Determinant functions

Sergi Boquera

November 2024

1 Determinant funcitions

In this section, we will dive deep into the concept of determinant functions and its applications. First, we will start with some simple definitions.

1.1 The concept of determinant function

We begin this section introducing the concept of determinant function and the basic related definitions.

Definition 1 *Let F be a field, n a positive integer and D, a function that assigns to each nxn matrix A over F a scalar D(A) over F. We say that D is n-linear if for each $1 \leq i \leq n$, D is linear function of the i-th row when the other (n-1) rows are held fix.*

Definition 2 *Let D be a n-linear function. We say that D is alternating if the following two conditions are satisfied:*

- $D(A) = 0$*, whenever two rows of A are equal.*
- *if A' is a matrix obtained from A by interchanging two rows of A, then* $D(A') = -D(A)$.

Definition 3 *Let F be a field and let n be a positive integer. Suppose D is a function from nxn matrices over F into F. We say that D is a determinant function if D is n-linear, alternating and D(I) = 1.*

1.2 Facts about the determinant function

We are now on position to prove the basic properties of determinant functions.

Lemma 1 *A linear combination of n-linear functions is n-linear.*

proof. It suffices to show that the linear combination of two n-linear functions is n-linear. Let D and E be n-linar functions, $a, b \in F$ and G = $(aD+bE)(A)$. $Noticethat$G = aD(A) + bE(A) hence if we held fix all rows of A except α_i, $(aD + bE)(\alpha_i + \alpha) = aD(\alpha_i + \alpha) + bE(\alpha_i + \alpha) = aD(\alpha_i) + aD(\alpha) + bE(\alpha_i) + bE(\alpha) =$ which proves that G is n-linear.