- Two equations

$$\hat{\psi}_{se} = \begin{cases} \psi_{in} & \text{at } x=0 \\ \psi_a & \text{at } x \in (0,1] \end{cases}$$

$L\psi$

$-L\psi \cdot R$

$L\psi = q - R$

$$\frac{\partial \psi}{\partial x} + \sigma\psi = 0, \quad \psi(0) = \psi_{in}$$

- If we add residual, we get back what we want, i.e.

$$\frac{\partial \hat{\psi}_1}{\partial x} + \sigma\hat{\psi}_1 = -\hat{R} = -(\psi_a - \psi_{in})\delta(x^+) + \sigma\psi_a$$

- will give back $\psi = \hat{\psi}$

$$\psi_1 e^{\sigma x} - \cancel{\psi_1(0)}^{\psi_{in}} = \int_0^x e^{\sigma x'} dx' (\psi_a - \psi_{in})\delta(x^+) + \psi_a e^{\sigma x'}\Big]_0^{\cancel{x}}$$

$$\psi_1 e^{\sigma x} \cancel{-\psi_{in}} = (\psi_a - \cancel{\psi_{in}}) + \psi_a(e^{\sigma x} - 1)$$

$$\psi_1 = (\psi_a - \psi_a) + \psi_a e^{\sigma x}(e^{-\sigma x}) = \psi_a \checkmark$$

- Error equation is the same, but no B.C.:

$$\frac{\partial \epsilon}{\partial x} + \sigma\epsilon = \hat{R}$$

$$\Rightarrow \quad \epsilon(x) = \psi_{in} e^{-\sigma x} - \psi_a$$

$$\epsilon(\cancel{x}) = (\psi_{in} - \psi_a) e^{-\sigma x} + \psi_a e^{-\sigma x} - \psi_a + \epsilon(0) e^{-\sigma \cancel{x}}$$

$\frac{\partial \psi}{\partial x} + \sigma \psi = 0, \ \psi(0) = 1$ $\quad R = -\sigma \psi - \frac{\partial \psi}{\partial x}$ $\quad \bar{\psi} = \psi_{in}$ at $x = 0$

$= -\sigma \psi_a - (\psi_a - \psi_{in})\delta(x)$ $\quad \bar{\psi} = \psi_a$ at $x \in (0, 1]$

$$\frac{\partial \epsilon}{\partial x} + \sigma \epsilon = R = -\sigma \psi_a + (\psi_{in} - \psi_a)\delta(x), \quad \epsilon(0) = 0$$

$$\frac{\partial}{\partial x}\left(\epsilon e^{\sigma x}\right) = \left[-\sigma \psi_a + (\psi_{in} - \psi_a)\delta(x)\right] e^{\sigma x}$$

$$\epsilon e^{\sigma x} - 0 = -\psi_a e^{\sigma x} + \psi_a + (\psi_{in} - \psi_a) e^{0}$$

$$\epsilon(x) = -\psi_a + \cancel{\psi_a e^{-\sigma x}} + \cancel{(\psi_{in} - \psi_a) e^{-\sigma x}}$$

$$\epsilon(x) = \psi_{in} e^{-\sigma x} - \psi_a$$

$$\int_0^1 \epsilon(x) = \int_0^1 \left(\psi_{in} e^{-\sigma x} - \psi_a\right) dx = -\psi_a + \frac{\psi_{in}(1 - e^{-\sigma x})}{\sigma}$$

$$\bar{\psi}_{a,c} = \cancel{\psi_a} + \frac{\psi_{in}(1 - e^{-\sigma x})}{\sigma} \cancel{- \psi_a}$$

$$\psi(x)e^{\sigma_t x} - \cancel{\psi_M e^{\sigma_t x_{i-1/2}}} = (\psi_L - \cancel{\psi_M})e^{\sigma_t x_{i+1/2}} + \left(\frac{\psi_R - \psi_L}{\sigma_t h}\right)\left[e^{\sigma_t x} - e^{\sigma_t x_{i-1/2}}\right]$$

$$\psi(x) = \psi_L e^{-\sigma_t \Delta x} + \left(\frac{\psi_R - \psi_L}{\sigma_t h}\right)\left[1 - e^{-\sigma_t \Delta x}\right]$$

$$+ \cancel{\psi_R b_R} - \cancel{\frac{\psi_R}{\sigma h}} + \cancel{\frac{\psi_R e^{-\sigma_t \Delta x}}{\sigma h}} \cancel{+\psi_L b_L} - \psi_L e^{-\sigma_t \Delta x} + \cancel{\frac{\psi_L}{\sigma h}} - \cancel{\frac{\psi_L e^{-\sigma_t \Delta x}}{\sigma h}}$$

$$= \cancel{\psi_L e^{-\sigma_t \Delta x}} + \cancel{\left(\frac{\psi_R - \psi_L}{\sigma_t h}\right)\left[1 - e^{-\sigma_t \Delta x}\right]} + \psi_R b_R + \psi_L b_L - \cancel{\left(\frac{\psi_R - \psi_L}{\sigma h}\right)\left[1 - e^{-\sigma_t \Delta x}\right]}$$

$$- \cancel{\psi_L e^{-\sigma_t \Delta x}}$$

$$\psi(x) = \boxed{\psi_L b_L + \psi_R b_R} \checkmark$$