Incrementally Reduced Angle Difference and Polar Volterra Series for Power Amplifier Modeling

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***Abstract*—Improving the trade-off between modeling accuracy and computational complexity is a subject of high interest in power amplifier (PA) behavioral modeling. In discrete-time feed-forward models that are linear in the parameters, the instantaneous output is projected into a basis composed of functions dependent on the present and past inputs. A non-structured pruning strategy called incremental approach was recently introduced to select a reduced set of input functions. The core objective of this work is to assess the performance of the incremental approach when applied to three distinct input bases. One input basis, called polar Volterra series (PVS), adopts a number of positive input angles that is always equal to the number of negative input angles plus one. The other two input bases handle differences between two input angles: the angle difference series (ADS) retains only the differences between two consecutive samples and the modified angle difference series (MADS) takes into account all differences. The incremental approach is applied to three input bases for the direct and inverse modeling of a GaN class AB PA. Matlab simulation results indicate that the choice of input basis has a significant impact on the accuracy of the incremental approach and that the PVS has a superior performance than ADS and MADS.**

Keywords*—Modeling, Power amplifier, Radiofrequency, Volterra series.

I. INTRODUCTION

The power amplifier (PA) is an essential piece of modern wireless data transmission structures [1]. However, strict regulation on how linear its output has to be limits utilization in the energy-efficient region [2].

Using digital treatment to linearize PA signals in its most energy-efficient operation state has been common practice in modern radiofrequency (RF) systems [3]. Polar Volterra series (PVS) [4] and neural networks (NNs) [5] are very popular and reliable models for such digital treatment. In a PVS, each contribution uses the product of several input amplitude components together with a number of input angles components. No constraint is imposed on the amplitudes, but in a PVS the number of negative angles must be set equal to the number of positive angles minus one. Conversely, NNs can only manipulate angle differences and amplitudes. The PVS is linear in its parameters, whereas the NN is nonlinear in its parameters. The property of being linear in the parameters is the major advantage of the PVS in comparison with the NN. However, the number of parameters in a PVS increases very rapidly with the polynomial order and memory length. Hence, a topic of high interest in literature is the search for complexity-reduced models that can provide models with high fidelity. An approach called incremental algorithm is introduced in [6] to select the model contributions that are most effective in reducing the modeling error. Given an arbitrary basis composed of input contributions, the incremental approach individually projects the output into each function present on the basis and those contributions which provide the best projections are chosen as the most effective ones. The incremental approach can only be applied to models that are linear in the parameters, like the PVS. Motivated by the way angle components are manipulated by NNs, this work introduces two novel series that are linear in the parameters, called angle difference series (ADS) and modified angle difference series (MADS). The study hereafter reported compares the modeling accuracy of incrementally reduced PVS, ADS and MADS.

This paper is divided as follows: Section II addresses the models utilized in the study. Section III reviews the algorithm for complexity reduction. Section IV shows and discusses the obtained results. Section V presents a discussion on the achieved results.

II. DIRECT AND INVERSE PA BEHAVIORAL MODELING

From the perspective of a linearization designer, the PA can be seen as a single-input-single-output nonlinear dynamic system. The dynamic behaviors, or memory effects, are attributed to the non-flat frequency response of the passive frequency-selective circuitries inside the PA, especially the biasing and matching networks. The nonlinearities are seen as power gain compression and saturation of the active device due to silicon limitations. An accurate discrete-time feed-forward behavioral model for the PA direct or inverse transfer characteristics must formulate the instantaneous complex-valued output envelope as a nonlinear function of the present and past samples of the complex-valued input envelope. The following three distinct approaches are addressed in this work: the polar Volterra series (PVS) introduced in [4] is revisited in Subsection II.A and the two series proposed in this work, namely the ADS and MADS, are presented in Subsections II.B and II.C, respectively.

A. Polar Volterra series

The PVS is a mathematical model with memory, nonlinear but with linear coefficients [4]. This particular model is very efficient to approximate PA behavior because of its nature to correctly model causal, stable and of fading memory systems. Its constitutive equation is given by:

$$\tilde{y}(n)=\sum_{p_1=1}^{P_1}\sum_{p_2=1}^{P_2}\sum_{m_1=0}^{M}\cdots\sum_{m_{p_1}=m_{p_1-1}}^{M}\sum_{l_1=0}^{L}\cdots\sum_{l_{p_2}=l_{p_2-1}}^{L}\sum_{l_{p_2+1}=0}^{L}\cdots\sum_{l_{2p_2-1}=l_{2p_2-2}}^{L}$$
$$[h_{p_2,2p_2-1}\left(m_1\ldots,m_{p_1},l_1,\ldots,l_{p_2-1}\right)\times\ldots a\left(n-m_1\right)\times \quad , \quad (1)$$
$$a\left(n-m_{p_1}\right)e^{j\varphi(n-l_1)}\ldots e^{j\varphi\left(n-l_{p_2}\right)}\times e^{-j\varphi\left(n+l_{p_2+1}\right)}\ldots e^{-j\varphi\left(n-l_{2p_2-1}\right)}]$$

where h is the PVS complex-valued parameter, $\tilde{y}(n)$ is the complex-valued output envelope at instantaneous time sample n, $a(n\text{-}m)$ is the amplitude component of the complex-valued input envelope at time sample $n\text{-}m$ (with m ranging from 0 to the amplitude memory length M) and $\varphi(n\text{-}l)$ is the phase component of the complex-valued input envelope at time sample $n\text{-}l$ (with l ranging from 0 to the phase memory length L). The output is nonlinearly dependent on the input by means of two polynomial approximations. The output is a power series function of the input amplitude truncated to the polynomial order P_1. The output is also dependent on the product of up to ($2P_2$ -1) input phase contributions that must comply with a constraint. Specifically, any input phase information must be included as the imaginary part of an exponential operator and, moreover, exactly half minus one input phase contributions must be accompanied by a negative sign.

B. Angle Difference Series (ADS)

Motivated by the way NN-based PA behavioral models handle the input phase information [5], this subsection introduces a novel nonlinear series with memory, but linear in the parameters. The proposed ADS utilizes the same input amplitude dependency as the PVS model but changes the manner in which the input phase affects the output. Each input phase information still needs to be included as the positive or negative imaginary part of an exponential operator. But now, the power series expansion is imposed on the difference between two consecutive input phases (or angles), namely $\varphi\,(n\text{-}l)$ - $\varphi\,(n\text{-}l\text{-}1)$ with l ranging from 0 to L-1. Besides, such angle difference is truncated to the polynomial order P_2 and there is also mandatory to multiply every contribution to the exponential of the imaginary unit (j) times the instantaneous input phase. By doing that, the ADS model is represented by:

$$\tilde{y}(n) = \sum_{p_1=1}^{P_1}\sum_{p_2=0}^{P_2}\sum_{m_1=0}^{M}\cdots\sum_{m_{p_1}=m_{p_1-1}}^{M}\sum_{l=0}^{L-1}$$
$$[\tilde{h}_{p_1,2p_2+1}\left(m_1,\ldots,m_1,l_1,\ldots,l_{p_2-1}\right)a\left(n-m_1\right)\cdot \ldots a\left(n-m_{p_1}\right)\left(e^{j\varphi(n-l)}e^{-j\varphi(n-l-1)}\right)^{p_2}e^{j\varphi(n)}] \tag{2}$$

C. Modified Angle Difference Series (MADS)

A closer look into the ADS model indicates that some phase combinations from (1) cannot be accomplished by (2). For example, assuming P_2 = 1 and L =1 in (1), among the resulting phase contributions is $\varphi\,(n\text{-}1)$. However, the same $\varphi\,(n\text{-}1)$ phase contribution cannot be generated by (2), no matter the particular choices for P_2 and L. Such observation inspires the introduction of the MADS that arranges the phase differences in a broader way by allowing access to each possible angle difference and, more important, can yield every contribution of (1). The MADS constitutive equation is given by:

$$\tilde{y}(n) = \sum_{p_1=1}^{P_1}\sum_{p_2=0}^{P_2}\sum_{m_1=0}^{M}\cdots\sum_{m_{p_1}=m_{p_1-1}}^{M}\sum_{l_1=0}^{L}\sum_{l_2=0}^{L}$$
$$[\tilde{h}_{p_1,2p_2+1}\left(m_1,\ldots,m_1,l_1,\ldots,l_{p_2-1}\right)a\left(n-m_1\right)\cdot \ldots a\left(n-m_{p_1}\right)\left(e^{j\varphi(n-l_1)}e^{-j\varphi(n-l_2)}\right)^{p_2}e^{j\varphi(n)}] \tag{3}$$

For instance, if in (3) the truncation factors are set to $P_2 \geq 2$ and $L \geq 1$, then the $\varphi\,(n\text{-}1)$ phase contribution is obtained when the summation lower indexes assume the values $p_2 = 1$, $l_1 = 1$ and $l_2 = 0$.

III. Incremental Approach for Complexity Reduction

The PVS, ADS and MADS are linear in their parameters. The definition of such models can be expanded to a version where it encapsulates many elements in time, using matrix notation:

$$Y = XH\,, \tag{4}$$

where Y is a N x 1 vector of estimated outputs, X is a N x Q matrix whose elements are obtained from the applied inputs, H is a Q x 1 vector containing the coefficients, N is the number of collected time samples and Q is the number of coefficients.

The number of coefficients of PVS, ADS and MADS becomes extremely high for larger values of P_1, P_2, M, and L, making necessary the reduction of such amount. Using the coefficient incrementor method developed in [6] the coefficients that contribute the most can be selected and used to build a reduced model.

In this method, single coefficients were selected and the normalized mean square error (NMSE) contribution that single coefficients awarded was measured. Then, the desired number of coefficients were selected as ascending contribution order, and the NMSE for the increasing number of coefficients was calculated [7]:

$$NMSE = 10\log_{10}\left\{\frac{\sum_{n=1}^{N}\left|y_n^{des}-y_n^{cal}\right|^2}{\sum_{n=1}^{N}\left|y_n^{des}\right|^2}\right\}\,, \tag{5}$$

where y_n^{des} indicates the desired (measured) output at time sample n and y_n^{cal} indicates the estimated output at time sample n calculated from PVS, ADS or MADS models.

IV. Results

In this section, the incremental approach is applied to the PVS, ADS and MADS for the modeling of the direct and inverse transfer characteristics from a PA under study. The inverse modeling has application in predistortion schemes. The input and output data were measured by a Rohde & Schwarz FSQ vector signal analyzer with the sampling frequency set to 30.72 MHz. The measured PA was a GaN class AB, stimulated by a carrier at 900 MHz, modulated by a 3.84 MHz WCDMA envelope.

In the PVS, ADS and MADS models, the truncation factors are set to M = 2, L = 1, P_1 = 4 and P_2 = 2. The incremental approach is executed in Matlab software through least squares algorithms using floating-point double-precision arithmetic. Two independent sets of input-output data are used: one for parameter identification and one for model validation. Figures 1 and 2 report the NMSE values as a function of the number of parameters for the modeling of direct and inverse transfer characteristics, respectively. To evaluate the trade-off between modeling accuracy, measured by the NMSE results, and computational complexity, related to the number of coefficients, two scenarios are possible. In the first scenario, the NMSE results from models with the same number of coefficients are compared and the model with the best performance is the one with the lowest NMSE value. In the second scenario, the number of coefficients from models providing the same NMSE results is compared and the model with the best performance is the one with the lowest number of coefficients. Figures 1 and 2 show the clear difference in accuracy as a function of the number of coefficients for the PVS in comparison to the other selected models. It can be clearly evaluated that the PVS model has way superior results in comparison with both ADS and MADS models. For direct modeling with 85 coefficients, the PVS model provides an NMSE of -34 dB, while the MADS model obtained an NMSE result of -27 dB and the ADS model obtained an NMSE value of -18 dB. For inverse modeling with 85 coefficients, the PVS model provides an NMSE of -27 dB, while the MADS model obtained an NMSE result of -24 dB and the ADS model obtained an NMSE value of -16 dB. Decreasing values of NMSE are observed for increasing amounts of coefficients in each of the models, for direct and inverse data entries, up to a point in which further increasing the number of parameters does not decrease the NMSE values. The NMSE in the ADS stabilizes with a lower amount of coefficients (around 30) in comparison with the PVS, which requires about 70 coefficients to stabilize. However, such faster convergence does not imply in a superior performance of the ADS, because in practice a single model realization is chosen based on the compromise between NMSE and number of coefficients associated to it. Moreover, it is not possible to define a target NMSE that will be acceptable for any PA modeling. In fact, the obtained NMSE values would be significantly dispersed if the same model with the same number of parameters is employed to model different PA circuitries or even to model the same PA circuitry but operating at distinct compression levels. It is expected that any improvement in the modeling accuracy will be somehow translated into a superior performance in any practical application of the developed model, for instance, to improve the linearization capability in predistortion applications.

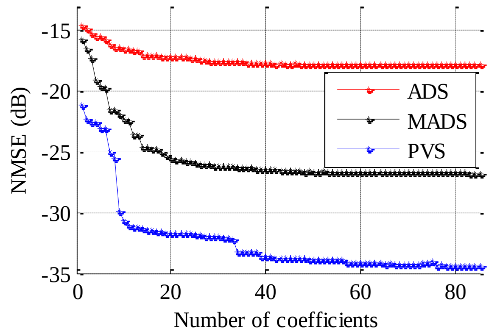

Fig. 1. NMSE *versus* the number of coefficients for direct modeling.

Figures 3 and 4 show the relationship between the estimated phase characteristic of every model with 100 coefficients in comparison with the measured data, whereas Figs. 5 and 6 show the same comparison but now for the amplitude characteristic. PVS and MADS models show high similitude with measured data and hence high capability to model the nonlinearities and the dispersions caused by fading memory. Such characteristic is very important for the intended purpose of utilization of the models. However, a closer match to the measured data is provided by the PVS, especially in the phase characteristic. Besides, it can be clearly visualized that the ADS model is more dispersed than the other two models. The more spread out distribution of points seen in the ADS explains why its results were not as satisfying as the PVS and MADS.

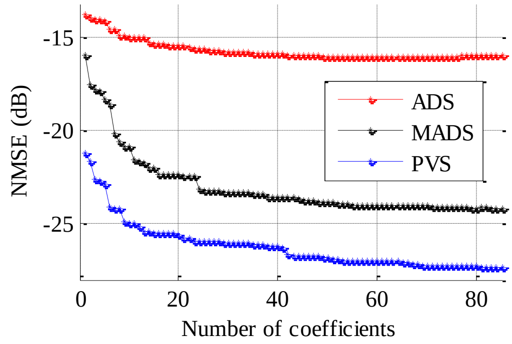

Fig. 2. NMSE *versus* the number of coefficients for inverse modeling.

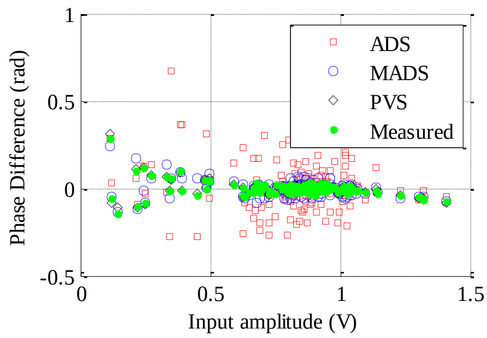

Fig. 3. AM-PM characteristics for direct modeling with 100 coefficients.

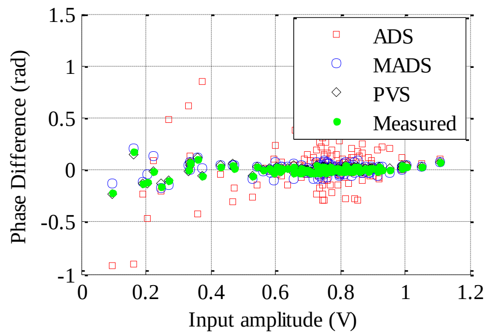

Fig. 4. AM-PM characteristics for inverse modeling with 100 coefficients.

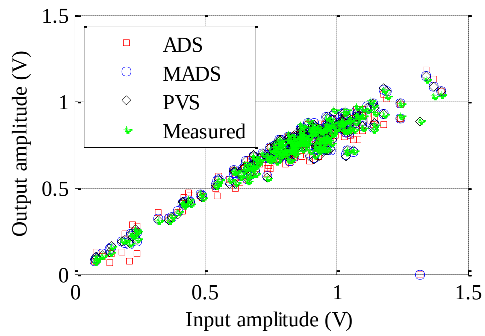

Fig. 5. AM-AM characteristics for direct modeling with 100 coefficients.

Further inspection, as seen in Figs. 7 and 8, shows that error amplitude average values, as well as error amplitude peak values, are clearly higher in the ADS model. This occurs due to the way the ADS model handles phase information. In comparison with the other two models, the ADS model has access to a narrower array of options.

Furthermore, lower errors are obtained for the PVS than the MADS, which is in agreement with the NMSE results shown in Figs. 1 and 2.

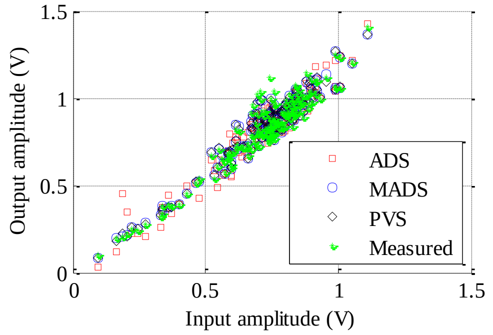

Fig. 6. AM-AM characteristics for inverse modeling with 100 coefficients.

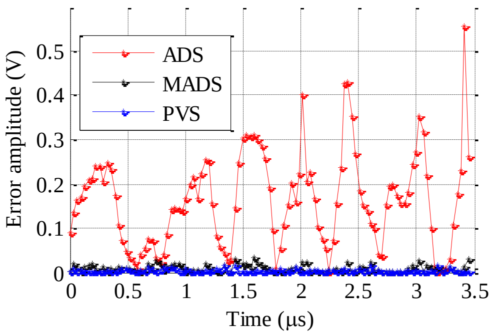

Fig. 7. Error amplitude *versus* time for direct modeling.

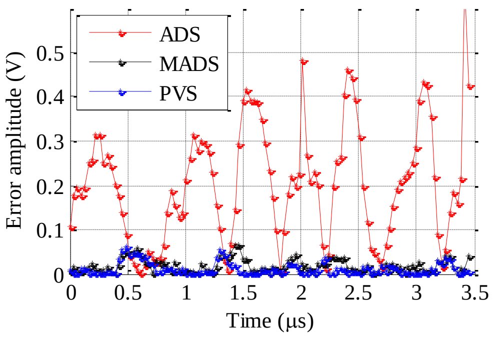

Fig. 8. Error amplitude *versus* time for inverse modeling.

V. Discussion

When modeling both direct and indirect PA transfer characteristics, the complex-valued envelope signals must comply with a fundamental constraint related to the phase components: the exponential operator must manipulate positive and negative angle components, in a way that the amount of positive angles must exceed the amount of negative angles by exactly one. Such fundamental constrained related to the passband behavior of PAs can easily be fulfilled in Volterra-based models because in such models the user has complete control of how the model manipulates the input envelope. On the other hand, NNs are treated as completed black-box tools in which the user has little or no control on the mechanisms by which the network processes the input signals. For instance, NNs take into account nonlinearities by employing sigmoid functions that can be expanded into a Taylor series exhibiting non-null contributions for every odd power of the complex-valued signals applied at their inputs. As an example, if the PVS angle components are applied as inputs of an NN-based model, the presence of the third-order contribution inside the network will provide an output contribution in which the exponential operator receives as argument the sum of three positive angle components. Hence, modifications on the signals applied as NN inputs are compulsory to comply with the bandpass constrained. Since Volterra and NN-based models deal differently with the phase components of the PA model, this work aims to introduce a comparative analysis as fair as possible among the distinct ways to handle the angle components. A similar comparison in the NN framework would not be fair because without performing a phase difference, the NN would provide a significant amount of useless contributions that do not comply with the bandpass constrained. Indeed, in the polynomial-based comparative analysis reported in this work, all models do comply with the bandpass constrained and therefore do provide only useful contributions. However, the introduced ADS and MADS polynomial-based series inspired by the NN approach, unfortunately, lose in performance compared to the traditional PVS. The better performance of the PVS can be justified by its simplicity because it handles the bandpass constrained in exactly the same manner such requirement is stated. In other words, the ADS and MADS, due to their inspiration on NN models, apply non-obvious and more complicated approaches based on phase differences to comply with the bandpass constrained. By the authors' point of view, the major contribution of the study introduced in here is to evidence that there is still available margin for improving the performance of NN-based models, because this work illustrates that the available models complying with the bandpass constrained in NN-based approaches, namely ADS and MADS, have modeling accuracies far away from the one provided by the PVS, which by its turn cannot be directly applied to NN-based models since it does not comply with the bandpass constrained.

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