

A High-Order Low-Order Algorithm with Exponentially Convergent Monte Carlo for Thermal Radiative Transfer

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Background on Thermal Radiative Transfer

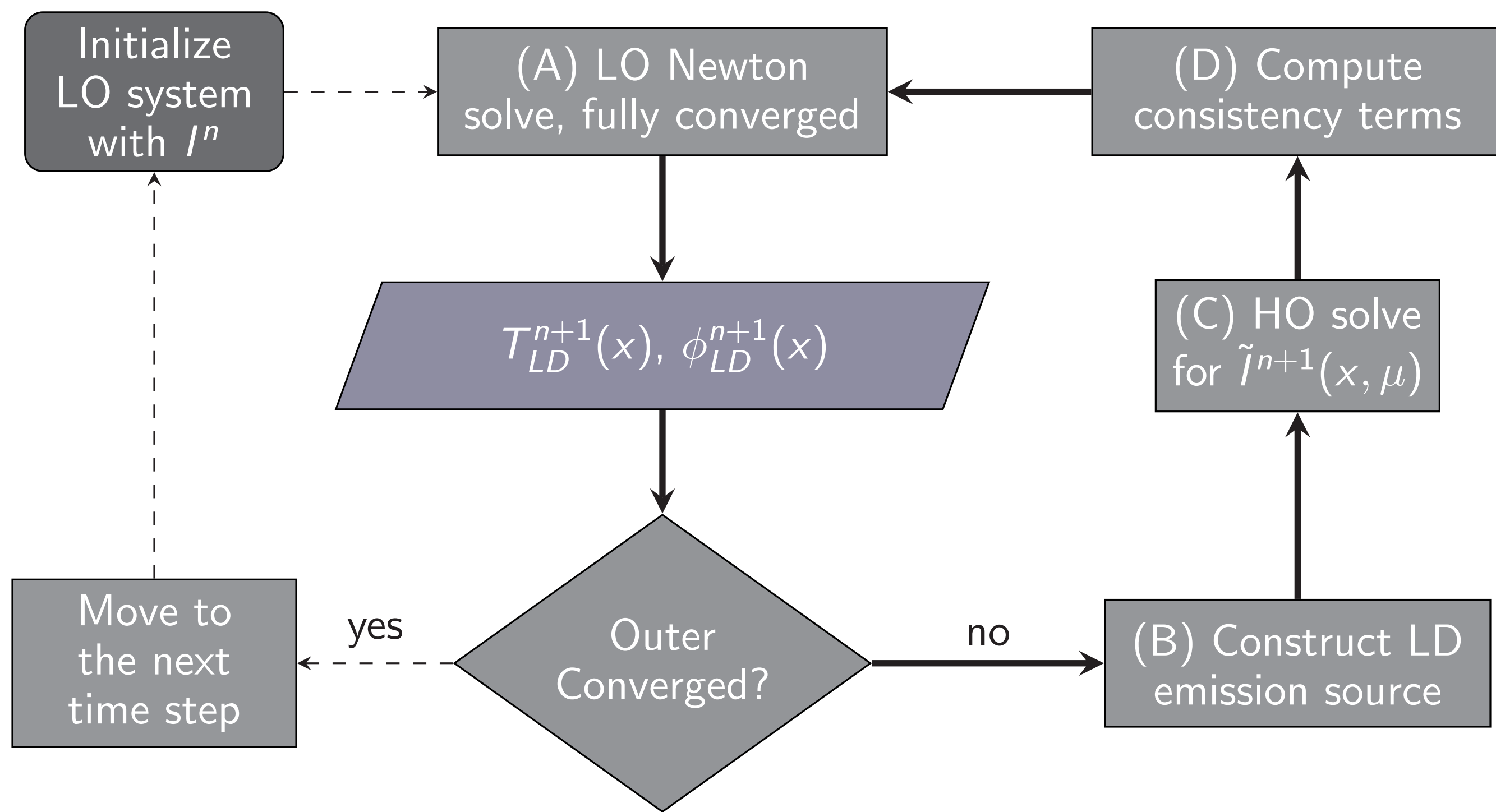
- The time-discretized radiation and material energy equations are

$$\frac{I^{n+1} - I^n}{c\Delta t} + \mu \frac{\partial I^{n+1}}{\partial x} = \frac{1}{2} \sigma_a a c (T^4)^{n+1} - \sigma_a I^{n+1} \quad (1)$$

$$\rho c_v \frac{T^{n+1} - T^n}{\Delta t} = \sigma_a \phi^{n+1} - \sigma_a a c (T^4)^{n+1}. \quad (2)$$

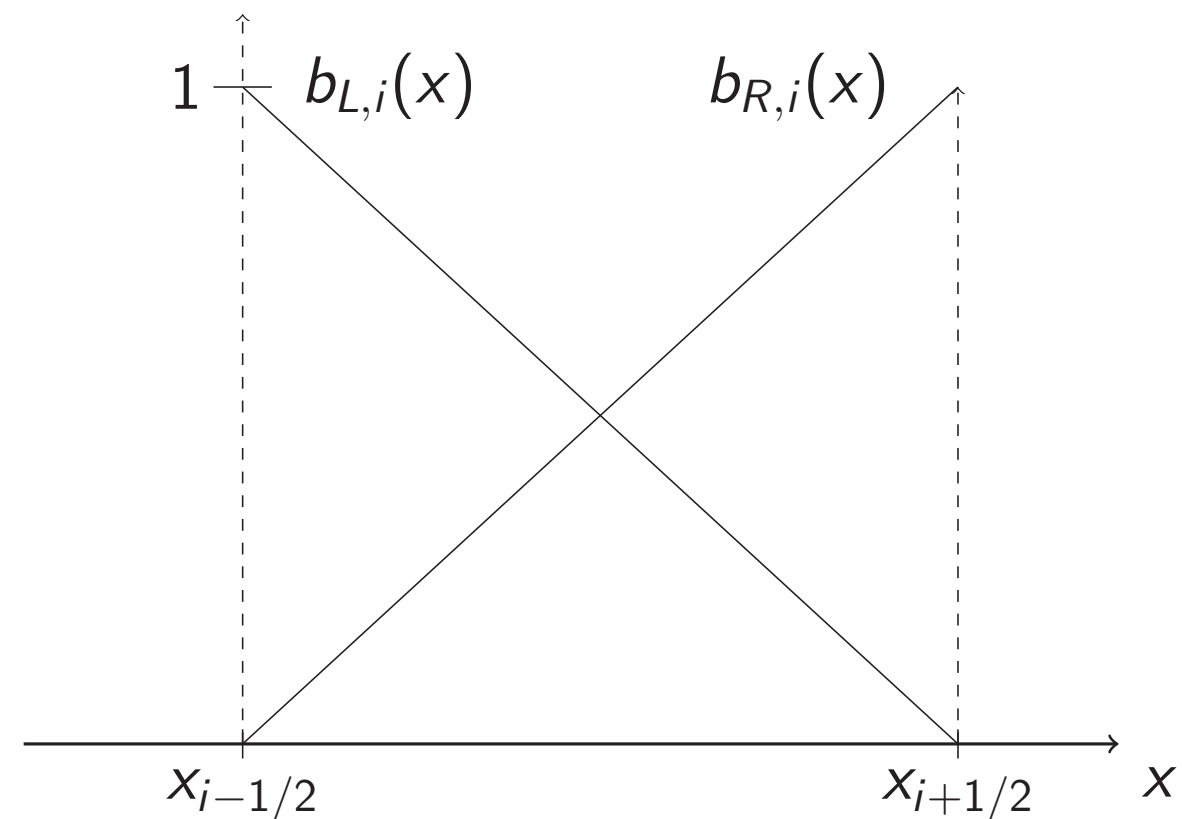
- The radiation intensity $I(x, \mu, t^{n+1})$ and material temperature $T(x, t^{n+1})$ are **coupled** through photon **emission** $\sigma_a a c T^4$ and **absorption** $\sigma_a \phi$.
- We form a **lower-dimensional** system which can efficiently **resolve non-linearities** in the problem. The low-order (LO) equations contain consistency terms that preserve accuracy of **efficient**, high-order (HO) Monte Carlo simulations.

Algorithm



Details of the algorithm

- (A) The LO equations are formed by taking spatial and angular moments of Eq. (1) & (2), defined locally over the i -th spatial finite element (FE).



Spatial moments

$$\langle \cdot \rangle_{L/R,i} = \frac{2}{h_i} \int_{x_{i-1/2}}^{x_{i+1/2}} b_{L/R,i}(x) (\cdot) dx$$

Angular half-ranges

$$\phi^\pm(x) = \pm \int_0^{\pm 1} I(x, \mu) d\mu$$

Based on the current HO estimate of consistency terms, the non-linearities in T are fully resolved for each LO solve, using **Newton's method**.

- (B) The HO emission source is constructed from the previous LO solution, producing a **fixed source, pure absorber** HO problem.
- (C) For each HO solve, multiple exponentially-convergent Monte Carlo (ECMC) batches are performed to solve a pure absorber transport problem.
- Each ECMC batch **tallies the error** in the latest estimate of the solution. By initializing the first estimate of \tilde{I}^{n+1} to \tilde{I}^n , **very few histories are needed** because we are only estimating the change in I over a time step.
 - ECMC uses a **projection of the exact solution** onto a linear discontinuous (LD) finite element, **space-angle** mesh denoted \tilde{I}^{n+1} .
- (D) We use the latest ECMC solution for \tilde{I}^{n+1} to evaluate **angular consistency terms** in the LO equation, e.g.,

$$\bar{\mu}_L^+ = \frac{\langle \mu I^{HO} \rangle_L^+}{\langle I^{HO} \rangle_L^+}$$

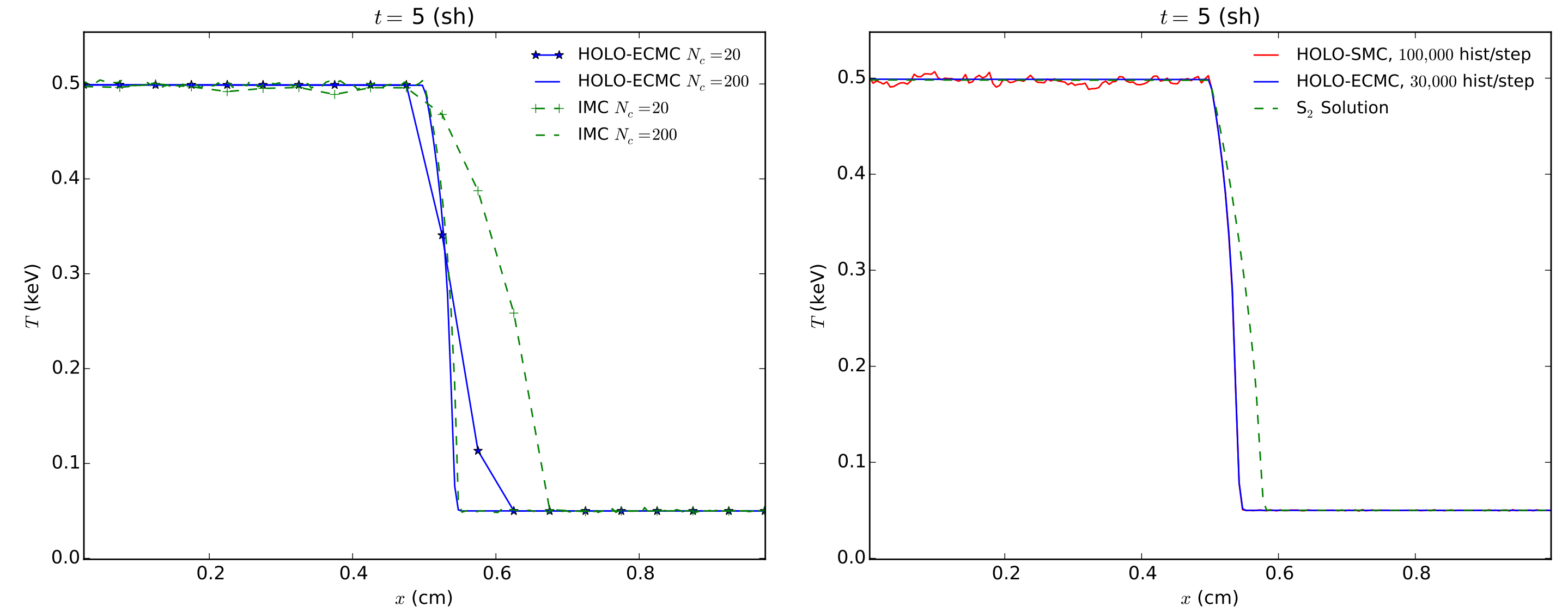
Diffusion Synthetic Acceleration (DSA) of LO equations

- We perform DSA-accelerated source iterations on the effective scattering source resulting from linearization of the LO equations.
- A spatially **continuous** discretization of the diffusion equation is applied to the residual equation for scattering iterations. The estimated error is mapped onto the moment unknowns based on a **discontinuous** discretization of the P_1 equations over each cell.
- In a modified version of the two material problem, DSA **reduced the spectral radius of iterations from 0.993 to <0.1**.

Results for two Marshak wave test problems

- For all HOLO results only one outer iteration is performed, per time step. Figures depict radiation temperature $T_R = \sqrt[4]{\phi/(ac)}$.

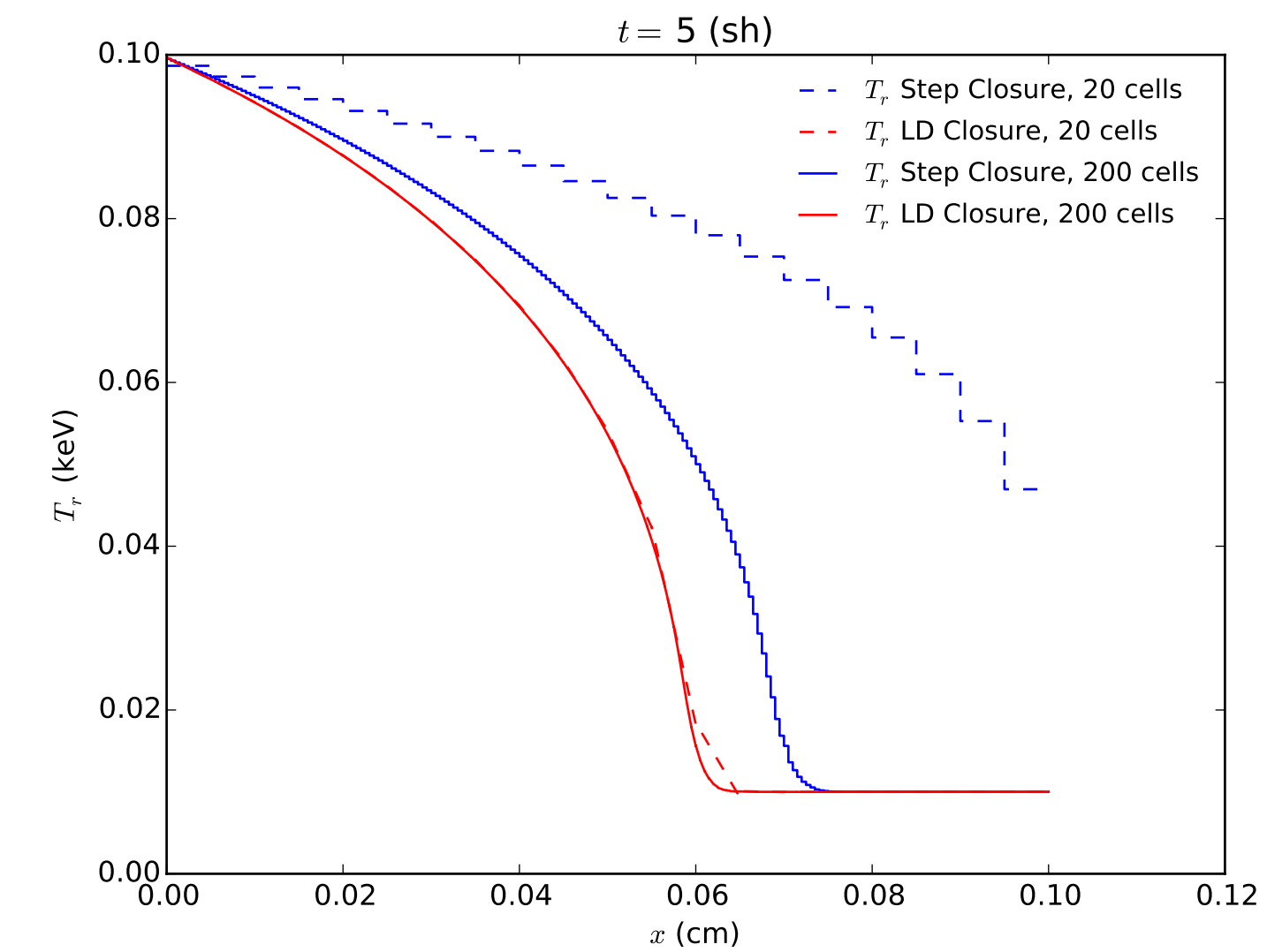
Two Material Problem: Optically thin (left) and thick (right) regions



(a) Spatial mesh convergence.

(b) ECMC vs Standard MC as HO solver

Equilibrium Diffusion Limit Problem: large σ_a and small C_v .

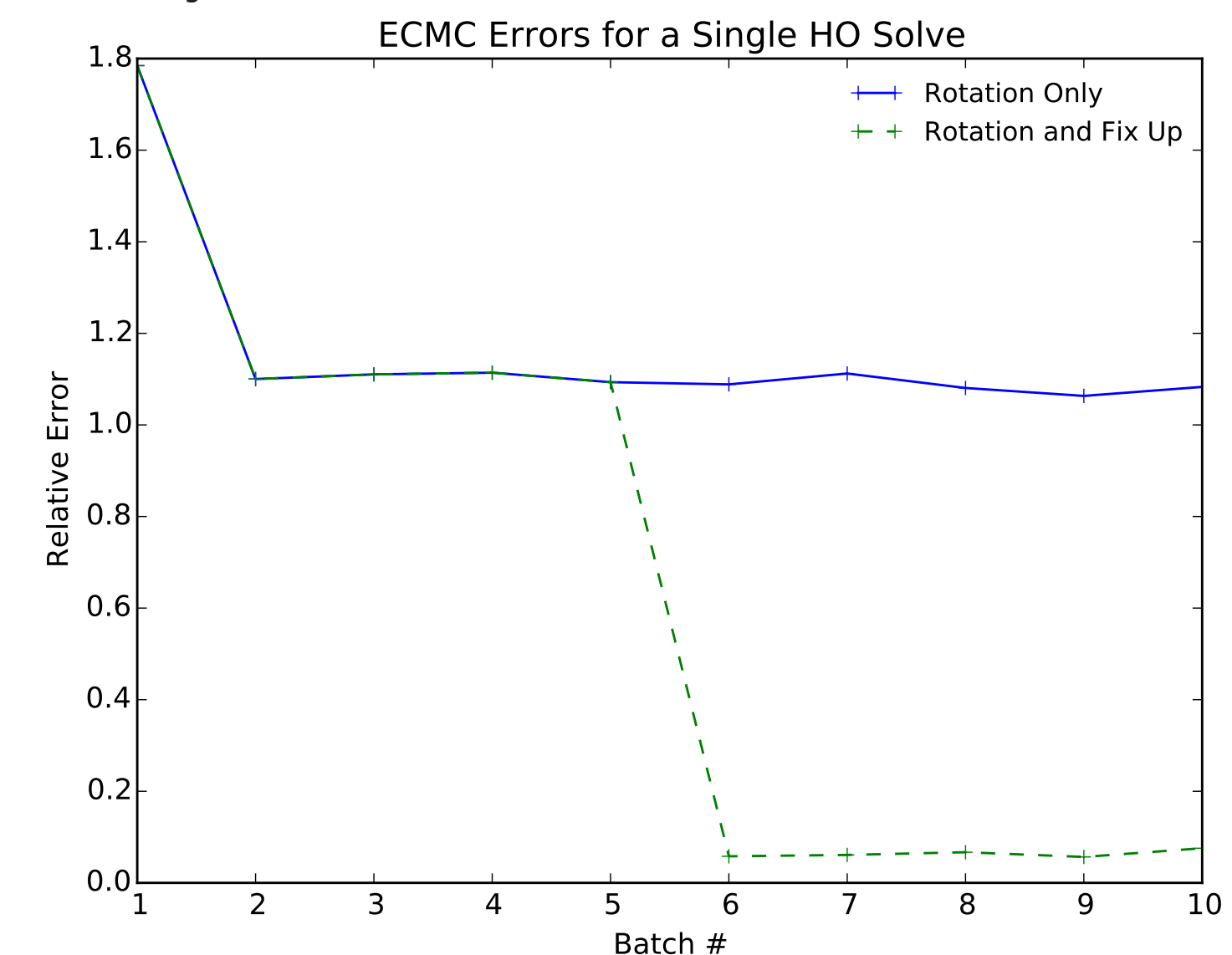


(a) Comparison of HOLO using LD and step discretizations in LO equations.

A fixup is necessary near strong solution gradients

- In strongly absorbing regions, the linear discontinuous spatial representation can lead to **unphysical negative solutions** in certain cells
- In the LO system, we set a strict floor value and preserve energy conservation to determine the cell average.
- For the HO system, we **rotate** $\tilde{I}(x, \mu)$ to be strictly positive after each batch. However, $\tilde{I}_{\text{rotated}}(x, \mu)$ doesn't satisfy the original residual equation, so the ECMC convergence quickly **stagnates**. Thus, we add an artificial source $\tilde{\delta}(x, \mu)$, which is estimated *iteratively* as

$$\tilde{\delta}(x, \mu) = \mathbf{L}(\tilde{I}^{n+1} - \tilde{I}_{\text{rotated}}^{n+1})$$
 where \mathbf{L} is the continuous streaming plus removal operator. The solution will now converge towards the **strictly positive** projection $\tilde{I}_{\text{rotated}}^{n+1}$.
- We also allow the outflow from the cell to be **discontinuous**, allowing for greater angular accuracy on faces



Ongoing work

- We are working towards **time-continous** transport for ECMC to produce high temporal accuracy in optically thin regions, requiring additional consistency terms in the LO equations.
- Further investigation of accuracy, robustness, and consistency in fixup regions is necessary.

Acknowledgements

