Newton's Method

~~Damping~~

$$\cancel{\phi^{k+1} = \phi^{k} - \left(\frac{\partial\phi}{\partial T}\right)^{-1}\phi}$$

$$T^{n+1} = \left(T^{*} + \delta T\right)$$

$$\frac{\phi^{n+1} - \phi^{n}}{c\Delta t} + \sigma_a\phi = \sigma_a acT^{4,n+1}$$

$$\rho C_v \frac{T^{n+1} - T^{n}}{\Delta t} = \sigma_a\phi^{n+1} - \sigma_a acT^{4,n+1}$$

$$\delta x = -\frac{f(x_0)}{f'(x_0)}$$

$$X^{n+1} = X^{*} + \delta X$$

- Linearize:

$$J(f(X_0))\Delta x = -f(X_0)$$

$$f_1 = \sigma_a acT^{4,n+1} - \sigma_a\phi^{n+1} + \frac{\phi^{n} - \phi^{n+1}}{c\Delta t}$$

$$f_2 = \sigma_a\phi^{n+1} - \sigma_a acT^{4,n+1} - \rho C_v\left(\frac{T^{n+1} - T^{n}}{\Delta t}\right)$$

$$J = \begin{pmatrix} -\sigma_a - \frac{1}{c\Delta t} & 4\sigma_a acT^{3} \\ \sigma_a & -4\sigma_a acT^{3} + \frac{\rho C_v}{\Delta t} \end{pmatrix} \begin{pmatrix} \delta\phi^{n+1} \\ \delta T^{n+1} \end{pmatrix} = \begin{pmatrix} -\sigma_a acT^{4*} + \sigma_a\phi^{n+1*} + \frac{\phi^{n} - \phi^{n+1*}}{c\Delta t} \\ -\sigma_a\phi^{n+1} + \sigma_a acT^{4,*} + \rho C_v\left(\frac{T^{*} - T^{n}}{\Delta t}\right) \end{pmatrix}$$

- Solving:

$$\frac{\phi^{*} - \phi^{n}}{c\Delta t} + \sigma_a\phi^{n+1*} + \sigma_a\delta\phi^{n+1} + \frac{1}{c\Delta t}\delta\phi^{n+1} = \sigma_a acT^{4*} + 4\sigma_a acT^{3}\delta T^{n+1}$$

$$\rho C_v\left(\frac{T^{*} + \delta T^{n+1} - T^{n}}{c\Delta t}\right) = \sigma_a(\phi^{*} + \delta\phi) - \sigma_a\left(acT^{4,*} + 4\sigma_a acT^{3}\delta T^{n+1}\right)$$

- To damp need to multiply each δ by α
- In usual form, $\phi^{n+1,l+1} = \phi^{*} + \delta\phi^{n+1}$

- <u>After elimination:</u>

$$\phi^{n+1}\left(\sigma_a + \frac{1}{c\Delta t} + \cdots\right) = \frac{\sigma_s \phi^{n+1} + \sigma_a(1-f)\phi^{n+1}}{4\pi} + \frac{\sigma_a a c f T_*^4}{4\pi} + \frac{\rho c_v}{\Delta t}\frac{(1-f)(T^n - T^*)}{4\pi}$$

- <u>Leave in expansion form, solve for δT</u>

$$\delta T\left(\frac{\rho c_v}{\Delta t} + 4\sigma_a a c T^3\right) = T^n \frac{\rho c_v}{\Delta t} + \sigma_a(\phi^* + \delta\phi) - \sigma_a a c T^{4,*} - T^* \frac{\rho c_v}{\Delta t}$$

$$f = \left(\frac{1}{1 + \frac{4\sigma_a a c T^3 \Delta t}{\rho c_v}}\right)$$

$$\delta T = \frac{T^n \frac{\rho c_v}{\Delta t}}{(\cdots)} + \frac{\sigma_a(\phi^* + \delta\phi)}{(\rho c_v + \cdots)} - \frac{\sigma_a a c T^{4,*}}{(\cdots)}$$

ϕ eq:

$$\frac{(\phi^* + \delta\phi) - \phi^n}{c\Delta t} + \sigma_a(\phi^* + \delta\phi) = \sigma_a a c T^{4*} + \sigma_a a c 4 T^3 \delta T$$

- For ex:

$$\frac{\sigma_a(\phi^* + \delta\phi)}{\frac{\rho c_v}{\Delta t} + 4\sigma_a a c T^3} \cdot \left(4\sigma_a a c T^3\right) = (1-f)\sigma_a(\phi^* + \delta\phi)$$

$$\frac{(\phi^* + \delta\phi) - \phi^n}{c\Delta t} + \sigma_a(\phi^* + \delta\phi) = \sigma_a a c T^{4,*} + (1-f)\frac{\rho c_v}{\Delta t} T^n + \sigma_a(1-f)(\phi^* + \delta\phi)$$

$$(-(1-f) + 1)\sigma_a a c T^{*4}$$

$$\frac{(\phi^* + \delta\phi) - \phi^n}{c\Delta t} + \sigma_a(\phi^* + \delta\phi) = \sigma_a a c T^{4,*} + (1-f)\frac{\rho c_v T^n}{\Delta t} + \sigma_a(1-f)(\phi + \delta\phi)$$
$$+ (1-f)\frac{\rho c_v (T^*)}{\Delta t} + f\sigma_a a c T^{*,4}$$

- Solves for $\phi^* + \delta\phi$. $\phi^{n+1} = \phi^* + \alpha\delta\phi$
- so take $(\phi^*) + \big[\underbrace{(\phi^* + \delta\phi)}_{\text{from solver}} - \phi^*\big]\alpha = \phi^{n+1}$

$\phi^*(1-\alpha) + (\phi^* + \delta\phi)\alpha$

- Temperature update

$$T^{n+1} = (T^* + \delta T) = \frac{\Delta t}{\rho c_v}\left[f\sigma_a(\phi + \delta\phi\alpha) + \sigma_a a c T^{4*}\right]$$
$$+ (1-f)T^* + fT^n$$

- Subtract T^*:

$$\delta T = \frac{\Delta t}{\rho c_v}\left[\ \text{- - - - -}\ \right] \text{"}$$
$$+ f(T^n - T^*)$$

- Multiply this all by α?

$$\delta T = \frac{\alpha\Delta t}{\rho c_v}\left[\ \text{- - -}\ \right] + \alpha f(T^n - T^*)$$

$$T^{n+1} = T^* + \frac{\alpha\Delta t}{\rho c_v}\left[f\sigma_a(\phi + \delta\phi\alpha) + \sigma_a a c T^{4,*}\right]$$
$$+ f(T^n - T^*)\alpha$$

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