- Velocity is important as to change requires fuel

→ using ellipse properties & cons. of energy, we can find velocity for any point in orbit.

Specific Energy of orbit, $\varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r} = (e^2-1)\frac{\mu^2}{2h^2}$

only valid for ellipses

for an ellipse $h^2 = \mu a(1-e^2)$

$$\therefore \quad \varepsilon = (e^2-1)\frac{\mu^2}{2[\mu a(1-e^2)]} = \frac{\mu}{2a}\overset{=-1}{\cancel{\frac{e^2-1}{1-e^2}}} = \frac{-\mu}{2a}$$

→ We arrive at the Vis-Viva equation

given $\boxed{\varepsilon = \frac{1}{2}v^2 - \frac{\mu}{r} = -\frac{\mu}{2a}}$ → $\boxed{v = \sqrt{\mu\left[\frac{2}{r} - \frac{1}{a}\right]}}$

→ useful when calculating delta velocity budgets.

→ note, the velocity changes as you go around orbit, but wouldn't cost any delta V to stay in orbit in this ideal case.

- For orbiting bodies around earth, due to luni-solar perturbations, e.g solar radiation pressure, a small amount of delta V required to maintain orbit.

Circular Orbits:

- Circular orbit is particular case of ellipse where $r \equiv a$

$$\therefore \quad v_c^2 = \mu\left[\frac{2}{r} - \frac{1}{r}\right] = \frac{\mu}{r}$$

→ $\therefore \; v_c \propto \frac{1}{\sqrt{r}}$

$r = r_e + h$

velocity proportional to square root of altitude

Parabolic Orbits :

$$v_{esc} = \sqrt{\frac{2\mu}{r}}$$

if you compare v_{esc} with v_c, there is only $\sqrt{2}$ difference
$\therefore$ necessary Δv to escape on a parabolic orbit from a circular orbit is $\boxed{(\sqrt{2}-1)v_c}$

parabola is shape of orbit we need as minimum to escape a planet's gravity

- Parabola is the demarcation between negative & positive energy orbits

'boundary'

elliptical or circular orbit that's transversed repeatedly

object has great enough v to escape gravity and follows a hyperbolic trajectory

The Kepler Equation :

- Position in orbit as a function of t.

eccentricity

$$\boxed{M = n(t - t_0) = E - e\sin E}$$

mean anomaly (rad)
angle from periapsis if body in circular orbit had same period

mean motion (rad/s)
average angular velocity needed to complete one orbit

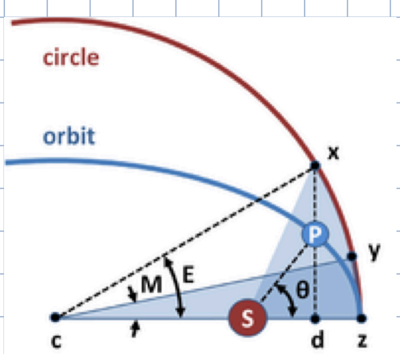
$$\boxed{n = \sqrt{\frac{\mu}{a^3}}}$$

time since periapsis (s)

Eccentric anomaly (rad)
defined by diagram below

related to true anomaly θ by :

$$\boxed{\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}}$$



Don't need to know diagram for exam

Manoeuvres :

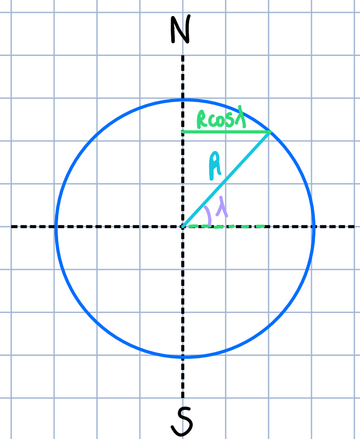
Gravity turn - rocket turned from vertical to horizontal only using gravitational force.

Launch Losses :

Gravity losses : loss in net performance of rocket whilst thrusting in gravitational field.
↳ ~ 15 % of fuel

Drag losses ~ 0.5 % of fuel

Effect of Earth's Rotation :



λ = latitude

$$\Delta V = \frac{\text{distance}}{\text{time}} = V\cos\lambda$$

$$\Delta V = \frac{2\pi \times 6378000}{86164} \cos\lambda$$

sidereal day (s)

$$\boxed{\Delta V = 465 \cos\lambda \text{ m/s}}$$

- Rotational velocity of earth at launch site gives lanuch vehicle a extra increment (or undesirible decrement if required orbit is retrograde) of velocity.
- Effect greatest at low latitude → equator max.
- Launch site latitude limits inclination of satellite's orbit.
  ↳ min - inclination = latitude of launch site
    ↳ can be changed during flight via orbit plane change