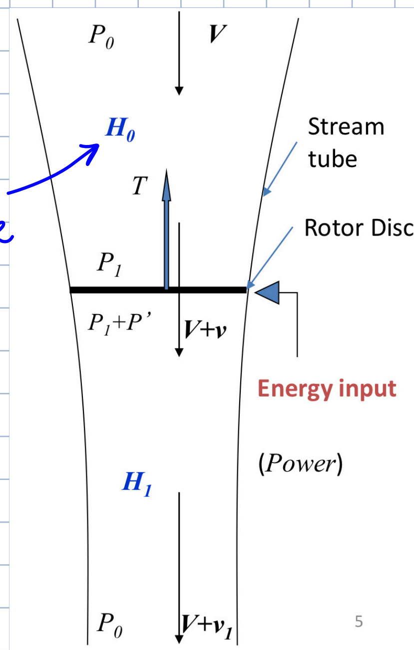
In its most simple form, a lifting rotor is a propeller.

→ Mechanical energy (rotating blades) used to accelerate (a) a mass (m) of air.
→ N3 : equal and opposite force on rotor : thrust.

Applying Bernoulli's Equation in streamtube :



total pressure

$$H_0 = P_0 + \tfrac{1}{2}\rho V^2 = P_1 + \tfrac{1}{2}\rho (V+v)^2$$

ambient static pressure

V = onset velocity (copter velocity)

Induced velocity

Beyond rotor blades :

$$H_1 = P_0 + \tfrac{1}{2}\rho (V+v_1)^2 = P_1 + P' + \tfrac{1}{2}\rho (V+v)^2$$

additional downstream flow velocity from rotor flow

Change in total pressure

$$\boxed{H_1 - H_0 = \tfrac{1}{2}\rho\left(2Vv_1 + v_1^2\right) = P'} \qquad (1)$$

Also, thrust = change of axial momentum per unit time :

$$\boxed{\frac{T}{A} = P' = \rho (V+v) v_1} \qquad (2)$$

rotor disk area

From equation (1) & (2) we can obtain :

$$\frac{v_1}{2} = v \quad \text{or} \quad \boxed{v_1 = 2v}$$

(additional downstream velocity is twice the induced velocity at the blades)

$$\boxed{\text{Thrust : } \quad T = 2\rho A (V+v) v}$$

$$\boxed{\text{Power : } \quad P = T(V+v)}$$

Momentum Theory in Hover :

In hover , $V = 0$

Induced velocity in hover, $\boxed{v_h = \sqrt{\dfrac{T}{2\rho A}}}$

Induced power in hover , $P_h = T v_h$ $\quad \therefore \quad \boxed{P_h = \dfrac{T^{3/2}}{\sqrt{2\rho A}}}$

Induced Velocity in Axial Flight :

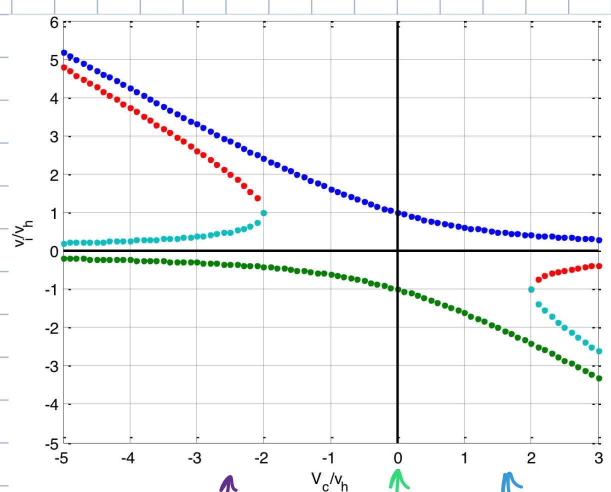
- In steady state axial flight
  - vertical acceleration = 0
    - climb with constant speed
    - descent with constant speed
    - hover

$$T_{hover} = T_{axial\,flight} = T$$

$$2\rho A v_h^2 = 2\rho A (V + v) v$$

$\div$ by v_h^2 and plotting variation of $\dfrac{v}{v_h}$ as function of $\dfrac{V}{v_h}$:

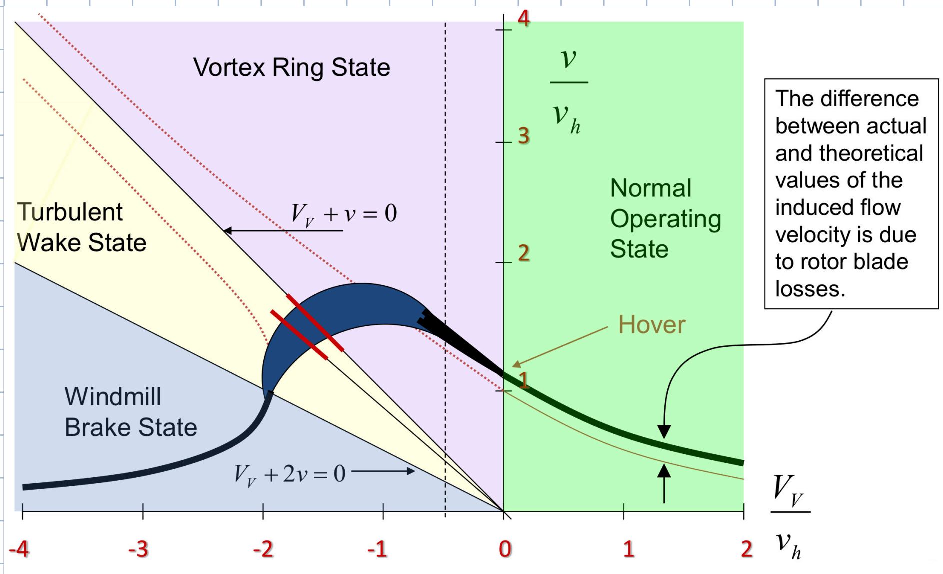
$$\frac{v}{v_h} = -\frac{V_c}{2v_h} \pm \sqrt{\left(\frac{V_c}{2v_h}\right)^2 \pm 1}$$



Descent $\quad$ Hover $\quad$ Climb

Universal Induced Velocity Diagram :

Momentum theory applies here



When $(V + v) = 0$, we have autorotation

$\rightarrow \quad P = T(V + v) \quad \therefore \quad T \neq 0$, but $P = 0$