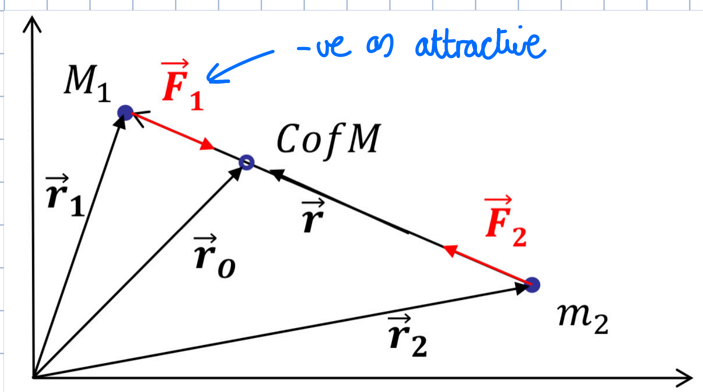
Escape Velocity : min. v needed to escape gravitational force

From conservation of energy :

$(K+U)_{initial} = (K+U)_{final} = 0$ (in absence of friction) (potential -ve)

$$\boxed{E = \frac{1}{2}mv^2 - \frac{GMm}{r}} \qquad \frac{1}{2}mv^2 = \frac{GMm}{r} \qquad \boxed{v_{esc} = \sqrt{\frac{2GM}{r}}}$$

Two Body Problem :



If $M_1 > M_2$ CoM nearer to M_1 than M_2

Coordinate system defined such that :

$$\vec{r} = \vec{r}_1 - \vec{r}_2 = \vec{r}_{21}$$

$$\hat{r}_{21} = \frac{\vec{r}_{21}}{|\vec{r}_{21}|} \qquad r = |\vec{r}|$$

$$\boxed{F_{12} = M_1 \ddot{\vec{r}}_1 = - \frac{GM_1 m_2}{r^2}\hat{r}_{12}} \qquad \ddot{\vec{r}} = \frac{d^2\vec{r}}{dt^2}$$

$$F_{21} = m_2 \ddot{\vec{r}}_2 = - \frac{GM_1 m_2}{r^2}\hat{r}_{21} = \frac{GM_1 m_2}{r^2}\hat{r}_{12} \qquad \text{and } F = m\ddot{\vec{r}}$$

as $\hat{r}_{12} = -\hat{r}_{21}$ $\therefore$ $\boxed{\vec{F}_{12} = -\vec{F}_{21}}$

$$\boxed{\ddot{\vec{r}} = \ddot{\vec{r}}_1 - \ddot{\vec{r}}_2} = - \frac{G\cancel{M_1} m_2}{\cancel{M_1} r^2}\hat{r}_{12} - \left(\frac{GM_1 \cancel{m_2}}{\cancel{m_2} r^2}\hat{r}_{12} \right)$$

$$\boxed{\ddot{\vec{r}} = - \frac{G(M_1 + m_2)}{r^2}\hat{r}_{12}}$$

$G(M_1 + m_2) = \mu =$ gravitational parameter

$\rightarrow$ if $M_1 >> m_2$, $\mu \approx GM_1$

$$\boxed{\ddot{\vec{r}}_1 = - \frac{Gm_2}{r^2}\hat{r}_{12} \quad , \quad \ddot{\vec{r}}_2 = - \frac{GM_1}{r^2}\hat{r}_{21}}$$

Barycentre : centre of mass of two or more bodies that orbit eachother, ∴ the point about which the bodies orbit.

$\rightarrow$ defined as point where the weighted sum of the two position vectors of the two masses relative to the bary centre is 0.

$\rightarrow \quad M_1(\vec{r}_1 - \vec{r}_0) + m_2(\vec{r}_2 - \vec{r}_0) = 0$

$\therefore$ Centre of Mass $= \boxed{\vec{r}_0 = \dfrac{M_1\vec{r}_1 + m_2\vec{r}_2}{M_1 + m_2}}$ or $\dfrac{1}{M}\sum_i m_i r_i$

- $\rightarrow$ if $M_1 >> m_2$, $\vec{r}_0 \approx \vec{r}_1$
- $\rightarrow$ $\dot{\vec{r}}_0$ = constant $\therefore$ no net force acts upon it
- $\rightarrow$ $\ddot{\vec{r}}_0 = 0$, therefore CoM can be found from initial conditions

Remember to set $\vec{r}_x$ at zero if placed at origin.