Solution:

- First, expand scattering kernel in Legendre Polynomials.
- Begin w/ Legendre generating fn:

$$\frac{1}{\sqrt{1-2xh+h^2}} = \sum_{n=0}^{\infty} P_n(x)h^n \qquad (1) \text{ [wiki]}$$

- Take a derivative w.r.t. x

$$\frac{h}{(1-2xh+h^2)^{3/2}} = \sum_{n=0}^{\infty} P_n'(x)h^n \qquad (2)$$

- Multiply both sides by $\frac{(1-h^2)}{2h}$:

$$K(x) = \frac{1}{2}\frac{(1-h^2)}{(1-2xh+h^2)^{3/2}} = \frac{1}{2}\sum_{n=0}^{\infty} P_n'(x)h^{n-1}(1-h^2) \qquad (3)$$

- (3) is our scattering kernel. Now we need to eliminate the derivative in terms of $P_n(x)$. Rewrite sum by letting $n \to n+1$

$$\frac{1}{2}\sum_{n=0}^{\infty} P_n'(x)h^{n-1}(1-h^2) = \frac{1}{2}\sum_{n=0}^{\infty} P_{n+1}'(x)h^n(1-h^2) \qquad (4)$$

- Note that the sum still starts at zero because $P_0'(x)=0$, so we can trivially add this term. From Abromowitz:

$$P_{n+1}'(x) = \sum_{m \text{ even}}^{n} (2m+1)P_m(x) \qquad (5)$$

- (5)→(4) and write as two sums:

$$K(x) = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m \text{ even}}^{n} (2m+1)P_m(x)h^n - \frac{1}{2}\sum_{n=0}^{\infty}\sum_{m \text{ even}}^{n} (2m+1)P_m(x)h^{n+2} \qquad (6)$$

* Note: Here orthogonality is defined as $\int P_m P_n = \frac{\delta_{mn}\, 2}{2n+1}$ *

- Shift the second infinite sum by $n' = n+2$

$$K = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{\substack{m\\ \text{even}}}^{n}(2m+1)P_m(x)h^n - \frac{1}{2}\sum_{n'=2}^{\infty}\sum_{\substack{m\\ \text{even}}}^{n'-2}(2m+1)P_m(x)h^{n'}$$

- Combine the two series, writing out $n=0,1$ & letting $n' \to n$

$$K = \frac{1}{2}\sum_{n=0}^{1}(2n+1)P_n(x)h^n + \frac{1}{2}\sum_{n=2}^{\infty}\left(\sum_{\substack{m\\ \text{even}}}^{n}(2m+1)P_m(x) - \sum_{\substack{m\\ \text{even}}}^{n-2}(2m+1)P_m(x)\right)h^n$$

- For the inner sums all terms but n cancel, adding back energy:

$$\boxed{K(\mu_0, \nu' \to \nu) = \frac{1}{2}\sum_{n=0}^{\infty}(2n+1)P_n(\mu_0)h^n\delta(\nu'-\nu)} \qquad (7)$$

- Taking legendre moments:

$$K_\ell(\nu' \to \nu) = \frac{\delta(\nu'-\nu)}{2}\sum_{n=0}^{\infty}(2n+1)h^n\int_{-1}^{1}d\mu_0\, P_n(\mu_0)P_\ell(\mu_0)$$

$$K_\ell(\nu' \to \nu) = \delta(\nu'-\nu)\sum_{n=0}^{\infty}\left(\frac{2n+1}{2}\right)h^n\frac{\delta_{n\ell}\,2}{(2\ell+1)}$$

$$\boxed{K_\ell(\nu' \to \nu) = h^\ell}$$

- Noting $\int_0^\infty d\nu\int_{-1}^{1}d\mu_0(\cdot) = \int_0^\infty d\nu\int_{-1}^{1}P_0(\mu_0)(\cdot)$; $\int_0^\infty d\nu\int_{-1}^{1}d(\mu_0)\,K(\mu_0, \nu' \to \nu) = h^0 = 1$,

So normalization is correct.

$$\boxed{\begin{aligned} K_0 &= 1\,\delta(\nu-\nu') \\ K_1 &= h\,\delta(\nu-\nu') \\ K_2 &= h^2\,\delta(\nu-\nu') \end{aligned}}$$