Inverse of a square matrix is the transpose of the matrix of cofactors divided by the determinant:

$$\boxed{A^{-1} = \frac{1}{\det(A)} \operatorname{adj}(A)}$$

if $|A| = 0$, matrix singular $\therefore$ no inverse

1. Find determinant, $|A|$ or $\det(A)$.

for 2x2

$$\begin{pmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{pmatrix} = a_{1,1}a_{2,2} - a_{1,2}a_{2,1}$$

for 3x3

$$\boxed{+ \quad - \quad + \quad - \quad \dots}$$

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} = a_{1,1}\begin{vmatrix} a_{2,2} & a_{2,3} \\ a_{3,2} & a_{3,3} \end{vmatrix} - a_{1,2}\begin{vmatrix} a_{2,1} & a_{2,3} \\ a_{3,1} & a_{3,3} \end{vmatrix} + a_{1,3}\begin{vmatrix} a_{2,1} & a_{2,2} \\ a_{3,1} & a_{3,2} \end{vmatrix}$$

↳ • take values in any row* in matrix
• 'cross out' row and column the value lies in and multiply value by remaining 2x2 matrix determinant

* can use any row for determinant
→ if row has 0's it makes it simpler

e.g for $a_{1,2}$

$$\begin{pmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ a_{3,1} & a_{3,2} & a_{3,3} \end{pmatrix} \rightarrow a_{1,2}\boxed{\begin{vmatrix} a_{2,1} & a_{2,3} \\ a_{3,1} & a_{3,3} \end{vmatrix}}$$

minor of $a_{1,2}$

Cofactor:
minor multiplied by appropiate sign

$$\begin{vmatrix} + & - & + & - & \dots \\ - & + & - & + & \dots \\ + & - & + & - & \dots \\ \dots & + & - & + & \dots \\ & \dots & \dots & \dots & \end{vmatrix} \qquad \therefore a_{1,2} \text{ cofactor} = -\boxed{\begin{vmatrix} a_{2,1} & a_{2,3} \\ a_{3,1} & a_{3,3} \end{vmatrix}}$$

2. Find adj (A) - transpose of cofactors

Cofactors found using above method : (Minor x appropriate sign) :

e.g cofactor of value 8

$$\begin{pmatrix} 1 & \cancel{2} & 3 \\ 4 & \cancel{5} & 6 \\ \cancel{7} & \cancel{8} & \cancel{9} \end{pmatrix} \begin{pmatrix} + & - & + & - & \cdots \\ - & + & - & + & \cdots \\ + & - & + & - & \cdots \\ \cdots & + & - & + & \cdots \\ & \cdots & \cdots & \cdots & \end{pmatrix} = - \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -(6-12) = 6$$

→ cofactor matrix $\begin{pmatrix} \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & 6 & \cdots \end{pmatrix}$

- Do this for all values to get cofactor matrix.
- Transpose cofactor matrix
  → 1^{st} row becomes 1^{st} column etc.

3. Use formula from beginning.

Remember :

- Determinant multiples minor by it's value, but cofactor is just minor x appropriate sign.