A HIGH-ORDER LOW-ORDER ALGORITHM WITH EXPONENTIALLY-CONVERGENT MONTE CARLO FOR THERMAL RADIATIVE TRANSFER PROBLEMS

A Dissertation

by

SIMON RAY BOLDING

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Chair of Committee,	Jim Morel
Committee Members,	Ryan McClarren
	Jean Ragusa
	Jean-Luc Guermond
Head of Department,	Yassin Hassan

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ABSTRACT

We have implemented a new high-order low-order (HOLO) algorithm for solving thermal radiative transfer problems. The low-order (LO) system is based on spatial and angular moments of the transport equation and a linear-discontinuous finite-element spatial representation, producing equations similar to the standard S_2 equations. The LO solver is fully implicit in time and efficiently resolves the non-linear temperature dependence at each time step. The HO solver utilizes exponentially-convergent Monte Carlo (ECMC) to give a globally accurate solution for the angular intensity to a fixed-source, pure absorber transport problem. This global solution is used to compute consistency terms, which require the HO and LO solutions to converge towards the same solution. The use of ECMC allows for the efficient reduction of statistical noise in the MC solution, reducing inaccuracies introduced through the LO consistency terms. We compare results with an implicit Monte Carlo (IMC) code for one-dimensional, gray test problems and demonstrate the efficiency of ECMC over standard Monte Carlo in this HOLO algorithm.

DEDICATION

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NOMENCLATURE

B/CS	Bryan/College Station
HSUS	Humane Society of the United States
P	Pressure
T	Time
TVA	Tennessee Valley Authority
TxDOT	Texas Department of Transportation

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0.1 MC Estimation of the Spatial Closure

The goal of this section is to use the HO solution to estimate the spatial closure for the LO system. A half-range balance equation for $\mu > 0$ is formed by adding the exact L and R moment equations given by Eq. (**??**) and (**??**), i.e.,

$$\overline{\mu}^+_{i+1/2}\phi^+_{i+1/2} - \overline{\mu}^+_{i-1/2}\phi^+_{i+1/2} + \frac{\sigma_{a,i}h_i}{2}\phi^+_i = \frac{h_i}{2}q_i, \tag{1}$$

where q is a general, isotropic source. The angular consistency terms can be estimated with the HO solution and the inflow term $\phi^+_{i-1/2}$ is upwinded from the previous cell. An additional equation is needed to eliminate $\phi^+_{i+1/2}$ to produce an equation for a single unknown ϕ^+_i. The outflow can be eliminated as a parametric combination of the other variables, i.e.,

$$\phi^+_{i+1/2} = f(\gamma_i^{HO}, \phi^+_i, \phi^+_x, \phi^+_{i-1/2}), \tag{2}$$

where γ_i^{HO} is some constant estimated with the HO solution and f is some function of some number of input variables. If the problem is linear, i.e., q is known a priori, then application of this closure will ensure that the HO and LO equations exactly preserve the same moments. As the problem is typically non-linear (e.g., scattering or thermal emission are included in q), then this will not be strictly true.

We will explore two different closure options: a scaled slope, i.e.,

$$\phi^\pm_{i\pm1/2} = \phi^+_i \pm \gamma_i\phi^+_x \tag{3}$$

and a scaled average

$$\phi^\pm_{i\pm1/2} = \gamma_i\phi^+_i \pm \phi^+_x, \tag{4}$$

where a value of $\gamma_i = 1$ produces the standard linear discontinuous expressions for the extrapolated outflows. The closures are only needed to eliminate the extrapolated face values, not the inflow values for the particular direction equation.

We now use the HO solution to estimate γ_i. For example,

$$\gamma_i^{+,HO} = \frac{\phi_{i+1/2}^{+} - \phi_x^{+}}{\phi_i^{+}} \tag{5}$$

in the scaled slope case. For this closure, as the slope goes to zero this expression becomes undefined. In cells where the slope is $O(10^{-13}\psi_i)$, we use $\gamma_i = 1$. No problems have been observed with the fact that at relatively modest slopes γ becomes very large because the solution is changing minimally in such sections. The main reason for using this closure is it allows for values of γ that are equivalent to step and lumping.

The expression for the outflow face term is substituted in each equation, using the γ estimated from a HO solution. For instance, the positive balance equation becomes

$$\overline{\mu}_{i+1/2}^{+}\left(\gamma_i^{+,HO}\phi_i^{+} + \phi_x^{+}\right) - \overline{\mu}_{i-1/2}^{+}\phi_{i+1/2}^{+} + \frac{\sigma_{a,i}h_i}{2}\phi_i^{+} = \frac{h_i}{2}q_i, \tag{6}$$

noting that ϕ_i^{+} and ϕ_x^{+} remain as unknowns. The MC solution can provides the face values.

Our solution is in terms of the L and R moments, however.

By introducing a LDD trial space into the LO equations, we introduce an additional unknown in each half-range equation. This extra unknown is eliminated as a function of the interior moments. However, in the HOLO context, the HO solution used a lagged source q_i.

In theory, if the problem were linear, or the nonlinear problem was fully converged, then the HO and LO solutions would produce exactly the same moments. There are several issues with ECMC that cause this to not be true, even for a linear problem. With ECMC, global energy balance is not preserved, but local energy balance is also not preserved. There are source biasing techniques for standard MC (systematic sampling) that would exactly preserve the zeroth moment of the source [**?**]). It is a requirement that the HO solution satisfies the zeroth moment equation. If the closure relation also uses the first moment, then it must also satisfy the first spatial moment equation. However, because we have to reconstruct the bilnear moment of x and μ, the consistency terms do not exactly preserve the first moment equation. One final reason is that the analog treatment of absorption (at low weights as discussed in Sec. **??**) results in the fact that the product of ϕ_i (as estimated via path-length estimators) and the removal cross section flux does not exactly equal the absorption density of weight within that cell, due to statistical noise.

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APPENDIX A

FIRST APPENDIX

Text for the Appendix follows.

Figure A.1: TAMU figure

APPENDIX B

SECOND APPENDIX WITH A LONGER TITLE - MUCH LONGER IN FACT

Text for the Appendix follows.

Figure B.1: TAMU figure

B.1 Appendix Section