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Cycle 1

P1 = Compute Hydro Variable Slopes

P2 = Advect Hydro Variable to $\frac{\Delta t}{4}$

$$u_i^* = u_i^n - \frac{\Delta t}{4\Delta x}\left(F_{R,i}^n - F_{L,i}^n\right)$$

P3 =

$$4\rho_i^{n+\frac{1}{4}}\frac{\left(u_i^{n+\frac{1}{4},k} - u_i^*\right)}{\Delta t} = \frac{1}{2}\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)\right]_{i+\frac{1}{2}}^{n} + \frac{1}{2}\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)\right]_{i+\frac{1}{2}}^{n+\frac{1}{4},k} \Bigg\}^{n+\frac{1}{8}}$$

P4 =

$$\frac{4}{\Delta t}\left(E_r^{n+\frac{1}{4},k+1} - E_r^n\right) = -\frac{1}{2}\left(\frac{\partial F_r^{n+\frac{1}{4},k+1}}{\partial x} + \frac{\partial F_r^n}{\partial x}\right)$$

$$+ \frac{\sigma_a^{n+\frac{1}{4},k} c}{2}\left[a\left(T^{n+\frac{1}{4},k+1}\right)^4 - E_r^{n+\frac{1}{4},k+1}\right]$$

$$+ \frac{\sigma_a^n c}{2}\left[a\left(T^n\right)^4 - E_r^n\right]$$

$$- \left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)u\right]^{n+\frac{1}{8},k}$$

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$$\frac{1}{c^2}\left(\frac{F_r^{n+1/4} - F_r^{n}}{\Delta t}\right) + \left[\frac{1}{3}\frac{\partial E_r^{n+\frac{1}{4},k+1}}{\partial x} + \frac{1}{3}\frac{\partial E_r^{n}}{\partial x}\right]\frac{1}{2}$$

$$+ \frac{\sigma_t^{n+\frac{1}{4}}}{2c}\left(F^{n+\frac{1}{4},k+1} - \frac{4}{3}E_r^{n+\frac{1}{4},k}u^{n+\frac{1}{4},k}\right) + \frac{\sigma_t^{n}}{2c}\left(F^{n} - \frac{4}{3}E_r^{n}u^{n}\right)$$

$$\frac{4}{\Delta t}\left(E^{n+1,k+1} - E^{n}\right) = -\frac{\sigma_a^{n+\frac{1}{4}}c}{2}\left[a(T^{n+\frac{1}{4},k+1})^4 - E_r^{n+\frac{1}{4},k+1}\right]$$

$$-\frac{\sigma_a^{n}c}{2}\left[a(T^{n})^4 - E_r^{n}\right] + \left[\frac{\sigma_t}{c}(F - \frac{4}{3}E_r u)u\right]^{n+\frac{1}{2},k}$$

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Cycle 1

C1: Advect Hydro Variables

$$u_i^{**} = u_i^n - \frac{\Delta t}{2\Delta x}\left(F_{i+\frac{1}{2}}^{n+\frac{1}{2}} - F_{i-\frac{1}{2}}^{n+\frac{1}{2}}\right)$$

C2: Update Momentum deposition

$$2\rho_i^{n+\frac{1}{2}}\frac{\left(u_i^{n+\frac{1}{2},k+1} - u_i^{**}\right)}{\Delta t} = \frac{\sigma_t}{c}\left(F - \frac{4}{3}E u\right)_{i+\frac{1}{2}}^{n+\frac{1}{4},k}$$

C3: $$\frac{2}{\Delta t}\left(E_r^{n+\frac{1}{2},k+1} - E_r^n\right) = -\frac{1}{2}\left(\frac{\partial F_r^{n+\frac{1}{2},k+1}}{\partial x} + \frac{\partial F_r^n}{\partial x}\right)$$

$$+\left[\sigma_a c\left(a T^4 - E_r\right)\right]^{n+\frac{1}{4},k+1} - \left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}E u\right)u\right]^{n+\frac{1}{2},k}$$

$$\frac{2}{c^2}\frac{\left(F_r^{n+\frac{1}{2},k+1} - F_r^n\right)}{\Delta t} + \frac{1}{3}\frac{\partial E_r}{\partial x}^{n+\frac{1}{4},k+1} +$$

$$\left[\frac{\sigma_t}{c}\left(F^{k+1} - \frac{4}{3}E u^k\right)\right]^{n+\frac{1}{4}} = 0$$

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$$\frac{2}{\Delta t}\left(E^{n+\frac{1}{2},k+1} - E^{**}\right) = -\sigma_a c\left(aT^4 - E_r\right)^{n+\frac{1}{2},k+1}$$

$$+\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}E_r u\right)u\right]^{n+\frac{1}{2},k}$$

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Cycle 2

Predictor = just like Cycle 1 predictor

except $n \to n+\frac{1}{2}$

C1: Advect Hydro Variables

$$u_i^{**} = u_i^{n+\frac{1}{2}} - \frac{\Delta t}{2\Delta x}\left(F_{i+\frac{1}{2}}^{n+3/4} - F_{i-\frac{1}{2}}^{n+\frac{3}{4}}\right)$$

C2: Update momentum Equation:

$$2\rho_i^{n+1}\left(u_i^{n+1,k+1} - u_i^{**}\right) = \left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)\right]^{n+\frac{3}{4},k}$$

C3: $$\frac{E_r^{n+1,k+1} - E_r^{n}}{\Delta t} = -\frac{1}{3}\left(\frac{\partial F_r^{n+1,k+1}}{\partial x} + \frac{\partial F_r^{n+\frac{1}{2}}}{\partial x} + \frac{\partial F_r^{n}}{\partial x}\right)$$

$$\frac{2}{3}\left[\sigma_a c\left(aT^4 - E_r\right)\right]^{n+\frac{1}{4}} + \frac{1}{3}\left[\sigma_a c\left(a\left(T^{n+1,k+1}\right)^4 - E_r^{n+1,k+1}\right)\right]$$

$$-\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)u\right]^{n+\frac{1}{4}} - \left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)u\right]^{n+\frac{3}{4}}$$

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$$\frac{2}{c^2}\left(\frac{F_r^{n+1,k+1} - F_r^{n}}{\Delta t}\right) + \frac{1}{9}\left(\frac{\partial E_r}{\partial x}^{n+1,k+1} + \frac{\partial E_r}{\partial x}^{n+k} + \frac{\partial E_r}{\partial x}^{n}\right) =$$

$$-\frac{2}{3}\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}E u\right)\right]^{n+\frac{1}{4}} - \frac{1}{3}\left[\frac{\sigma_t^{n+1,k}}{c}\left(F^{n+1,k+1} - \frac{4}{3}E^{k} u^{k}\right)\right]$$

$$\frac{E^{n+1,k+1} - E^{n}}{\Delta t} = -\frac{2}{3}\left[\sigma_a c\left(aT^4 - E_r\right)\right]^{n+\frac{1}{4}}$$

$$-\frac{1}{3}\sigma_a^{n+\frac{1}{4},k} c\left(a\left(T^{n+1,k+1}\right)^4 - E_r^{n+1,k+1}\right)$$

$$-\frac{2}{3}\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)u\right]^{n+\frac{1}{4}} - \frac{1}{3}\left[\frac{\sigma_t}{c}\left(F - \frac{4}{3}Eu\right)u\right]^{n+1,k}$$