Presentation

Application

`OBJECTIVE`: Forecasting the levels of air pollution for the TableView station in Cape Town.
`RESPONSE`: Nitrogen dioxide
`COVARIATES`: Particulate matter, Sulfur dioxide, and Wind speed

Time series plot

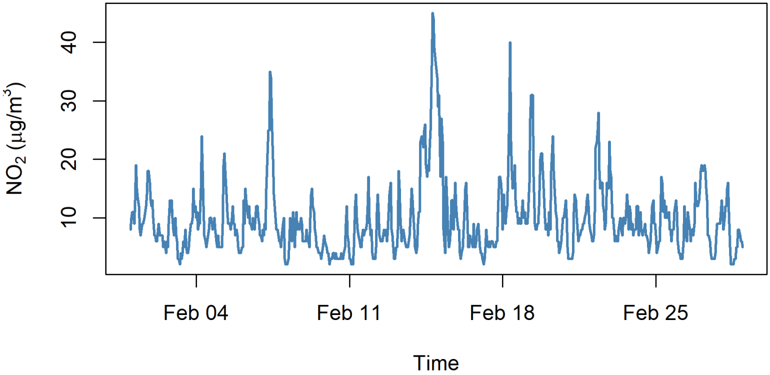

Figure 1: Time series of X_t

Components of the time series

`TREND COMPONENT`: Using the `ndiffs()` function in the `forecast` package, we obtain 1, suggesting that there is a linear trend in the time series.
`SEASONAL COMPONENT`: Using the `nsdiffs()` function in the `forecast` package, we obtain 0, suggesting that no seasonality is present in the time series.
`CYCLICAL COMPONENT`: There is no clear indication of a cyclical component in the time series.
`RANDOM COMPONENT`: Random variation is present in the time series.

Heteroscedasticity

$$X_t \not\sim \mathcal{N}(\mu,\, \sigma^2).$$

Using the `BoxCox.lambda()` function in the `forecast` package, we get that $\lambda \approx 0$, suggesting a natural log transformation.

$$\text{Let } Y_t = \ln(X_t),$$

$$Y_t \overset{approx}{\sim} \mathcal{N}(\mu,\, \sigma^2).$$

Fitted models

`AVERAGE`: The prediction is the average value.
`NAIVE`: The prediction is the last observed value.
`DRIFT`: The prediction is the last observed value adjusted for the average trend.
`AR(1)`: The prediction is based on a constant, plus a fraction of the previous value.

Gaussian process models

$$f_1(x) \sim \mathsf{GP}(0,\ \sigma^2\delta_{ij}),\ \delta_{ij} = 1 \text{ for } i = j, \text{ and } \delta_{ij} = 0 \text{ for } i \neq j.$$

$$f_2(x) \sim \mathsf{GP}(\mathbf{X}\beta,\ \sigma^2\delta_{ij}),\ \delta_{ij} = 1 \text{ for } i = j, \text{ and } \delta_{ij} = 0 \text{ for } i \neq j.$$

$$f_3(x) \sim \mathsf{GP}(0,\ \alpha^2 \mathsf{exp}[(\frac{x_i - x_j}{\rho})^2]).$$

$$f_4(x) \sim \mathsf{GP}(\mathbf{X}\beta,\ \alpha^2 \mathsf{exp}[(\frac{x_i - x_j}{\rho})^2]).$$

Results

Model	RMSE			MAE		
	Forecasts (h day time horizon)			Forecasts (h day time horizon)		
	24	168	744	24	168	744
Average	7.731	**5.397**	7.439	**6.042**	**4.254**	5.385
Naive	10.553	7.864	10.040	7.708	6.071	7.308
Drift	10.606	8.128	11.358	7.764	6.391	8.809
AR(1)	8.662	5.912	8.044	6.029	4.422	5.616
GP-0-WN	13.756	11.141	13.169	11.719	9.786	11.081
GP-0-SE						
GP-MLR-WN	**7.692**	6.134	**7.225**	6.119	4.490	**5.136**
GP-MLR-SE						

Figure 2: OOS performance of the fitted models.