A High-Order Low-Order Algorithm with Residual Monte Carlo Time Integration for Radiative Transfer

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Overview and background on thermal radiative transfer

- The continuous radiation and material energy equations are

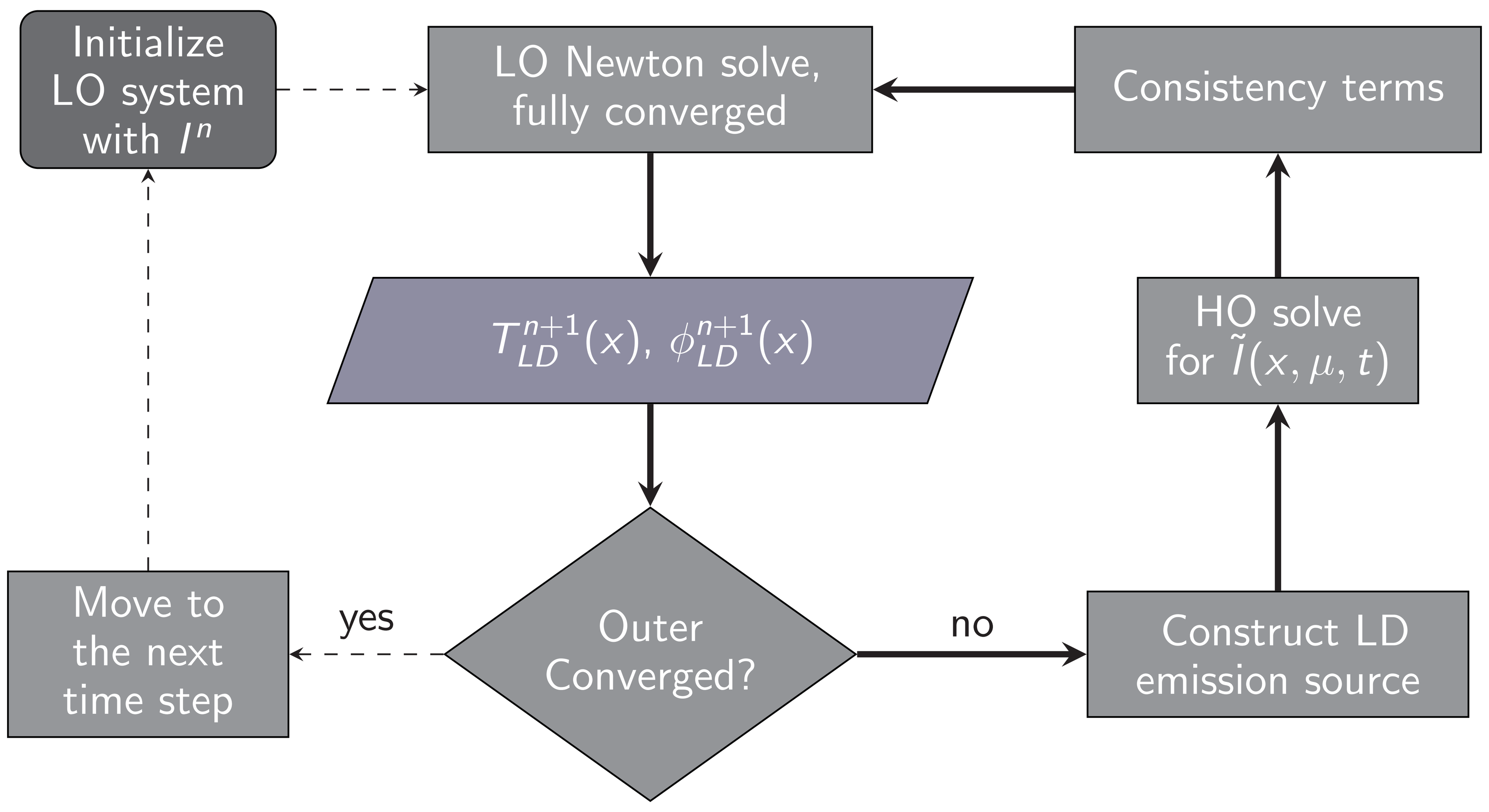
$$\frac{1}{c}\frac{\partial I}{\partial t} + \mu\frac{\partial I}{\partial x} + \sigma_t I(x,\mu,t) = \frac{\sigma_s\phi(x)}{4\pi} + \frac{1}{4\pi}\sigma_a acT^4, \qquad (1)$$

$$C_v\frac{\partial T(x,t)}{\partial t} = \sigma_a\phi(x,t) - \sigma_a acT^4 \qquad (2)$$

- The radiation intensity $I(x,\mu,t)$ and material temperature $T(x,t)$ are **coupled** through photon **emission** $\sigma_a acT^4$ and **absorption** $\sigma_a\phi$.
- We form a **lower-dimensional** system which can efficiently **resolve non-linearities** in the problem. The low-order (LO) equations contain consistency terms that preserve accuracy of **efficient**, high-order (HO) exponentially-convergent Monte Carlo (ECMC) simulations.

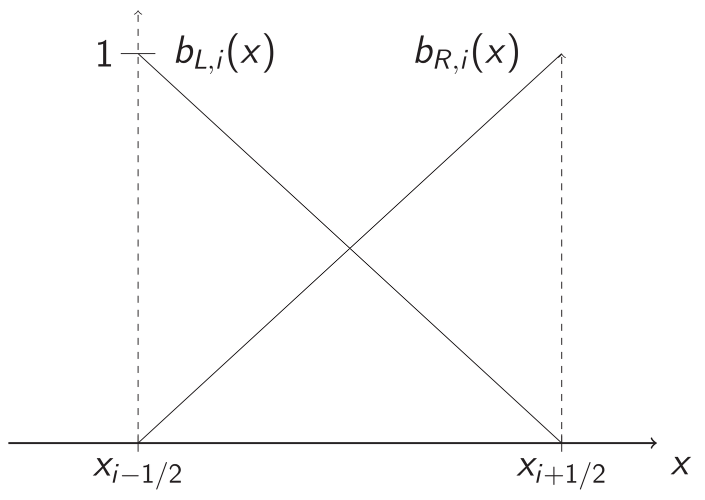
We now include the time variable t in the MC transport algorithm with a **consistent** LO temporal closure. This improves the accuracy of radiation wavefronts in optically thin problems.

The HOLO algorithm is applied for each time step



Details of the LO equations

- The LO equations are formed with spatial, angular, and temporal moments of Eq. (1) & (2) and algebraic manipulation. Moments are defined locally over the i-th spatial finite element (FE) and n-th time step.



Spatial moments

$$\langle\cdot\rangle_{L/R,i} = \frac{2}{h_i}\int_{x_{i-1/2}}^{x_{i+1/2}} b_{L/R,i}(x)(\cdot)\mathrm{d}x$$

Angular half-ranges

$$\phi^{\pm}(x) = \pm\int_0^{\pm 1} I(x,\mu)\mathrm{d}\mu$$

- The space-angle moment equations are exactly integrated over the n-th time step. Temperature unknowns must be discretized with linear-discontinuous finite elements (LDFE) in space and backward Euler in time.
- We use the latest ECMC solution for the **time-averaged projection** of $I(x,\mu,t)$ to evaluate angular consistency terms in the LO equations, e.g.,

$$\{\overline{\mu}\}^+_{L,i} = \frac{\langle\mu\,\overline{I}_{HO}(x,\mu)\rangle^+_{L,i}}{\langle\overline{I}_{HO}(x,\mu)\rangle^+_{L,i}}$$

- The extra end-of-time-step radiation moments, i.e., ϕ^{n+1}, are eliminated in terms of time-averaged moments, i.e., $\overline{\phi}$, with **parametric time closures**. The ECMC solution is used to estimate the local closures, e.g.,

$$\langle\phi\rangle^{n+1,+}_{L,i} = \gamma^{HO,+}_{L,i}\langle\overline{\phi}\rangle^+_{L,i}$$

where $\gamma^{HO,+}_{L,i}$ is a constant that is estimated with the above relation and the HO solution. There is a time closure for each radiation moment equation.

- The nonlinearities are fully resolved *for each LO solve* using Newton's method, based on the current HO consistency terms and time closure.
- The equations are solved in terms of time-averaged unknowns, then the solution is advanced to t^{n+1} for the next time step using the time closure.

Acknowledgements

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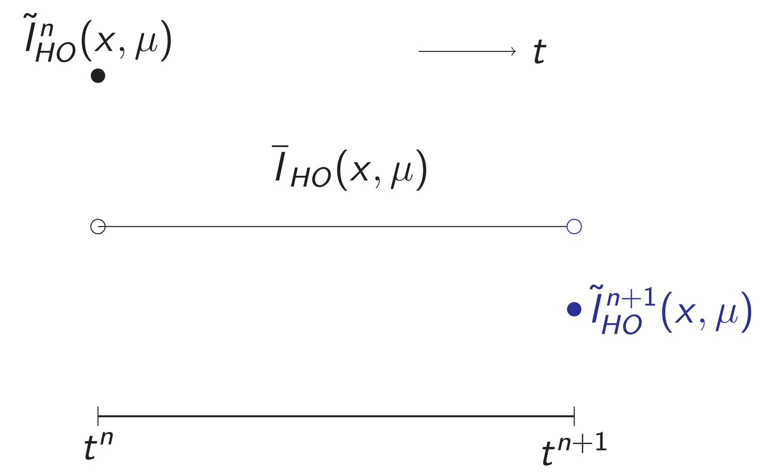
Details of ECMC Algorithm with MC time integration

- The HO emission source is constructed from the previous LO solution, producing a **fixed source, pure absorber** HO problem.
- ECMC requires a trial space representation $\tilde{I}(x,\mu,t)$ for the intensity. We use a step doubly discontinuous space in t, with an LDFE projection in x and μ:



- The continuous time derivative $\frac{1}{c}\frac{\partial}{\partial t}(\cdot)$ is included in the transport operator. With $T^4(x)$ implicit in time, the residual for $\tilde{I}(x,\mu,t)$ is

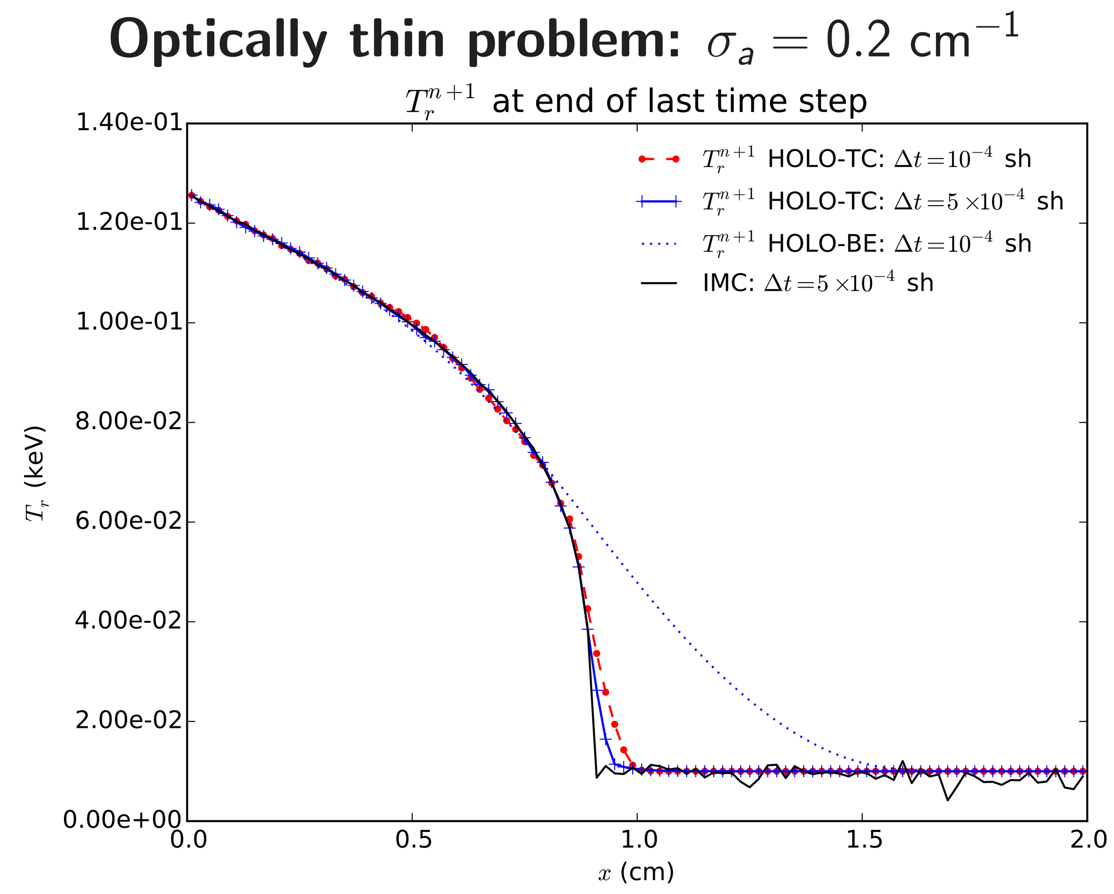
$$r(x,\mu,t) = \frac{1}{2}\sigma_a^{n+1}ac\left(T^{n+1}\right)^4 - \frac{1}{c}\frac{\partial\tilde{I}}{\partial t} - \mu\frac{\partial\tilde{I}}{\partial x} - \sigma_a\tilde{I}$$

- Particle histories are sampled and tracked in time to get a MC solution to the transport equation for the error in $\tilde{I}(x,\mu,t)$. The error is projected onto the trial space to accurately estimate the time-averaged and t^{n+1} intensities.

Computational Results

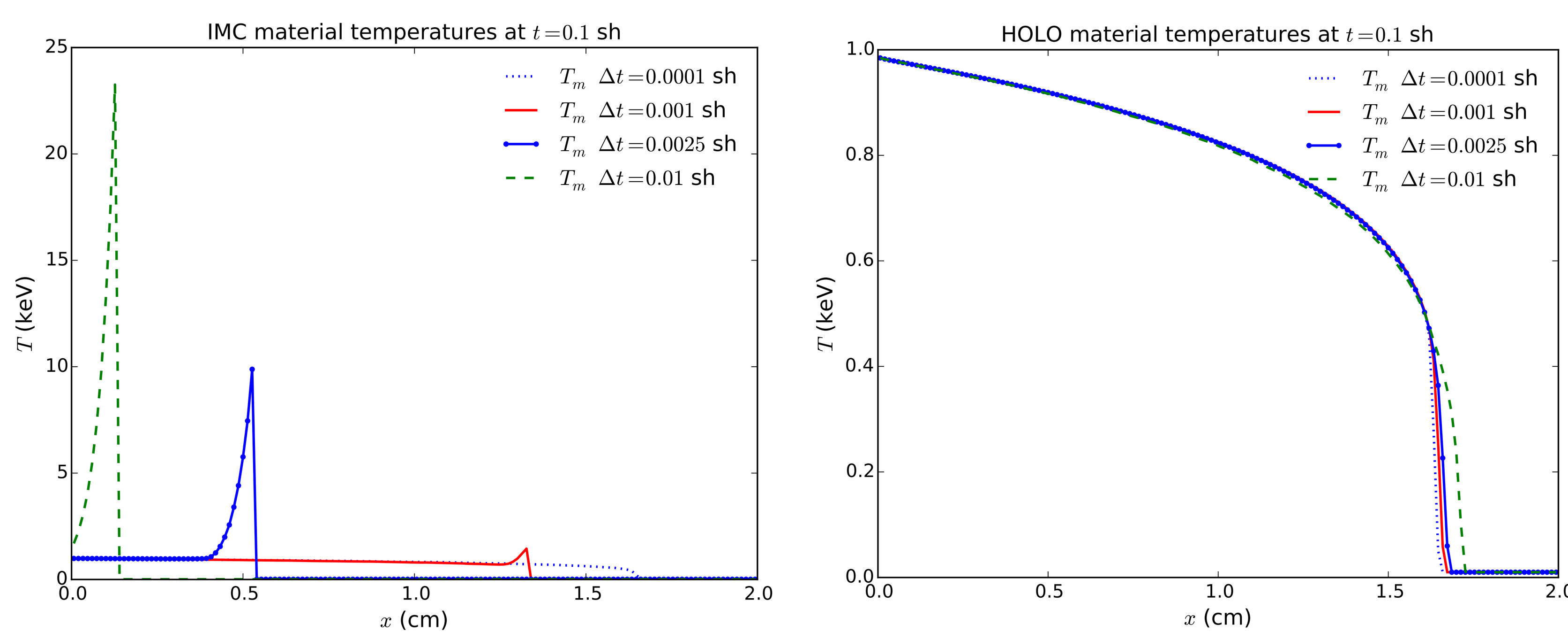
- The MC time integration with time closure (HOLO-TC) improves accuracy in optically thin problems compared to backward Euler time discretization (HOLO-BE). Figure depicts radiation temperature $T_R = \sqrt[4]{\phi/(ac)}$.

Optically thin problem: $\sigma_a = 0.2$ cm^{-1}



(a) Radiation temperatures $T_R = \sqrt[4]{\phi/(ac)}$

- The HOLO method converges nonlinearites with damped Newton iterations to prevent artificial "temperature spikes" that IMC demonstrates. Results below are the same problem but with different axis scales.



(a) IMC temperatures for different Δt.

(b) HOLO temperatures for different Δt.

Ongoing work

- We are working towards a linear-discontinuous trial space in time. ECMC will be used to estimate the slope of the intensity in time, which can be used to extrapolate the solution to t^{n+1}. This representation will improve the statistical efficiency over current tallies for $\tilde{I}^{n+1}(x,\mu)$ because all particles will contribute to the local slope.
- The LDFE x-μ-t space will require a more sophisticated sampling approach, which will be useful to investigate for extending ECMC to higher dimensions.

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