Homework 2

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1 Childs

(35 points) Compute three group cross-sections for a homogeneous mixture of graphite and natural uranium where the ratio of graphite to uranium is 150:1. You can assume the Watt-fission spectrum, and that the group bounds are $\{0, 1\,\mathrm{eV}, 100\,\mathrm{keV}, 20\,\mathrm{MeV}\}$.

2 Franklin

(40 points) The enclosed file gives the microscopic cross-sections for ^{1}H in units of barns for 5 groups as calculated by the code `NJOY`. Imagine we have a large, nearly infinite tank of high-pressure hydrogen at 30 atm next to a bare sphere of ^{235}U. Compute the scalar flux ϕ_g and the current $\vec{J}_g$ in the hydrogen using the separable, P1 equivalent, and extended Legendre approximations. Compare your solutions graphically.

3 Geer

(25 points) Find the solution to the diffusion equation for 1-group, slab geometry with a uniform source, where the material is a pure scatter and the slab width is X under the following conditions

1. Vacuum Marshak conditions
2. Vacuum Mark conditions
3. Vacuum Dirichlet conditions
4. Vacuum Dirichlet condition on the left and albedo on the right at $X/2$, and
5. Vacuum Dirichlet condition on the left and reflecting on the right at $X/2$.

Compare the solutions and comment on the similarities and differences.

3 Solution

- Since a pure scatterer, D.E. reduces to (for 1 speed, slabs)

$$-D\frac{d^2\phi}{dx^2} = Q \qquad x \in \left[-\frac{X}{2}, \frac{X}{2}\right] \qquad (1)$$

- w/ the general Brunner B.C.

$$A\phi + BD(\hat{n}\cdot\hat{i})\frac{\partial\phi}{\partial x} = C, \qquad x \in \partial V$$

- Solving for $\phi(x)$ on interior from (1):

$$\int -D\frac{d^2\phi}{dx^2}dx = \int Q\,dx$$

$$\frac{d\phi}{dx} = \frac{Qx}{D} + K_1$$

$$\phi(x) = \frac{Qx^2}{2D} + K_1x + K_2 \qquad (2)$$

- For all B.C. of interest, in Brunner form, $C=0$:

$$A_L\phi\left(-\frac{X}{2}\right) - B_L D\frac{d\phi}{dx}\bigg|_{x=-\frac{X}{2}} = 0 \qquad (3)$$

$$A_R\phi\left(\frac{X}{2}\right) + B_R D\frac{d\phi}{dx}\bigg|_{x=\frac{X}{2}} = 0$$

- (2) → (3)

$$A_L\left(\frac{QX^2}{8D} - \frac{K_1X}{2} + K_2\right) + B_L\left(\frac{QX}{2} - K_1D\right) = 0$$

$$A_R\left(\frac{QX^2}{8D} + \frac{K_1X}{2} + K_2\right) + B_R\left(\frac{QX}{2} + K_1D\right) = 0$$

or $\left(+\frac{A_L X}{2} + B_L D\right)K_1 - (A_L)K_2 = \frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2}$

$$\left(\frac{A_R X}{2} + B_R D\right)K_1 + A_R K_2 = -\left(\frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}\right)$$

- solving Algebraically gives:

$$K_1 = \frac{\left(\frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2}\right) - \left(\frac{A_L}{A_R}\right)\left(\frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}\right)}{\left(\frac{A_L X}{2} + B_L D\right) + \left(\frac{A_L}{A_R}\right)\left(\frac{A_R X}{2} + B_R D\right)} \tag{4}$$

$$K_2 = \frac{-\left(\frac{A_L Q X^2}{8D} + \frac{B_L Q X}{2} + \left(\frac{\frac{A_L X}{2} + B_L D}{\frac{A_R X}{2} + B_R D}\right)\left(\frac{A_R Q X^2}{8D} + \frac{B_R Q X}{2}\right)\right)}{A_L + \left(\frac{\frac{A_L X}{2} + B_L D}{\frac{A_R X}{2} + B_R D}\right) A_R} \tag{5}$$

- The above expressions are evaluated and simplified for each case, The below table summarizes results on the next page.

A summary of the solutions obtained for each of the boundary conditions is given in the table below. Each solution was checked to ensure they satisfy the boundary conditions.

Table 1: Solutions with different boundary conditions for a pure scatter for slab of width X centered at $x = 0$.

Left BC	Right BC	$\phi(x)$
Vacuum Marshak	Vacuum Marshak	$\phi(x) = Q\left(\frac{X^2}{8D} + X - \frac{x^2}{2D}\right)$
Vacuum Mark	Vacuum Marshak	$\phi(x) = Q\left(\frac{X^2}{8D} + \frac{X\sqrt{3}}{2} - \frac{x^2}{2D}\right)$
Vacuum Dirichlet	Vacuum Dirichlet	$\phi(x) = \frac{Q}{2D}\left(\frac{X^2}{4} - x^2\right)$
Vacuum Dirichlet	Albedo	$\phi(x) = -\frac{Qx^2}{2D} + QxX\left(\frac{1+\frac{(1-\alpha)}{2(1+\alpha)}\frac{X}{2D}}{\frac{(1-\alpha)}{2(1+\alpha)}X+D} - \frac{1}{2D}\right) + Q\frac{X^2}{2}\left(\frac{1+\frac{(1-\alpha)}{2(1+\alpha)}\frac{X}{2D}}{\frac{(1-\alpha)}{2(1+\alpha)}X+D} - \frac{1}{4D}$
Vacuum Dirichlet	Reflecting	$\phi(x) = \frac{Q}{2D}\left(\frac{3X^2}{4} + xX - x^2\right)$