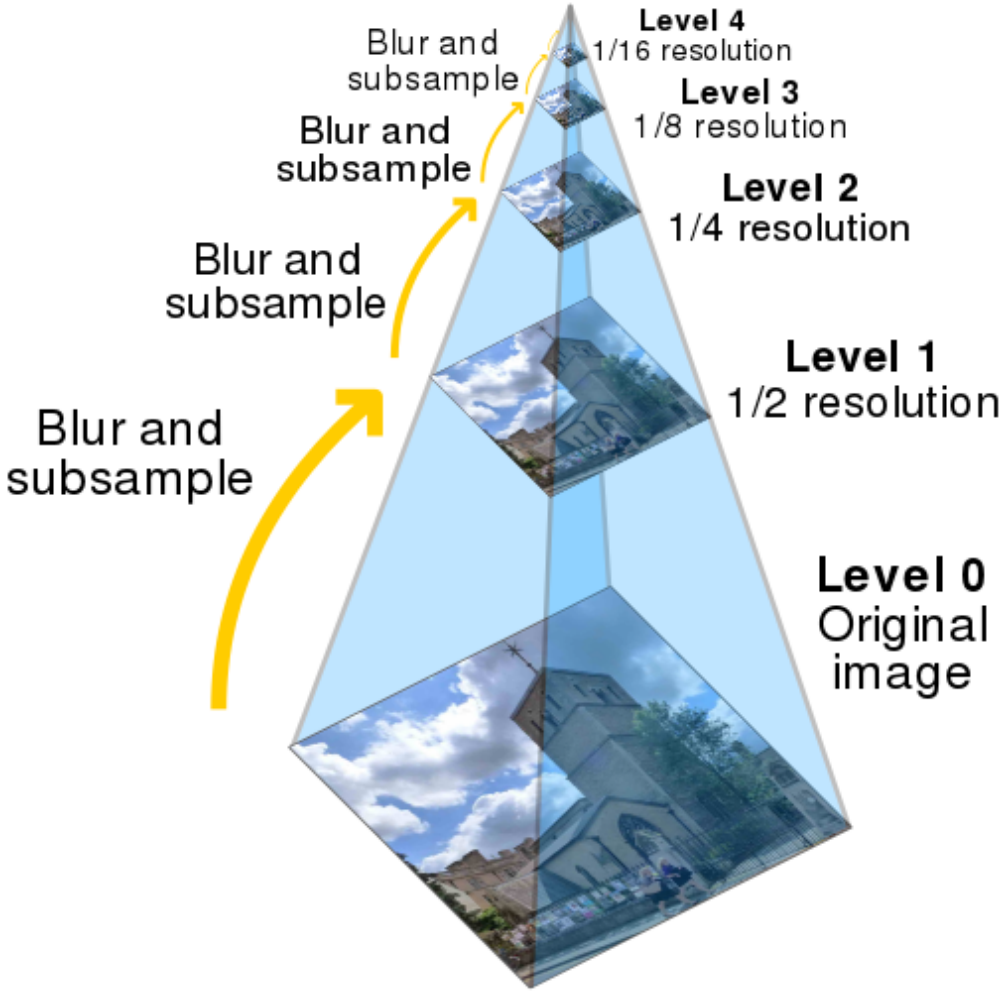
- Common approach: detect features at multiple scales using a Gaussian pyramid
- Example: Multiscale Oriented PatcheS descriptor (MOPS) (Brown, Szeliski, Winder, 2004)

Technical Report (Microsoft Research):

{MSR-TR-2004-133 | December 2004}

Key steps in MOPS

A) Construct Scale Space (Gaussian Pyramid)



- Each image I_s has scale $s \in \{\sigma_1, \ldots, \sigma_5\}$

Images are adapted from slides by Matthew Brown

Key steps in MOPS

B) Keypoints representation

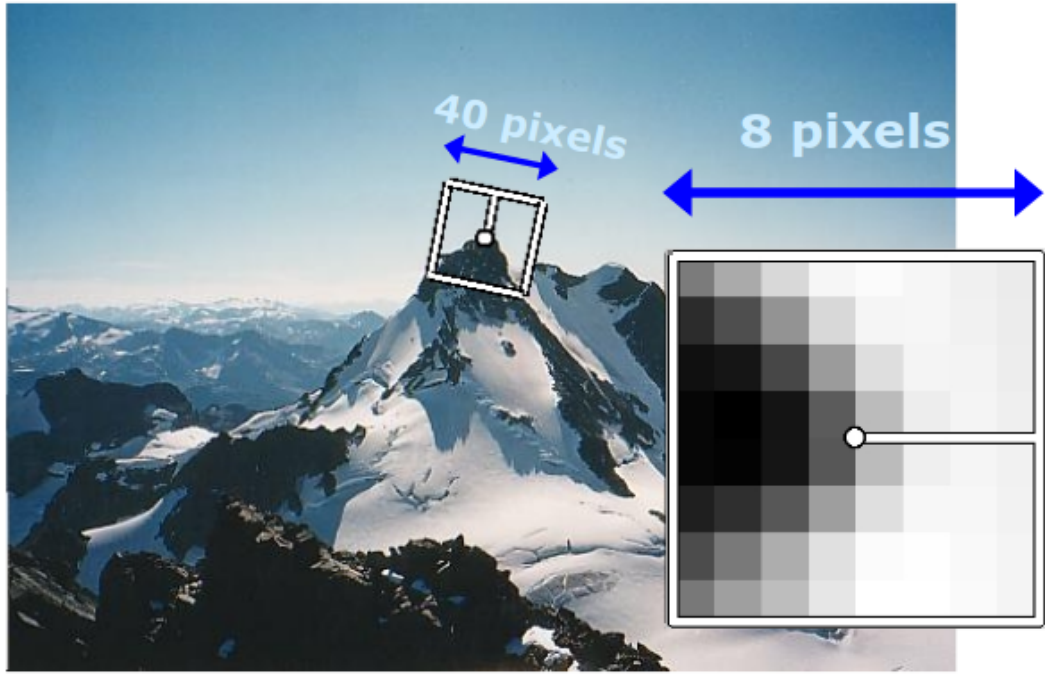
- Extract keypoints (corners) at the current scale $s \in \{\sigma_1, \sigma_2, .., \sigma_5\}$
- Find the dominant orientation θ of the keypoint
- Each keypoint now is represented by (x, y, s, θ)

Images are adapted from slides by Matthew Brown

Key steps in MOPS

C) Photometric normalization

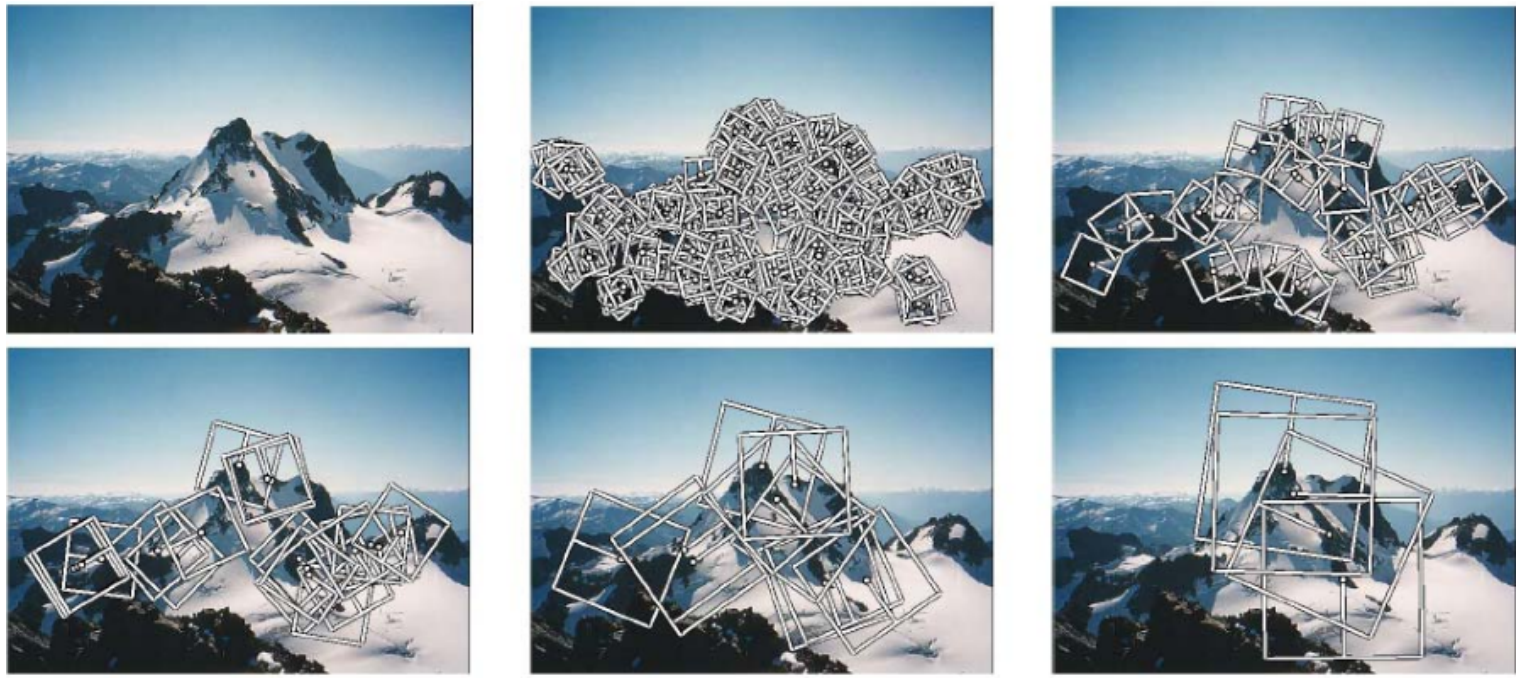
- Extract an **oriented** 40×40 patch around the keypoint
- Scale to 1/5 size (Gaussian smoothing+subsampling)
- Rotate to horizontal and sample 8x8 window.
- Estimate the local mean $\hat{x}$ and the std $\hat{s_x}$ of intensities in the 8x8.
- Normalize intensities $\hat{I} = \frac{I - \hat{x}}{\hat{s_x}}$



Images are adapted from slides by Matthew Brown

Key steps in MOPS

D) Extend to all scales



Note on matching feature descriptors in MOPS

To facilitate the search process:

- a Wavelet transform can be applied to the 8x8 patch.
- extracted coefficients = smaller fingerprint
- can be used to hash-searching other descriptors.

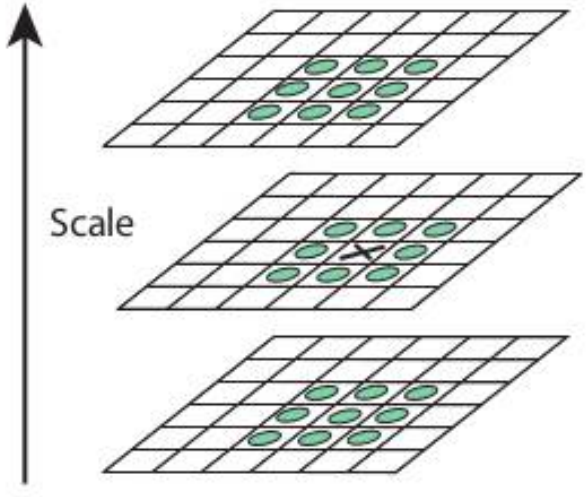
Images are adapted from slides by Matthew Brown

Scale Invariance 2: Capture the best features across scales

- MOPS represents features per-scale.
- In contrast, SIFT captures the best features along the scale axis.

Scale Invariant Feature Transform (SIFT)

- not a new way to find key-points or corners
- descriptor of detected corners of different image scales



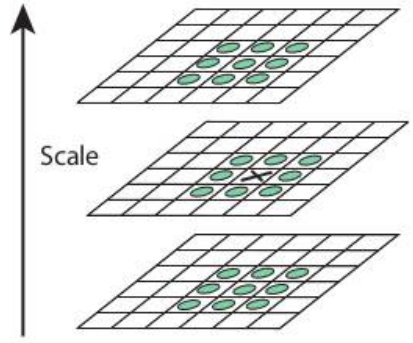
- Developed and patented by Lowe

  {Distinctive Image Featuresfrom Scale-Invariant Keypoints} {Patent}

SIFT

Key steps in SIFT

1. Scale-space construction
2. Scale-space extrema detection
    - find local maxima in DoG space

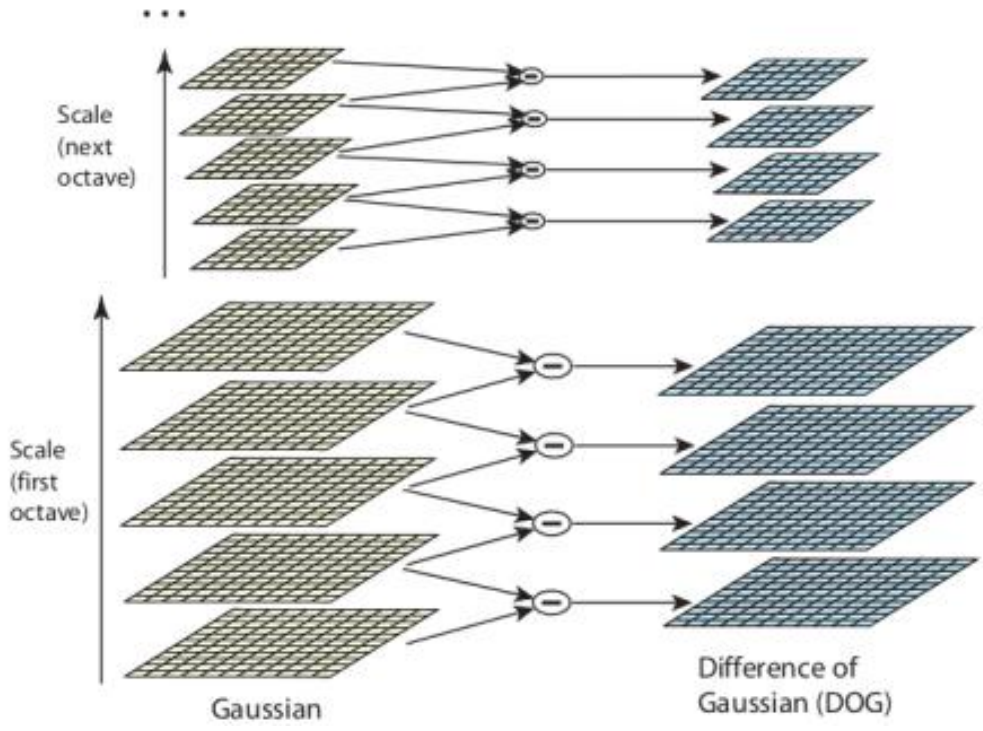


    - Refine the points (x, y, s) from (1).
    - Thresholding: by contrast and cornerness.
3. Orientation assignment
    - HoG
4. Keypoint descriptor
    - HoG

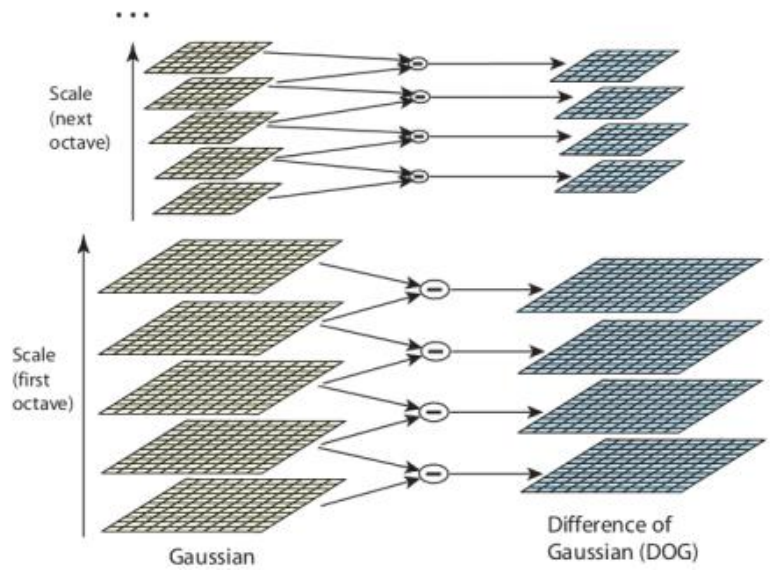
1. SIFT scale space (DOG)

In SIFT Pyramid we have

- Octaves: different levels of image resolutions (pyramids levels)
- Scales: different scales of window in each octave level (different σ of gaussian window)



1. SIFT scale space (DOG)



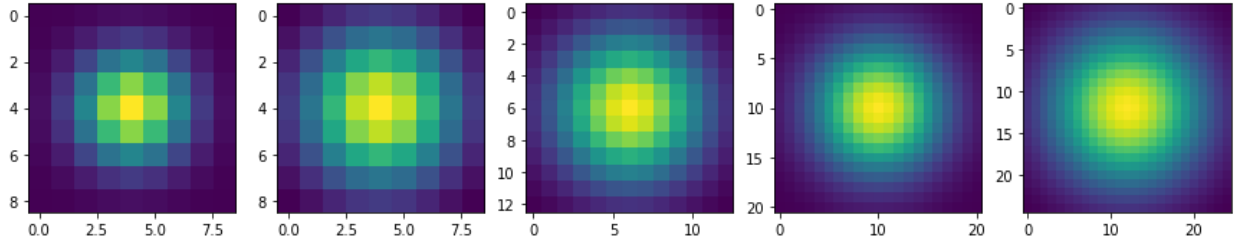
- First octave starts with the $\times 2$ upsampled input.
- Subsequent octaves $\times 2$ subsample their previous.
- Scales: $\sigma, k\sigma, k^2\sigma, k^3\sigma, k^4\sigma$

The author suggests using:

- $\sigma = 1.6$
- $k = \sqrt{2}$

1. SIFT scale space (DOG)

```
# The following are suggested by SIFT author
N_OCTAVES = 4
N_SCALES = 5
SIGMA = 1.6
K = sqrt(2)
# s: (s, √2s , 2s, 2√2 s , 4s )
SIGMA_SEQ = lambda s: [ (K**i)*s for i in range(N_SCALES) ]
# 1.6, 1.6√2, 3.2, 3.2√2, 6.4
SIGMA_SIFT = SIGMA_SEQ(SIGMA)
KERNEL_RADIUS = lambda s : 2 * int(round(s))
KERNELS_SIFT = [ gaussian_kernel2d(std = s,
                                   kernlen = 2 * KERNEL_RADIUS(s) + 1)
                 for s in SIGMA_SIFT ]
```



1. SIFT scale space (DOG)

Python's map, lambda, zip

```
a = [1,2,3,4,5,6]
b = list(map(lambda ai: ai**2, a))
print(b)
```

```
[1, 4, 9, 16, 25, 36]
```

```
a = [1,2,3,4,5,6]
b = list(map(lambda ai: ai**2, a))
print(list(zip(a,b)))
```

```
[(1, 1), (2, 4), (3, 9), (4, 16), (5, 25), (6, 36)]
```

```
print(a[-1])
print(a[:-1])
print(a[1:])
print(list(zip( a[:-1] , a[1:] )))
```

```
6
[1, 2, 3, 4, 5]
[2, 3, 4, 5, 6]
[(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)]
```

1. SIFT scale space (DOG)

```
def image_dog( img ):
    octaves = []
    dog = []
    base = rescale( img, 2, anti_aliasing=False)
    octaves.append([ convolve2d( base , kernel , 'same', 'symm')
                        for kernel in KERNELS_SIFT ])
    dog.append([ s2 - s1
            for (s1,s2) in zip( octaves[0][:-1], octaves[0][1:])])
    for i in range(1,N_OCTAVES):
        base = octaves[i-1][2][::2,::2] # 2x subsampling
        octaves.append([base] +
            [convolve2d( base , kernel , 'same', 'symm')
                        for kernel in KERNELS_SIFT[1:] ])
        dog.append([ s2 - s1
            for (s1,s2) in zip( octaves[i][:-1], octaves[i][1:])])
    return dog , octaves
```

1. SIFT scale space (DOG)

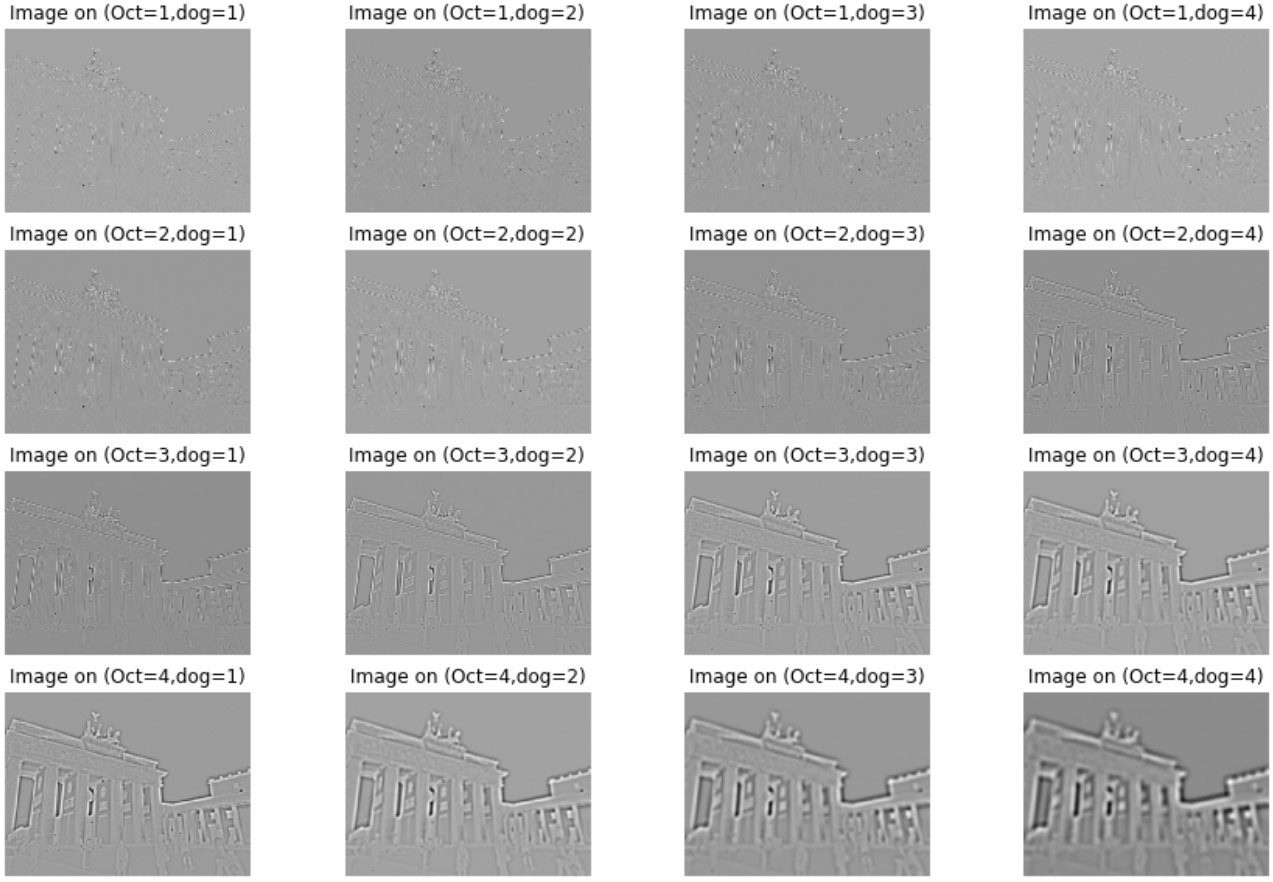
```
plt.imshow(imgs_rgb[0])
plt.axis('off'); plt.show()
```

1. SIFT scale space (DOG)

```
fig, ax = plt.subplots(N_OCTAVES,N_SCALES,figsize = (15, 10))
for octave_idx in range(N_OCTAVES):
    for scale_idx in range(N_SCALES):
        img_scale = img_octaves[octave_idx][scale_idx]
        ax[octave_idx,scale_idx].imshow(img_scale)
```

1. SIFT scale space (DOG)

```
fig, ax = plt.subplots(N_OCTAVES,N_SCALES-1,figsize = (15, 10))
for octave_idx in range(N_OCTAVES):
    for dog_idx in range(N_SCALES-1):
        img_dog = img_dogs[octave_idx][dog_idx]
        ax[octave_idx,dog_idx].imshow(img_dog, cmap = 'gray')
```



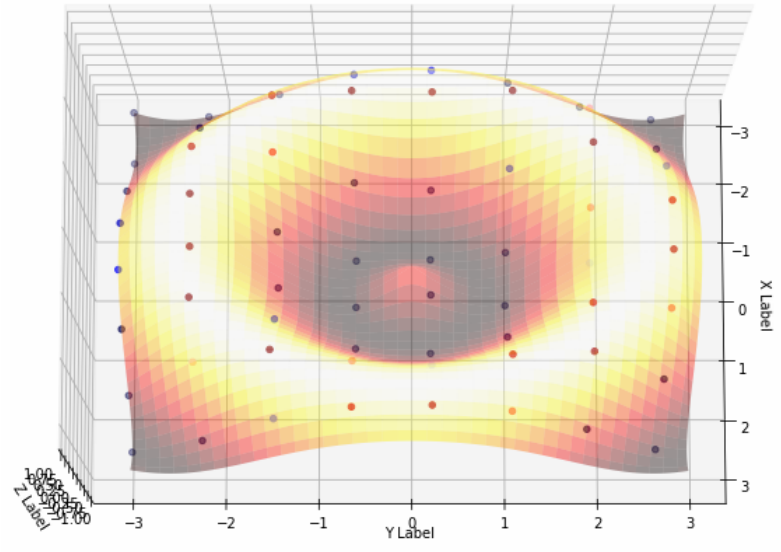
2. Scale-space extrema detection

- find local maxima in DoG space

  - Each pixel is compared with 26 neighbors.
  - 9 from upper scale+9 from lower scale+ 8 from current scale
  - Take if maximum or minimum.
  - For same iamge, it is not necessary for its corners to be localized at same scale.
  - Ignore first and last DoG (no enough neighbors)
- Refine the points $\{(x, y, s)\}$ from (1).
- Thresholding: by contrast and cornerness.

2. Scale-space extrema detection

- find local maxima in DoG space
- Refine the points (x, y, s) from (1).



  - Assume quadratic surface (3 terms from tylor expansion)
  - $D(x) = D + \frac{\partial D^T}{\partial \mathbf{x}}\mathbf{x} + \frac{1}{2}\mathbf{x}^T \frac{\partial^2 D}{\partial \mathbf{x}^2}\mathbf{x}$
  - $\mathbf{x} = (x, y, \sigma)^T$ is the offset from the sample
  - Differentiate and Equate to zero: $\hat{\mathbf{x}} = -\frac{1}{2}\frac{\partial^2 D^{-1}}{\partial \mathbf{x}^2}\frac{\partial D}{\partial \mathbf{x}}$
  - For this section, we will skip this refinement step (left as an exercise).
- Thresholding: by contrast and cornerness.

2. Scale-space extrema detection

- find local maxima in DoG space
- Refine the points $\{(x, y, s)\}$ from (1).
- Thresholding: by contrast and cornerness.
  - $\mathbf{H} = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix}$
  - $\mathrm{Tr}(\mathbf{H}) = D_{xx} + D_{yy} = \lambda_1 + \lambda_2$
  - $\mathrm{Det}(\mathbf{H}) = D_{xx}D_{yy} - D_{xy}^2 = \lambda_1\lambda_2$
  - $\lambda_1 = r\lambda_2$
  - $\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} = \frac{(\lambda_1+\lambda_2)^2}{\lambda_1\lambda_2} = \frac{(r\lambda_2+\lambda_2)^2}{r\lambda_2^2} = \frac{(r+1)^2}{r^2}$
  - Threshold: $\frac{\mathrm{Tr}(\mathbf{H})^2}{\mathrm{Det}(\mathbf{H})} < \frac{(r+1)^2}{r}$
  - Author recommends $r = 10$
  - Contrast thresholding: $|D| > t_c$
  - Author recommends $t_c = 0.03$

2. Scale-space extrema detection

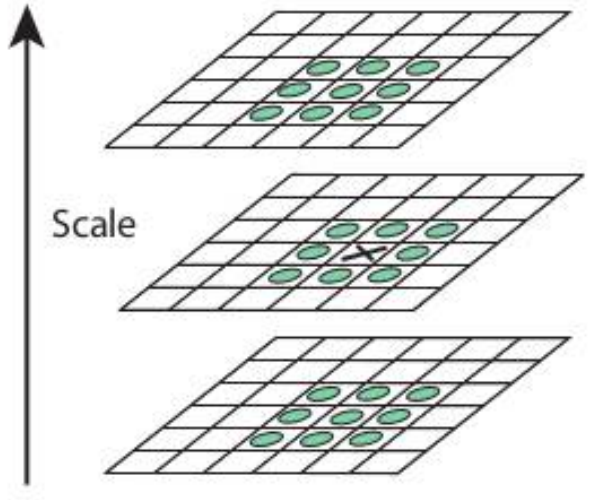
Corner & Contrast test

```
def corners( dog , r = 10.0 ):
    threshold = ((r + 1.0)**2)/r
    dx = np.array([-1,1]).reshape((1,2));  dy = dx.T
    dog_x = convolve2d( dog , dx , boundary='symm', mode='same' )
    dog_y = convolve2d( dog , dy , boundary='symm', mode='same' )
    dog_xx = convolve2d( dog_x , dx , boundary='symm', mode='same' )
    dog_yy = convolve2d( dog_y , dy , boundary='symm', mode='same' )
    dog_xy = convolve2d( dog_x , dy , boundary='symm', mode='same' )

    tr = dog_xx + dog_yy
    det = dog_xx * dog_yy - dog_xy ** 2
    response = ( tr ** 2 ) / det
    coords = list(map( tuple , np.argwhere( response < threshold ).tolist() ))
    return coords

def contrast( dog , img_max, threshold = 0.03 ):
    dog_norm = dog / img_max
    coords = list(map( tuple , np.argwhere( np.abs( dog_norm ) > threshold ).tolist() ))
    return coords
```

2. Scale-space extrema detection

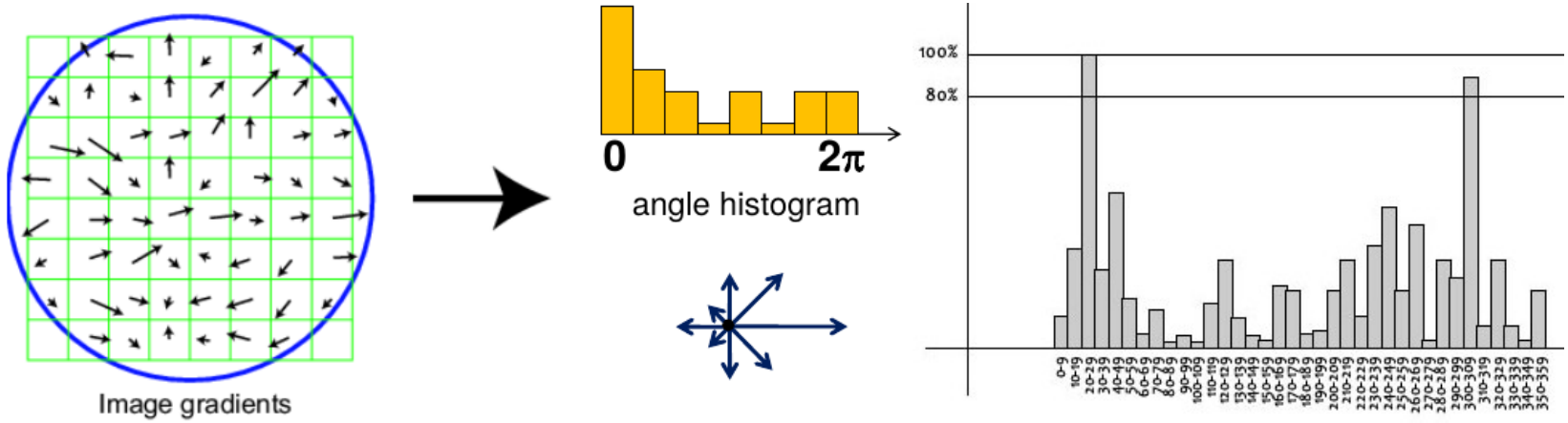



```
def cube_extrema( img1, img2, img3 ):
    value = img2[1,1]
    if value > 0:
        return all([np.all( value >= img ) for img in [img1,img2,img3]])
    else:
        return all([np.all( value <= img ) for img in [img1,img2,img3]])
```

2. Scale-space extrema detection

```
def dog_keypoints( img_dogs , img_max , threshold = 0.03 ):
    octaves_keypoints = []
    for octave_idx in range(N_OCTAVES):
        img_octave_dogs = img_dogs[octave_idx]
        keypoints_per_octave = []
        for dog_idx in range(1, len(img_octave_dogs)-1):
            dog = img_octave_dogs[dog_idx]; h, w = dog.shape
            keypoints = np.full( dog.shape, False, dtype = np.bool)
            candidates= set((i,j) for i in range(1,h-1) for j in range(1,w-1))
            candidates= candidates & set(corners(dog)) & set(contrast(dog,img_max,threshold))
            for i,j in candidates:
                slice1= img_octave_dogs[dog_idx -1][i-1:i+2, j-1:j+2]
                slice2= img_octave_dogs[dog_idx   ][i-1:i+2, j-1:j+2]
                slice3= img_octave_dogs[dog_idx +1][i-1:i+2, j-1:j+2]
                if cube_extrema( slice1, slice2, slice3 ):
                    keypoints[i,j] = True
            keypoints_per_octave.append(keypoints)
        octaves_keypoints.append(keypoints_per_octave)
    return octaves_keypoints
```

3. Keypoints Orientation

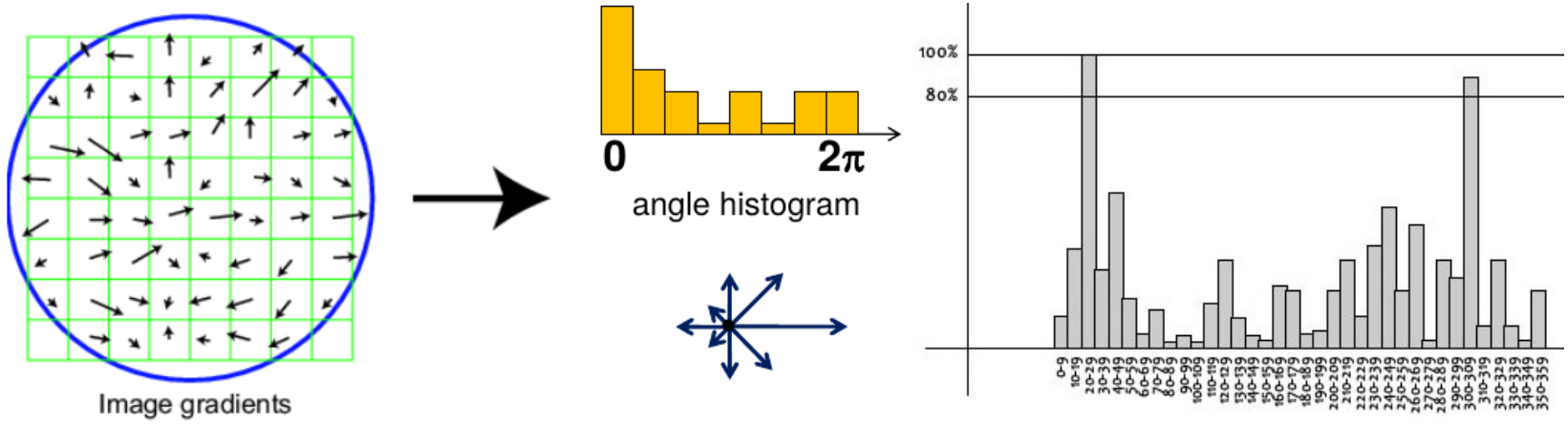


- Each keypoint should assigned a dominant direction (rotation invariance, later).
- $m(x,y) = \sqrt{(L(x+1,y) - L(x-1,y))^2 + (L(x,y+1) - L(x,y-1))^2}$
- $\theta(x,y) = tan^{-1}(\frac{L(x,y+1) - L(x,y-1)}{L(x+1,y) - L(x-1,y)})$

```
def sift_gradient(img):
    dx = np.array([-1,0,1]).reshape((1,3)); dy = dx.T
    gx = signal.convolve2d( img , dx , boundary='symm', mode='same' )
    gy = signal.convolve2d( img , dy , boundary='symm', mode='same' )
    magnitude = np.sqrt( gx * gx + gy * gy )
    direction = np.rad2deg( np.arctan2( gy , gx )) % 360
    return gx,gy,magnitude,direction
```

Image adapted from slides by David Lowe and {aishack.in}

3. Keypoints Orientation



- Get the gradient angles of the window and quantize them to **36** intervals (0, 10, 20, ..., 360).
- Use Gaussian kernel with ($\sigma_\theta = 1.5\sigma$)
- Kernel radius `r = 2 * int(round(sigma))`
- Any peaks above **80%** of the highest peak are converted into a new keypoint.

3. Keypoints Orientation

```
def dog_keypoints_orientations( img_gaussians , keypoints , num_bins = 36 ):
    kps = []
    for octave_idx in range(N_OCTAVES):
        for idx,scale_keypoints in enumerate(keypoints[octave_idx]):
            gaussian_img = img_gaussians[octave_idx][ scale_idx + 1 ]
            sigma = 1.5 * SIGMA * ( 2 ** octave_idx ) * ( K ** (scale_idx))
            radius = KERNEL_RADIUS(sigma)
            kernel = gaussian_kernel2d(std = sigma, kernlen = 2 * radius + 1)
            gx,gy,magnitude,direction = sift_gradient(gaussian_img)
            direction_idx = np.round( direction * num_bins / 360 ).astype(int)

            for i,j in map( tuple , np.argwhere( scale_keypoints ).tolist() ):
                window = [i-radius, i+radius+1, j-radius, j+radius+1]
                mag_win = padded_slice( magnitude , window ) * kernel
                dir_idx = padded_slice( direction_idx, window )

                hist = np.zeros(num_bins, dtype=np.float32)
                for bin_idx in range(num_bins):
                    hist[bin_idx] = np.sum( mag_win[ dir_idx == bin_idx ] )
                for bin_idx in np.argwhere( hist >= 0.8 * hist.max() ).tolist():
                    angle = (bin_idx[0]+0.5) * (360./num_bins) % 360
                    kps.append( (i,j,octave_idx,scale_idx,angle))
    return octaves_kps
```

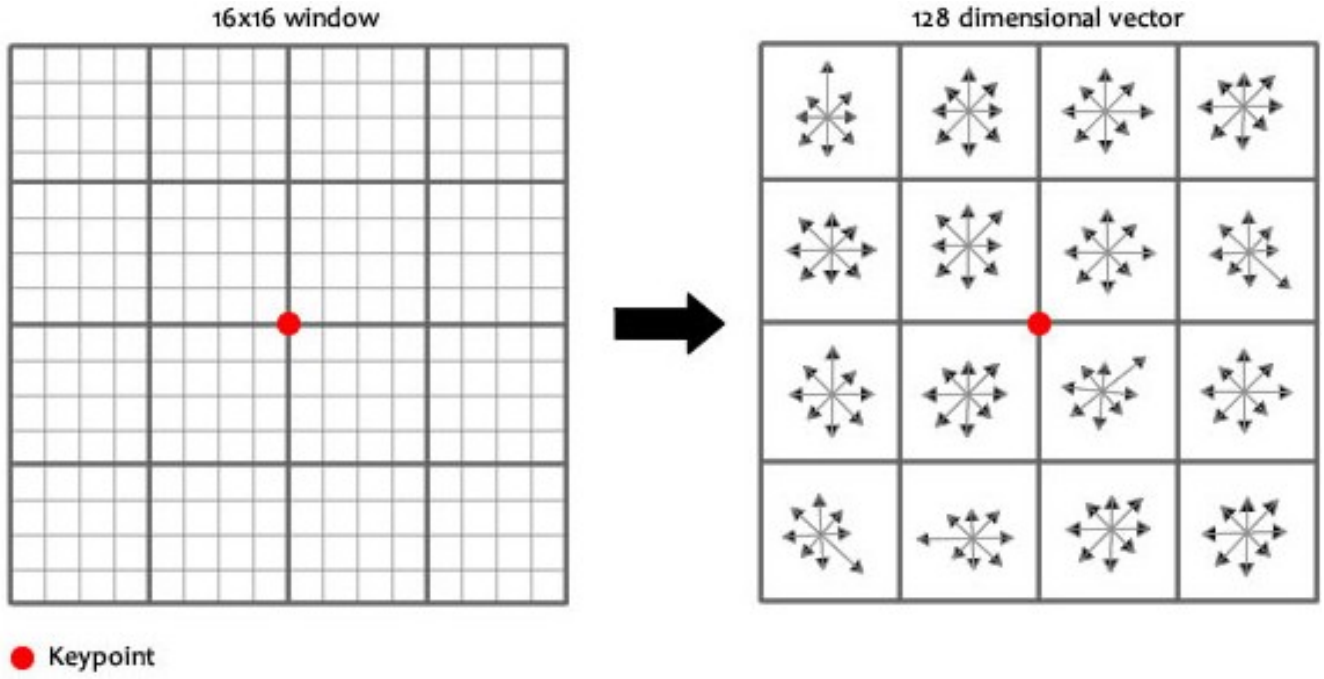
3. Keypoints Orientation

Corners + Orientation (Last Octave)

3. Keypoints Orientation

Corners + Orientation (all Octaves)

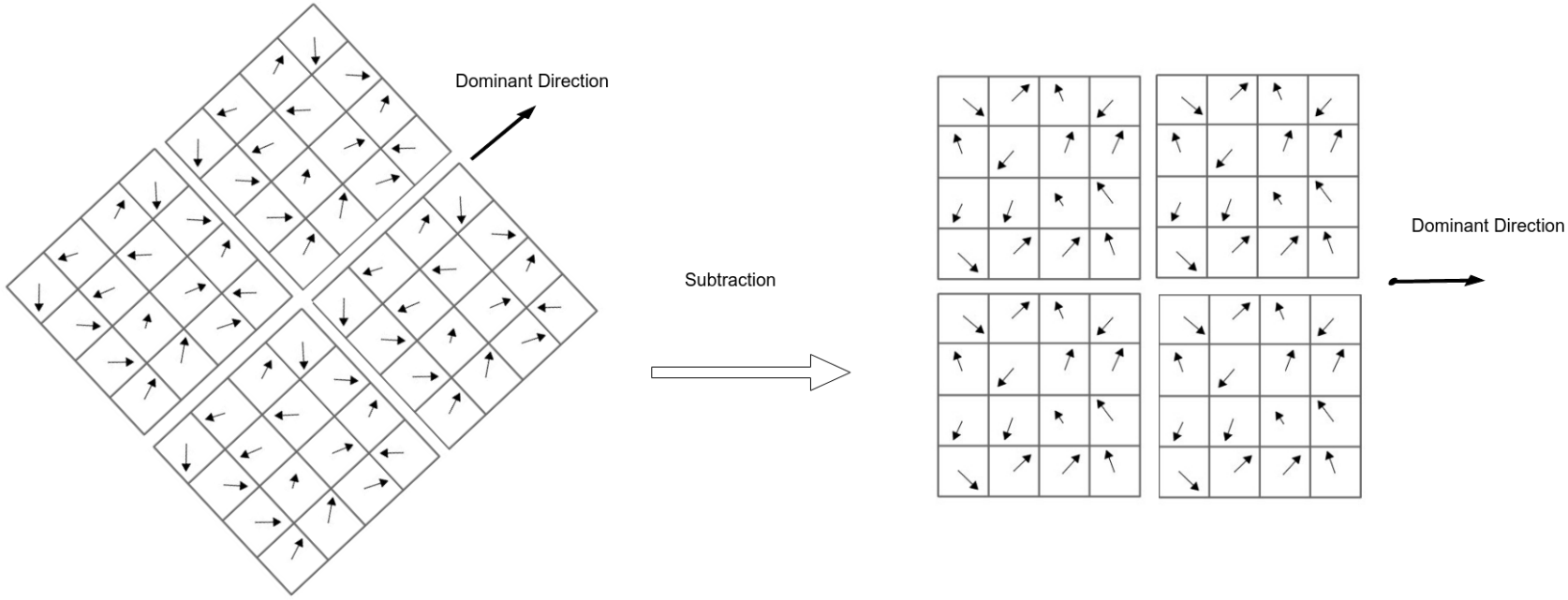
4. SIFT Descriptor



After localization of a keypoint and determination of dominant direction:

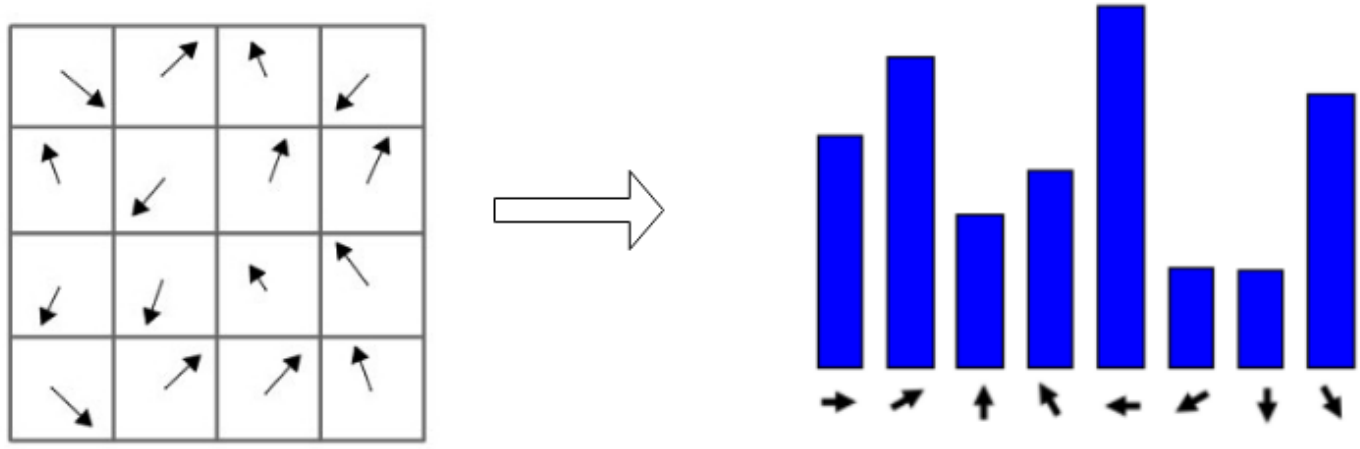
- Extract a 16×16 window centered by the keypoint.
- Get gradient magnitude and multiply it by a 16×16 gaussian window of $\sigma = 1.5$

4. SIFT Descriptor



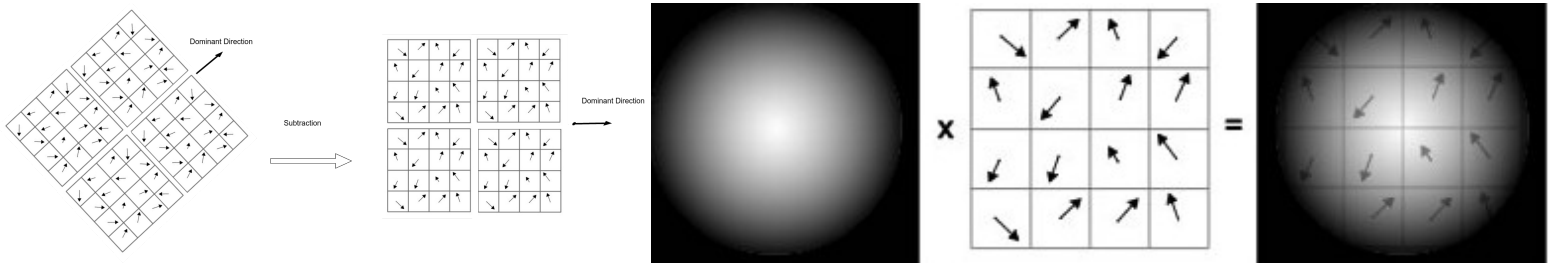
- Adjusting orientation (To be rotation invariant):
  - get the gradient angle of the window and Quantize them to 8 intervals (0, 45, 90, ..., 360)
  - subtract corner dominant direction from gradient angle.

4. SIFT Descriptor



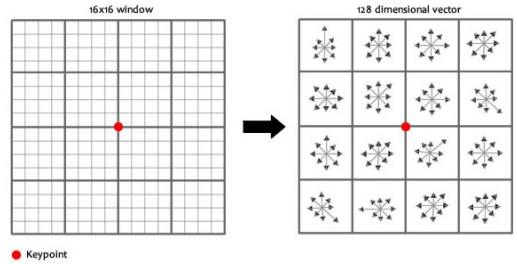
- Divide the 16×16 window into **16** subregions (4×4 each).
- For each region construct histogram for the quntized 8 directions.
- Each region is represented by **8 values**
- Combine all values into one vector S (size = $8 \times 16 = 128$)
- Unit normalize $S' = \frac{S}{||S||}$
- Clip values larger than 0.2 (overcome nonlinear illumination effect): $S'' = \text{Clip}(S', 0, 0.2)$
- Renormalize $S''' = \frac{S''}{||S''||}$

4. SIFT Descriptor




```
def extract_sift_descriptors128( img_gaussians, keypoints, num_bins = 8 ):
    descriptors = [] ; points = []; scale_data = {}
    for octave_idx in range(N_OCTAVES):
        for (i,j,o_idx,scale_idx, theta) in keypoints[octave_idx]:
            # A) Caching
            if 'index' not in data or data['index'] != (o_idx,scale_idx):
                data['index'] = (o_idx,scale_idx)
                gaussian_img = img_gaussians[octave_idx][ scale_idx ]
                sigma = 1.5 * SIGMA * ( 2 ** octave_idx ) * ( K ** (scale_idx))
                data['kernel'] = gaussian_kernel2d(std = sigma, kernlen = 16)
                _, _, data['magnitude'], data['direction'] =sift_gradient(gaussian_img)
            win_mag = rotated_subimage(data['magnitude'],(j,i),theta,16,16)* data['kernel']
            win_dir = rotated_subimage(data['direction'],(j,i),theta,16,16)
            win_dir = (((win_dir - theta) % 360) * num_bins / 360.).astype(int)
            # B) HoG "4x4" x 16 ...
            # C) Combine+Normalize ...
    return points , descriptors
```

4. SIFT Descriptor

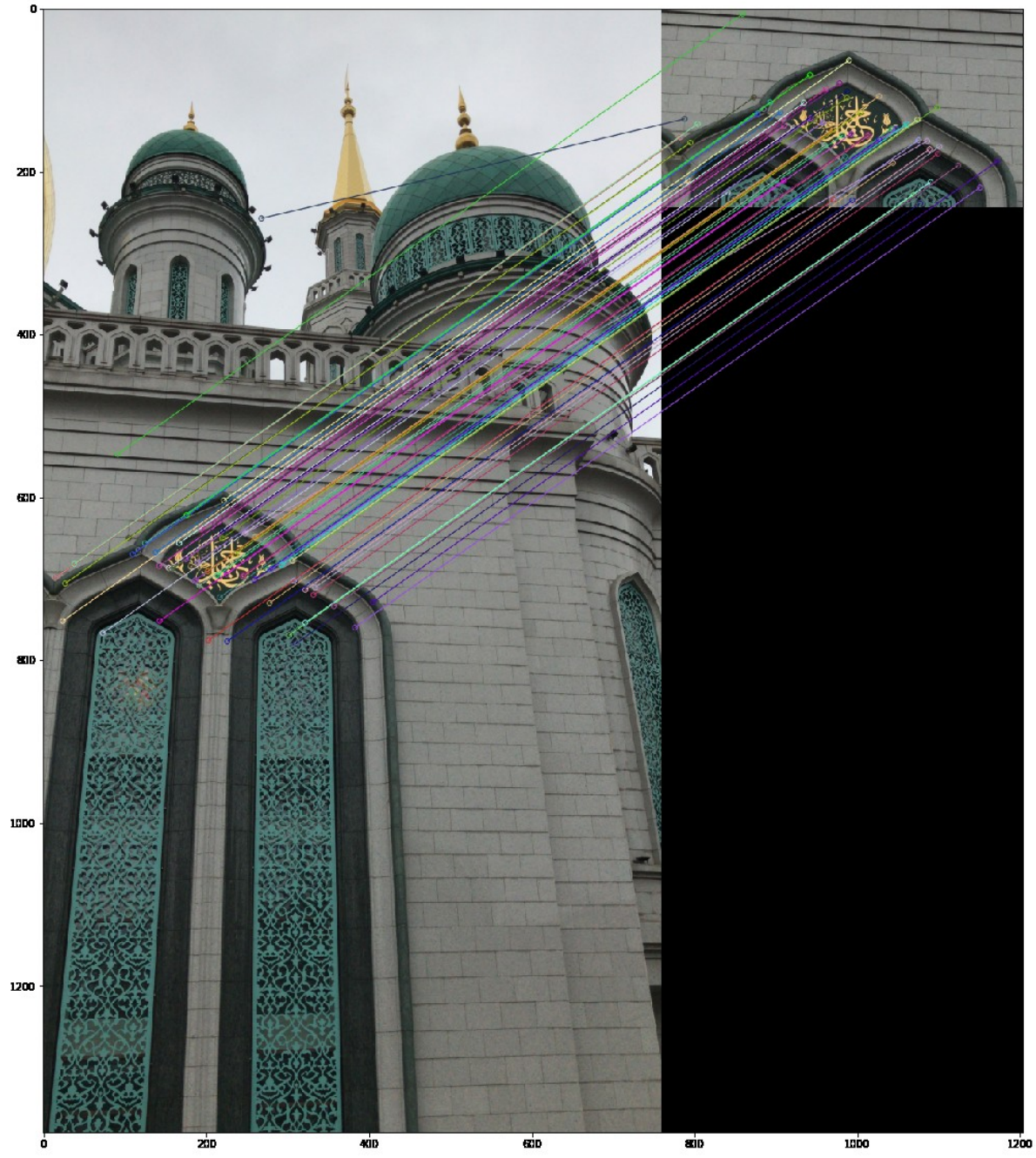



```
def extract_sift_descriptors128( img_gaussians, keypoints, num_bins = 8 ):
    descriptors = [] ; points = []; scale_data = {}
    for octave_idx in range(N_OCTAVES):
        for (i,j,o_idx,scale_idx, theta) in keypoints[octave_idx]:
            # A) Caching ...
            # B) HoG "4x4" x 16
            features = []
            for sub_i in range(4):
                for sub_j in range(4):
                    sub_weights = win_mag[sub_i*4:(sub_i+1)*4, sub_j*4:(sub_j+1)*4]
                    sub_dir_idx = win_dir[sub_i*4:(sub_i+1)*4, sub_j*4:(sub_j+1)*4]
                    hist = np.zeros(num_bins, dtype=np.float32)
                    for bin_idx in range(num_bins):
                        hist[bin_idx] = np.sum( sub_weights[ sub_dir_idx == bin_idx ] )
                    features.extend( hist.tolist())
            # C) Combine+Normalize ...
    return points , descriptors
```

4. SIFT Descriptor

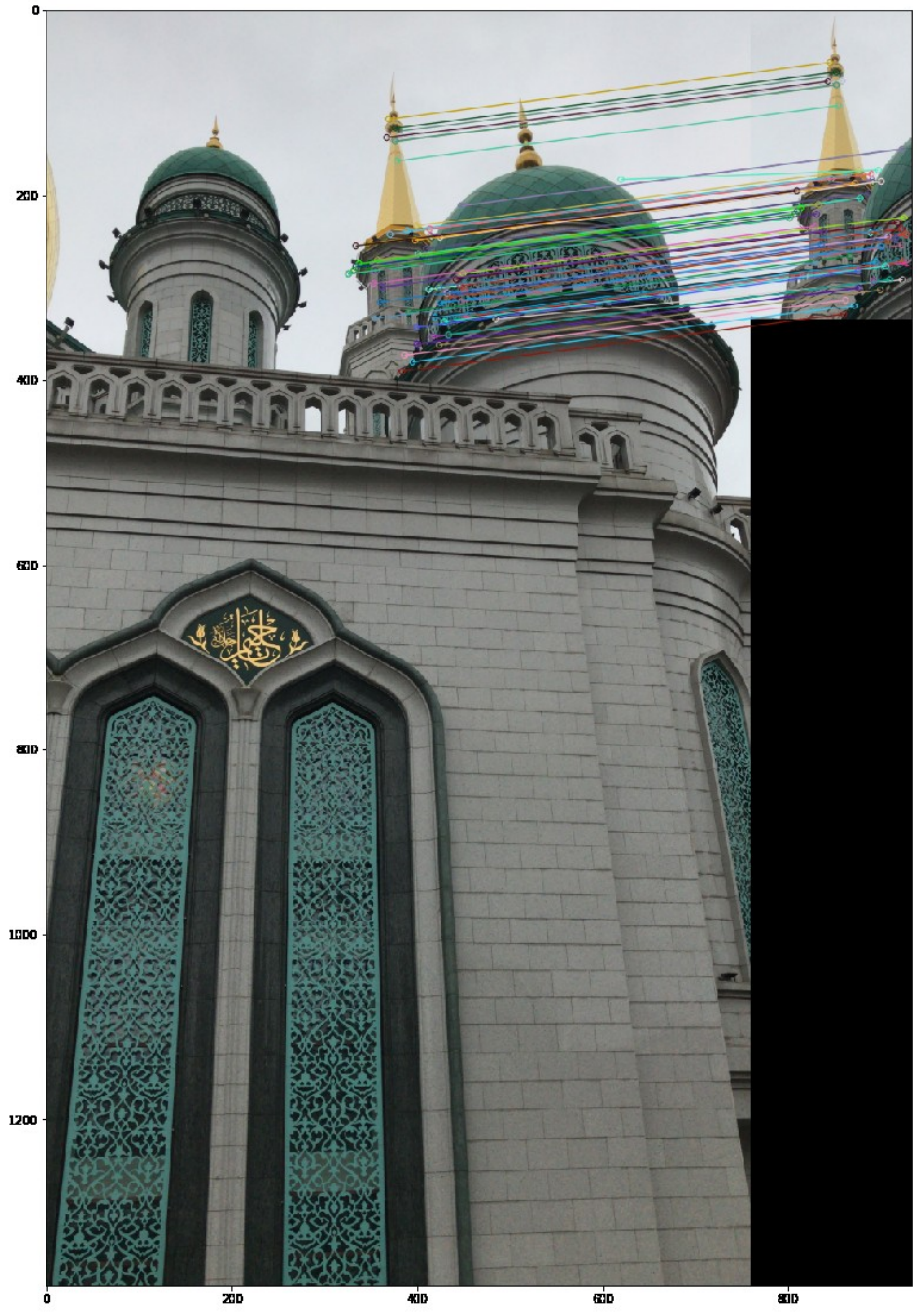
```
def extract_sift_descriptors128( img_gaussians, keypoints, num_bins = 8 ):
    descriptors = []
    points = []
    for octave_idx in range(N_OCTAVES):
        scale_data = {}
        for (i,j,o_idx,scale_idx, theta) in keypoints[octave_idx]:
            # Caching...
            # B) HoG "4x4" x 16...
            # C) Combine+Normalize
            features /= (np.linalg.norm(np.array(features)))
            features = np.clip(features , np.finfo(np.float16).eps , 0.2)
            features /= (np.linalg.norm(features))
            descriptors.append(features)
            points.append( (i ,j , octave_idx, scale_idx, theta))
    return points , descriptors
```

SIFT - Results

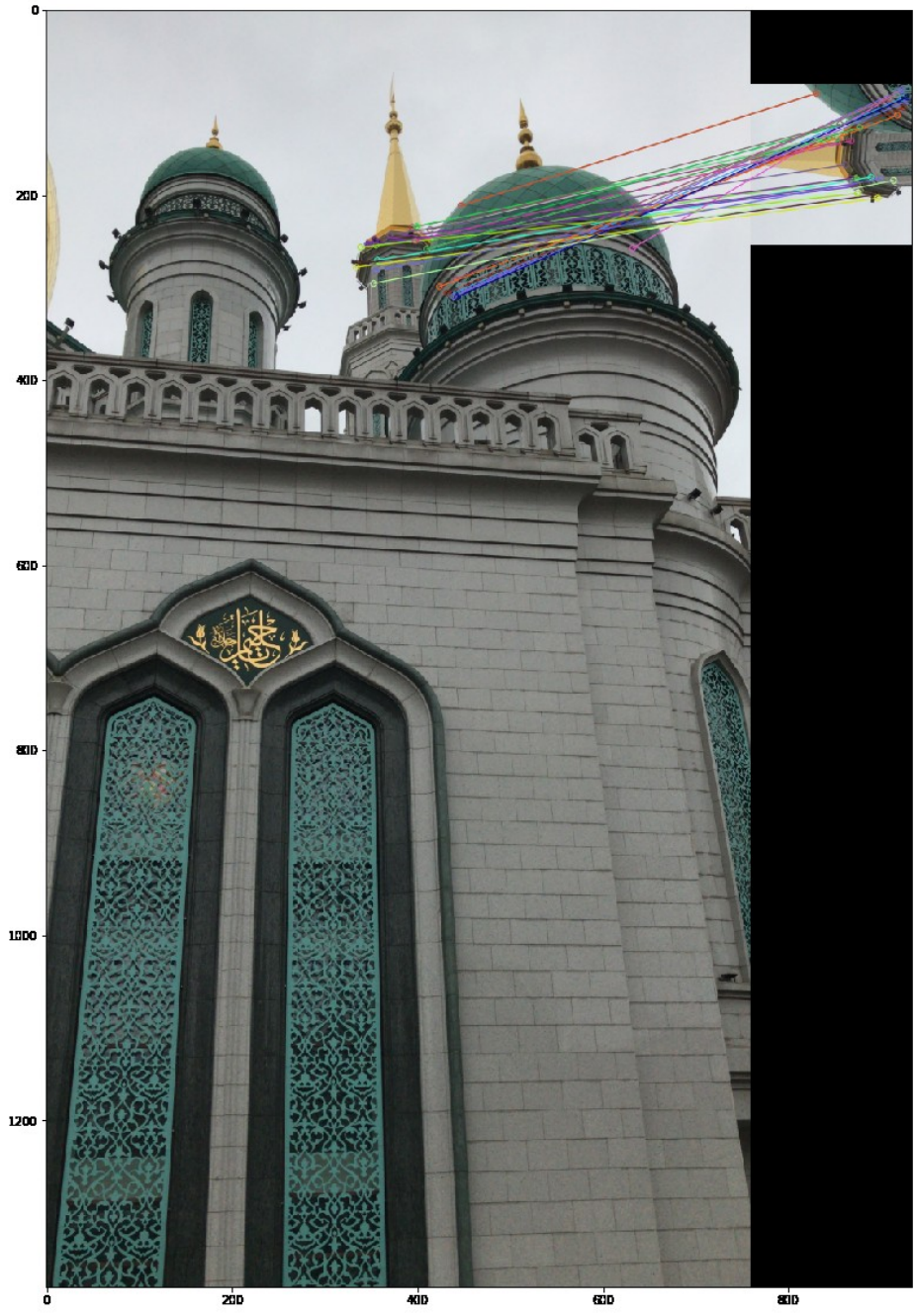


SIFT - Results

SIFT - Results



SIFT - Results



SIFT - Results

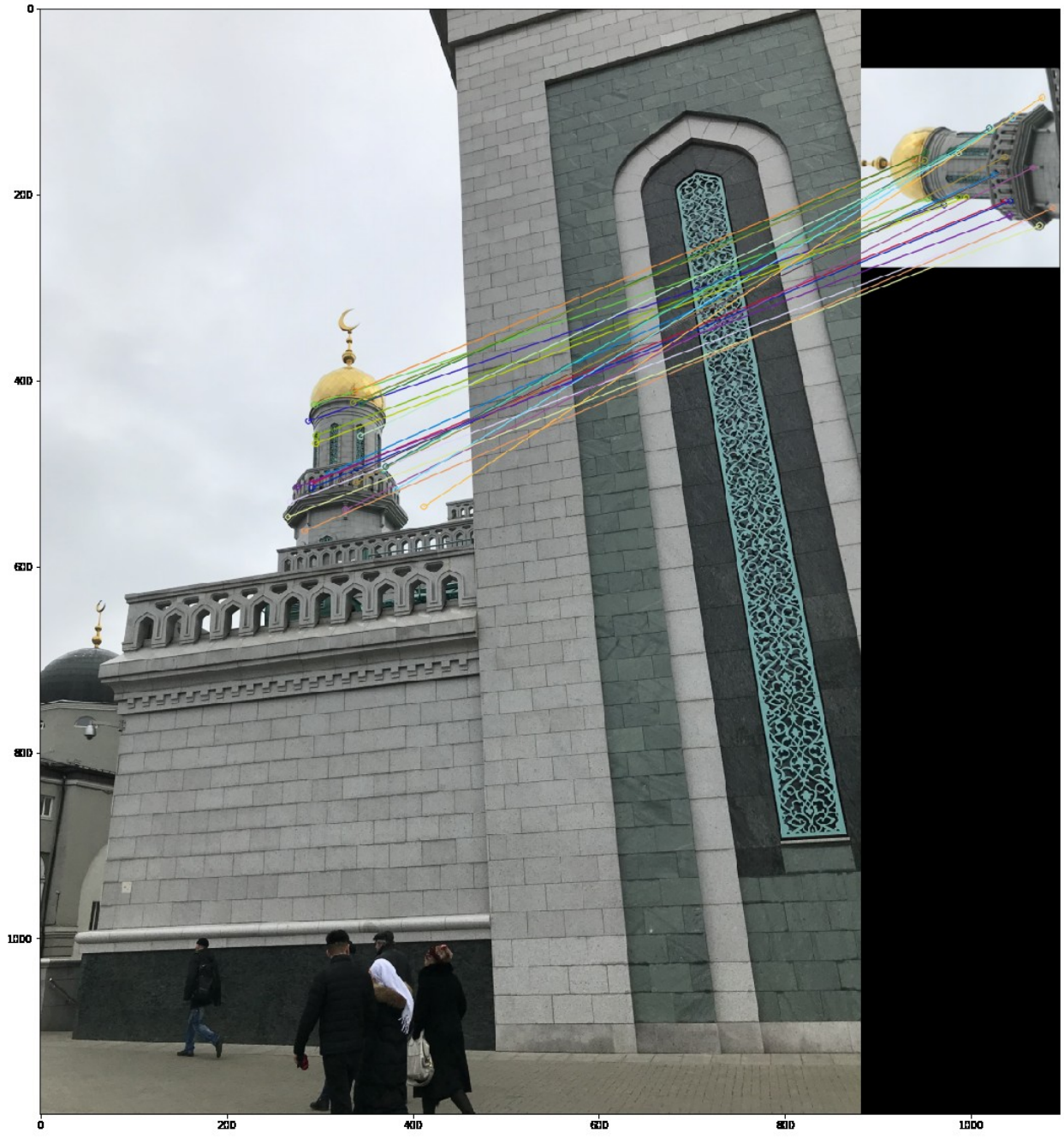
SIFT - Results

SIFT - Results

SIFT - Results

SIFT - Results

SIFT - Results



{scale_invariance.ipnyb}

Image Matching Challenge 2020
Sponsored by CVPR 2020

- Some local descriptors developed as early as 2004
- Several of them outperform the state-of-the-art machine learning approaches in object recognition

{vision.uvic.ca/image-matching-challenge}

{github.com/vcg-uvic/image-matching-benchmark}

Thank you