Manoeuvres :

1. Launch (and effect of earth's rotation)
2. In-Plane - change in semi-major axis, a.
3. Out-of-Plane - change in inclination, Ω & ω

General Principles of Manoeuvres :

- Burning prograde (forwards) increases 'a'.
- Burning retrograde (backwards) decreases 'a'
- Initial and final orbits intersect at point where impulse is applied
- Need to get to right point at right time
- Seperate manoeuvres can be combined vectorially
- All in-plane manoeuvres performed at peri or apoapsis

Changing Orbit Altitudes :

$$\boxed{E = -\frac{GMm}{2r}}$$

$\therefore$ to calculate energy required to change altitude, take final - initial energy:

$$\Delta E = \text{energy required} = \boxed{E_2 - E_1}$$

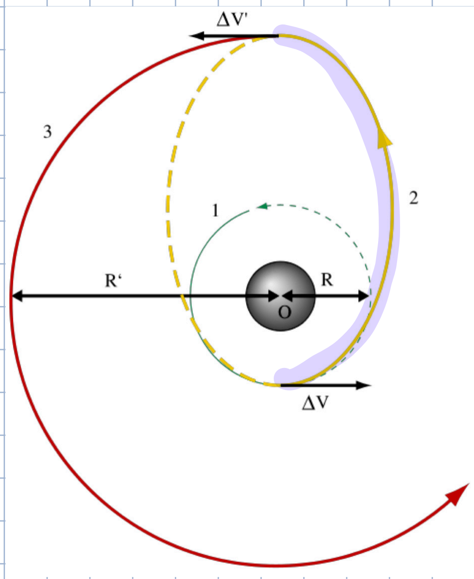
Hohmann Transfer : most common as it provides minimum delta V to change 'a'.

- Used to reduce / increase orbit altitude
- Used with coplanar circular orbits
- 2 burns

2nd at apoapsis → sets peri. to new $\therefore$ becomes new circular orbit

1st at periapsis → sets apoapsis to new alt.

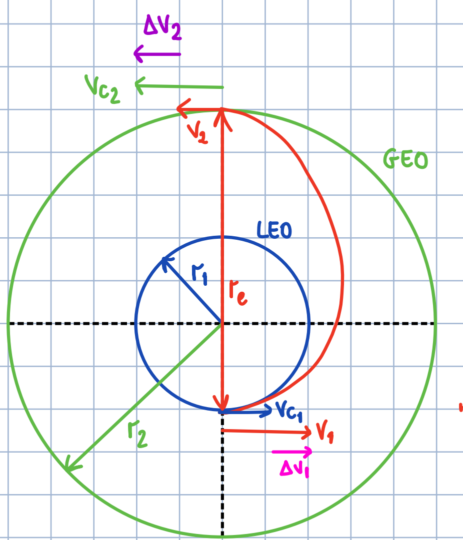
THINK HALF ORBIT AHEAD



Time of Flight

- half the period of the orbit

$$\boxed{TOF = \frac{T_{trans}}{2} \quad \text{and} \quad T_{trans} = \frac{2\pi}{\sqrt{\mu}} a_{trans}^{3/2}}$$



Initial LEO has radius r_1 & velocity V_{c_1} circular

$$V_{c_1} = \sqrt{\frac{\mu}{r_1}}$$

Desired GEO :

$$V_{c_2} = \sqrt{\frac{\mu}{r_2}}$$

Impulsive ΔV_1 for elliptical geostationary orbit at perigee :

$$a = \frac{r_1 + r_2}{2}$$

We leave LEO with V_1 :

$$V_1 = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)}$$

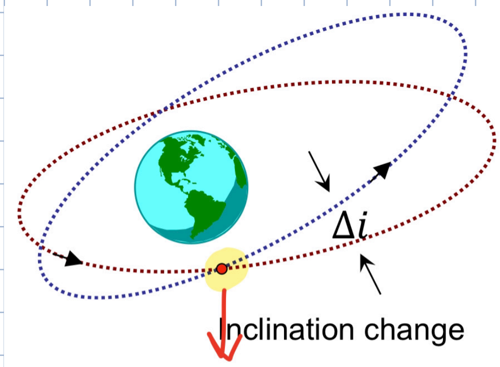
$$\Delta V_1 = V_1 - V_{c_1} = \sqrt{\mu\left(\frac{2}{r_1} - \frac{1}{a}\right)} - \sqrt{\frac{\mu}{r_1}}$$

Arrive at GEO with V_2 and need ΔV_2 to circularise :

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}} - \sqrt{\mu\left(\frac{2}{r_2} - \frac{1}{a}\right)}$$

Hohmann most efficient as transfers are tangential : any orbit transfers that are not tangential require additional propulsion to realign orbit path

Plane Change :



burning 'south' to increase inclination up

For no other elements to change, the burn must take place where the initial & target planes intersect (nodes)

Even small changes in plane requires ↑ ΔV

Less ΔV required at higher altitudes

Often combined with Hohmann or rendezvous.

From geometry :

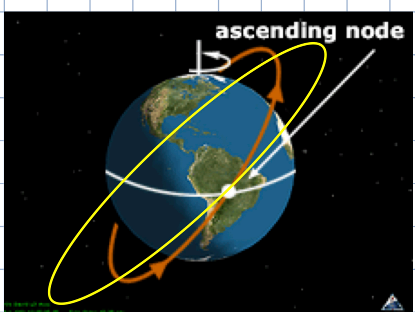
$$\boxed{\Delta V = 2V \sin\left(\frac{\Delta i}{2}\right)}$$

→ max velocity penalty for equatorial to polar (90°) plane change :

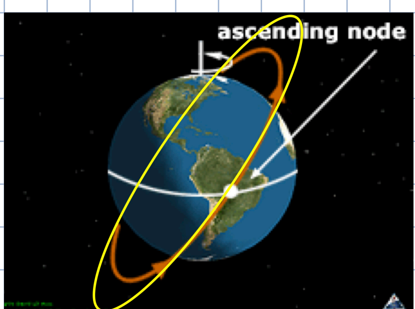
$$\Delta V = \sqrt{2}\, V$$

Rendevous Manoevres :

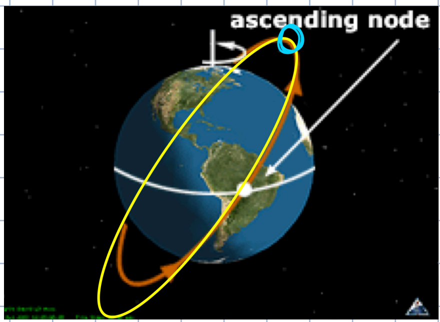
1. Launch into orbit with similar plane to target



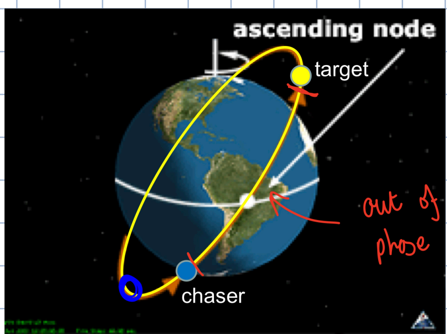
2. Match Inclination at nodes



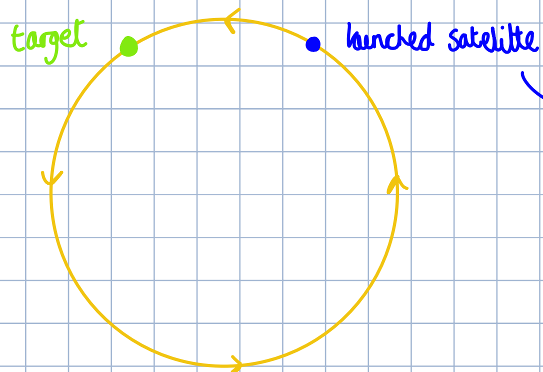
3. Match Apoapsis



4. Match Periapsis :



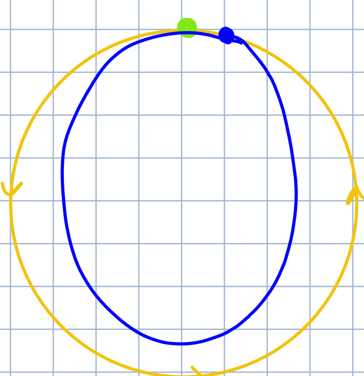
5. Co-orbital Rendevous :



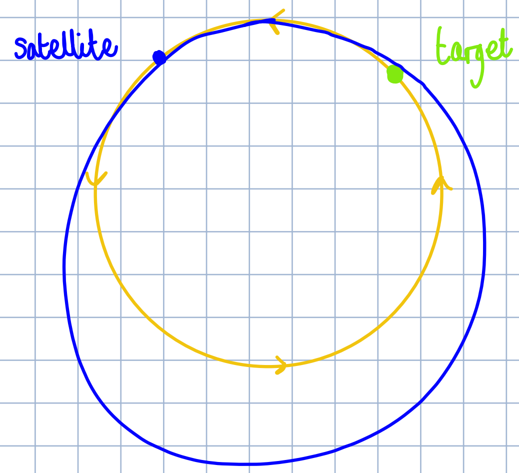
→ Satellite trailing target

Chaser retroburns → reducing apoapsis → smaller orbit = smaller period

↳ catches up



Alternatively : satellite ahead of target



Satellite burns prograde to increase period & slow down to meet target

↳ called phasing manoeuvre

6. Last 50m done at very low velocities (1-5 ms^{-1})