

# Combinational Angle Difference Series: A New Approach to Digital Predistortion

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**Abstract**—In the last couple of decades, wireless communication networks have become a vital asset for modern society. A key component in those systems is the power amplifier (PA). This tool has permitted the rise of many useful things, such as portable devices and the Internet of Things. In those applications, and in many others, efficiency becomes a key characteristic, due to physical restraints such as battery. Searching to improve the efficiency of PAs, this work studies implementation of different algorithms that model the transfer behavior of the PA in order to capture non-linearity and treat it in order to be able to operate in highly nonlinear, highly efficient points of operation, a process known as digital predistortion. The work introduces a new model, combinational angle difference series, as well as revisits an already studied model, the angle difference series, and compares them against the state of the art (Polar Volterra Series), showing promising results.

**Keywords**—digital predistortion, power amplifiers, radiofrequency, Volterra series

## I. INTRODUCTION

Power amplifiers (PAs) are essential to modern data transmission systems [1]. Raising power with a small non-linearity and high efficiency has always been a desired characteristic for the industry. However, due to the physical nature of PAs, those characteristics are often correlated, meaning that the higher the efficiency on a given operation point, the higher the non-linearity [2].

There are many approaches on how to handle such behavior in order to reduce non-linearity, without losing efficiency. Among those many, the one that will be addressed in this work is the very prominent digital predistortion (DPD). The polar Volterra series (PVS) and neural networks (NNs) are very popular methods for the implementation of DPD, however those methods usually are very computationally complex, which often renders the application unfeasible [3].

Motivated by those models, and in others already discussed in other works [4]-[5], this work discusses the implementation of another model, called Combinational Difference Angle Series (CADS), a different approach to the already studied model called Angle Difference Series (ADS).

## II. PA MODELING

### A. Polar Volterra series (PVS)

The polar Volterra series is a well known, popularized method to model PAs [5]. It is a very complicated algorithm

that handles with multiple sums and it is of high computational complexity. The PVS characteristic equation is:

$$\tilde{y}(n) = \sum_{p_1=1}^{P_1} \sum_{p_2=1}^{P_2} \sum_{m_1=0}^M \dots \sum_{m_{p_1}=m_{p_1-1}}^M \sum_{l_1=0}^L \dots \sum_{l_{p_2}=l_{p_2-1}}^L \sum_{l_{p_2+1}=0}^L \dots \sum_{l_{2p_2-1}=l_{2p_2-2}}^L h_{p_1,p_2}(m_1, \dots, m_{p_1}, l_1, \dots, l_{2p_2-1}) \prod_{k=1}^{p_1} a(n - m_k) \prod_{r=1}^{p_2} e^{j(\phi(n-l_r))} \prod_{s=p_2+1}^{2p_2-1} e^{-j(\phi(n-l_s))}, \quad (1)$$

where  $P_1$  and  $P_2$  are the powers by which the amplitude and phase information will be raised, respectively, and where  $M$  and  $L$  are the number of memory elements for amplitude and phase information, respectively. The input and output signals are represented by  $\tilde{x} = a.e^{j(\phi)}$  and  $\tilde{y}$ , respectively, and  $h$  indicates the Volterra kernels

### B. Angle difference series (ADS)

Suggested in the authors' previous work of [4], this model, inspired by the behavior of NNs, tries to handle the input phase information in a more simplistic manner compared to the commonly used PVS. The characteristic equation for this model is:

$$\tilde{y}(n) = \sum_{p_1=1}^{P_1} \sum_{p_2=0}^{P_2} \sum_{m_1=0}^M \dots \sum_{m_{p_1}=m_{p_1-1}}^M \sum_{l_1=0}^{L-1} h_{p_1,p_2}(m_1, \dots, m_{p_1}, l_1) \prod_{k=1}^{p_1} a(n - m_k) e^{j(\phi(n))} \{e^{j(\phi(n-l_1))} (e^{-j(\phi(n-l_1-1))})\}^{p_2}, \quad (2)$$

In every contribution of ADS, a positive instantaneous phase is always present, accompanied by a varying power of a single phase difference between two consecutive time instants.

### C. Combinational angle difference series (CADS)

This new model incorporates products among phase differences from distinct time samples, in order to increase the

variety of angle components that the algorithm can achieve. The characteristic equation for this model is:

$$\tilde{y}(n) = \sum_{p_1=1}^{P_1} \sum_{p_2=0}^{P_2} \sum_{m_1=0}^M \dots \sum_{m_{p_1}=m_{p_1}-1}^M \sum_{l_1=0}^{L-1} \sum_{l_2=l_1}^{L-1} \dots \sum_{l_{p_2}=l_{p_2}-1}^{L-1} h_{p_1,p_2}(m_1, \dots, m_{p_1}, l_1, \dots, l_{p_2}) \prod_{k=1}^{p_1} a(n - m_k) e^{j(\phi(n))} \prod_{i=1}^{p_2} \{e^{j(\phi(n-l_i))} (e^{-j(\phi(n-l_i-1))})\}, \quad (3)$$

### III. MODELING IDENTIFICATION AND ASSESSMENT

Many calculated elements can be arranged in vector, and be written in the following form:

$$Y^{cal} = XH, \quad (4)$$

where  $Y^{cal}$  is the  $N \times 1$  vector of the calculated outputs, using any of the algorithms,  $X$  is a  $N \times Q$  matrix whose elements are obtained from using a constitutive equation and taking the parameters,  $H$  is a  $Q \times 1$  vector containing the coefficients,  $N$  is the number of samples being used to model the system and  $Q$  is the number of the coefficients. The parameter identification is done by least squares. After  $Y^{cal}$  is properly calculated, we can use a metric, called normalized mean square error (NMSE) [6], to measure how good was the calculated value in comparison to the real output.

NMSE can be calculated by the following equation:

$$NMSE = 10 \log_{10} \left\{ \frac{\sum_{n=1}^N |y_n^{des} - y_n^{cal}|^2}{\sum_{n=1}^N |y_n^{des}|^2} \right\}, \quad (5)$$

where  $y_n^{des}$  is the measured output at the time sample  $n$  and  $y_n^{cal}$  is the estimated output at the time sample calculated from any of the given bases.

The result, in decibels, usually is a negative number. This number has to be as negative as possible, indicating that are little to no difference between the calculated and measured values. Comparing different NMSE will allow us to verify how different models performed.

Varying  $P_1$ ,  $P_2$ ,  $M$  and  $L$ , and calculating each of the combinations' NMSE for each model, will allow us to confirm if the model is in any way better than the current model.

### IV. RESULTS

By fixing  $P_1$  and  $M$ , which are variables that have the same behaviour in both models, it can be evaluated the performance of CADS in comparison to ADS by investigating how they perform for different values of  $P_2$  and  $L$ .

To ensure that our modeling was not biased or overfitted, the data samples are split into extraction and validation data, and the extraction data is used to calculate the coefficients  $H$  and the validation data is used to calculate values of NMSE for comparisons. To ensure our validation data was not being overfitted, NMSE of extraction and validation are

compared and if the  $|NMSE_{ext} - NMSE_{val}| > 2$  that model is discarded as overfitting. Also, both the direct and inverse (substituting the inputs for the outputs and calculating the inputs) modelings were done.

Input and output data was collected, with a vector signal analyzer from Rohde & Schwarz, from a GaN class AB PA, with a center frequency of 900 MHz and subjected to a WCDMA 3.84 MHz envelope. The sampling frequency is 30.72 MHz. The algorithms and calculations were implemented in Matlab R2018, in double floating point arithmetic.

Figure 1 and Figure 2 show the relationship between the estimated amplitude characteristic of every model, for the best found combinations of  $P_1$ ,  $P_2$ ,  $M$  and  $L$  in comparison with the measured data, for direct and inverse data, respectively, whereas Figure 3 and Figure 4 show the same comparisons but now for the phase characteristic.

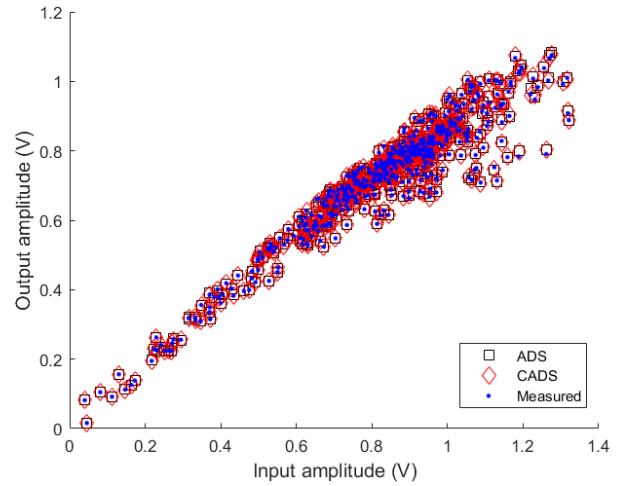


Fig. 1. AM-AM characteristics for direct modeling

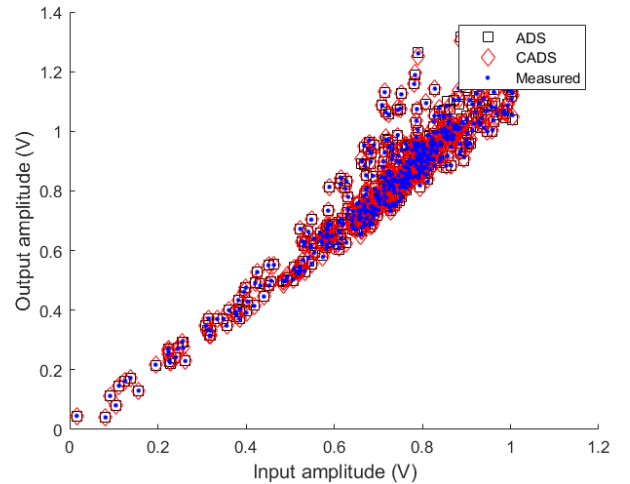


Fig. 2. AM-AM characteristics for inverse modeling

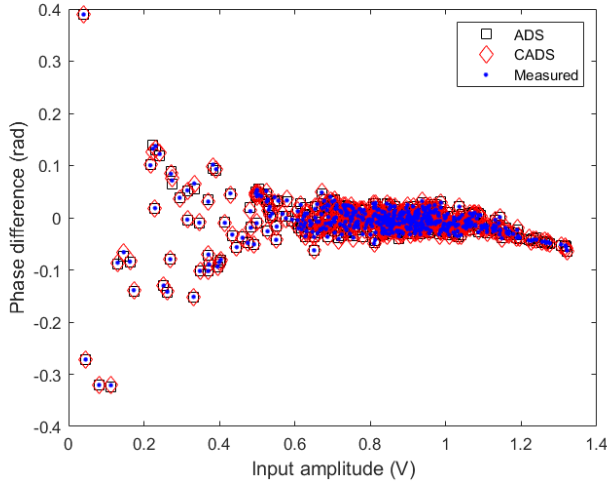


Fig. 3. AM-PM characteristics for direct modeling

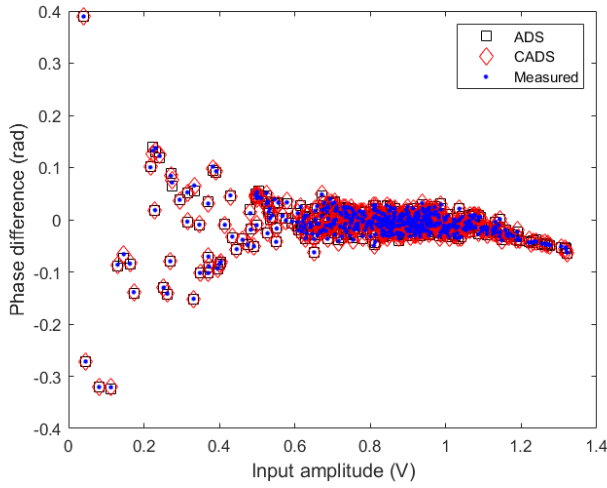


Fig. 4. AM-PM characteristics for inverse modeling

Tables I and II report the direct modeling results, whereas Tables III and IV contain the inverse modeling results.

TABLE I  
NMSE FOR  $P_1 = 3$ ,  $M = 3$  AND VARYING VALUES OF  $P_2$  AND  $L$  FOR DIRECT MODELING USING ADS (IN DECIBELS)

	$P_2=1$	$P_2=2$	$P_2=3$	$P_2=4$
$L=1$	-30.31	-30.66	-31.12	-31.56
$L=2$	-31.74	-32.50	-33.81	-35.23
$L=3$	-31.96	-32.99	-34.59	-36.37
$L=4$	-32.17	-33.48	-35.17	-37.38

It can be observed that, for same values of  $L$  and  $P_2$ , CADS achieves lower errors than ADS. As higher values of  $P_2$  and  $L$  are used, larger divergences between the two approaches are noticed. Specifically, the largest NMSE differences, equal to 4.38 dB and 3.98 dB for direct and inverse modeling, respectively, appears when  $L = 3$  and  $P_2 = 4$ . The increasing

TABLE II  
NMSE FOR  $P_1 = 3$ ,  $M = 3$  AND VARYING VALUES OF  $P_2$  AND  $L$  FOR DIRECT MODELING USING CADS (IN DECIBELS)

	$P_2=1$	$P_2=2$	$P_2=3$	$P_2=4$
$L=1$	-30.52	-30.89	-31.35	-31.74
$L=2$	-31.80	-33.11	-34.58	-36.68
$L=3$	-32.00	-33.88	-36.16	-40.75
$L=4$	-32.18	-34.89	-38.84	-

TABLE III  
NMSE FOR  $P_1 = 3$ ,  $M = 3$  AND VARYING VALUES OF  $P_2$  AND  $L$  FOR INVERSE MODELING USING ADS (IN DECIBELS)

	$P_2=1$	$P_2=2$	$P_2=3$	$P_2=4$
$L=1$	-37.72	-37.97	-38.08	-38.47
$L=2$	-38.05	-38.63	-39.02	-40.03
$L=3$	-38.17	-38.84	-39.44	-40.62
$L=4$	-38.31	-39.15	-39.95	-41.32

divergence arrives to the point where, even with a lower value of  $P_2$ , better NMSE results are observed with CADS in comparison with ADS having the same value for  $L$ . For instance, with  $L = 4$ , CADS with  $P_2 = 3$  improves the modeling accuracy of ADS with  $P_2 = 4$  by 1.46 dB and 2.19 dB in NMSE for direct and inverse modeling, respectively. It is also noted that for  $P_2$  and  $L$  equal 4, overfitting starts to occur in CADS. Table V reports the best NMSE achieved by ADS, CADS and PVS. Due to its much larger set of phase components, the modeling accuracy of PVS is severely compromised by overfitting, exhibiting NMSE values not better than -32 dB for direct modeling and -38 dB for inverse modeling.

TABLE IV  
NMSE FOR  $P_1 = 3$ ,  $M = 3$  AND VARYING VALUES OF  $P_2$  AND  $L$  FOR INVERSE MODELING USING CADS (IN DECIBELS)

	$P_2=1$	$P_2=2$	$P_2=3$	$P_2=4$
$L=1$	-37.83	-38.05	-38.13	38.53
$L=2$	-38.10	38.87	-39.76	-41.25
$L=3$	-38.20	39.39	-41.20	-44.60
$L=4$	-38.33	40.17	-43.51	-

TABLE V  
BEST NMSE VALUES FOR DIRECT AND INVERSE MODELINGS USING THE STUDIED SERIES ( IN DECIBELS)

	Direct	Inverse
PVS	-32	-38
ADS	-37.38	-41.32
CADS	-40.75	-44.60

## V. CONCLUSION

Digital predistortion is a powerful tool for the linearization of power amplifiers. Providing models of high fidelity and low complexity for the implementation of such techniques is essential.

In that regard, this work has managed to evaluate and compare simpler models (ADS and CADS) to the state of the art (PVS), and show that those models are viable options for implementation. As it can be seen from our measurements of NMSE, shown in Table V, both ADS and CADS have significantly less modeling error results, meaning that they can be utilized for DPD, same as PVS, with a less intensive computational load or with more margin for other kinds of error and noise. Given the results, it is demonstrable that it is feasible to adopt a CADS implementation to substitute PVS as a considerably simpler model.

It is good to remind the reader that in implementing those systems, one should be cautious to verify if the model is not being overfitted, and validating data before considering any system to have high fidelity.

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