#3 Solution

- Since a pure scatterer, D.E. reduces to (for 1-speed, slabs)

$$-D\frac{d^2\phi}{dx^2} = Q \qquad x \in \left[-\frac{X}{2}, \frac{X}{2}\right] \qquad (1)$$

- w/ the general Brunner B.C.

$$A\phi + BD(\hat{n}\cdot\hat{i})\frac{\partial\phi}{\partial x} = C, \qquad x \in \partial V$$

- Solving for $\phi(x)$ on interior from (1):

$$\int -D\frac{d^2\phi}{dx^2}dx = \int Q\,dx$$

$$\frac{d\phi}{dx} = -\frac{Qx}{D} + K_1$$

$$\phi(x) = -\frac{Qx^2}{2D} + K_1x + K_2 \qquad (2)$$

- For all B.C. of interest, in Brunner form, $C=0$:

$$A_L\phi\left(-\frac{X}{2}\right) - B_L D\frac{d\phi}{dx}\bigg|_{x=-\frac{X}{2}} = 0 \qquad (3)$$

$$A_R\phi\left(\frac{X}{2}\right) + B_R D\frac{d\phi}{dx}\bigg|_{x=\frac{X}{2}} = 0$$

- (2)→(3)

$$A_L\left(-\frac{QX^2}{8D} - \frac{K_1X}{2} + K_2\right) + B_L\left(-\frac{QX}{2} - K_1D\right) = 0$$

$$A_R\left(-\frac{QX^2}{8D} + \frac{K_1X}{2} + K_2\right) + B_R\left(-\frac{QX}{2} + K_1D\right) = 0$$

or:
$$\left(\frac{A_L X}{2} + B_L D\right)K_1 - (A_L K_2) = -\left(\frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2}\right)$$

$$\left(\frac{A_R X}{2} + B_R D\right)K_1 + (A_R K_2) = \frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}$$

- Solving algebraically

$$K_1 = \frac{-\left(\frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2}\right) + \left(\frac{A_L}{A_R}\right)\left(\frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}\right)}{\left(\frac{A_L X}{2} + B_L D\right) + \left(\frac{A_L}{A_R}\right)\left(\frac{A_R X}{2} + B_R D\right)}$$

$$K_2 = \frac{\frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2} + \left(\frac{\frac{A_L X}{2} + B_L D}{\frac{A_R X}{2} + B_R D}\right)\left(\frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}\right)}{A_L + \left(\frac{\frac{A_L X}{2} + B_L D}{\frac{A_R X}{2} + B_R D}\right)A_R}$$

- The above equations are evaluated and simplified for each case to get K_1 and K_2.