$$egin{aligned} x_{in} &= \int f(t)dt = \int lpha g(t)dt = lpha \int g(t)dt = lpha y_{in} \ lpha &= rac{x_{in}}{y_{in}} \int rac{dx}{rac{x_{in}}{y_{in}} - rac{x^2}{k}} = y_{in} \int rac{dx}{x_{in} - rac{y_{in}}{k}x^2} = 1 \ &\int rac{dx}{a - bx^2} = rac{1}{\sqrt{ab}} anh^{-1}igg(\sqrt{rac{b}{a}x}igg) + ext{ constant} \ \int_{x_0}^{x_{ ext{end}}} rac{dx}{x_{in} - rac{y_{in}}{k}x^2} = \sqrt{rac{k}{x_{in}y_{in}}} anh^{-1}igg(\sqrt{rac{y_{in}}{kx_{in}}x}igg)igg|_{x_0}^{x_{ ext{end}}} = 1 \ &x_{ ext{end}} = \sqrt{rac{kx_{in}}{y_{in}}} rac{e^2\sqrt{rac{x_{in}y_{in}}{k}} + c}{e^2\sqrt{rac{x_{in}y_{in}}{k}} - c} \quad c = rac{\sqrt{x_0y_{in}} - \sqrt{y_0x_{in}}}{\sqrt{x_0y_{in}} + \sqrt{y_0x_{in}}} \end{aligned}$$