- Art. Src Fixup

$$L\psi = q - r$$
$$L\psi_{pos} = q - r + \delta$$

$$\delta = L(\psi_{pos} - \psi)$$

- In code, initialize $\psi^{(m)}$ to ψ_{pos}

$$L\psi^{(m)} = q + \delta = q + L(\psi_{pos} - \psi)$$
$$\Rightarrow \boxed{\psi^{(m)} = L^{-1}q + (\psi_{pos} - \psi)}$$

- Residual: normally:

$\tilde{r}_{pos} = q - L\psi_{pos}$ $\Rightarrow$ give us back neg. ψ, doesn't satisfy equation.

- Residual now

$$\tilde{r}_{pos} = q - L\psi_{pos} + L(\psi_{pos} - \psi) = q - L\psi$$
$$\epsilon = L^{-1}(q - L\psi) = L^{-1}q - \psi$$
$$\psi^{(m)} = \psi_{pos} + (L^{-1}q - \psi) = \boxed{L^{-1}q + (\psi_{pos} - \psi)}$$

• Make residual change have no zeroth moment

$$\delta = L(\psi_{pos} - \psi)$$

• require ψ_{pos} is positive, and $\delta_a = 0$

$$\hat{\psi} = \psi_a + \frac{2}{h_x}\psi_x(x - x_i) + \frac{2}{h_\mu}\psi_\mu(\mu - \mu_j)$$

• Integrate δ_a

$$L\psi = \mu\frac{\partial \psi}{\partial x} + \sigma_t \psi$$

$$= \mu\,\delta^{+}(x_i - x_{i\pm})\left(\psi_{i+\frac{1}{2}}(\mu) - \psi_{i-\frac{1}{2}}(\mu)\right) + \mu\frac{2}{h_x}\psi_x + \sigma_t\tilde{\psi}_{int}$$

• Integrate over cell

$$dx\int \cancel{\delta(x,\mu)} = \mu\left(\psi_{pos}^{i+\frac{1}{2}} - \psi_{pos}^{i-\frac{1}{2}}\right) + \sigma_a\psi_a^{pos} - \mu\left(\psi^{i+\frac{1}{2}} - \psi_i^{i-\frac{1}{2}}\right) - \sigma_a\psi_a$$

• we assume: $\psi_{pos}^{i+\frac{1}{2}} = \psi^{i+\frac{1}{2}}$

$$\psi_a^{pos} = \psi_a$$

$$= \int \mu\left[\left(\psi_a + \psi_x^{pos} + \frac{2}{h_\mu}\psi_\mu(\mu - \mu_j)\right) - \left(\psi_a + \psi_x + \frac{2}{h_\mu}\psi_\mu^{i}(\mu - \mu_j)\right)\right]$$

$$\int_D \delta(x,\mu) = \int d\mu \left[\mu\left(\psi^{int}_{BCS} - \psi_{i-\frac{1}{2}}\right) + \mu\left(\psi^{int} - \psi_{i-\frac{1}{2}}\right) + \cancel{\sigma_t\left(\psi_a - \psi_n\right)} \right.$$

$$+ \frac{2}{h_x}\delta\psi_x \mu$$

$$\int \left(\mu \frac{\partial \psi}{\partial x} + \sigma_t \psi_n = q + \delta \right) dV$$

$$J_{i+\frac{1}{2}} - J_{i-\frac{1}{2}} + \sigma_a \phi = q \cdot V + \delta$$

$$\int_D \delta(x,\mu) = \int_D \mu\, \delta^+(x - x_{i-\frac{1}{2}})\left(\delta\psi^{int}(\mu)\right) + \frac{2}{hx}\delta\psi_x \mu$$

$$= \int_{\mu_-}^{\mu_+} \left(\mu\, \delta\psi^{int}(\mu) + 2\delta\psi_x \mu\right) d\mu$$

$$= \int_{\mu_-}^{\mu_+} \mu\left(-\delta\psi_x + \delta\psi_\mu \frac{2}{h_\mu}(\mu - \mu_i)\right) + 2\delta\psi_x \mu\, d\mu$$

$$= 2\delta\psi_x\left(\mu_+^2 - \mu_-^2\right) - \frac{\delta\psi_x}{2}\left(\mu_+^2 - \mu_-^2\right)$$