Gaussian Processes for Time Series Modelling

Azar Raphaela Gumede Sbonelo

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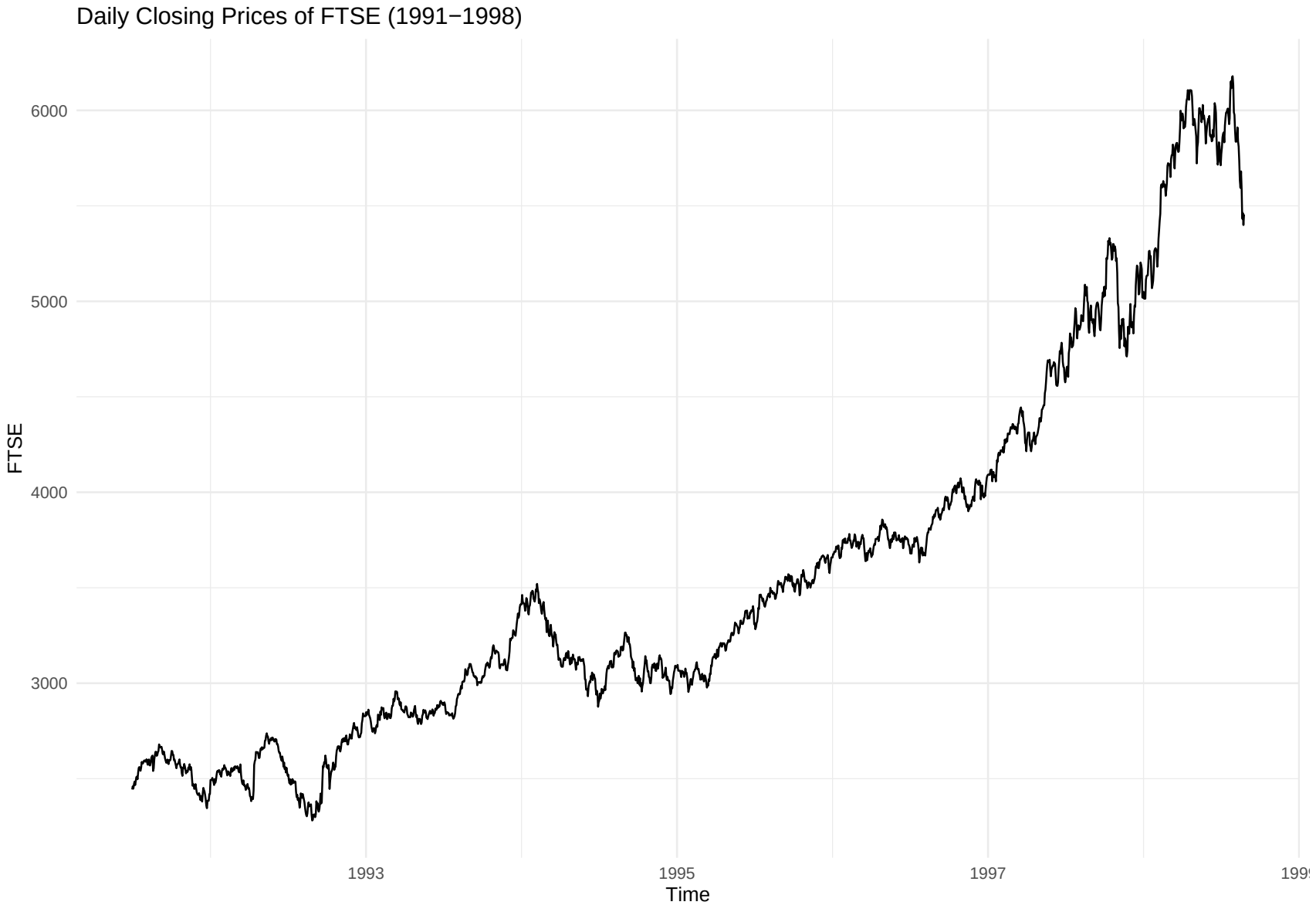
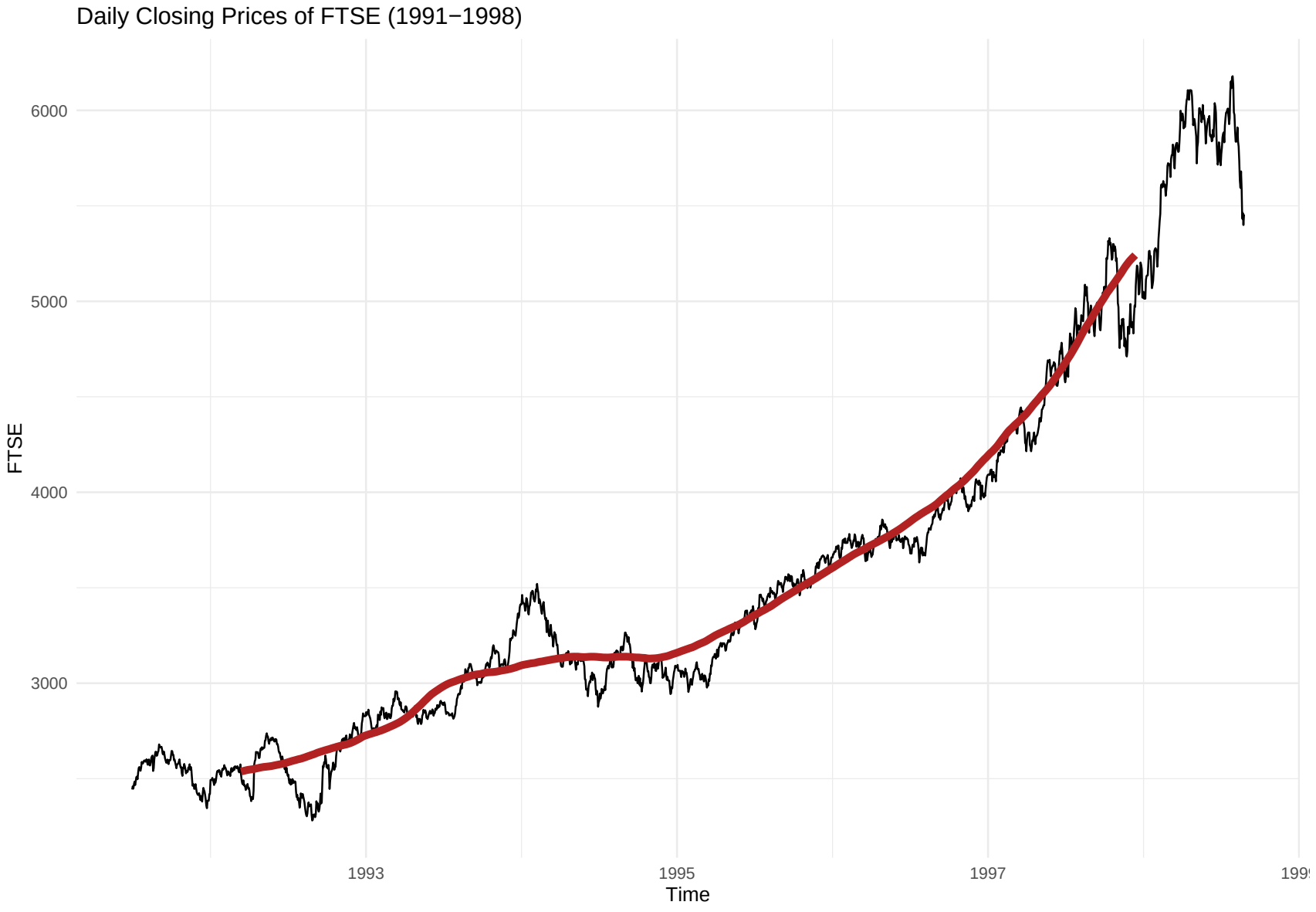
Dataset


Daily Closing Prices of FTSE (1991−1998)
FTSE
6000
5000
4000
3000
1993
1995
1997
Time

CMA(k = 365)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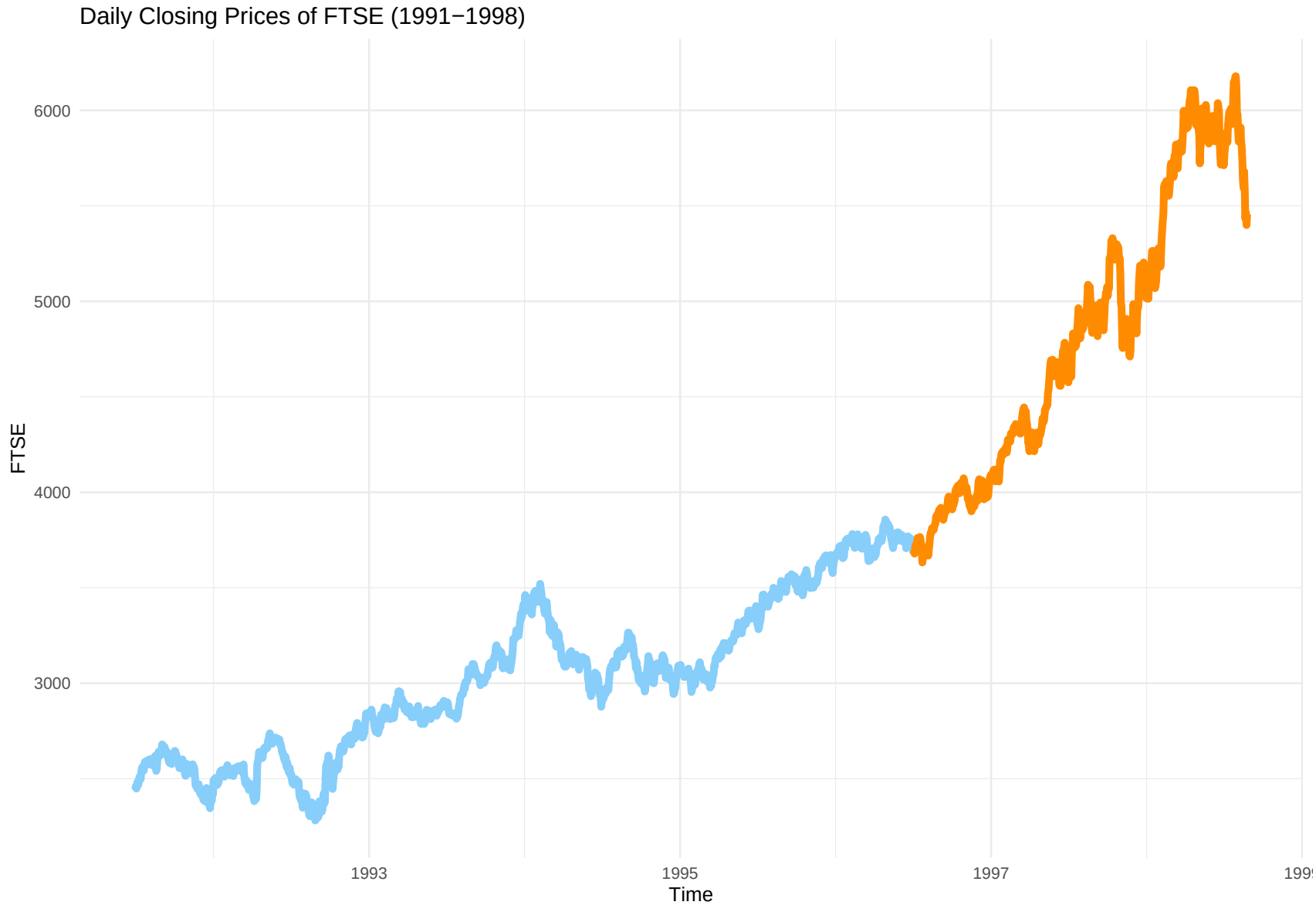
Discussion

There are infinite models that we could use. However, since we picked up the trend using CMA(k=365) we can make an inspired guess. A second degree polynomial spline seems to be appropriate.

Let us divide the data into a train and test set in order to compare model performance.

Partition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B-Spline

Base Case

$$B_{i,0}(t) := \begin{cases} 1, & \text{if } t_i \leq t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

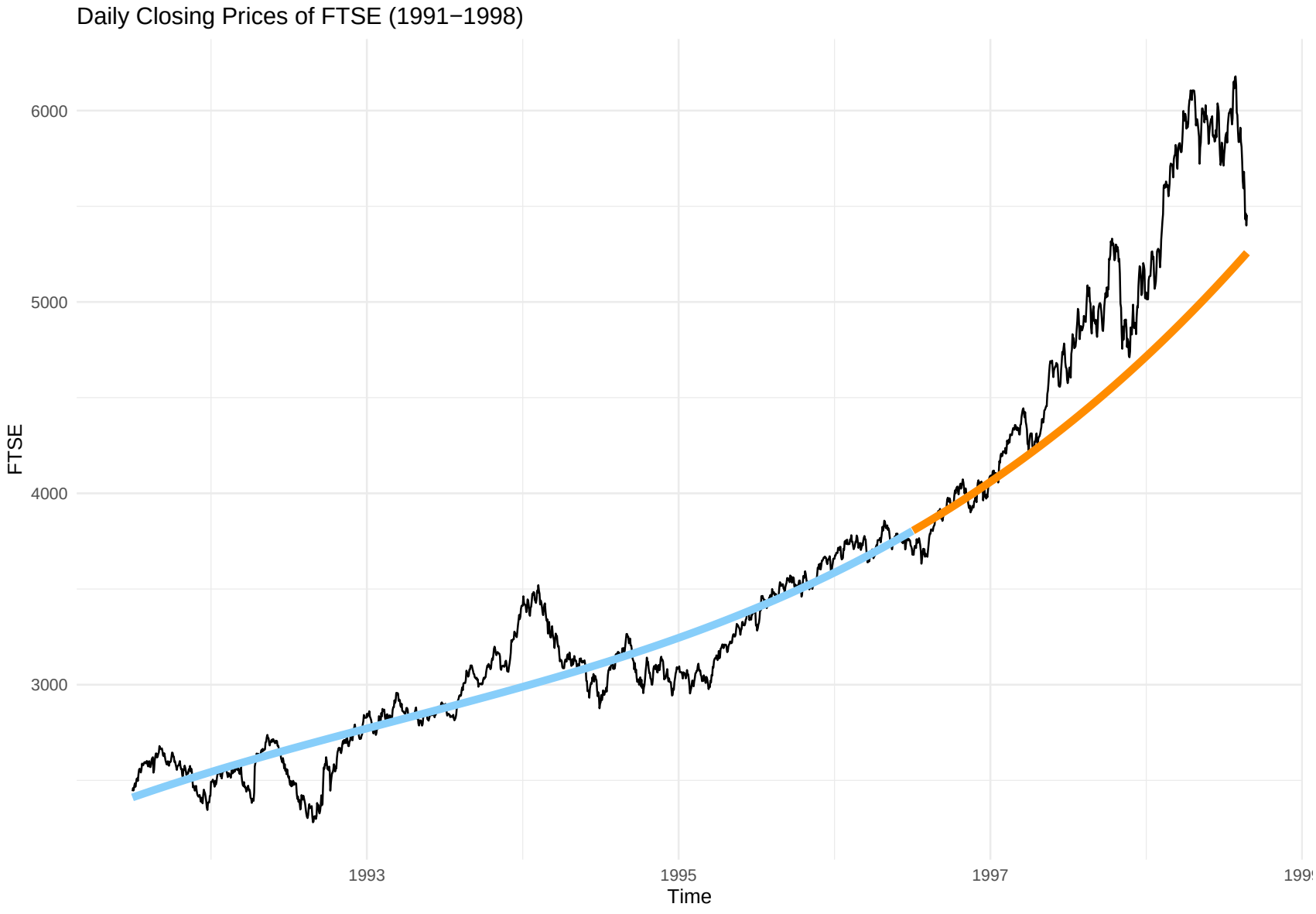
Recursive Step

$$B_{i,p}(t) := \frac{t - t_i}{t_{i+p} - t_i} B_{i,p-1}(t) + \frac{t_{i+p+1} - t}{t_{i+p+1} - t_{i+1}} B_{i+1,p-1}(t)$$

Where

t is the covariate and p is the degree of the polynomial.

Spline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Problem

Robust use of the polynomial model requires knowledge of how the coefficients interact to control functional behaviour, which becomes unmanageable as the order of the polynomial grows.

Solution

A Gaussian Process defines a probability distribution over functions; in other words, it is an entire function from the covariate space to the real-valued output space.

Gaussian Process