Sampling from tri-linear residual

- Usual expectation: $E[z] = \langle z \rangle$

$$E[z(x)] = \int z(x)\, f(x)\, dx$$

- Estimator: $\bar{z} = \frac{1}{N} \sum_{i=1}^{N} z_i(x)$

$$E[\bar{z}] = \frac{1}{N} \sum_{i=1}^{N} E[z_i(x)] = \frac{1}{N} \cdot N \langle z \rangle = \langle z \rangle$$

- Importance Sampling:

$$E[z(x)] = \int z(x) \left(\frac{f(x)}{f^*(x)} \right) f^*(x) dx$$

same expectation as $f(x)\, z(x)$

$$E[\bar{z}] = \frac{1}{N} \sum_{i=1}^{N} E\left[z(x) \left(\frac{f(x)}{f^*(x)} \right) \right] = \frac{1}{N} \cdot N \langle z \rangle = \langle z \rangle$$

- In residual MC, we compute $z(x)$ and then multiply by $\|r(x)\|_1$. Lets say $f(x) = \frac{r(x)}{\|r(x)\|_1} = \frac{r(x)}{R_1}$

- So:

$$\|r\|_1 \cdot E[z(x)] = \|r\|_1 \int z(x) f(x) dx = \int z(x) |r| f(x) dx$$

- In reality, $r(x)$ has a + & - part, so we use a weight that is negative 1 if $r(x) < 0$

$$\|r\|_1 \cdot E[z(x)] = \|r\|_1 \int z(x) \left(\frac{f(x)}{|f(x)|} \right) |f(x)| dx$$

* where $|f(x)|$ is positive, not a norm.

$$= \|r\|_1 \int z(x) f(x) dx$$

• what if we just move $r(x)$ inside weight

$$\|r\|_1 \cdot E[z(x)] \overset{?}{=} \int z(x)\left(\frac{r(x)}{|s(x)|}\right)|s(x)|\,dx$$

$$= \int z(x)\left(\frac{s(x)\cdot\|r_1(x)\|}{|s(x)|}\right)|s(x)|\,dx$$

$$\overset{\checkmark}{=} \|r(x)\|_1 \int z(x)\left(\frac{s(x)}{|s(x)|}\right)|s(x)|\,dx$$