Gaussian Processes for Time Series Modelling

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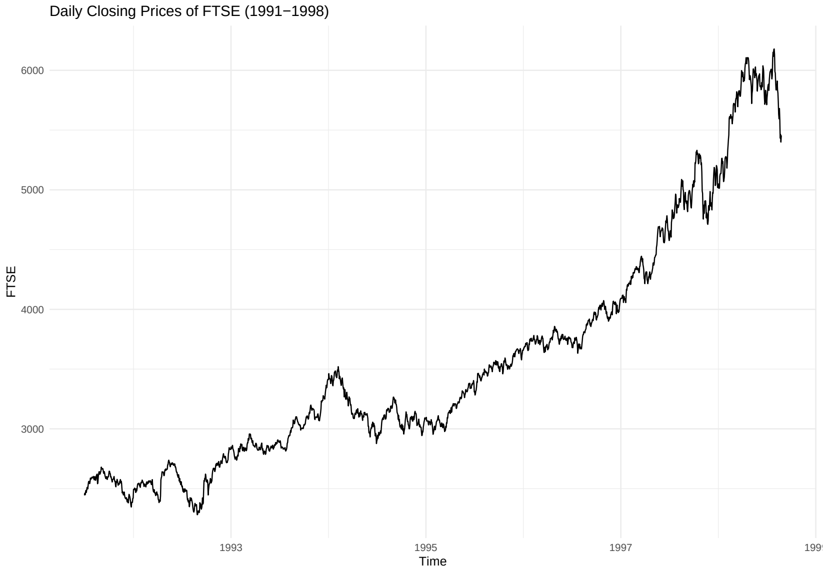
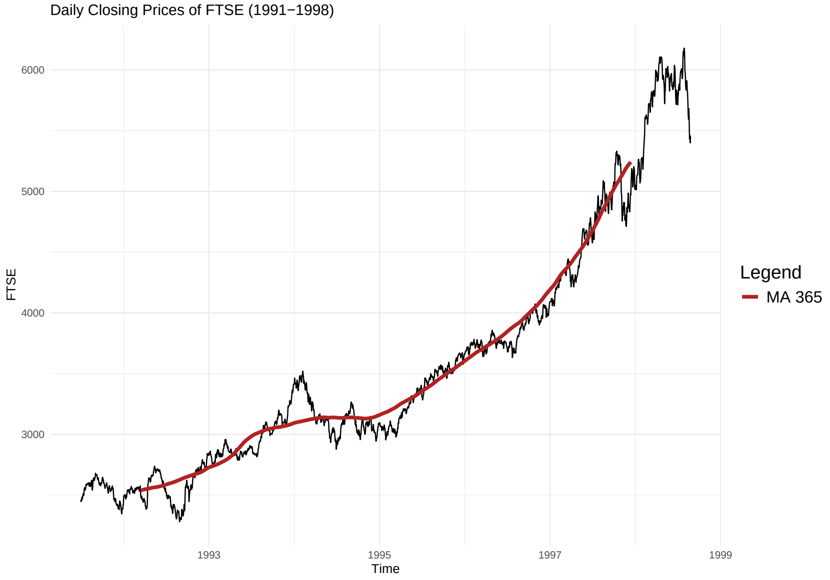
2025-04-28

Plot of the dataset



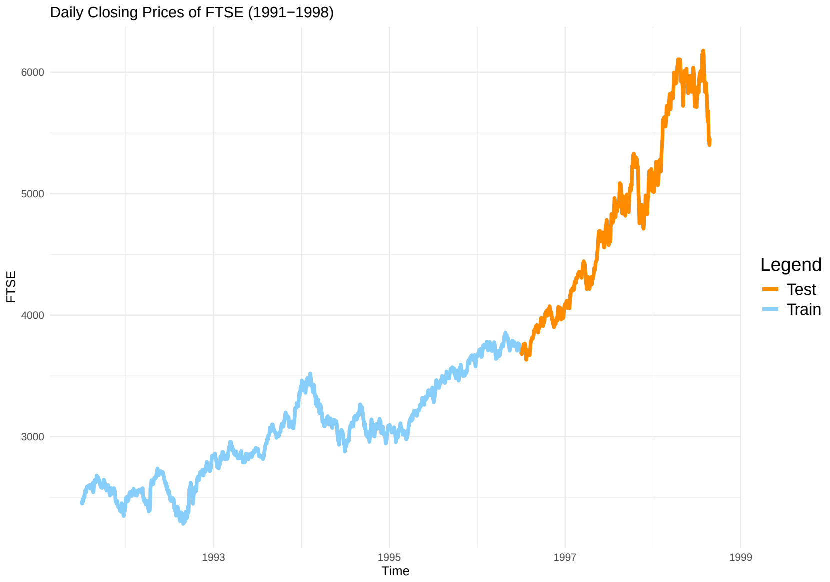
Moving average smooting



Discussion

There are infinite models that we could use. However, since we picked up the trend using CMA(k=365) we can make an inspired guess. A second degree polynomial spline seems to be appropriate.

Partition



Daily Closing Prices of FTSE (1991−1998)
6000
5000
4000
3000
FTSE
1993
1995
1997
1999
Time
Legend
Test
Train

B-splines

Base case

$$B_{i,0}(t) := \begin{cases} 1, & \text{if } t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

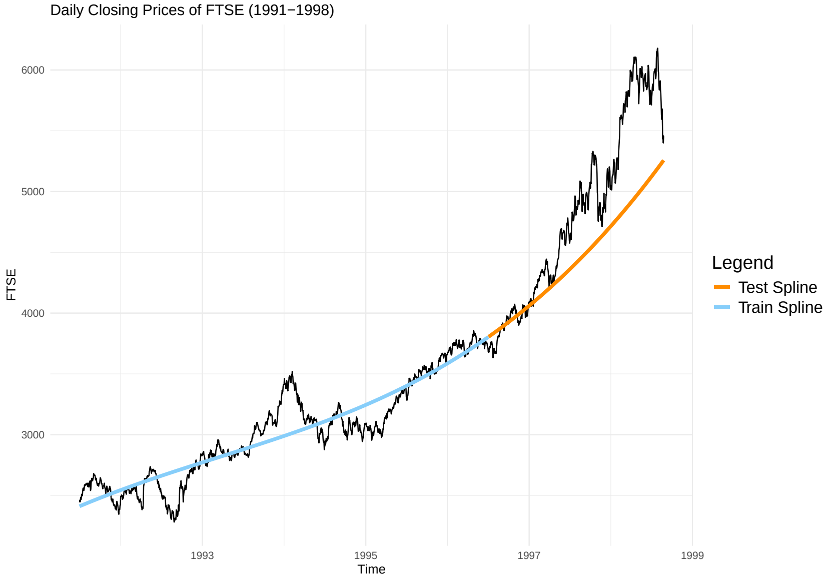
Recursive step

$$B_{i,p}(t) := \frac{t - t_i}{t_{i+p} - t_i} B_{i,p-1}(t) + \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} B_{i+1,p-1}(t)$$

Where

t is the covariate and p is the degree of the polynomial.

B-spline of order 2 fit



Discussion

Problem

Robust use of the polynomial model requires knowledge of how the coefficients interact to control functional behaviour, which becomes unmanageable as the order of the polynomial grows.

Solution

A Gaussian Process defines a probability distribution over functions; in other words, it is an entire function from the covariate space to the real-valued output space.

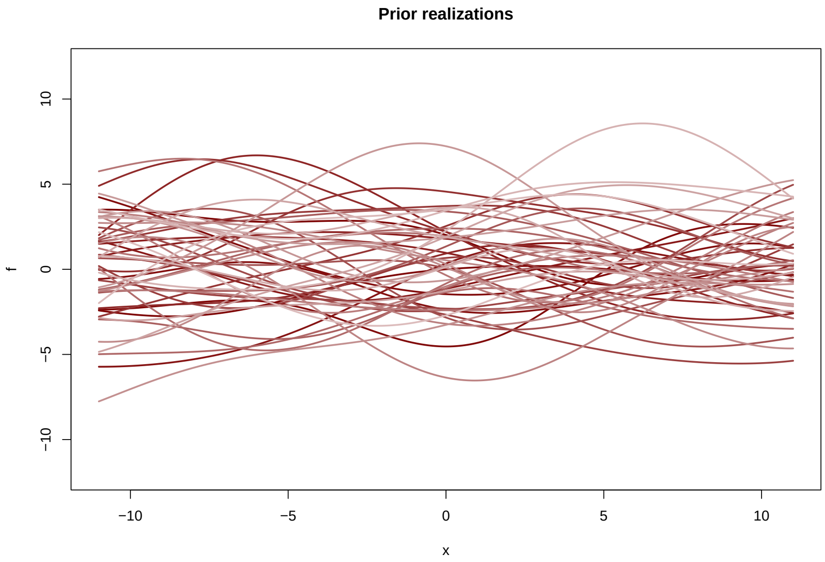
Gaussian process

A time continuous stochastic process $\{X_t;\ t \in T\}$ is Gaussian if and only if for every finite set of indices $t_1, ..., t_k$ in the index set T

$$\mathbf{X}_{t_1,...,t_k} = (X_{t_1}, ..., X_{t_k})$$

is a multivariate Gaussian random variable.

Prior realizations



Prior quantiles