Second-Order Discretization in Space and Time for Grey S_2-Radiation Hydrodynamics

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16 October 2015

CLASS seminar

Outline

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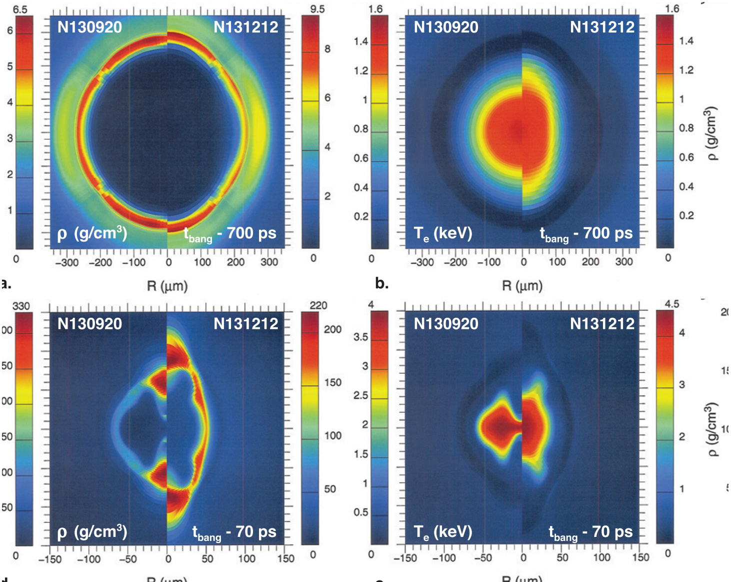
What is radiation hydrodynamics?

- Thermal radiative transfer coupled to material motion
  - Inertial confinement fusion (NIF) and astrophysics calculations



Example of a 1D Radiative Shock Solution

Goal of this project

- Extended work by Edwards and Morel for a method that is second-order in space and time
- We extended the method to S_2 equations which allows for conservation of momentum
  - Can be generalized to S_n equations, but would require some form of acceleration

The hydrodynamics equations

- The 1D Euler Equations

$$a\frac{\partial}{\partial t}\mathbf{U} + \frac{\partial}{\partial x}f(U) = \mathbf{Q} \tag{1}$$

The hydrodynamics equation governing equations continued

- Radiation transport equation, collocated to $\mu = \pm\frac{1}{\sqrt{3}}$

$$\frac{1}{c}\frac{\partial \psi^{\pm}}{\partial t} \pm \frac{1}{\sqrt{3}}\frac{\partial \psi^{\pm}}{\partial x} + \sigma_t \psi^{\pm} = \frac{\sigma_s}{4\pi} c E_r + \frac{\sigma_a}{4\pi} a c T^4 - \frac{\sigma_t u}{4\pi c} F_{r,0} \pm \frac{\sigma_t}{\sqrt{3}\pi} E_r u$$

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The MUSCL Hancock method

- The MUSCL Hancock method is a finite volume method that utitlized slope-reconstruction in space
  PICTURE OF SLOPE RECONSTRUCTION
- A predictor corrector in time is used for second order accuracy
- For example, from t_n to $t_{n+1/2}$
  - Step 1
    $$U^* = U^n + \frac{\Delta t}{4}(F(U^n))$$
  - Step 2
    $$U^{N+1} = U^n + \frac{\Delta t}{2}(F(U^*))$$

Linear Discontinuous Galerkin Spatial Discreziation for TRT

- We use a lumped linear discontinous trial space representation, with upwinding.
- There is no slope reconstruction
- Preserves the equilibrium diffusion limit

Operator splitting and general approach

Development of Algorithm

Non-linear iteration scheme for implicit solve

Extra stuff

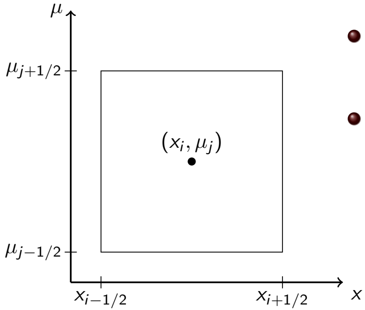
Code implemented in python
Energy slope stuff
Other things

Future Work

Coupling to a high-order system using hybrid-“S_2-like” equations.

Outline

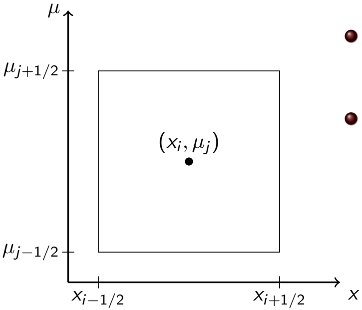
Space-Angle LDFE Mesh



- $\tilde{\psi}(x,\mu)$ is linear over each cell, preserving 0^{th} and 1^{st} moment in x and μ
- Use path-length estimators of flux to approximate moments e.g.

$$\langle \psi \rangle_{\mu,ij} = \frac{6}{h_\mu^2 h_x} \iint_{\mathcal{D}} (\mu - \mu_i)\psi(x,\mu)\mathrm{d}x\mathrm{d}\mu$$

Space-Angle LDFE Mesh



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- Use standard LD and upwinding to get face terms

Questions?

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The full equations

- Material balance equations

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$
$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) = \frac{\sigma_t}{c} F_{r,0}$$
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left[(E+p)\, u\right] = -\sigma_a c\left(a T^4 - E_r\right) + \frac{\sigma_t u}{c} F_{r,0}$$

- Radiation transport equation, collocated to $\mu = \pm\frac{1}{\sqrt{3}}$

$$\frac{1}{c}\frac{\partial \psi^{\pm}}{\partial t} \pm \frac{1}{\sqrt{3}}\frac{\partial \psi^{\pm}}{\partial x} + \sigma_t \psi^{\pm} = \frac{\sigma_s}{4\pi} c E_r + \frac{\sigma_a}{4\pi} a c T^4 - \frac{\sigma_t u}{4\pi c} F_{r,0} \pm \frac{\sigma_t}{\sqrt{3}\pi} E_r u$$