Homework 4

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NUEN 629

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NUEN 629, Homework 4

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Solve the following problem and submit a detailed report, including a justification of why a reader should believe your results and a description of your methods and iteration strategies.

1 Vaquer

(150 points + 50 points extra credit) In class we discussed the diamond-difference spatial discretization. Another discretization is the step discretization (this has several other names from other disciplines). It writes the discrete ordinates equations with isotropic scattering as, for $\mu_n > 0$ to

$$\mu_n \frac{\psi_{i,n} - \psi_{i-1,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2}, \tag{1}$$

and for $\mu_n < 0$

$$\mu_n \frac{\psi_{i+1,n} - \psi_{i,n}}{h_x} + \Sigma_t \psi_{i,n} = \frac{\Sigma_s}{2}\phi_i + \frac{Q}{2}. \tag{2}$$

You should be able to modify the codes I have already provided to implement this discretization.

1. (50 points) Your task is to solve a problem with uniform source of $Q = 0.01$, $\Sigma_t = \Sigma_s = 100$ for a slab in vacuum of width 10 using step and diamond difference discretizations. Use 10, 50, and 100 zones ($h_x = 1, 0.02, 0.01$) and your expert choice of angular quadratures. Discuss your results and how the two methods compare at each number of zones.

2. (10 points) Discuss why there is a different form of the discretization for the different signs of μ.

3. (40 points) Plot the error after each iteration using a 0 initial guess for the step discretization with source iteration and GMRES.

4. (50 points) Solve Reed's problem (see finite difference diffusion codes). Present convergence plots for the solution in space and angle to a "refined" solution in space and angle.

5. (50 points extra credit) Solve a time dependent problem for a slab surrounded by vacuum with $\Sigma_t = \Sigma_s = 1$ and initial condition given by $\phi(0) = 1/h_x$. Plot the solution at $t = 1$ s using step and diamond difference. The particles have a speed of 1 cm/s. Which discretization is better with a small time step? What do you see with a small number of ordinates compared to a really large number (100s)?

Solution 1-1:

To modify the provided code to use the step discretization, essentially only the 1DSweep function needs to be modified. For example, for a positive direction of μ_n the flux in the i-th cell is, for the $k-th$ sweep,

$$\psi_{i,n}^{(k+1)} = \frac{\frac{1}{2}\left(\phi_i^{(k)} + Q\right) + \frac{\mu_n}{h_x}\psi_{i-1,n}}{\Sigma_t + \frac{\mu_n}{h_x}}, \tag{1}$$

where $\psi_{i-1,n}$ is either defined by the boundary conditon, i.e., $\psi_0 = f(\mu_n)$, or is known from solution of the previous cell in the sweep. The negative direction sweep is defined analogously.

For the given problem parameters, source iteration was too slow to converge as $c = 1$, so the GMRES solver was used for both spatial discretizations. This was verified to converge to the same solution as source iteration for a sanity check. A plot of the different solutions for the different resolutions and spatial discretizations is given below, for $N = 16$ angles, in Fig. **??**. On the finest mesh, increasing to $N = 24$ had no visible effect on the solution. The solutions do not agree well between different refinements, or between step and the diamond difference (DD) solutions. This is due to the large, pure scattering cross sections. Also plotted below is a diffusion solution to the same problem using Mark Boundary conditions for 100 cells. It is noted that for this problem diffusion theory should be fairly accurate, particuarly away from the boundaries. The difference in solutions is due to the fact that neither step nor DD preserve the diffusion limit.

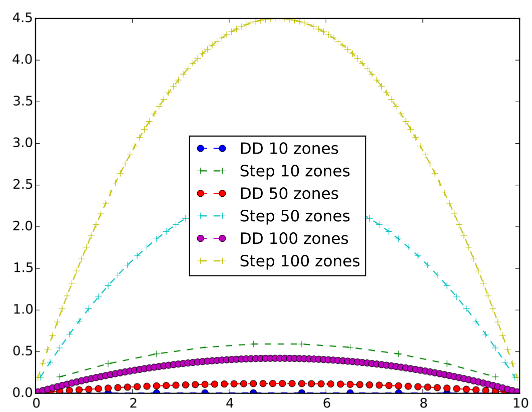

Figure 1: Comparison of step and DD spatial discretizations for different numbers of zones.

Solution 1-2:

The different froms of the discretization are result of the solution being generally undefined at the faces of cells, due to the discontinuity of the solution at cell edges. A closure

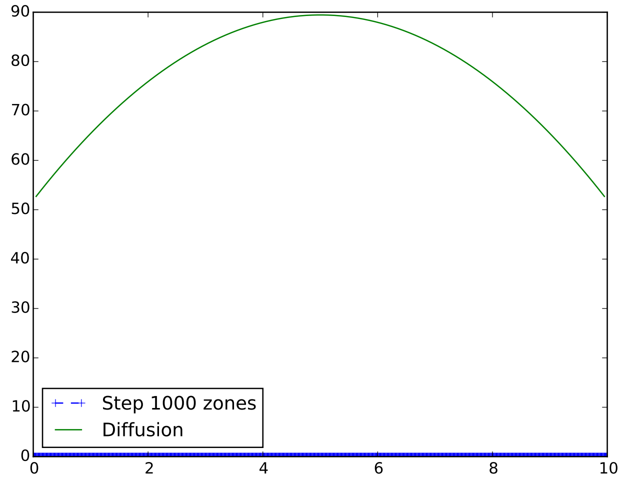

Figure 2: Comparison of diffusion and very fine mesh step solution.

of some kind must be defined because even in the weak-form we need a value for ψ on the face. The given equations define ψ on the face using the upwind closure, which attempts to numerically propogate information in the physical direction of flow, based on the characteristic flow of information in each direction. This, in theory, resolves strong spatial gradients with higher physical accuracy. This closure also provides stability to the equations, which could demonstrate oscillations with a poor choice of closure.

Code