Balance for zero residual

$$\tilde{\psi} = \psi_a + \frac{2}{h_x}\psi_x(x-x_i) + \frac{2}{h_\mu}\psi_\mu(\mu-\mu_i)$$

$$\int \mu\frac{\partial\tilde{\psi}}{\partial x} + \sigma\tilde{\psi} = q$$

$$\int_{\mu}^{d\mu} \mu\psi_{i+1/2} - \mu\psi_{i-1/2} + \sigma\psi_{a,i}h_{x,i} = q_i h_{x,i}$$

$$\tilde{J}_{i+1/2} - \tilde{J}_{i-1/2} + \sigma\psi_{a,i}h_{x,i}h_{\mu,j} = q_i h_{x,i}h_{\mu,j}$$

- What is $\tilde{J}_{i+1/2}$?

$$\psi_{i+1/2} = \psi^a_{i+1/2} + \frac{2}{h_\mu}(\mu-\mu_j)\psi^\mu_{i+1/2}$$

$$\tilde{J}_{i+1/2} = \int_{\mu_j-h/2}^{\mu_j+h/2} \mu\left(\psi^a_{i+1/2} + \frac{2}{h_\mu}(\mu-\mu_j)\psi^\mu\right)d\mu$$

$$= \left[\psi^a_{i+1/2} - \mu_j\frac{2}{h_\mu}\psi_\mu\right]\frac{\mu^2}{2}\Big|_{\mu_j-h/2}^{\mu_j+h/2} + \frac{2}{h_\mu}\psi^\mu\frac{\mu^3}{3}\Big|_{\mu_j-h/2}^{\mu_j+h/2}$$

$$= \left(\psi^a_{i+1/2} - \mu_j\frac{2}{h_\mu}\psi^\mu\right)\left[(\mu_j h)\right] + \frac{2}{h_\mu}\psi^\mu\left(\mu_j^2 h_j + \frac{h_j^3}{12}\right)$$

$$= \psi^a_{i+1/2}\mu_j h_j + \frac{2}{h_\mu}\psi^\mu\left[\mu_j^2 h_j - \mu_j^2 h + \frac{h_j^3}{12}\right]$$

$$\tilde{J}_{i+1/2} = \boxed{\psi^a_{i+1/2}\mu_j h_j + \frac{\psi^\mu h_j^2}{6}}$$

$$\boxed{\psi_{a_i} = \frac{q_i - \frac{\tilde{J}_{i+1/2} - \tilde{J}_{i-1/2}}{h_x h_\mu}}{\sigma_t}}$$