Magnitude in Component Form:

$$\hat{r} = \frac{r}{|r|} = \frac{xi + yj + zk}{\sqrt{x^2 + y^2 + z^2}}$$

'r hat'

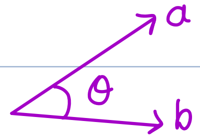
$$= \frac{x}{\sqrt{x^2+y^2+z^2}}i + \frac{y}{\sqrt{x^2+y^2+z^2}}j + \frac{z}{\sqrt{x^2+y^2+z^2}}k$$

Scalar (λ)

- Parallel when $a = \lambda b$ ie $\lambda > 0$ parallel
- Antiparallel when $a = -\lambda b$ $\lambda < 0$ antiparallel

Scalar (inner, dot) product:

$$a \cdot b = |a||b| \cos\theta$$



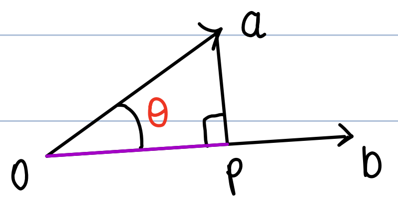
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

If a and b perpendicular, $a \cdot b = 0$

$$\rightarrow i \cdot i + j \cdot j + k \cdot k = 0$$

$$a \cdot a = |a|^2$$

Projection of one vector onto another :

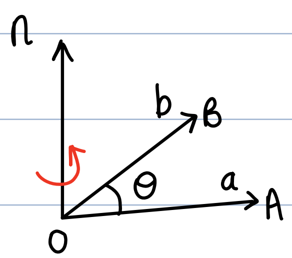


Component of a in direction of b

$$|\overrightarrow{OP}| = |a| \cos\theta$$

also, $a \cdot b = |a||b|\cos\theta$ $\therefore$ $\boxed{|\overrightarrow{OP}| = \dfrac{a \cdot b}{|b|}}$

Cross Product :



$$a \times b = |a||b|\sin\theta\, n$$

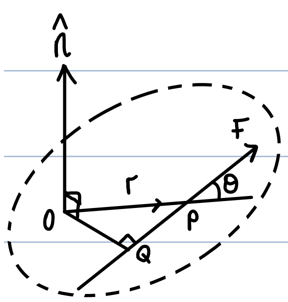
cross not multiply

⊥ to both vectors

n perpendicular to a, b plane and direction given by right - handed / screw rule.

→ put screw perp. and rotate a → b

↳ this example CCW so up.



$$M = r \times F = |r||F|\sin\theta\,\hat{n} = OQ\,|F|\,\hat{n}$$

Cover up approach

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

i. $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ Cover i column, cross multiply:

$(a_2 b_3 - a_3 b_2)\, i$

j. $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ For j, cross multiply reversed:

$(a_3 b_1 - a_1 b_3)\, j$

k. $\begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ For k, do as i:

$(a_1 b_2 - a_2 b_1)\, k$

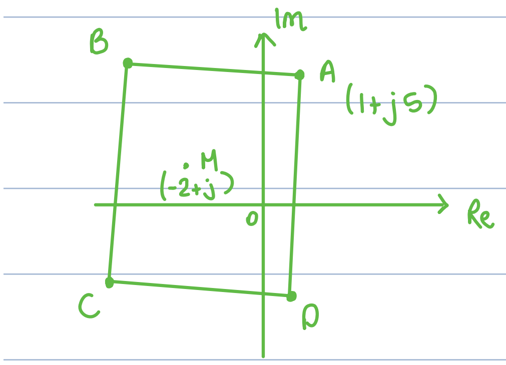
Learn:

$\lambda(a \times b) = \lambda a \times b = a \times \lambda b$	$a \cdot b = b \cdot a$
$a \times (b+c) = (a \times b) + (a \times c)$	$a \cdot \lambda b = \lambda a \cdot b = \lambda(a \cdot b)$
$a \times b = 0$ is equivalent to $a \parallel b$	$a \cdot (b+c) = a \cdot b + a \cdot c$
$a \times a = 0$	$a \cdot a = a_1^2 + a_2^2 + a_3^3 = \lvert a \rvert^2$
$a \times b = -b \times a$	

Complex Numbers as Vectors:

- Represented similarly to cartesian vectors, but just in real and imaginary axes.

M is the centre of square with vertices A, B, C, D taken anticlockwise in order. If, in the Argand diagram, M and A are represented by the complex numbers $-2+j$ and $1+j5$ respectively, find the complex numbers represented by B, C and D:



$$\overrightarrow{MA} = \overrightarrow{OA} - \overrightarrow{OM}$$
$$= \begin{pmatrix} 1 \\ 5j \end{pmatrix} - \begin{pmatrix} -2 \\ j \end{pmatrix} = \begin{pmatrix} 3 \\ 4j \end{pmatrix}$$

angle between $= \frac{2\pi}{4} = \frac{\pi}{2}$

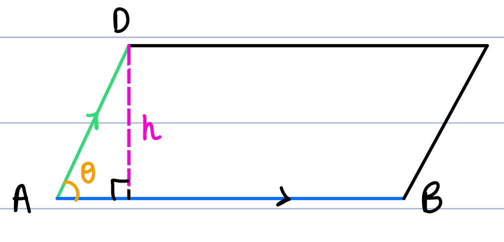
Multiplying a complex number by j rotates it through $\frac{\pi}{2}$ radians in anticlockwise direction:

$$\therefore \quad \overrightarrow{MB} = j\,\overrightarrow{MA} = j\begin{pmatrix} 3 \\ 4j \end{pmatrix} = -4 + 3j$$
$$\overrightarrow{OB} = \overrightarrow{OM} + \overrightarrow{MB} = \begin{pmatrix} -2 \\ j \end{pmatrix} + \begin{pmatrix} -4 \\ 3j \end{pmatrix} = \begin{pmatrix} -6 \\ j4 \end{pmatrix}$$

etc. for rest.

Area of Parallelogram:

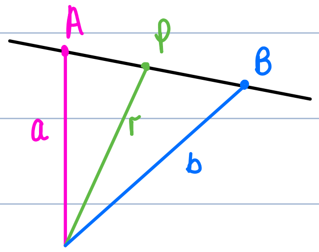
$$h\,|\overrightarrow{AB}| = |\overrightarrow{AD}|\sin\theta\,|\overrightarrow{AB}| = |\overrightarrow{AD} \times \overrightarrow{AB}|$$



Area of Triangle ABD:

$$\frac{1}{2}(AD)(AB)\sin\theta$$

Vector Equation of Line :



scalar along line

$$r = a + t\,\overrightarrow{AB} = a + t\,(b - a)$$

$$r = (1-t)\,\underset{\sim}{a} + t\,\underset{\sim}{b}$$

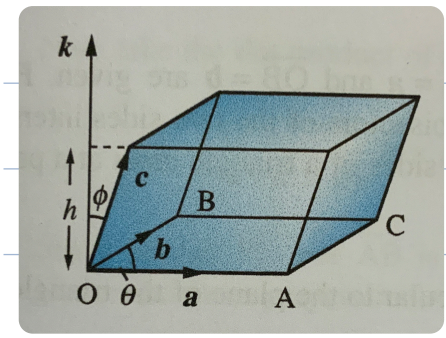
* $b - a = c$

Cartesian : $r = a + tc$*

$$\frac{x - a_1}{c_1} = \frac{y - a_2}{c_2} = \frac{z - a_3}{c_3} \quad (=t)$$

e.g $c_1 = b_1 - a_1$

Triple Scalar Product : $(a \times b) \cdot c$



$$a \times b = |a|\,|b| \sin\theta\, k$$

$$= (\text{Area of parallelagram } OACB)\, k$$

$$(a \times b) \cdot c = (\text{area } OACB)\, k \cdot c$$

$$= (\text{area } OACB)\, |k|\,|c| \cos\phi$$

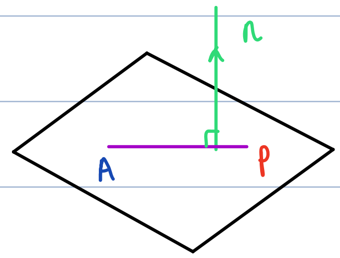
$$= (\text{area } OACB)\, h$$

$$= \text{Volume of parallelepiped}$$

$\therefore$ $(a \times b) \cdot a = 0$, $(a \times b) \cdot b = 0$ $\Big\}$ parallepiped 'collapsed'

if 3 vectors coplanar $(a \times b) \cdot c = 0$

Vector Equation of a Plane:



vector n $\perp$ to plane

A and P lie on plane

position vector of any point P on plane given by r

$$\therefore \ \vec{AP} = r - a$$

so the dot product of $(r - a)$ and n is 0 (cos90)

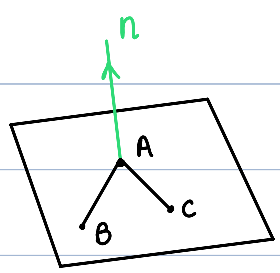
$$(r - a) \cdot n = 0$$

so $\quad r \cdot n = a \cdot n \quad$ or $\quad r \cdot n = p$

(both $\perp$)

perp. distance from origin to plane

e.g find plane passing through A(1,1,1), B(0,1,2), C(-1,1,-1)



a-b in plane = (1,0,-1)

a-c in plane = (2,0,2)

$\therefore$ cross product = vector n perp.

$$= \begin{matrix} 1 & 0 & -1 \\ 2 & 0 & 2 \end{matrix} = (0,-4,0)$$

using $\quad r \cdot n = a \cdot n \implies r \cdot (0,-4,0) = (1,1,1) \cdot (0,-4,0)$

$$= r \cdot (0,-4,0) = -4$$

$$= (x, y, z) \cdot (0,-4,0) = -4 \implies y = 1$$