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Question 3

a. In the formula φG, for the vertex cover of a graph G, the number of clauses is given by *m* + *t*.

   For each edge in the graph, you add a clause that ensures at least one of the two vertices connected by that edge is in the vertex cover. This gives a total of *m* clauses, where m is the number of edges. Additionally, for each triangle in the graph, you add a clause that ensures at least one of the three vertices in the triangle is not in the vertex cover. This adds *t* clauses, where *t* is the number of triangles. Therefore the total number of clauses in φG is *m* + *t*.

b. In a graph G, a triangle is made up of three vertices where each pair is connected by an edge, so connected by three edges. To estimate the maximum number of triangles, we look at the pairs of edges because each pair can help form a triangle. The number of edge pairs in a graph with m edges in calculated using the formula
   t ≤ M choose 2 = $\frac{m(m-1)}{2}$. This formula is used to count the number of unique pairs of edges in a graph with m edges. Since each triangle in a graph is made up of three edges and each pair of edges can be part of only one triangle, this formula helps determine the maximum number of possible triangles by counting all potential pairs of edges.

c. Number of clauses $= \ m \ + \ \frac{m(m-1)}{2}$.
   Each edge contributes exactly one clause to the logical formula φG, resulting in m edge clauses. Additionally to account for the triangles, we include clauses that represent the absence of these triangles. The maximum number of these triangle clauses in given by $\frac{m(m-1)}{2}$, which accounts for all possible pairs of edges in the graph. Hence, the total number of clauses in the sum of edge clauses and triangles, giving the result of $m \ + \ \frac{m(m-1)}{2}$.