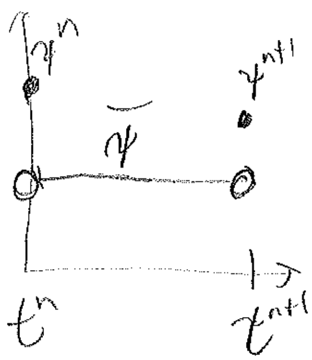
DBL Disc in time



- Residual is same as if there were no disc, it only affects solution at t^{n+1}
- The sol^{n} at t^{n+1} is $\psi^{n+1(\ell+1)} = \psi^{n+1(\ell)} + \epsilon^{(\ell+1)} + \hat\epsilon^{(\ell+1)}$

$$\epsilon = L^{-1}(R)\Big|_{t=t^{n+1}}$$

- Source comes from $-\frac{1}{c}\frac{\partial\psi}{\partial t}$ term. Look at $\hat R^{n+1}$, only source at t^{n+1}

$$\hat R^{n+1} = -\frac{\psi^{n+1}-\bar\psi}{c}\,\delta(t^{n+1}-t)$$

- Integrate to get total strength

$$\left|\hat R^{n+1}\right| = \left|\frac{\psi^{n+1}-\bar\psi}{c}\right|$$

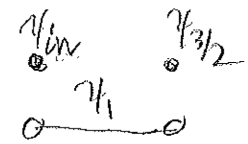
- In terms of transport, this source would be added to residual in each cell, but particles are immediately killed, so

$$\psi^{n+1,\ell+1} = \psi^{(n+1)(\ell)} + \epsilon^{(\ell+1)} + \underbrace{\left(\frac{\psi^{n+1(\ell)}-\bar\psi}{c}\right)c}_{\hat\epsilon^{(n+1)}}$$

$$\boxed{\psi^{n+1,\ell+1} = \bar\psi^{\ell} + \epsilon^{(\ell+1)}}$$

$\psi_1 e^{\sigma x_{1/2}} - (\psi_1 - 1) - \psi_1 e^{\sigma x_{1/2}} = (\ldots,\ x_{1/2} = 0$

• One cell D.D.-space



$$\frac{\partial \psi}{\partial x} + \sigma\psi = 0,$$

$$\frac{\partial \epsilon}{\partial x} + \sigma\epsilon = \tilde{R} = -\frac{\partial \tilde{\psi}}{\partial x} - \sigma\tilde{\psi} = -(\psi_1 - \psi_{in})\delta(x_{1/2}) - \sigma\psi_1 - (\psi_{3/2} - \psi_1)\delta(x_{3/2})$$

$$\epsilon e^{\sigma x_{3/2}} - 0 = \int_0^{x_{3/2}} \tilde{R} e^{\sigma x} dx = -(\psi_1 - \psi_{in})\cancel{e^{0}} - \psi_1 \frac{e^{\sigma x}}{\cancel{\sigma}}\Big]_0^{x_{3/2}} - (\psi_{3/2} - \psi_1) e^{\sigma x_{3/2}}$$

$$\epsilon e^{\sigma x_{3/2}} = \psi_{in} - \cancel{\psi_1} - \cancel{\psi_1 e^{\sigma x_{3/2}}} + \cancel{\psi_1} - \psi_{3/2} e^{\sigma x_{3/2}} + \cancel{\psi_1 e^{\sigma x_{3/2}}}$$

$$\epsilon = \psi_{in} e^{-\sigma x_{3/2}} - \psi_{3/2}$$

$$\psi_{3/2}^{exact} = \psi_{3/2} - \cancel{\psi_{3/2}} + \psi_{in} e^{-\sigma x_{3/2}} = \boxed{\psi_{in} e^{-\sigma x_{3/2}}}$$

• Neglecting $R_{3/2}$; then add it back on

$$\epsilon = \psi_{in} e^{-\sigma x_{3/2}} - \psi_1 + \psi_1 e^{-\sigma x_{3/2}} + \int_0^{x_{3/2}} \psi_1 e^{-\sigma x} \delta(x_{3/2})(\psi_1 - \psi_{3/2})$$

$$\frac{1}{c}\frac{\partial \hat{\epsilon}}{\partial t} + \mu\frac{\partial \hat{\epsilon}}{\partial x} + \sigma_t \hat{\epsilon} = \left(\frac{\bar{\psi} - \psi^{n+1}}{c}\right)\delta(t^{n+1} - t)$$

- Integrate from $-\delta + t^{n+1}$ to t^{n+1}, take limit as $\delta \to 0$

$$\frac{1}{c}\int_{t^{n+1}-\delta}^{t^{n+1}} \frac{\partial \hat{\epsilon}}{\partial t} + \underbrace{\int_{t^{n+1}-\delta}^{t^{n+1}} \left(\mu\frac{\partial \hat{\epsilon}}{\partial x} + \sigma_t \hat{\epsilon}\right) dt}_{=0,\ \text{smooth in time}} = \frac{1}{c}\left(\bar{\psi} - \psi^{n+1}\right)$$

$$\frac{\hat{\epsilon}^{n+1} - \cancel{\hat{\epsilon}(t^{n+1}-\delta)}^{\,0}}{\cancel{c}} = \frac{1}{\cancel{c}}\left(\bar{\psi} - \psi^{n+1}\right)$$

$$\boxed{\hat{\epsilon}^{n+1} = \bar{\psi} - \psi^{n+1}}$$