$$x_{in} = \int f(t)dt = \int \alpha g(t)dt = \alpha \int g(t)dt = \alpha y_{in}$$

$$\alpha = \frac{x_{in}}{y_{in}} \quad \int \frac{dx}{\frac{x_{in}}{y_{in}} - \frac{x^2}{k}} = y_{in} \quad \int \frac{dx}{x_{in} - \frac{y_{in}}{k}x^2} = 1$$

$$\int \frac{dx}{a - bx^2} = \frac{1}{\sqrt{ab}} \tanh^{-1}\left(\sqrt{\frac{b}{a}}x\right) + \text{ constant}$$

$$\int_{x_0}^{x_{\text{end}}} \frac{dx}{x_{in} - \frac{y_{in}}{k}x^2} = \sqrt{\frac{k}{x_{in}y_{in}}} \tanh^{-1}\left(\sqrt{\frac{y_{in}}{kx_{in}}}x\right)\Bigg|_{x_0}^{x_{\text{end}}} = 1$$

$$x_{\text{end}} = \sqrt{\frac{kx_{in}}{y_{in}}} \frac{e^{2\sqrt{\frac{x_{in}y_{in}}{k}}} + c}{e^{2\sqrt{\frac{x_{in}y_{in}}{k}}} - c} \quad c = \frac{\sqrt{x_0 y_{in}} - \sqrt{y_0 x_{in}}}{\sqrt{x_0 y_{in}} + \sqrt{y_0 x_{in}}}$$