$$\mu \frac{\partial \psi}{\partial x} + \sigma \psi = q \qquad \psi_i^+ = \frac{\langle \psi \rangle_L^+ + \langle \psi \rangle_R^+}{2}$$

- Take ~~moments~~, positive $\mu^+ = \frac{1}{\sqrt{3}}$

$$\mu^+ \frac{2}{h_i}\left(\psi_{i+\frac{1}{2}}^+ - \psi_i^+\right) + \sigma \langle \psi \rangle_R^+ = \langle q \rangle_R^+$$

$$\mu^+ \frac{2}{h_i}\left(\psi_{i+\frac{1}{2}}^+ - \left(\frac{\langle \psi \rangle_L^+ + \langle \psi \rangle_R^+}{2}\right)\right) + \sigma \langle \psi \rangle_R^+ = \langle q \rangle_R^+$$

$$\mu^+ \frac{2}{h}\left(\frac{\langle \psi \rangle_L^+ + \langle \psi \rangle_R^+}{2} - \psi_{i-\frac{1}{2}}^+\right) + \sigma \langle \psi \rangle_L^+ = \langle q \rangle_L^+$$

- Eliminate $\psi_{i+\frac{1}{2}}^+$ in terms of $\langle \psi \rangle_R^+$ & $\langle \psi \rangle_L^+$, $\psi_{i-\frac{1}{2}}^+$ is upwinded

- Standard LD:

$$\psi_{i+\frac{1}{2}} = 2\langle \psi \rangle_R^+ - \langle \psi \rangle_L^+$$

- If sol^n is linear, $\psi_{i+\frac{1}{2}}^+ = 2\left(\frac{2}{3}\psi_R^+ + \frac{1}{3}\psi_L^+\right) - \left(\frac{1}{3}\psi_R^+ + \frac{2}{3}\psi_L^+\right) = \psi_R^+$

Plug in everything:

$$\mu^+ \frac{2}{h_i}\left(\frac{\psi_L^+ + \psi_R^+}{2} - \psi_{i-1,R}^+\right) + \sigma\left(\frac{2}{3}\psi_L + \frac{1}{3}\psi_R\right) = \left(\frac{2}{3}q_L^+ + \frac{1}{3}q_R^+\right)$$

$$\mu^+ \frac{2}{h}\left(\psi_{R,i}^+ - \frac{\psi_L^+ + \psi_R^+}{2}\right) + \sigma\left(\frac{1}{3}\psi_L^+ + \frac{2}{3}\psi_R^+\right) = \left(\frac{1}{3}q_L^+ + \frac{2}{3}q_R^+\right)$$

- Standard LD

- Lumping? $\psi^+_{i+1/2} = \langle\psi\rangle^+_R$

$$\mu^+ \frac{2}{h_i}\left(\langle\psi\rangle^+_R - \frac{\langle\psi\rangle^+_L + \langle\psi\rangle^+_R}{2}\right) + \sigma\langle\psi\rangle^+_R = \langle q\rangle^+_R$$

$$\mu^+ \frac{2}{h_i}\left(\frac{\langle\psi\rangle^+_L + \langle\psi\rangle^+_R}{2} - \langle\psi\rangle^+_{(i-1,R)}\right) + \sigma\langle\psi\rangle^+_L = \langle q\rangle^+_L$$

- Same equations but w/ $\langle\psi\rangle_{R/L}$ instead of $\psi_{R/L}$
- Now if we plug in Linear relation instead:

$\langle\psi\rangle^+_R = \frac{2}{3}\psi_R + \frac{1}{3}\psi_L$:

$$\mu^+ \frac{2}{h_i}\left(\frac{2}{3}\psi^+_R + \frac{1}{3}\psi^+_L - \frac{\psi^+_L + \psi^+_R}{2}\right) + \sigma\left(\frac{2}{3}\psi_R + \frac{1}{3}\psi_L\right) = \left(\frac{2}{3}q_R + \frac{1}{3}q_L\right)$$

- Not Lumped Eq's?

- If we define $\langle\psi\rangle^+_R = \psi^+_R$, $\langle\psi\rangle^+_L = \psi^+_L$, we do get lumped equations.