Name: Srividya Bobba
StudentID: 33899207

Question 5:

(a) $\varphi X = (v_1 \lor v_2) \land (v_1 \lor v_3) \land (v_2 \lor v_3) \land (v_4 \lor v_5) \land (v_4 \lor v_7) \land (v_5 \lor v_6) \land (v_6 \lor v_7) \land (\neg v_1 \lor \neg v_2 \lor \neg v_3)$

(b) Base Case:

cycle of length 3 (forms triangle)

vertex cover clauses: 3 clauses (bc one for each edge)

triangle free clauses: 1 clause (to prevent triangle)

Total Clauses $= 3+1 = 4$ clauses.

So for $n = 3$:

$$\frac{4(3)}{3} = 4$$

$\therefore$ base case holds

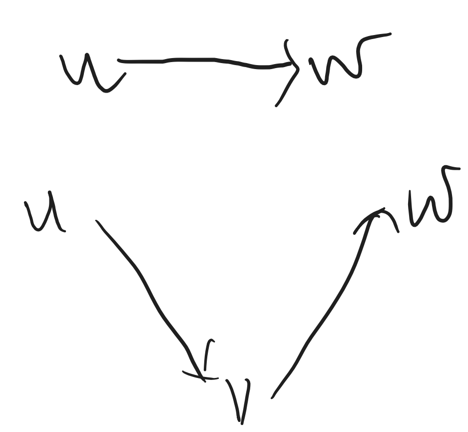
Inductive Step:

Assume for any 2-regular graph with k $(k \le n)$ vertices, the boolean expression φG has at most $\frac{4k}{3}$ clauses.

Need to prove that for a 2-regular graph with $k+1$ vertices the boolean expression φG has at most $\frac{4(k+1)}{3}$ clauses

2-regular graph G with n vertices. To form a 2-regular graph G' with $n+1$ vertices, adding new vertex 'v' and connect to existing vertices u and w (ensuring remains 2-R).

- replaces one existing $(u \to w)$ edge with 2 new edges $(u \to v)$ and $(v \to w)$
- addition of 'v' and transformation of edge can add at most 1 new clause to φG.
  bc new edges are connected to only 2 existing vertices.



which means:

$$\frac{4k}{3} + 1 \le \frac{4(k+1)}{3}$$

$$\frac{4k+3}{3} \le \frac{4k+4}{3}$$

since $\frac{4k+3}{3}$ is less, this holds $\therefore$ by induction, it is seen that the boolean expression φG for any 2-regular graph G with n vertices has at most $\frac{4n}{3}$ clauses.