A High-Order Low-Order Algorithm with Exponentially Convergent Monte Carlo for Thermal Radiative Transfer

Simon R. Bolding & Jim E. Morel

Department of Nuclear Engineering, Texas A&M University – CERT Project

Background on Thermal Radiative Transfer

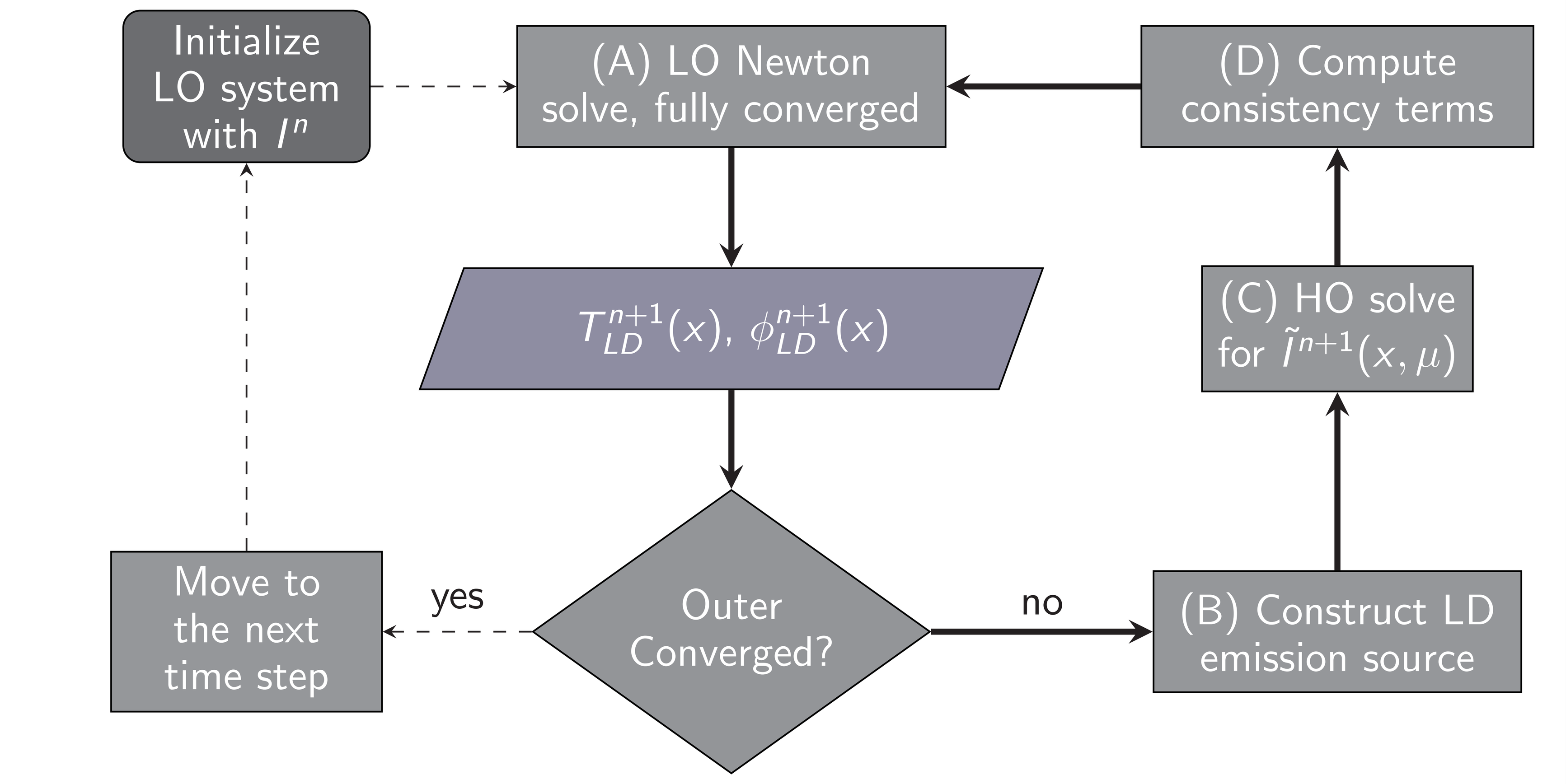
- The time-discretized radiation and material energy equations are

$$\frac{I^{n+1}-I^{n}}{c\Delta t}+\mu\frac{\partial I^{n+1}}{\partial x}=\frac{1}{2}\left(\sigma_a acT^4\right)^{n+1}-\sigma_a I^{n+1} \qquad (1)$$

$$\rho c_v\frac{T^{n+1}-T^{n}}{\Delta t}=\sigma_a\phi^{n+1}-\sigma_a ac(T^4)^{n+1}. \qquad (2)$$

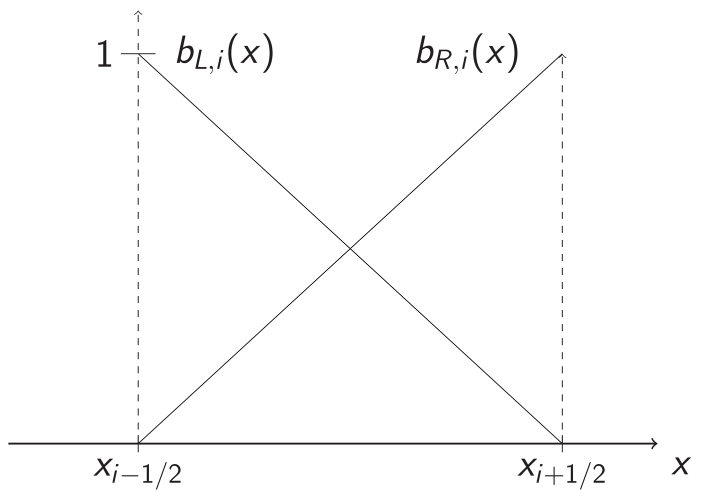
- The radiation intensity $I(x,\mu,t^{n+1})$ and material temperature $T(x,t^{n+1})$ are **coupled** through photon **emission** $\sigma_a acT^4$ and **absorption** $\sigma_a\phi$.
- We form a **lower-dimensional** system which can efficiently **resolve non-linearities** in the problem. The low-order (LO) equations contain consistency terms that preserve accuracy of **efficient**, high-order (HO) Monte Carlo simulations.

Algorithm



Details of the algorithm

(A) The LO equations are formed by taking spatial and angular moments of Eq. (1) & (2), defined locally over the i-th spatial finite element (FE).



Spatial moments

$$\langle\cdot\rangle_{L/R,i}=\frac{2}{h_i}\int_{x_{i-1/2}}^{x_{i+1/2}} b_{L/R,i}(x)(\cdot)\mathrm{d}x$$

Angular half-ranges

$$\phi^{\pm}(x)=\pm\int_0^{\pm1} I(x,\mu)\mathrm{d}\mu$$

Based on the current HO estimate of consistency terms, the non-linearities in T are fully resolved *for each LO solve*, using Newton's method.

(B) The HO emission source is constructed from the previous LO solution, producing a **fixed source, pure absorber** HO problem.

(C) *For each HO solve*, multiple exponentially-convergent Monte Carlo (ECMC) batches are performed to solve a pure absorber transport problem.

- Each ECMC batch tallies the error in the latest estimate of the solution. By initializing the first estimate of $\tilde{I}^{n+1}$ to $\tilde{I}^{n}$, **very few histories are needed** because we are only estimating the change in I over a time step.
- ECMC uses a **projection of the exact solution** onto a linear discontinuous (LD) finite element, space-angle mesh denoted $\tilde{I}^{n+1}$.

(D) We use the latest ECMC solution for $\tilde{I}^{n+1}$ to evaluate angular consistency terms in the LO equation, e.g.,

$$\overline{\mu}_L^{+}=\frac{\langle\mu\, I^{HO}\rangle_L^{+}}{\langle I^{HO}\rangle_L^{+}}$$

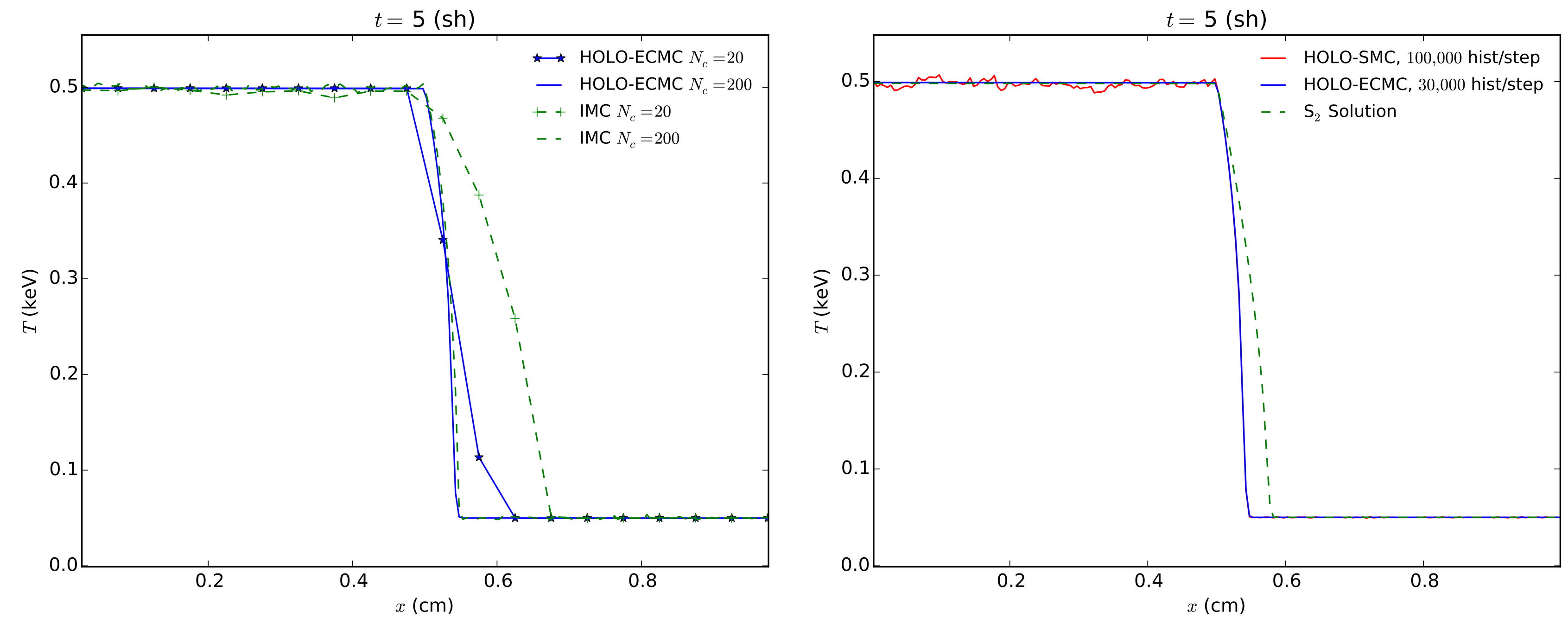
Diffusion Synthetic Acceleration (DSA) of LO equations

- We perform DSA-accelerated source iteration on the (effective) scattering source resulting from linearization of the LO equations.
- A spatially continuous discretization of the diffusion equation is applied to the residual equation for scattering iterations. The estimated error is mapped onto the moment unknowns based on a discontinuous discretization of the P_1 equations over each cell.
- In a modified version of the two material problem, DSA **reduced the spectral radius from 0.993 to** $<$**0.1.**

Results for two Marshak wave test problems

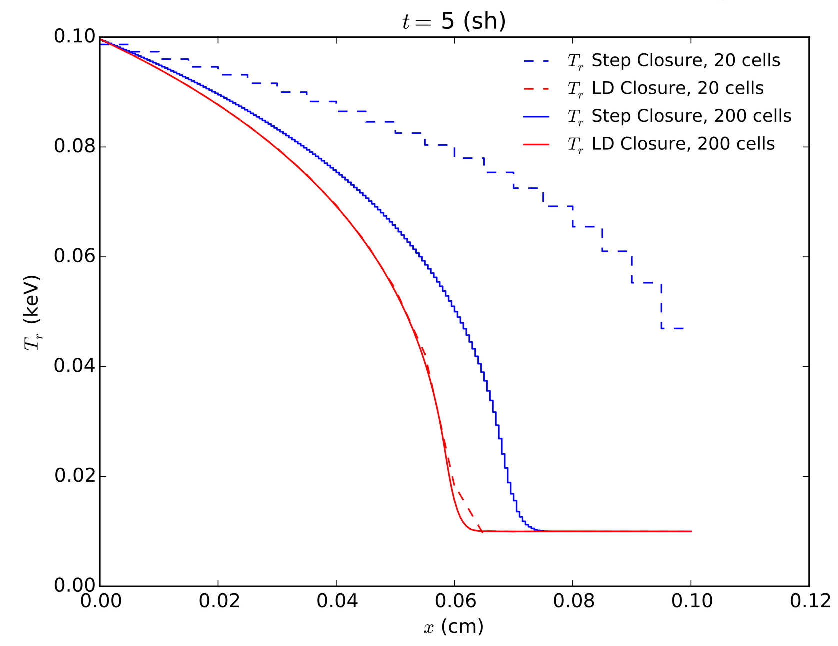
- For all HOLO results only one outer iteration is performed, per time step. Figures depict radiation temperature $T_R=\sqrt[4]{\phi/(ac)}$.

Two Material Problem: Optically thin (left) and thick (right) regions



(a) Spatial mesh convergence. (b) ECMC vs Standard MC as HO solver

Equilibrium Diffusion Limit Problem: large σ_a and small C_v.



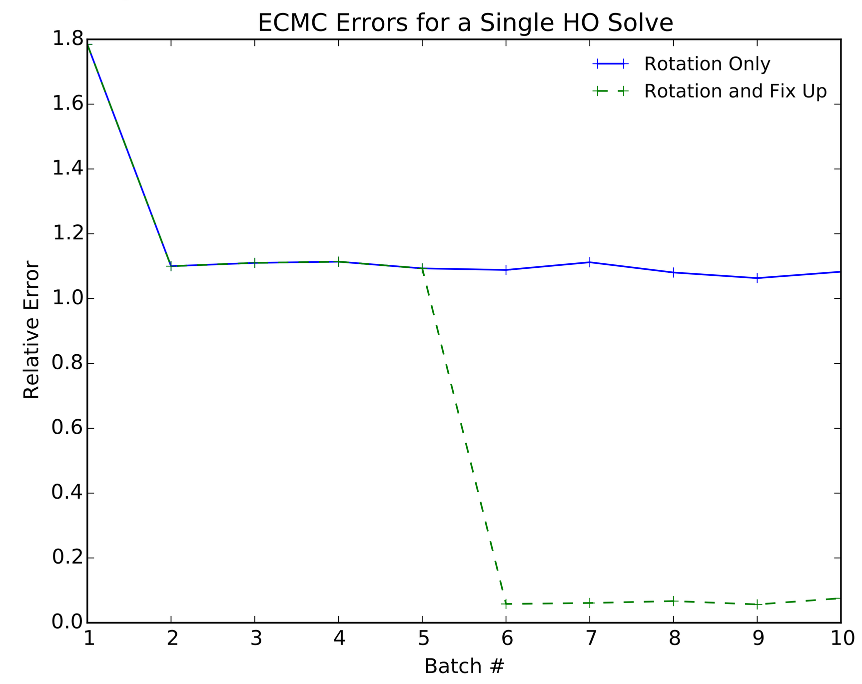
(a) Comparison of HOLO using LD and step discretizations in LO equations.

A fixup is necessary near strong solution gradients

- In strongly absorbing regions, the linear discontinuous spatial representation can lead to unphysical negative solutions in certain cells
- In the LO system, we set a strict floor value and preserve energy conservation to determine the cell average.
- For the HO system, we **rotate** $\tilde{I}(x,\mu)$ to be strictly positive after each batch. However, $\tilde{I}_{\text{rotated}}(x,\mu)$ doesn't satisfy the original residual equation, so the ECMC convergence quickly stagnates. Thus, we add an artifical source $\tilde{\delta}(x,\mu)$, which is estimated *iteratively* as

$$\tilde{\delta}(x,\mu)=\mathbf{L}(\tilde{I}^{n+1}-\tilde{I}^{n+1}_{\text{rotated}})$$

where $\mathbf{L}$ is the continuous streaming plus removal operator. The solution will now converge towards the strictly positive projection $\tilde{I}^{n+1}_{\text{rotated}}$.
- We also allow the outflow from the cell to be **discontinuous**, allowing for greater angular accuracy on faces



Ongoing work

- We are working towards time-continous transport for ECMC to produce high temporal accuracy in optically thin regions, requiring additional consistency terms in the LO equations.
- Further investigation of accuracy, robustness, and consistency in fixup regions

Acknowledgements

CERT
Center for Exascale Radiation Transport