Rocket : any motor that carries its own reaction mass and its own oxidant.

$\hookrightarrow$ works in vacuum

Rocket = launcher / subsystem
Propellant = fuel / oxidiser

Rocket Propulsion :

- $F = ma = \dfrac{d(mv)}{dt}$ 'time derivative of linear momentum'

$\hookrightarrow$ if you small amount of mass at high v, larger rocket mass will accelerate slightly in opposite direction

Mass m, velocity V

discharge with mass dm & relative velocity Ve

$$\Sigma F = \frac{d(mV)}{dt} = m\frac{dV}{dt} + V\frac{dm}{dt}$$

for rocket there are no external forces acting

$$\hookrightarrow m\frac{dV}{dt} + V\frac{dm}{dt} = 0$$

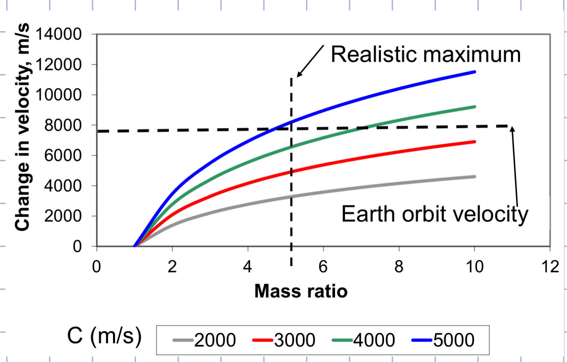
$$\hookrightarrow dV = -V\frac{dm}{m}$$

$\hookrightarrow$ V assumed constant and is effective exhaust v. relative to rocket, often written as C.

Integrating :

$$\boxed{\Delta V = C.\ln\left(\frac{m_0}{m_0 - \Delta m}\right)}$$

$$\text{mass ratio} = \frac{m_0}{m_0 - \Delta m} = \frac{\text{initial } m}{\text{final } m}$$



where mass ratio $= e^{\frac{dV}{C}}$ by rearranging above & taking exp.

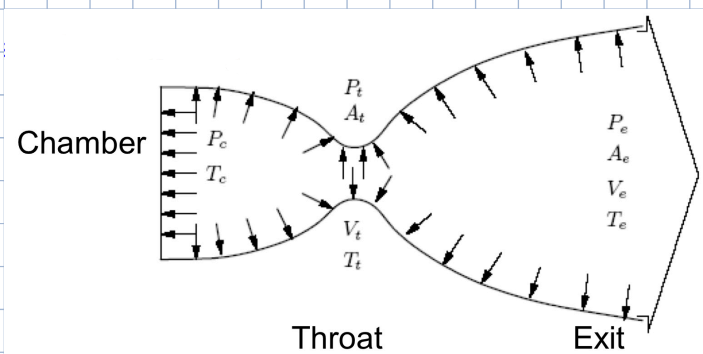
Specific Impulse :

Impulse = F (thrust) x time = $F\Delta t$

$\therefore$ $\boxed{\text{Specific Impulse} = \frac{F\Delta t}{\Delta m} = \frac{F}{\dot{m}}}$

units $\frac{Ns}{kg}$ or $\frac{m}{s}$ or $\frac{Isp}{g} = s$

Rocket Thrust :



Integrating forces acting on thrust chamber :

$$\boxed{F = \dot{m}V_e + A_e(P_e - P_a) = \dot{m}C}$$

C is exhaust v
but is generalised
'effective' velocity that
captures several losses

Total impulse $I_t = I_{sp}\Delta m = F\Delta t = \dot{m}C\Delta t$

$\therefore$ $\boxed{C = \frac{F}{\dot{m}} = \text{Specific impulse}}$ $\boxed{\text{exhaust } V = Isp}$

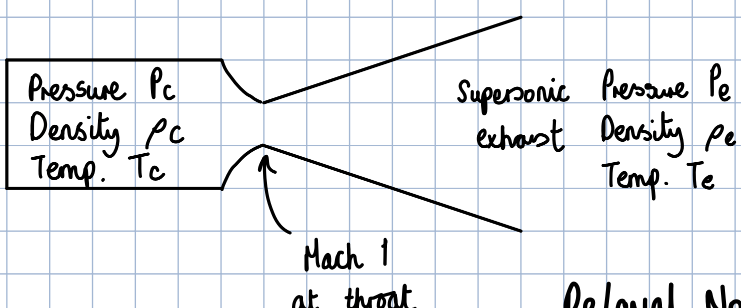
We are focusing on chemical rockets

↳ potential energy stored in chemical bonds released via reaction, heating up a gas

Types of propellant :

- Liquid mono, bi, tri propellants
- Solids
- Hybrids

How Chemical Rocket Works :

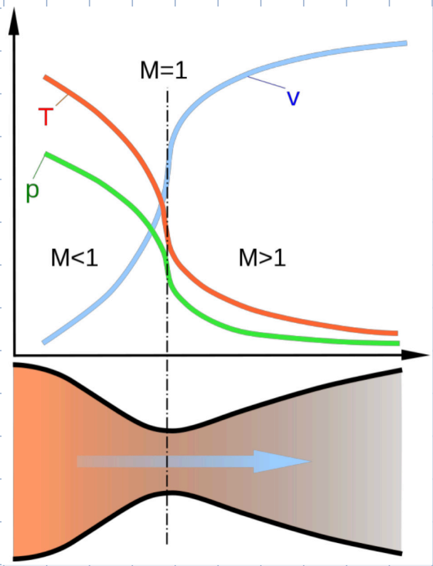


Flow escapes down the nozzle, accelerating but cooling as it goes.

Delaval Nozzle - nozzle with convergent & divergent section.

Nozzle converts:

↓V, ↑P, ↑T gas in combustion chamber ⟶ ↑V, ↓P, ↓T at exit



Terms:

p = pressure
V = volume of container
v = velocity
M = molecular mass of propellant (kg/kmol)

T = absolute temperature
R = universal gas constant : 8314 J/kmol.K
h = sp. enthalpy
γ = ratio of $\frac{C_p}{C_v}$

From 1st law of thermodynamics & ideal gas laws:

$$h_i - h = \frac{1}{2}v^2 - \frac{1}{2}v_i^2 = \frac{\gamma . R(T_i - T)}{(\gamma - 1)M}$$

initial

Rearranging for $$\boxed{v = \left[\frac{2\gamma . RT_i}{(\gamma - 1)M}\left(1 - \frac{T}{T_i}\right) + v_i^2\right]^{1/2}}$$ ①

$$\boxed{\frac{P_e}{P_c} = \left(\frac{T_e}{T_c}\right)^{\frac{\gamma}{\gamma - 1}} \quad \text{or} \quad \frac{T_e}{T_c} = \left(\frac{P_e}{P_c}\right)^{\frac{\gamma - 1}{\gamma}}}$$ ② using isentropic flow

where e = exhaust, c = chamber

also $\dot{m} = \rho A v$ = constant

subbing ① into ② $$\boxed{v_e^2 = \frac{2\gamma . RT_c}{(\gamma - 1)M}\left(1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma - 1}{\gamma}}\right)}$$

- We want $\uparrow V_e$, so we need $\frac{P_e}{P_c} \to 0$, $M\downarrow$, $T_c\uparrow$

- We can group into two two numbers characterising the engine : C^* & C_F

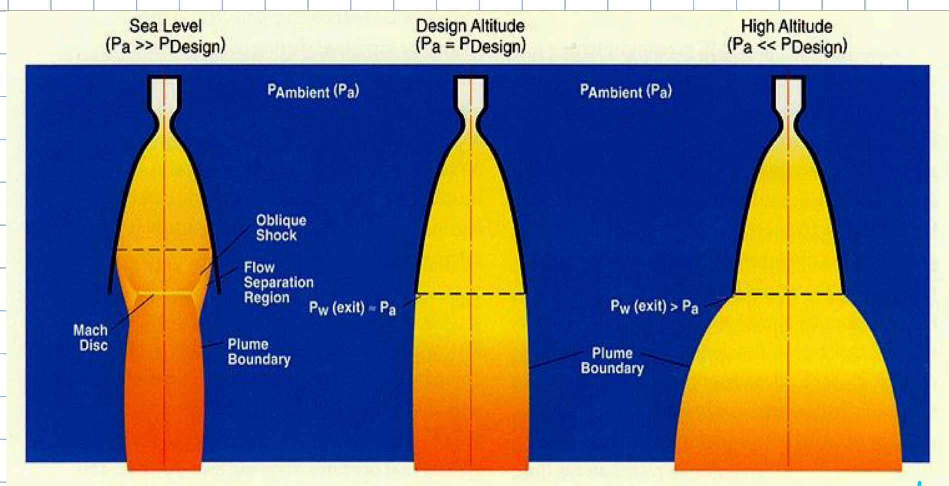
Thrust Coefficient $\boxed{C_F = \frac{F}{P_c A_t} = \frac{\dot{m} V_e + A_e (P_e - P_a)}{P_c A_t}}$

Characteristic Exhaust Velocity C^* $\boxed{C^* = \frac{C}{C_F} = \frac{P_c A_t}{\dot{m}}}$

$\dot{m} \to \rho A V \to$ dependent on $\frac{R}{M}$, T_c, γ

propellant parameters but independent of nozzle performance

$\uparrow T_c$ value desirable for $\uparrow I_{sp}$, but gives problems with heat transfer into case walls and dissociation of combustion products.
Practical limit 2750 - 3500 K



Over-Expanded

- $P_a > P_e$

→ pressure term -ve

→ suction & shocks with seperation

→ most nozzles designed not to seperate

Correctly Expanded

- $P_e = P_a$

Under-expanded

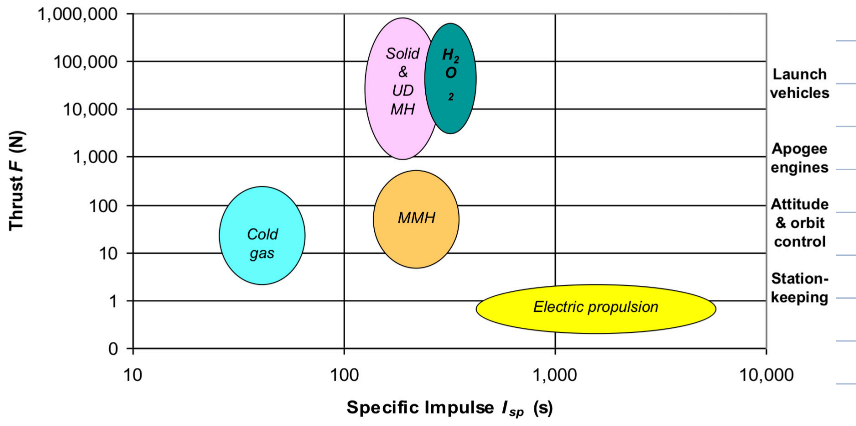
- $P_a < P_e$

→ flow continues to expand outward after it has exited the nozzle

→ this energy is not converted into thrust & is lost

V_e equation implies low P_e useful, but if P_a high (low altitude), pressure term becomes -ve and ∴ get suction ⟶ seperation if severe ⟶ unstable & dangerous.

Thrust vs. Isp for Various Propellants:

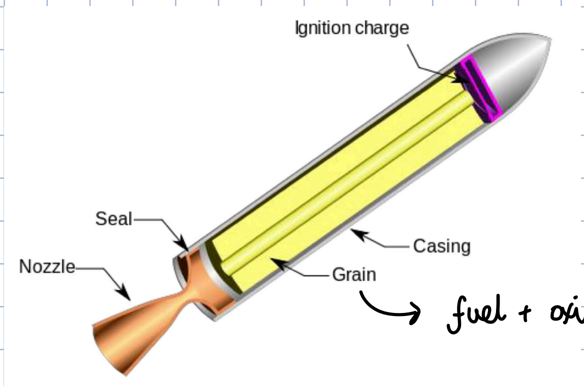


↑Isp means more force per amount of propellant per unit time.

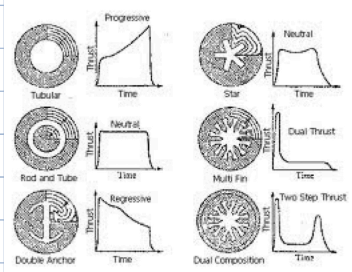
Three Main Types of Chemical Propellants:

Solids: 'motors'

- Motor case less massive than liquid
- Propellant must burn at correct rate to maintain pressure
- ↑$\dot{m}$, ↑F, ↓Isp (~2000-2500)

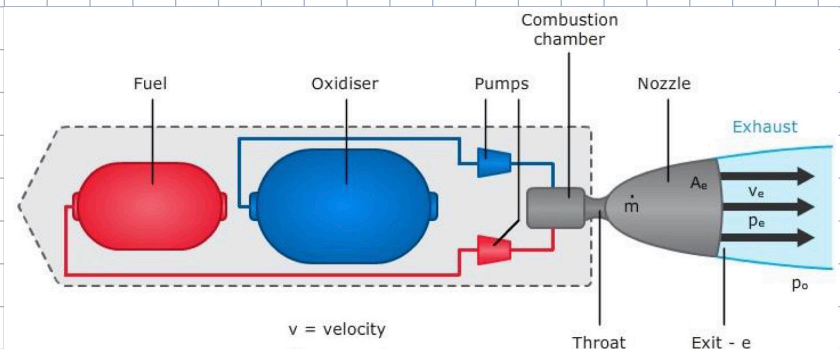


→ fuel + oxidiser



→ grain can be shaped to control burn rate

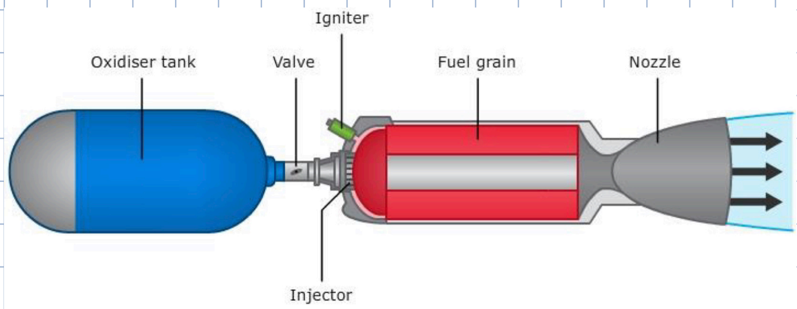
Liquids: 'engine'



- Liquids give
  → thrust control
  → restartable engines
- if biprop, thrust > solids
- lower reliability, problems w. temp due to propellant lines freezing up

Hypergolic propellants : ignite upon contact → no ignition required

Hybrids :



- Normally solid fuel, liquid oxidiser
- Rocket can be turned on & off
- Can be throttled
- Relatively safe
- Similar perf. to solids but ↑ reliability
- Lower Isp than liquids but more reliable
- Some fuels used are toxic

Example Isp Values :

Class	Propellant	Isp (m/s)
Solid	Rubber	2200-2800
Liquid monopropellant	Hydrazine	1600-1900
Liquid bipropellant	Liquid Oxygen/Liquid Hydrogen	3800-4500
	Fluorine/Hydrogen	3800-4500
	Liquid oxygen/Hydrazine	3200-3800
	Liquid oxygen/Kerosene (RP-1)	2900-3400
Hybrid	Nitrous oxide/rubber (HTPB)	2400-2800

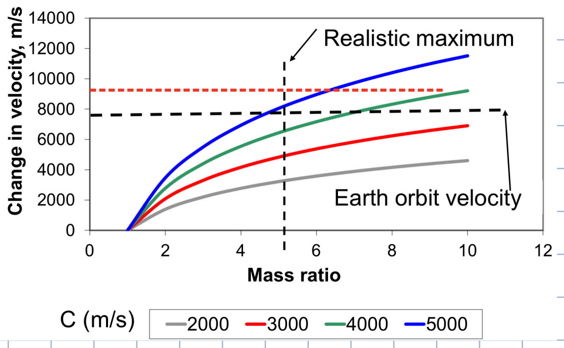
Launch Losses : and percentage of total energy

- Potential energy - energy required to raise object from one altitude to another 15-20%
- Gravity losses - large horizontal impulse takes finite time so extra fuel required to counteract 10 - 15 %
- Drag losses - due to atmosphere 0.5 %
- Steering losses - due to axis of rocket not aligned with V > 0.5 %

Summary of losses for ΔV for LEO (185km):

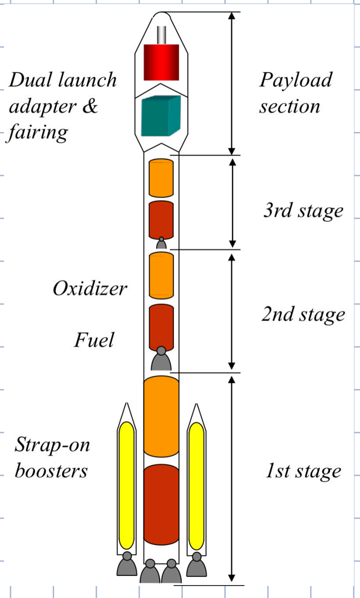
Orbital Velocity	7.7 Km/s
Get to Altitude (P.E.)	1.3 Km/s
Gravity Losses (finite time)	0.7 Km/s
Atmosphere losses (drag)	0.1 Km/s
Earth's Rotation (varies)	-0.5 Km/s
Total	9.3 Km/s

→ ΔV required without losses



if we take normal $I_{sp} = 3000$ m/s for solid booster, we can see mass ratio isn't achievable.

∴ stages used



→ remove dry mass as you go

→ upper stages can use motors optimised for vacuum

→ ΔV changes are cumulative

↳ Calculate using:

$$\Delta V = I_{sp} \ln\left(\frac{M_{start}}{M_{end}}\right)$$

Propellant mass fraction, $f_p = \frac{\Delta m}{\Delta m + m_d}$

mass propellant

dry mass

$$\therefore m_d = \Delta m\left(\frac{1}{f_p} - 1\right)$$

Multistages: $M_{start} = \underbrace{m_1 + m_2 + m_3 \ldots}_{\text{wet masses (dry mass + propellant)}} + \text{payload mass}$