Pure absorber LD in Angle

- Analytic solution

$$\psi(x,\mu) = \psi_0 e^{-\frac{\sigma x}{\mu}}, \quad \mu > 0 \qquad x \in (0,T)$$

- Take LD angular moment over a cell

$$\psi^{\mu} = \frac{6}{h_\mu} \int_{\mu_L}^{\mu_R} \left(\frac{\mu - \mu_i}{h_\mu}\right) \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu$$

$$\cancel{= \frac{6}{h_\mu^2} \int_{\mu_L}^{\mu_R} \mu \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu - \int_{\mu}^{\mu_R} \mu_i \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu}$$

- change vars to $v = \frac{\mu - \mu_L}{h_\mu}$ $\quad v(\mu_L) = 0, \; v(\mu_R) = 1$

$$\frac{\psi^{\mu}}{6} = \int_0^1 v \psi_0 e^{-\frac{\sigma x}{\mu}} dv + \int_{\mu_L}^{\mu_R} \left(\frac{h}{2}\right)\left(\frac{1}{h_\mu}\right) \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu$$

- General is too hard, case of $\mu \in [0,1]$:

$$E_n(x) = \int_0^1 \mu^{n-2} e^{-x/\mu} d\mu$$

$$\frac{\psi^{\mu}}{6} = \frac{1}{h_\mu} \int_0^1 \left(\frac{\mu - 0.5}{h_\mu}\right) \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu$$

$$= \int_0^1 \mu \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu - 0.5 \psi_0 \int_0^1 \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu$$

$$\psi^{\mu} = 6\left[E_3(\sigma x) - \frac{1}{2} E_2(\sigma x)\right]$$

Average over ψ

$$\psi^{\mu} = \frac{6}{h_x}\int_{x_L}^{x_R} \left(E_3(\sigma x) - \frac{1}{2}E_2(\sigma x) \right)$$

$dz = \sigma dx$

$\frac{dz}{\sigma} = dx$

$\frac{n-1}{n} E_n(x) < E_{n+1}(x) < E_n(x)$

But $E_n'(z) = -E_{n-1}(z)$

$$= \frac{6}{\sigma h_x}\int_{x_L}^{x_R} \left(-E_4'(z) + \frac{1}{2}E_3'(z) \right) dz$$

$$\psi^{\mu} = \boxed{\frac{6}{\sigma h_x}\left(\frac{1}{2}E_3(z) - E_4(z) \right) \Bigg]_{\tau_L}^{\tau_R} \psi_0}$$

Average:

$$\psi_a = \frac{1}{h_x h_\mu}\int_{x_L}^{x_R} \psi_0 e^{-\frac{\sigma x}{\mu}} d\mu = \psi_0 E_3(\sigma x)\Big]_{x_L}^{x_R} = \psi_0 E_3(\sigma x)$$

$$\boxed{\psi_a = \frac{\psi_0}{h_y \sigma} E_3(z) \Big]_{\tau_L}^{\tau_R}}$$

$$\psi_x = \frac{6}{h_x^2}\int_{x_L}^{x_R}\left(\frac{x-x_i}{h_x}\right)\psi_0\int_0^1 e^{-\frac{\sigma x}{\mu}}\,d\mu$$

$$= \frac{6}{h_x^2}\int_{x_L}^{x_R}\left(\frac{x-x_i}{h}\right)E_2(\sigma x)\,dx$$

- Most negative corner $= \psi_a + \psi_x - \psi_\mu$
- Look at just $\psi_a - \psi_\mu$:

$$\left(\psi_a - \psi_\mu\right) = \frac{1}{\sigma h_x}\left[4E_3(z) - 6E_4(z)\right]\Bigg|_{\tau_L}^{\tau_R}\psi_{inc}$$