1 Cycle 1

1.1 Cycle 1, 1st Step

$$\mathbf{U}_{L,i}^{n} = \mathbf{U}_{i}^{n} - \frac{\Delta_{i}^{n}}{2}, \quad \mathbf{U}_{R,i}^{n} = \mathbf{U}_{i}^{n} + \frac{\Delta_{i}^{n}}{2}, \tag{1}$$

$$\mathbf{U}_{i}^{n+\frac{1}{4},*} = \mathbf{U}_{i}^{n} - \frac{\frac{1}{4}\Delta t}{\Delta x}\left(\mathbf{F}_{R,i}^{n} - \mathbf{F}_{L,i}^{n}\right) \tag{2}$$

$$\frac{\rho_{i}^{n+\frac{1}{4}}\left(u_{i}^{n+\frac{1}{4},k} - u_{i}^{n+\frac{1}{4},*}\right)}{\frac{1}{4}\Delta t} = \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i}^{n} + \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i}^{n+\frac{1}{4},k-1} \tag{3}$$

$$\begin{aligned}\frac{\mathcal{E}^{n+\frac{1}{4},k} - \mathcal{E}^{n}}{\frac{1}{4}\Delta t} =& -\frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^{n} - \frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^{n+\frac{1}{4},k} \\ &+ \frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]^{n} + \frac{1}{2}\sigma_a^{n+\frac{1}{4},k-1} c\left[aT^4 - \mathcal{E}\right]^{n+\frac{1}{4},k} \\ &- \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n} - \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{4},k-1}\end{aligned} \tag{4}$$

$$\begin{aligned}\frac{1}{c^2}\frac{\mathcal{F}^{n+\frac{1}{4},k} - \mathcal{F}^{n}}{\frac{1}{4}\Delta t} =& -\frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^{n} - \frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^{n+\frac{1}{4},k} \\ &- \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n} - \frac{1}{2}\frac{\sigma_t^{n+\frac{1}{4},k-1}}{c}\left(\mathcal{F}^{n+\frac{1}{4},k} - \frac{4}{3}\mathcal{E}^{n+\frac{1}{4},k-1}u^{n+\frac{1}{4},k-1}\right)\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{E_{i}^{n+\frac{1}{4},k} - E_{i}^{n+\frac{1}{4},*}}{\frac{1}{4}\Delta t} =& -\frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]_{i}^{n} - \frac{1}{2}\left[\sigma_a^{n+\frac{1}{4},k-1} c\left(a(T^{n+\frac{1}{4},k})^4 - \mathcal{E}^{n+\frac{1}{4},k}\right)\right]_{i} \\ &+ \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i}^{n} + \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_{i}^{n+\frac{1}{4},k-1}\end{aligned} \tag{6}$$

1.2 Cycle 1, 2nd Step

$$\mathbf{U}_i^{n+\frac{1}{2},*} = \mathbf{U}_i^n - \frac{\frac{1}{2}\Delta t}{\Delta x}\left(\mathbf{F}_{i+\frac{1}{2}}^{n+\frac{1}{4}} - \mathbf{F}_{i-\frac{1}{2}}^{n+\frac{1}{4}}\right) \tag{7}$$

$$\frac{\rho_i^{n+\frac{1}{2}}\left(u_i^{n+\frac{1}{2},k} - u_i^{n+\frac{1}{2},*}\right)}{\frac{1}{2}\Delta t} = \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^n + \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2},k-1} \tag{8}$$

$$\begin{aligned}\frac{\mathcal{E}^{n+\frac{1}{2},k} - \mathcal{E}^n}{\frac{1}{2}\Delta t} =& -\frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^n - \frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^{n+\frac{1}{2},k} \\ &+ \frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]^n + \frac{1}{2}\sigma_a^{n+\frac{1}{2},k-1} c\left[aT^4 - \mathcal{E}\right]^{n+\frac{1}{2},k} \\ &- \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^n - \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{2},k-1}\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{1}{c^2}\frac{\mathcal{F}^{n+\frac{1}{2},k} - \mathcal{F}^n}{\frac{1}{2}\Delta t} =& -\frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^n - \frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^{n+\frac{1}{2},k} \\ &- \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^n - \frac{1}{2}\frac{\sigma_t^{n+\frac{1}{2},k-1}}{c}\left(\mathcal{F}^{n+\frac{1}{2},k} - \frac{4}{3}\mathcal{E}^{n+\frac{1}{2},k-1}u^{n+\frac{1}{2},k-1}\right)\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{E_i^{n+\frac{1}{2},k} - E_i^{n+\frac{1}{2},*}}{\frac{1}{2}\Delta t} =& -\frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]_i^n - \frac{1}{2}\left[\sigma_a^{n+\frac{1}{2},k-1} c\left(a(T^{n+\frac{1}{2},k})^4 - \mathcal{E}^{n+\frac{1}{2},k}\right)\right]_i \\ &+ \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^n + \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2},k-1}\end{aligned} \tag{11}$$

2 Cycle 2

2.1 Cycle 2, 1st Step

$$\mathbf{U}_{L,i}^{n+\frac{1}{2}} = \mathbf{U}_i^{n+\frac{1}{2}} - \frac{\Delta_i^{n+\frac{1}{2}}}{2}, \quad \mathbf{U}_{R,i}^{n+\frac{1}{2}} = \mathbf{U}_i^{n+\frac{1}{2}} + \frac{\Delta_i^{n+\frac{1}{2}}}{2}, \tag{12}$$

$$\mathbf{U}_i^{n+\frac{3}{4},*} = \mathbf{U}_i^{n+\frac{1}{2}} - \frac{\frac{1}{4}\Delta t}{\Delta x}\left(\mathbf{F}_{R,i}^{n+\frac{1}{2}} - \mathbf{F}_{L,i}^{n+\frac{1}{2}}\right) \tag{13}$$

$$\frac{\rho_i^{n+\frac{3}{4}}\left(u_i^{n+\frac{3}{4},k} - u_i^{n+\frac{3}{4},*}\right)}{\frac{1}{4}\Delta t} = \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2}} + \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{3}{4},k-1} \tag{14}$$

$$\begin{aligned}\frac{\mathcal{E}^{n+\frac{3}{4},k} - \mathcal{E}^{n+\frac{1}{2}}}{\frac{1}{4}\Delta t} =& -\frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^{n+\frac{1}{2}} - \frac{1}{2}\frac{\partial \mathcal{F}}{\partial x}^{n+\frac{3}{4},k} \\ &+ \frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]^{n+\frac{1}{2}} + \frac{1}{2}\sigma_a^{n+\frac{3}{4},k-1} c\left[aT^4 - \mathcal{E}\right]^{n+\frac{3}{4},k} \\ &- \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{2}} - \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{3}{4},k-1}\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{1}{c^2}\frac{\mathcal{F}^{n+\frac{3}{4},k} - \mathcal{F}^{n+\frac{1}{2}}}{\frac{1}{4}\Delta t} =& -\frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^{n+\frac{1}{2}} - \frac{1}{6}\frac{\partial \mathcal{E}}{\partial x}^{n+\frac{3}{4},k} \\ &- \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{2}} - \frac{1}{2}\frac{\sigma_t^{n+\frac{3}{4},k-1}}{c}\left(\mathcal{F}^{n+\frac{3}{4},k} - \frac{4}{3}\mathcal{E}^{n+\frac{3}{4},k-1}u^{n+\frac{3}{4},k-1}\right)\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{E_i^{n+\frac{3}{4},k} - E_i^{n+\frac{3}{4},*}}{\frac{1}{4}\Delta t} =& -\frac{1}{2}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]_i^{n+\frac{1}{2}} - \frac{1}{2}\left[\sigma_a^{n+\frac{3}{4},k-1} c\left(a(T^{n+\frac{3}{4},k})^4 - \mathcal{E}^{n+\frac{3}{4},k}\right)\right]_i \\ &+ \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2}} + \frac{1}{2}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{3}{4},k-1}\end{aligned} \tag{17}$$

2.2 Cycle 2, 2nd Step

$$\mathbf{U}_i^{n+1,*} = \mathbf{U}_i^{n+\frac{1}{2}} - \frac{\frac{1}{2}\Delta t}{\Delta x}\left(\mathbf{F}_{i+\frac{1}{2}}^{n+\frac{3}{4}} - \mathbf{F}_{i-\frac{1}{2}}^{n+\frac{3}{4}}\right) \tag{18}$$

$$\frac{\rho_i^{n+1}\left(u_i^{n+1,k} - u_i^{n+1,*}\right)}{\frac{1}{2}\Delta t} = \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2}} + \frac{1}{2}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+1,k-1} \tag{19}$$

$$\begin{aligned}\frac{\mathcal{E}^{n+1,k} - \mathcal{E}^n}{\Delta t} =& -\frac{1}{3}\frac{\partial \mathcal{F}}{\partial x}^n - \frac{1}{3}\frac{\partial \mathcal{F}}{\partial x}^{n+\frac{1}{2}} - \frac{1}{3}\frac{\partial \mathcal{F}}{\partial x}^{n+1,k} \\ &+ \frac{1}{3}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]^n + \frac{1}{3}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]^{n+\frac{1}{2}} + \frac{1}{3}\sigma_a^{n+1,k-1} c\left[aT^4 - \mathcal{E}\right]^{n+1,k} \\ &- \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^n - \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{2}} - \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+1,k-1}\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{1}{c^2}\frac{\mathcal{F}^{n+1,k} - \mathcal{F}^n}{\Delta t} =& -\frac{1}{9}\frac{\partial \mathcal{E}}{\partial x}^n - \frac{1}{9}\frac{\partial \mathcal{E}}{\partial x}^{n+\frac{1}{2}} - \frac{1}{9}\frac{\partial \mathcal{E}}{\partial x}^{n+1,k} \\ &- \frac{1}{3}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^n - \frac{1}{3}\left[\frac{\sigma_t}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]^{n+\frac{1}{2}} - \frac{1}{3}\frac{\sigma_t^{n+1,k-1}}{c}\left(\mathcal{F}^{n+1,k} - \frac{4}{3}\mathcal{E}^{n+1,k-1}u^{n+1,k-1}\right)\end{aligned} \tag{21}$$

$$\begin{aligned}\frac{E_i^{n+1,k} - E_i^n}{\Delta t} =& -\frac{1}{3}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]_i^n - \frac{1}{3}\left[\sigma_a c\left(aT^4 - \mathcal{E}\right)\right]_i^{n+\frac{1}{2}} - \frac{1}{3}\left[\sigma_a^{n+1,k-1} c\left(a(T^{n+1,k})^4 - \mathcal{E}^{n+1,k}\right)\right]_i \\ &+ \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^n + \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+\frac{1}{2}} + \frac{1}{3}\left[\sigma_t \frac{u}{c}\left(\mathcal{F} - \frac{4}{3}\mathcal{E}u\right)\right]_i^{n+1,k-1}\end{aligned} \tag{22}$$