A High-Order Low-Order Algorithm with Exponentially Convergent Monte Carlo for Thermal Radiative Transfer

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Background on Thermal Radiative Transfer

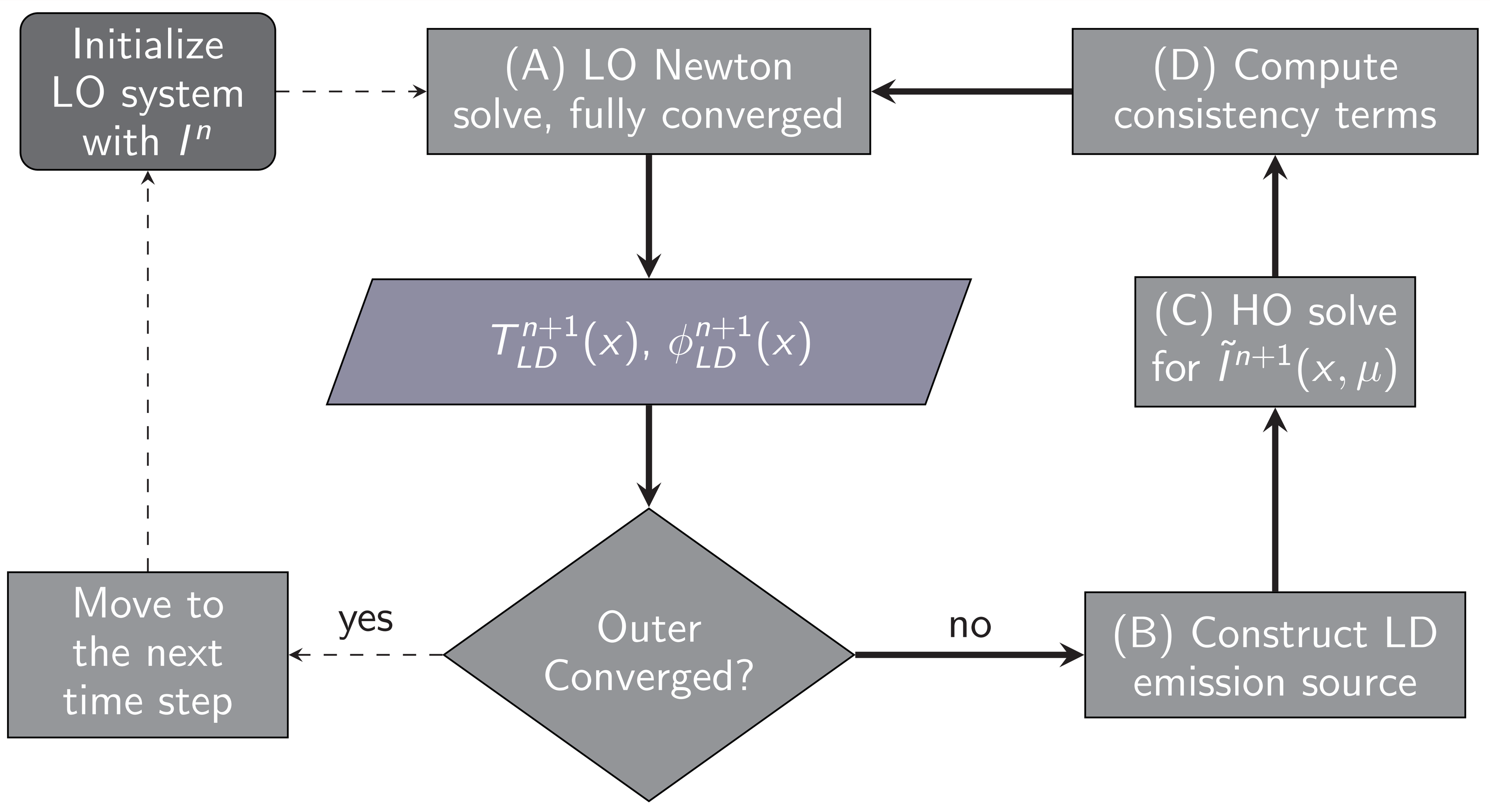
- The continuous radiation and material energy equations are

$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma_t I(x,\mu,t) = \frac{\sigma_s\phi(x)}{4\pi} + \frac{1}{4\pi}\sigma_a acT^4, \qquad (1)$$

$$C_v\frac{\partial T(x,t)}{\partial t} = \sigma_a\phi(x,t) - \sigma_a acT^4 \qquad (2)$$

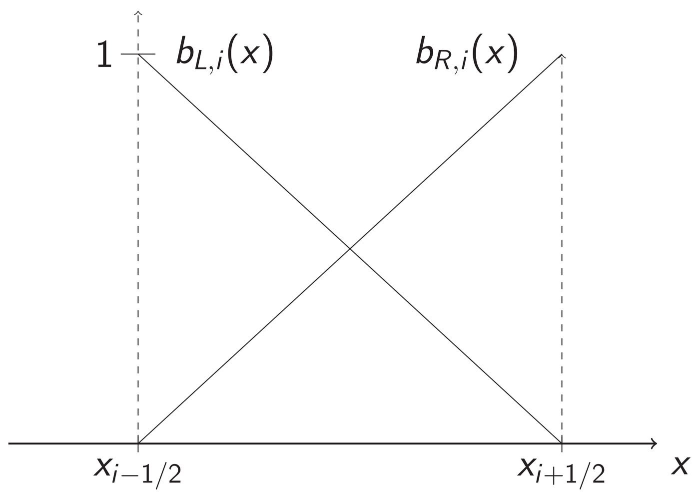
- The radiation intensity $I(x,\mu,t)$ and material temperature $T(x,t)$ are **coupled** through photon **emission** $\sigma_a acT^4$ and **absorption** $\sigma_a\phi$.
- We form a **lower-dimensional** system which can efficiently **resolve non-linearities** in the problem. The low-order (LO) equations contain consistency terms that preserve accuracy of **efficient**, high-order (HO) Monte Carlo simulations.
- We recently have included the time variable in the Monte Carlo transport algorithm, improving accuracy in optically thin problems.

Algorithm



Details of the algorithm

(A) The LO equations are formed by taking spatial, angular, and temporal moments of Eq. (1) & (2), defined locally over the i-th spatial finite element (FE).



Spatial moments

$$\langle\cdot\rangle_{L/R,i} = \frac{2}{h_i}\int_{x_{i-1/2}}^{x_{i+1/2}} b_{L/R,i}(x)(\cdot)\mathrm{d}x$$

Angular half-ranges

$$\phi^{\pm}(x) = \pm\int_0^{\pm1} I(x,\mu)\mathrm{d}\mu$$

Based on the current HO estimate of consistency terms, the non-linearities in T are fully resolved *for each LO solve*, using Newton's method.

(B) The HO emission source is constructed from the previous LO solution, producing a **fixed source, pure absorber** HO problem.

(C) *For each HO solve*, multiple exponentially-convergent Monte Carlo (ECMC) batches are performed to solve a pure absorber transport problem.

- Each ECMC batch tallies the error in the latest estimate of the solution. By initializing the first estimate of $\tilde{I}^{n+1}$ to $\tilde{I}^n$, **very few histories are needed** because we are only estimating the change in I over a time step.
- ECMC uses a **projection of the exact solution** onto a linear discontinuous (LD) finite element, space-angle mesh denoted $\tilde{I}^{n+1}$.

(D) We use the latest ECMC solution for $\tilde{I}^{n+1}$ to evaluate angular consistency terms in the LO equation, e.g.,

$$\overline{\mu}_L^+ = \frac{\langle\mu\, I^{HO}\rangle_L^+}{\langle I^{HO}\rangle_L^+}$$

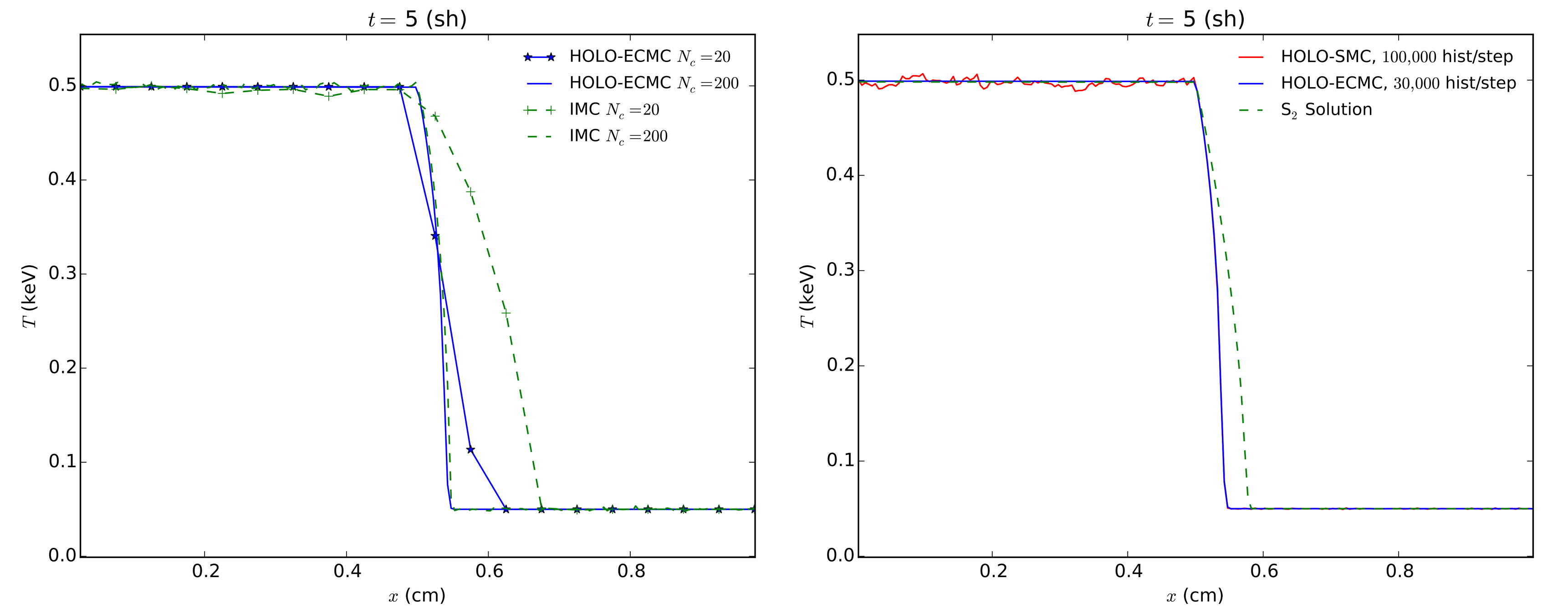
Diffusion Synthetic Acceleration (DSA) of LO equations

- We perform DSA-accelerated source iterations on the effective scattering source resulting from linearization of the LO equations.
- A spatially continuous discretization of the diffusion equation is applied to the residual equation for scattering iterations. The estimated error is mapped onto the moment unknowns based on a discontinuous discretization of the P_1 equations over each cell.
- In a modified version of the two material problem, DSA **reduced the**

Results for two Marshak wave test problems

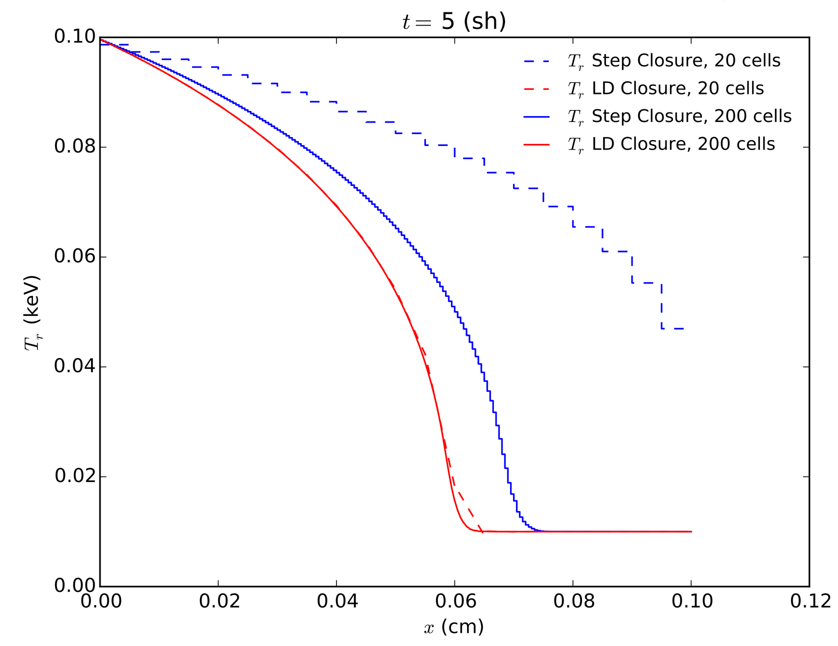
- For all HOLO results only one outer iteration is performed, per time step. Figures depict radiation temperature $T_R = \sqrt[4]{\phi/(ac)}$.

Two Material Problem: Optically thin (left) and thick (right) regions



(a) Spatial mesh convergence. (b) ECMC vs Standard MC as HO solver

Equilibrium Diffusion Limit Problem: large σ_a and small C_v.



(a) Comparison of HOLO using LD and step discretizations in LO equations.

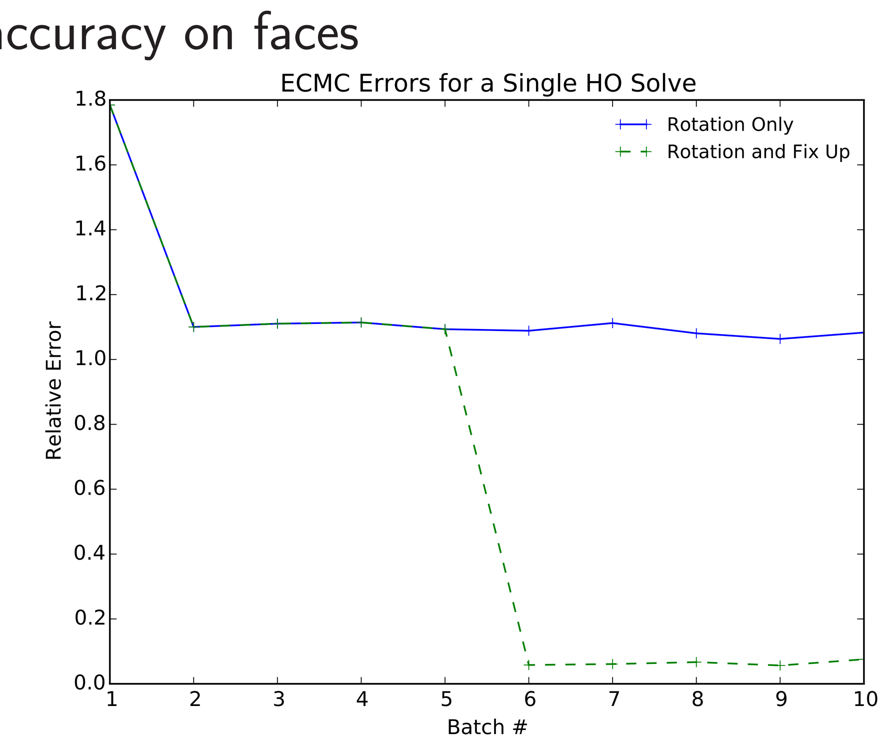
A fixup is necessary near strong solution gradients

- In strongly absorbing regions, the linear discontinuous spatial representation can lead to unphysical negative solutions in certain cells
- In the LO system, we set a strict floor value and preserve energy conservation to determine the cell average.
- For the HO system, we **rotate** $\tilde{I}(x,\mu)$ to be strictly positive after each batch. However, $\tilde{I}_{\text{rotated}}(x,\mu)$ doesn't satisfy the original residual equation, so the ECMC convergence quickly stagnates. Thus, we add an artifical source $\tilde{\delta}(x,\mu)$, which is estimated *iteratively* as

$$\tilde{\delta}(x,\mu) = \mathbf{L}(\tilde{I}^{n+1} - \tilde{I}^{n+1}_{\text{rotated}})$$

where $\mathbf{L}$ is the continuous streaming plus removal operator. The solution will now converge towards the strictly positive projection $\tilde{I}^{n+1}_{\text{rotated}}$.

- We also allow the outflow from the cell to be **discontinuous**, allowing for greater angular accuracy on faces



Ongoing work

- We are working towards a linear-discontinuous trial space in time. ECMC will be used to estimate the slope of the intensity in time, which can be used to extrapolate to the end of the time step. This representation will improve the statistical efficiency of tallies used to estimate $\tilde{I}(x,\mu)$ at the end of the time step because all particles contribute to the local slope
- The LD time space will require a different sampling approach, which will be useful to investigate for extending ECMC to higher spatial dimensions.

Acknowledgements

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