# Sets

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May 28, 2025

## 1 Basics

**Set:** A set is a well-defined collection of unordered elements. Sets can be defined by either listing each element or by defining the property for which an element is included. For example, S = $\{1, 2, 3, 4\}$ and S = $\{s|s \in \mathbb{R} \ \ \& \ \ s < 2\}$ are both valid set definitions. To show an element x is in a set S, we write x $\in$ S. For the opposite, x $\notin$ S.

**Subset:** If for two sets A and B, when x $\in$ A, x $\in$ B, then A is a subset of B. This can be written A $\subseteq$ B. A subset is proper if A $\neq$ B.

**Equality:** Two sets A and B are equal if A $\subseteq$ B and B $\subseteq$ A.

**Empty/Null Set:** A set with no elements is called empty of null, denoted with $\emptyset$' or $\{\}$

**Cardinality:** The cardinality of a set A, written as $|A|$, is the number of elements within the set.

**Disjoint:** Two sets A and B are disjoint if A $\cap$ B = $\emptyset$. Similarly n sets A$_1$, A$_2$, ..., A$_n$ are disjoint if $\bigcap\limits_{i=1}^{n} A_i = \emptyset$. N sets A$_1$, A$_2$, ..., A$_n$ are pairwise disjoint if for every i $\neq$ j A$_i$ and A$_j$ are disjoint.

**Ordered n-Tuples:** An ordered pair or ordered triple is two or three objects in a specified order, for example (a, b) and (a, b, c), respectively. An ordered n-tuple is n objects with a specified order, such as ($a_1$, $a_2$, ..., $a_n$).

## 2 Set Operations

**Union:** The union of two sets A and B is the set consisting of all elements in A or B, written as A $\cup$ B.

**Intersection:** The intersection of two sets A and B is the set consisting of all elements which are in both A and B, written as A $\cap$ B.

**Difference:** The difference of two sets A and B, written as A - B, is the set of all elements in A that are not in B.

**Complement:** The complement of a set A or $A^c$ is the set of all elements which are not in A.

> **Example:** Let A be the set of all jackets and let B be the set of all leather jackets. The intersection of A and B would be B, the set of all leather jackets, since it is a subset of A. For similar reasons, the union of these two sets would be A, since all of be is already contained in A.

**Cartesian Product:** The Cartesian product of two sets A and B, A $\times$ B, is the set of all ordered pairs (a, b) such that a $\in$ A and b $\in$ B. The Cartesian product of n sets $A_1$, $A_2$, ..., $A_n$ is the set of all ordered n-tuples ($a_1$, $a_2$, ..., $a_n$) such that $a_1 \in A_1$, $a_2 \in A_2$, ..., $a_n \in A_n$.

**Powerset:** The powerset of a set A is the set of all subsets of A. For a set A $= \{a, b, c\}$, P(A) $=$ $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$.

## 3 Connection to Probability

**Sets in Probability:** The sample space of an experiment, which contains all its possible outcomes, is a set. The event, which is a selected outcome or collection of outcomes, is a subset of the sample space.

**Sets Operations in Probability:** We use set operations when finding the probability of either event A or event B occurring P(A $\cup$ B), and the probability of an event A not occurring P($A^c$). The inclusion-exclusion formula uses both unions and intersections of events to ensure each possible probability is only included once. In conditional probability, the probability of the intersection of two events is used to calculate the necessary value.