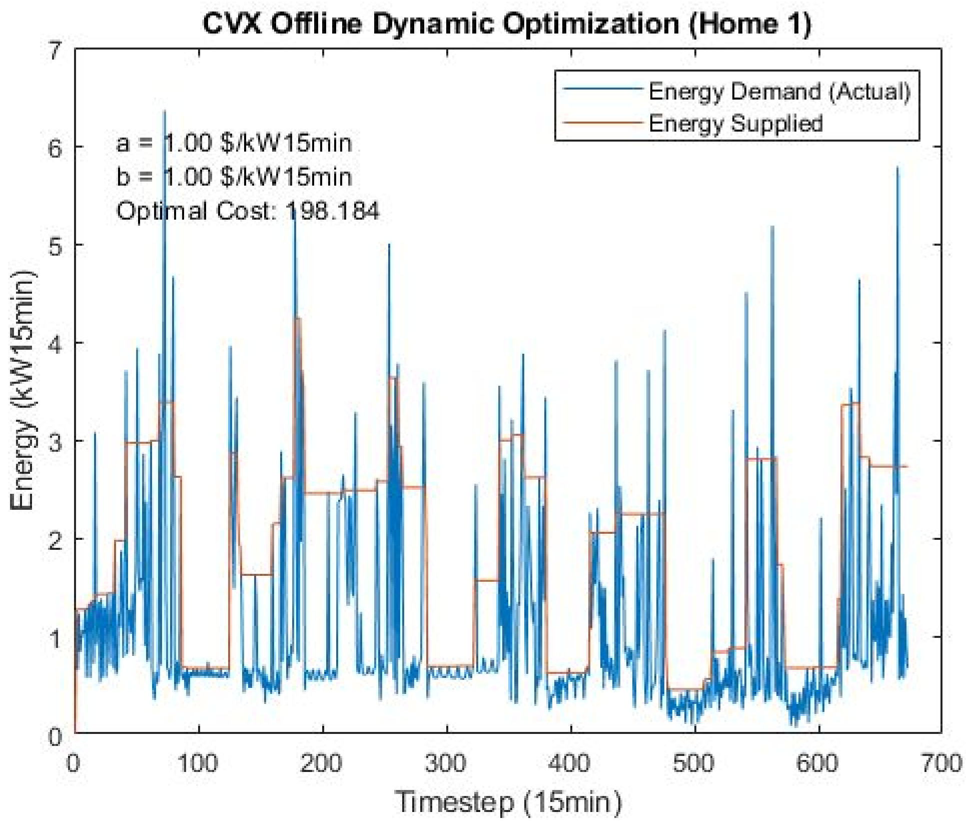
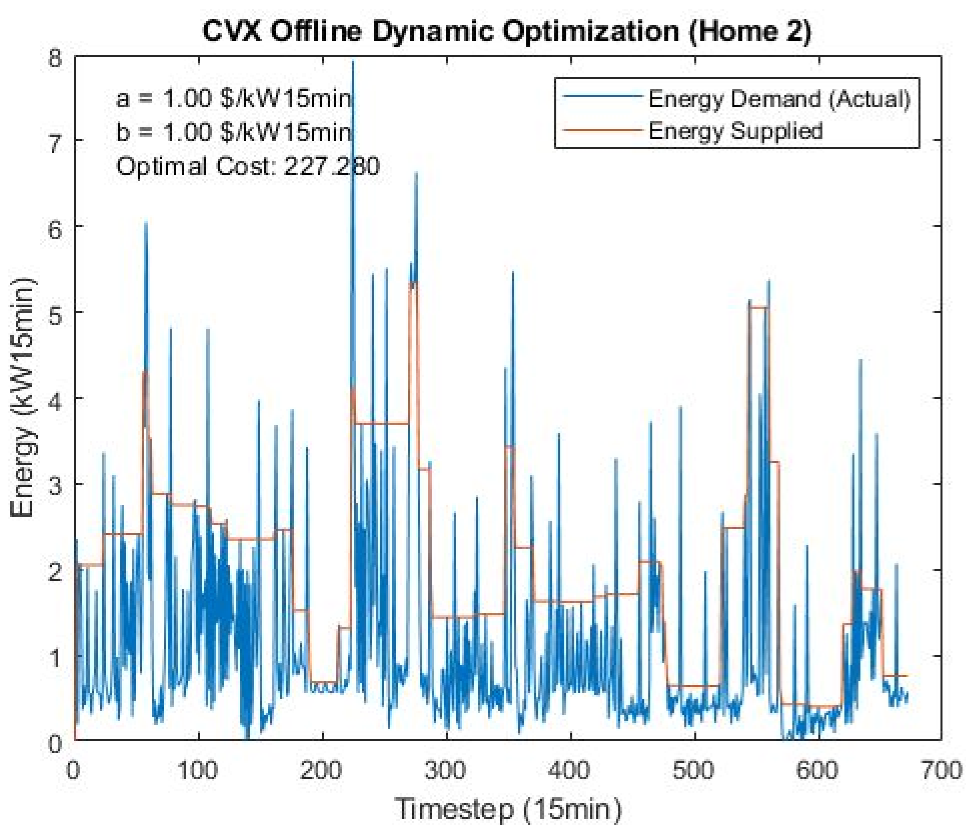
Group 2 (John Buckheit, Nikhil Siddhartha, Shivasagar Boraiah)
AMS 559
Homework 2
Due: December 3rd, 2018

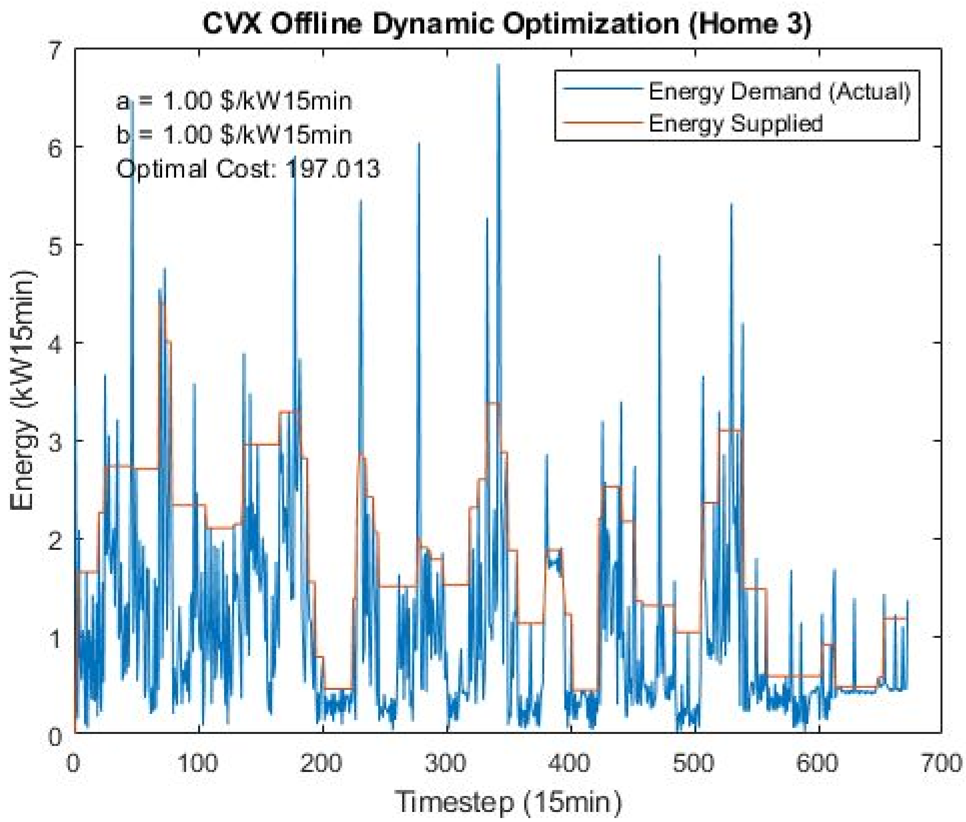
1. MATLAB CVX: Offline Dynamic Optimization

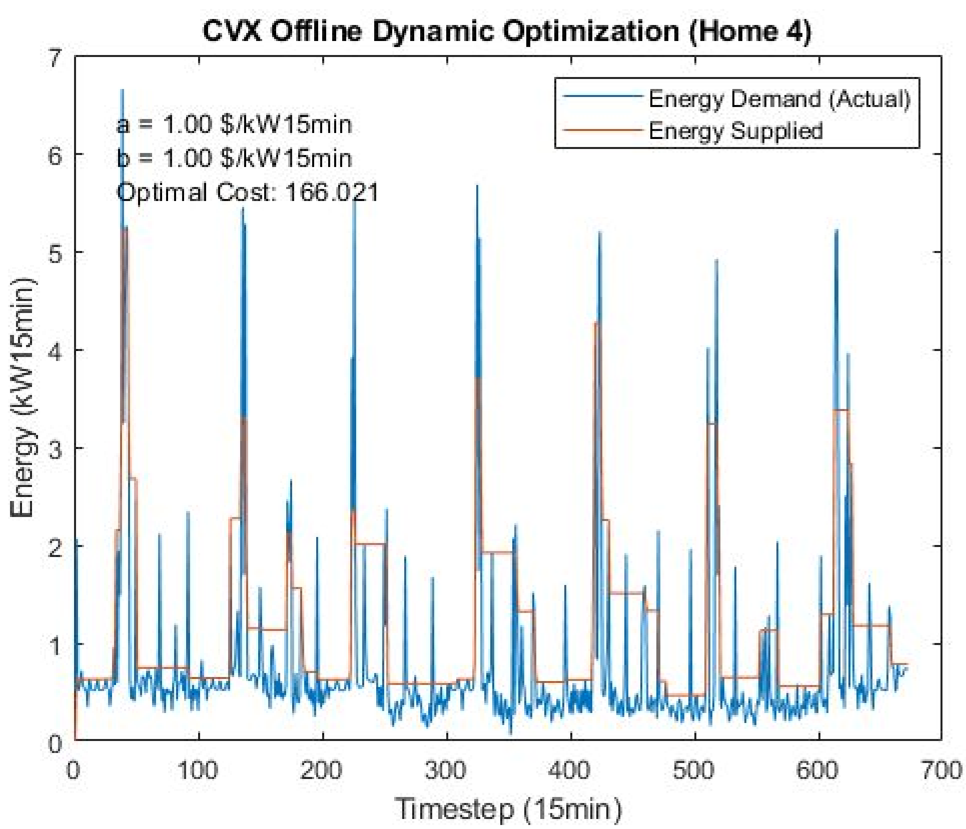
Using the CVX package in Matlab, offline dynamic and static optimizations were performed. The interval of interest is the week of 11/01/2015 - 11/07/2015 (672 time steps) for energy demand from each home (1-10). Cost and Penalty values of 0.4kWh, 4kWh, and 4kWh were used for p,a, and b, respectively. These were divided by 4 to get kilowatts per 15 minutes since those are the time steps we are working with. This gives the standard values used in the code of p = 0.1, a = 4, and b = 4. The following table and figures represent the offline results.

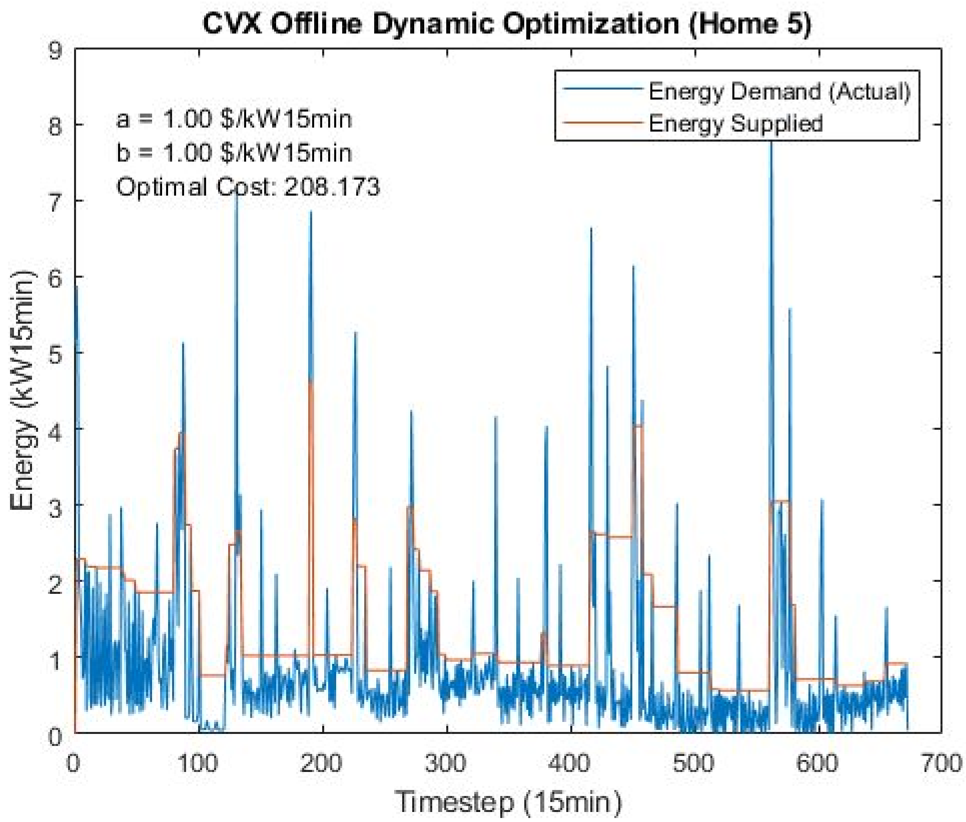
CVX Offline Dynamic	
Home Number	Optimal Cost
1	198.184
2	227.280
3	197.013
4	166.021
5	208.173
6	246.040
7	245.767
8	251.024
9	183.391
10	170.900

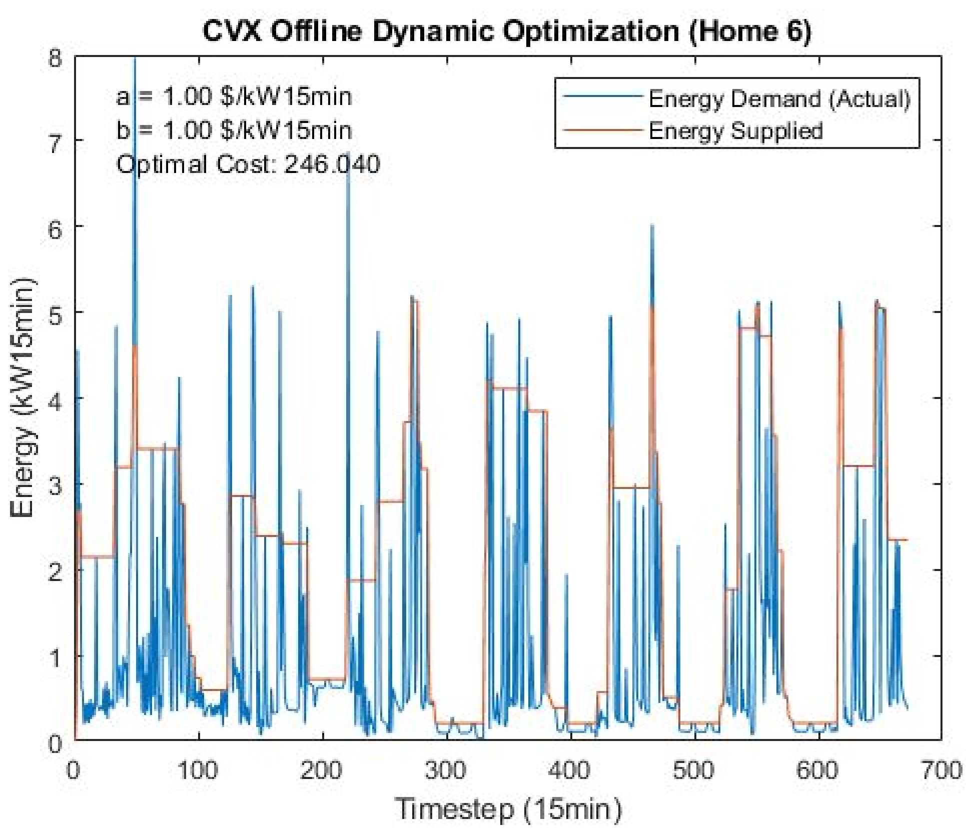

CVX Offline Dynamic Optimization (Home 1)
Energy Demand (Actual)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 198.184
Energy (kW15min)
Timestep (15min)

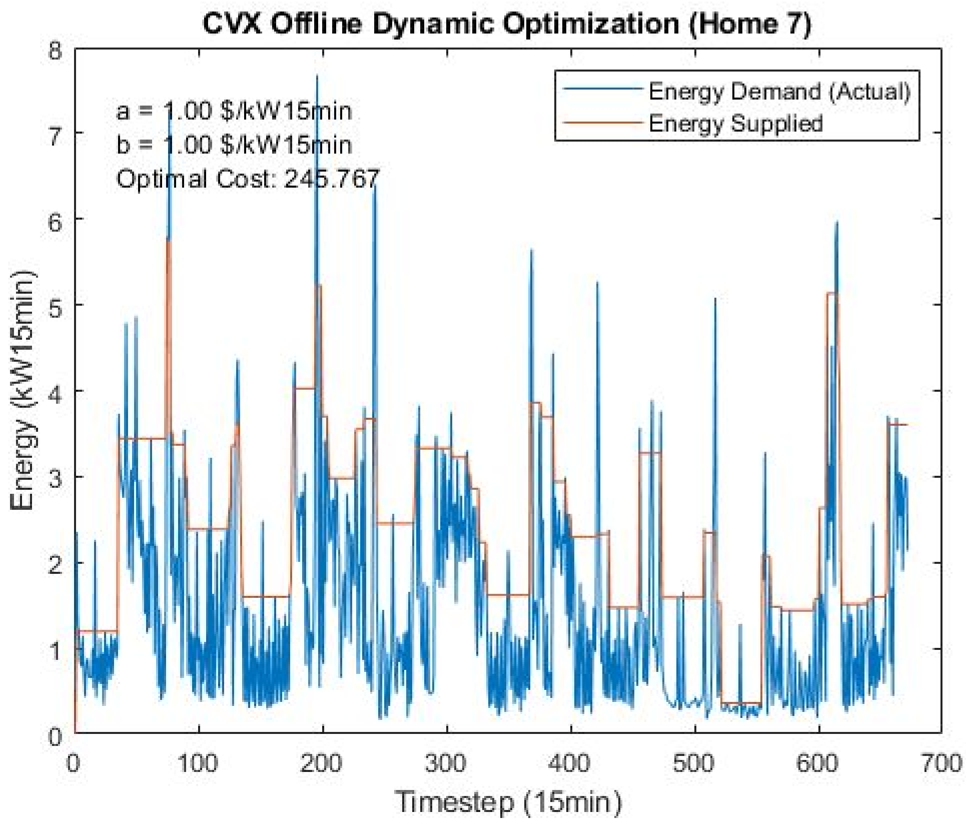

CVX Offline Dynamic Optimization (Home 2)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 227.280
Energy Demand (Actual)
Energy Supplied
Energy (kW15min)
Timestep (15min)

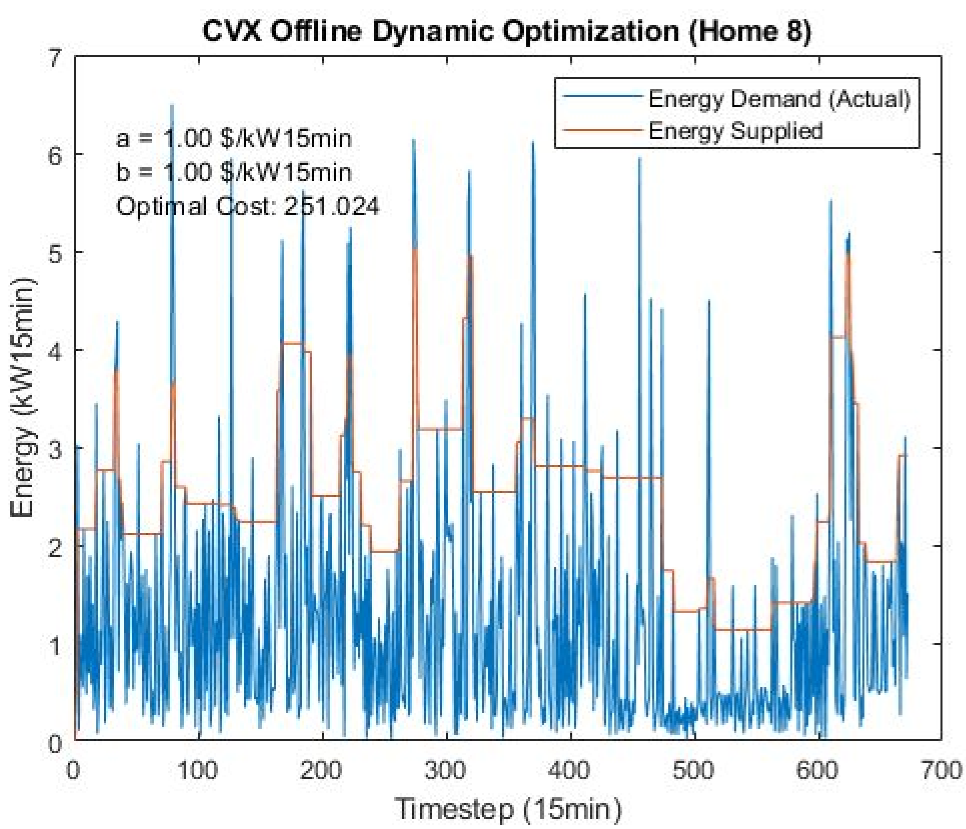

CVX Offline Dynamic Optimization (Home 3)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 197.013
Energy Demand (Actual)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
100
200
300
400
500
600
700
0
1
2
3
4
5
6
7

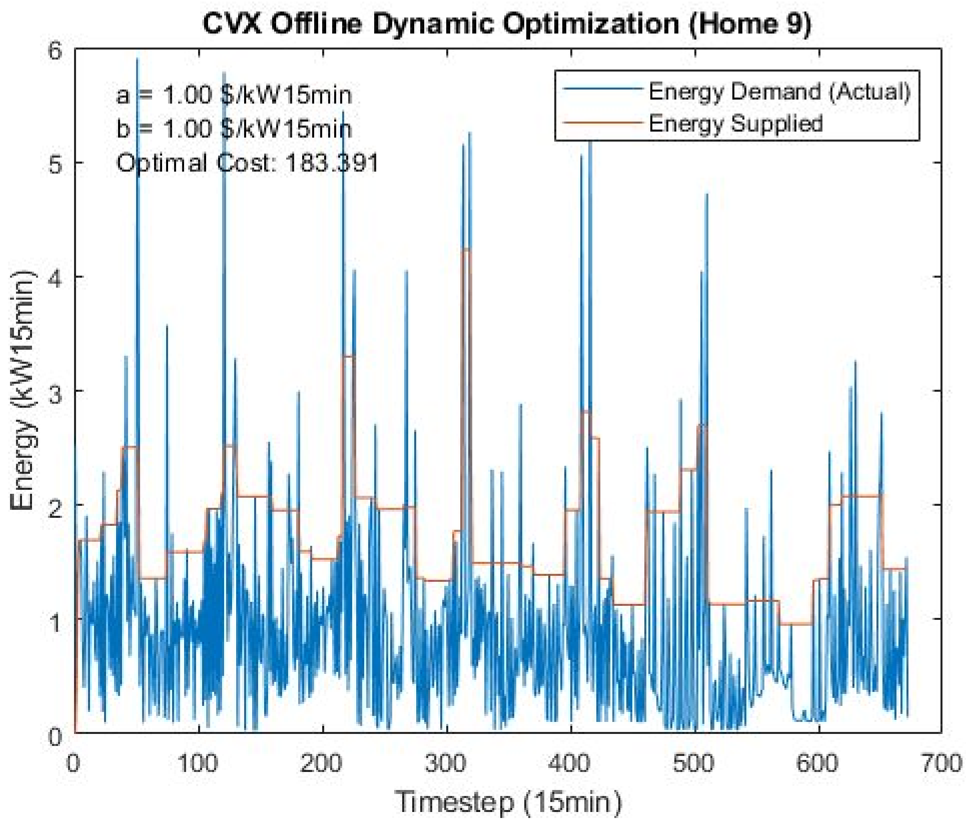

CVX Offline Dynamic Optimization (Home 4)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 166.021
Energy Demand (Actual)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
100
200
300
400
500
600
700
0
1
2
3
4
5
6
7


CVX Offline Dynamic Optimization (Home 5)
Energy Demand (Actual)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 208.173
Energy (kW15min)
Timestep (15min)

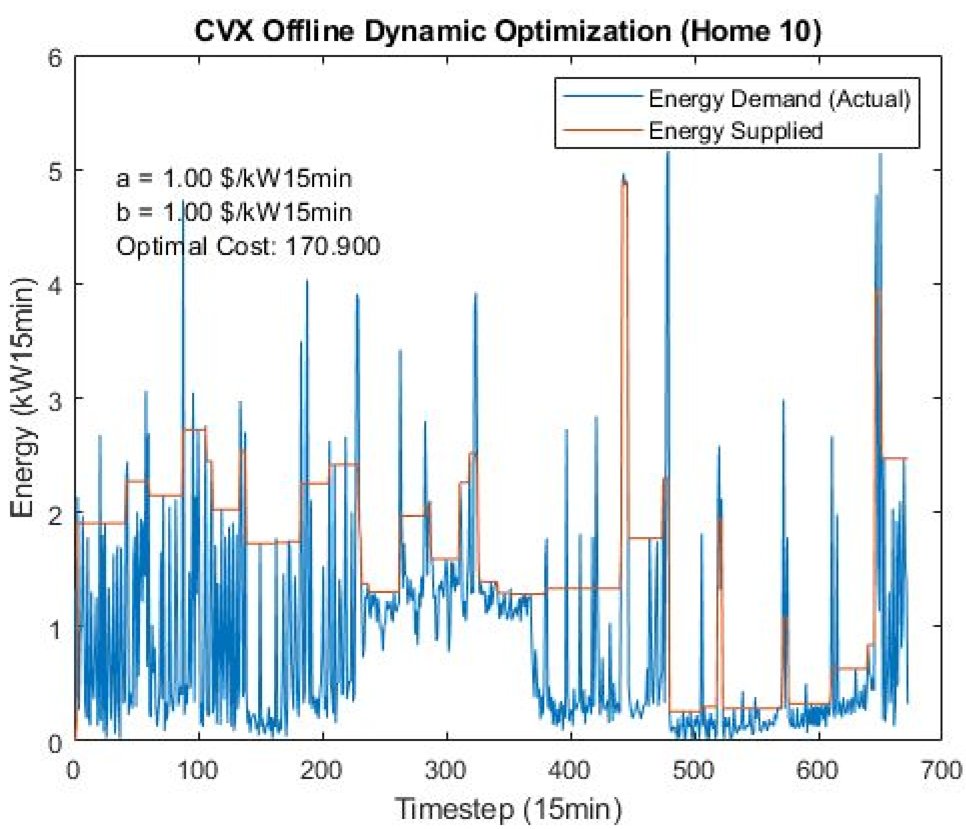

CVX Offline Dynamic Optimization (Home 6)
Energy Demand (Actual)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 246.040
Energy (kW15min)
Timestep (15min)


CVX Offline Dynamic Optimization (Home 7)
Energy Demand (Actual)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 245.767
Energy (kW15min)
Timestep (15min)


CVX Offline Dynamic Optimization (Home 8)
Energy Demand (Actual)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 251.024
Energy (kW15min)
Timestep (15min)



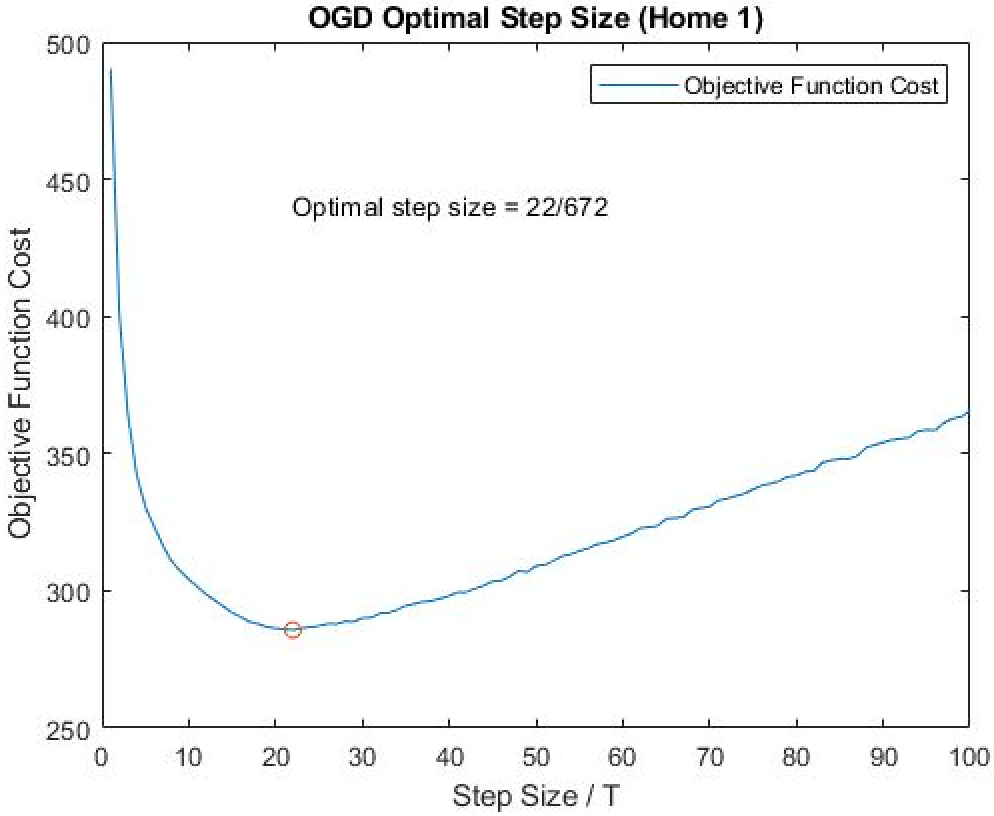
CVX Offline Dynamic Optimization (Home 9)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 183.391
Energy Demand (Actual)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
1
2
3
4
5
6
100
200
300
400
500
600
700



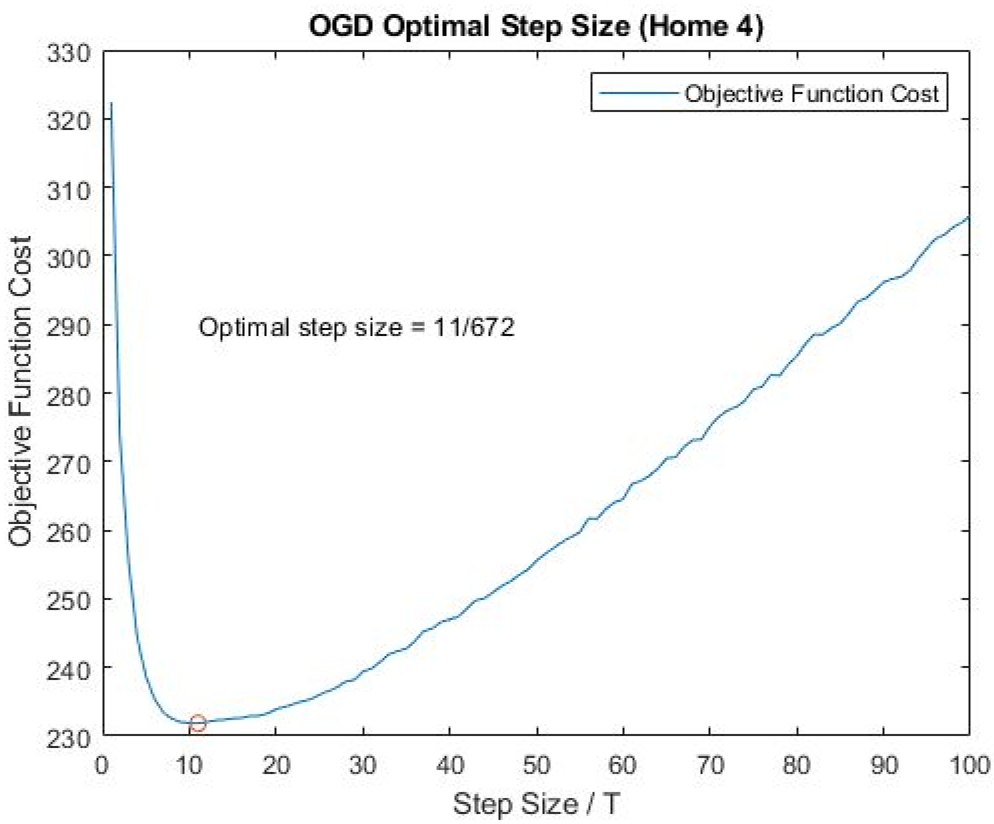
CVX Offline Dynamic Optimization (Home 10)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 170.900
Energy Demand (Actual)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
1
2
3
4
5
6
100
200
300
400
500
600
700

2. Online Optimisation Methods

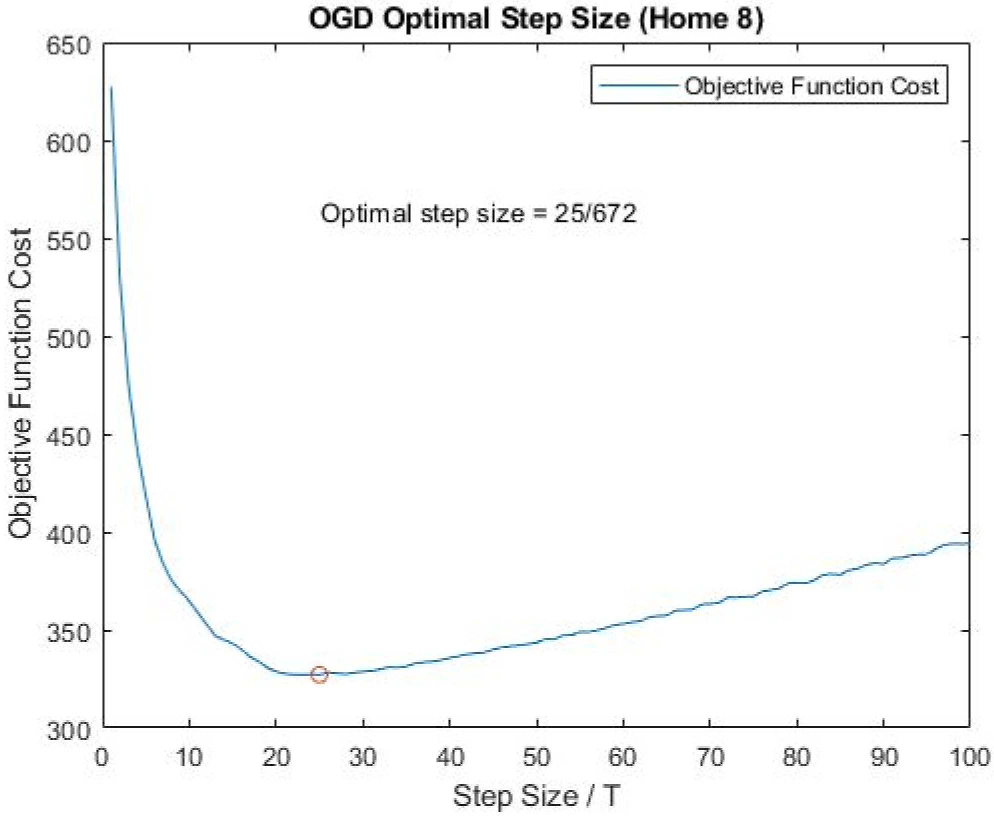
Three optimisation methods: Online Gradient Descent (OGD), Receding Horizon Control (RHC), and Commitment Horizon Control (CHC), were used to find the optimal supply x(t). These being online optimisations, predicted values were used for y(t). These predicted values were obtained from the neural network used in HW1. The same values of p,a,b are used as in part 1. For OGD, RHC, and CHC, stepsize, window size, and commitment level are varied to find the best conditions, respectively. When the optimal step size is found for OGD, this becomes the fixed step size for RHC when finding window size, and it follows that optimal window size is then fixed when finding optimal commitment level. This is done for three homes (home1, home 4, and home 8) which all had different optimal conditions. The following plots and tables show these optimal values and the iterations used to find them.

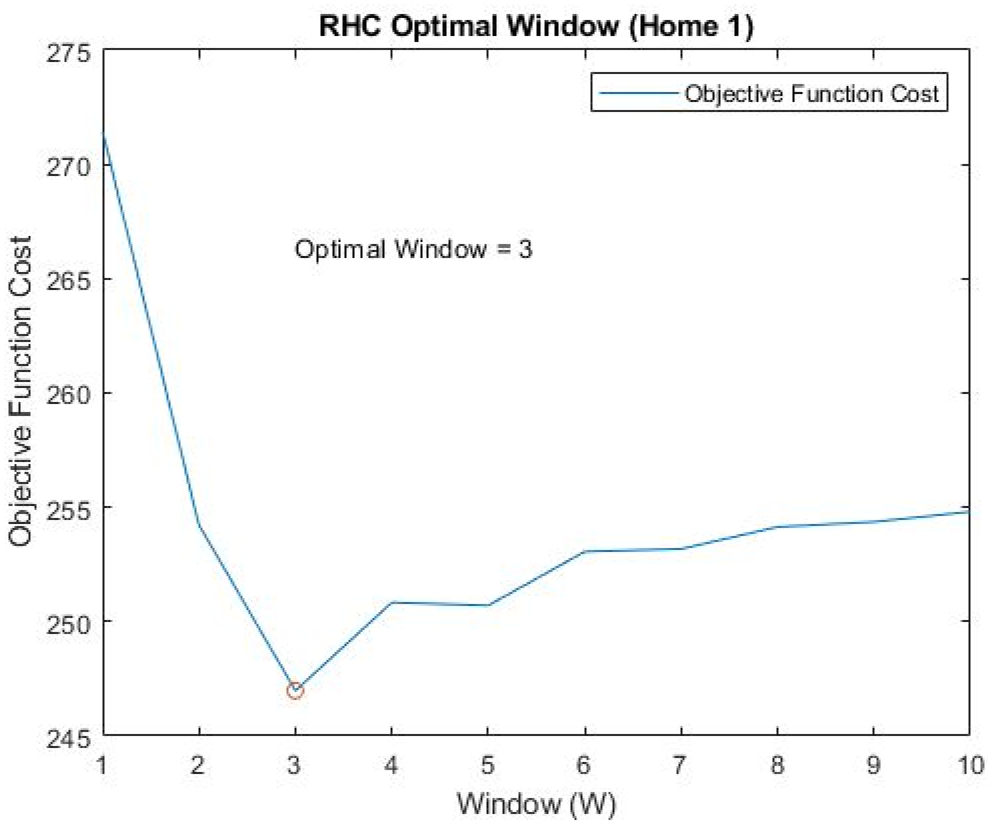


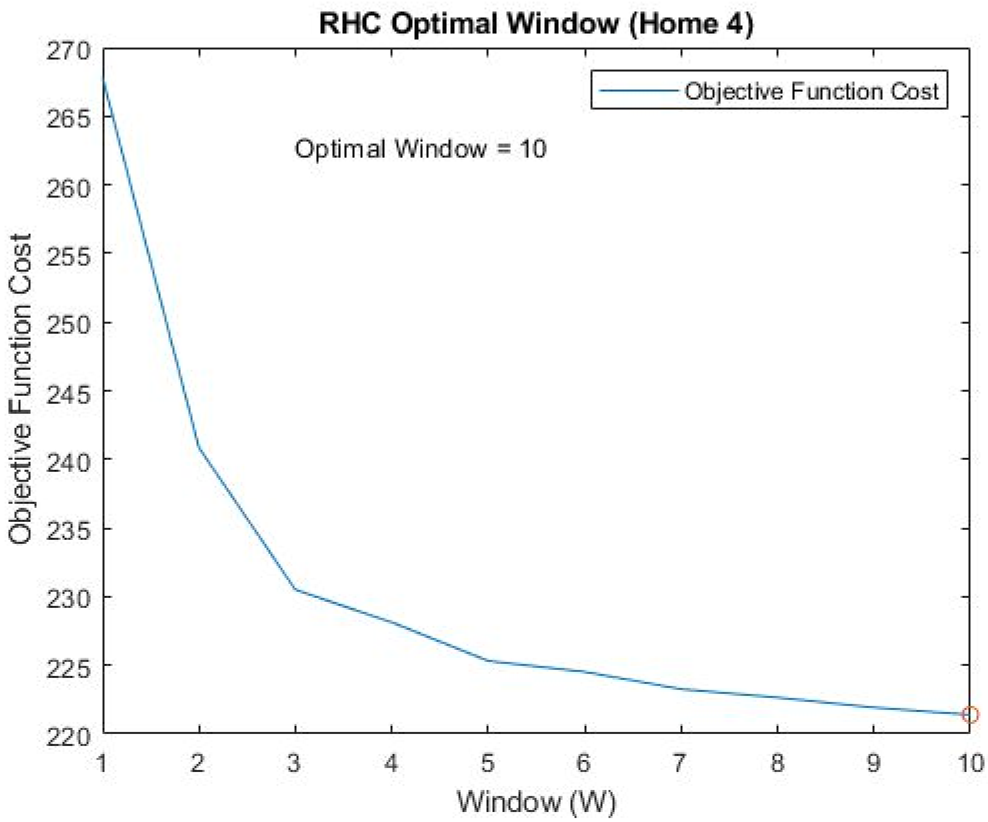
OGD Optimal Step Size (Home 1)
Objective Function Cost
Optimal step size = 22/672
Objective Function Cost
Step Size / T
500
450
400
350
300
250
0
10
20
30
40
50
60
70
80
90
100

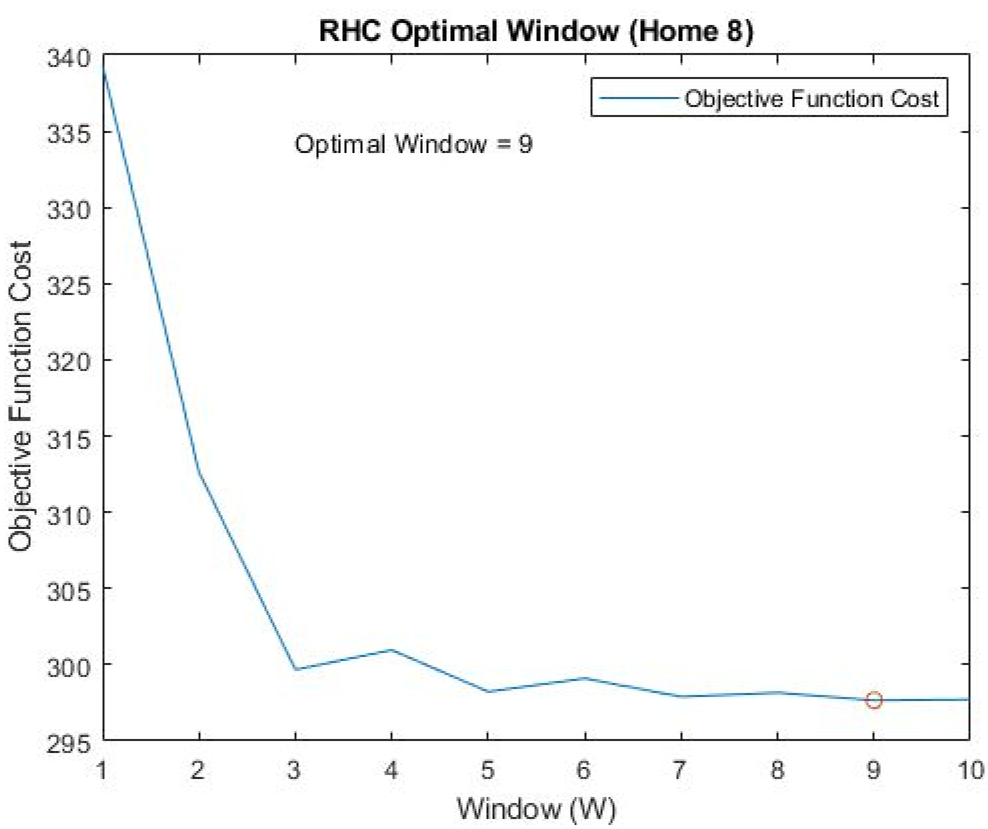


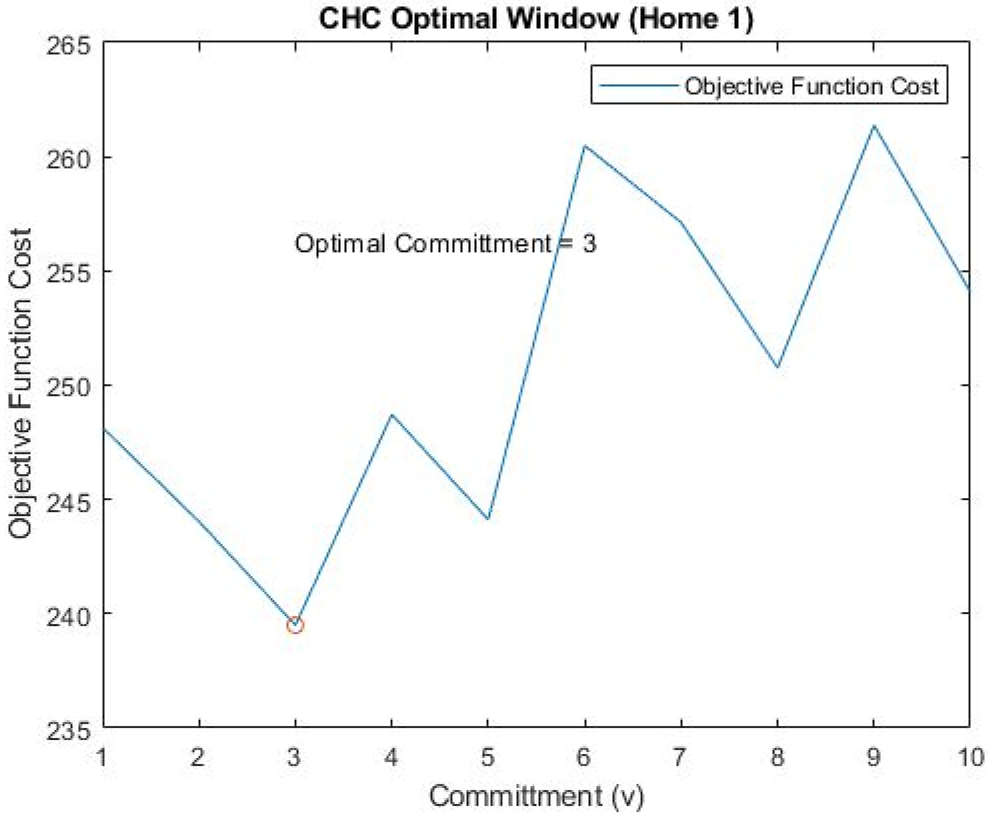
OGD Optimal Step Size (Home 4)
Objective Function Cost
Optimal step size = 11/672
Objective Function Cost
Step Size / T
330
320
310
300
290
280
270
260
250
240
230
0
10
20
30
40
50
60
70
80
90
100

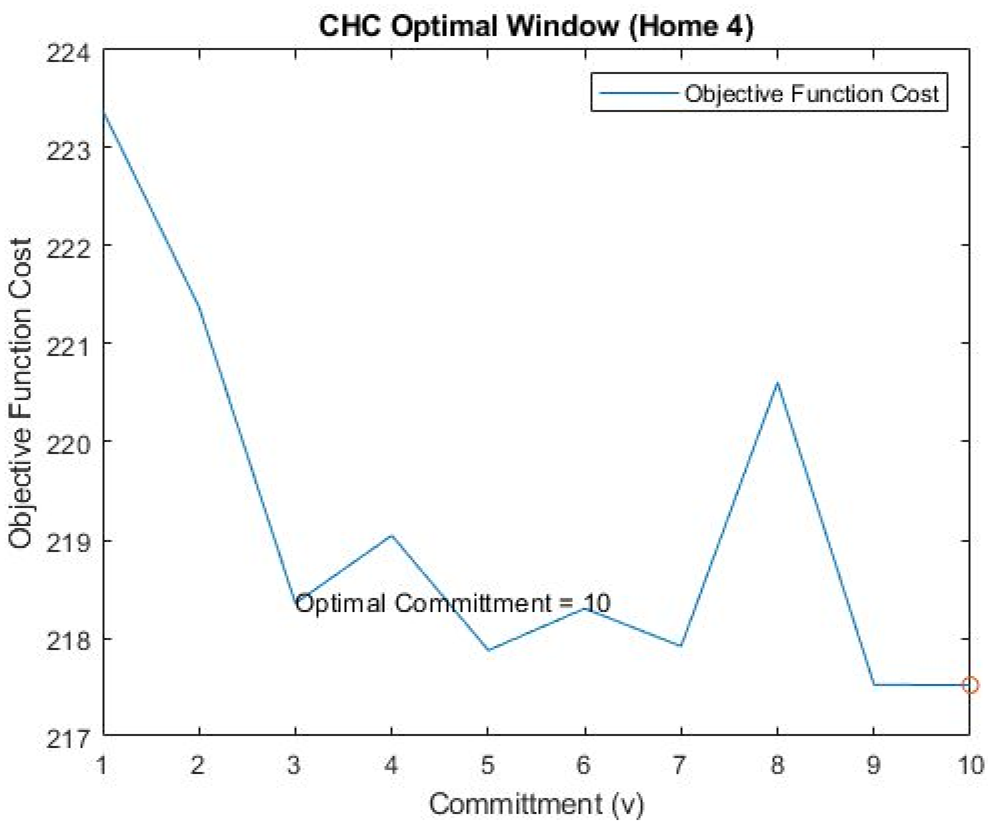

OGD Optimal Step Size (Home 8)
Objective Function Cost
Optimal step size = 25/672
Objective Function Cost
Step Size / T
650
600
550
500
450
400
350
300
0
10
20
30
40
50
60
70
80
90
100

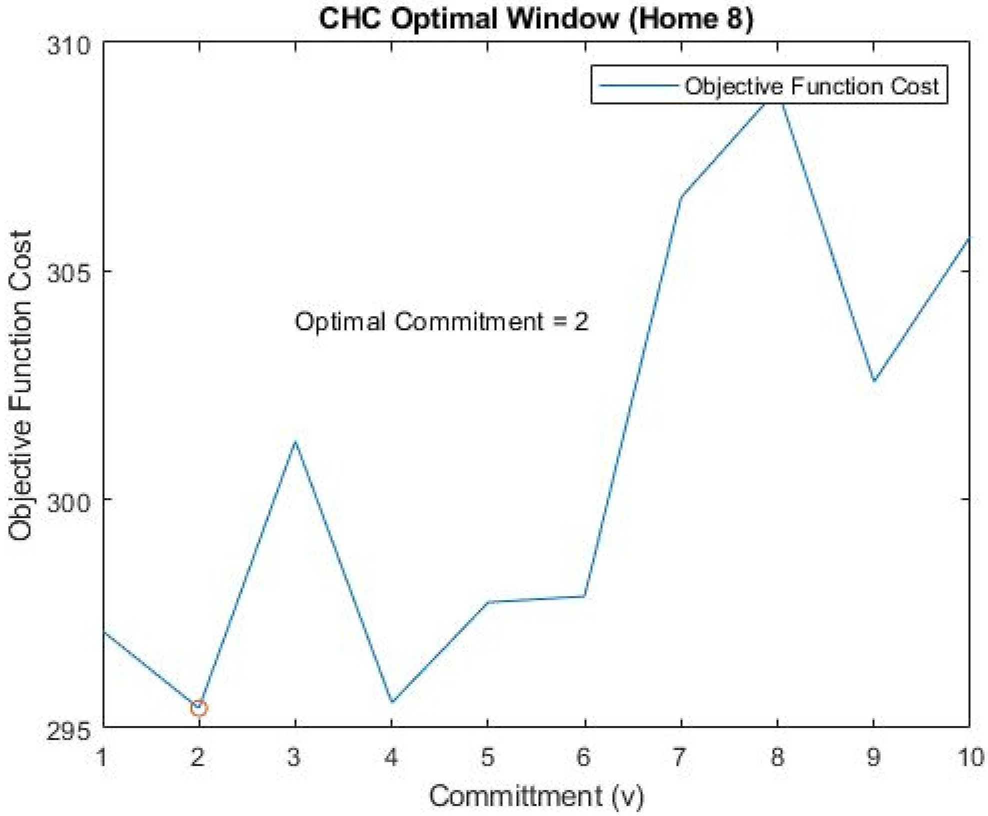

RHC Optimal Window (Home 1)
Objective Function Cost
Optimal Window = 3
Objective Function Cost
Window (W)
275
270
265
260
255
250
245
1
2
3
4
5
6
7
8
9
10


RHC Optimal Window (Home 4)
Objective Function Cost
Optimal Window = 10
Objective Function Cost
Window (W)

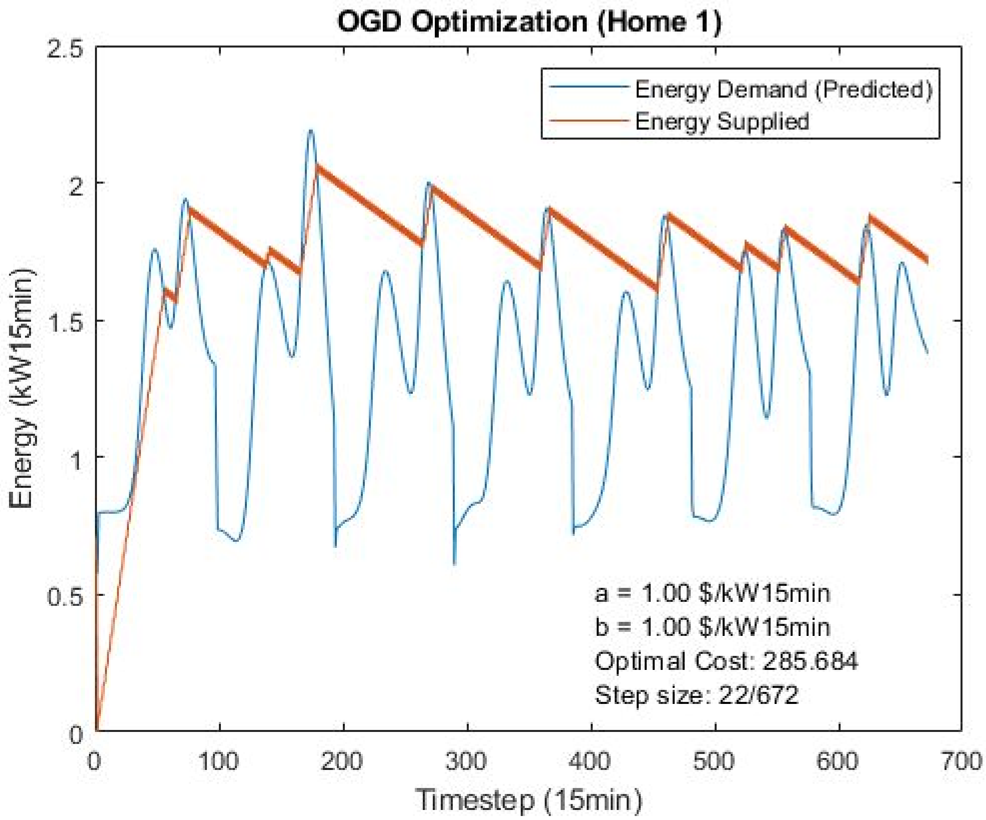

RHC Optimal Window (Home 8)
Objective Function Cost
Optimal Window = 9
Objective Function Cost
Window (W)

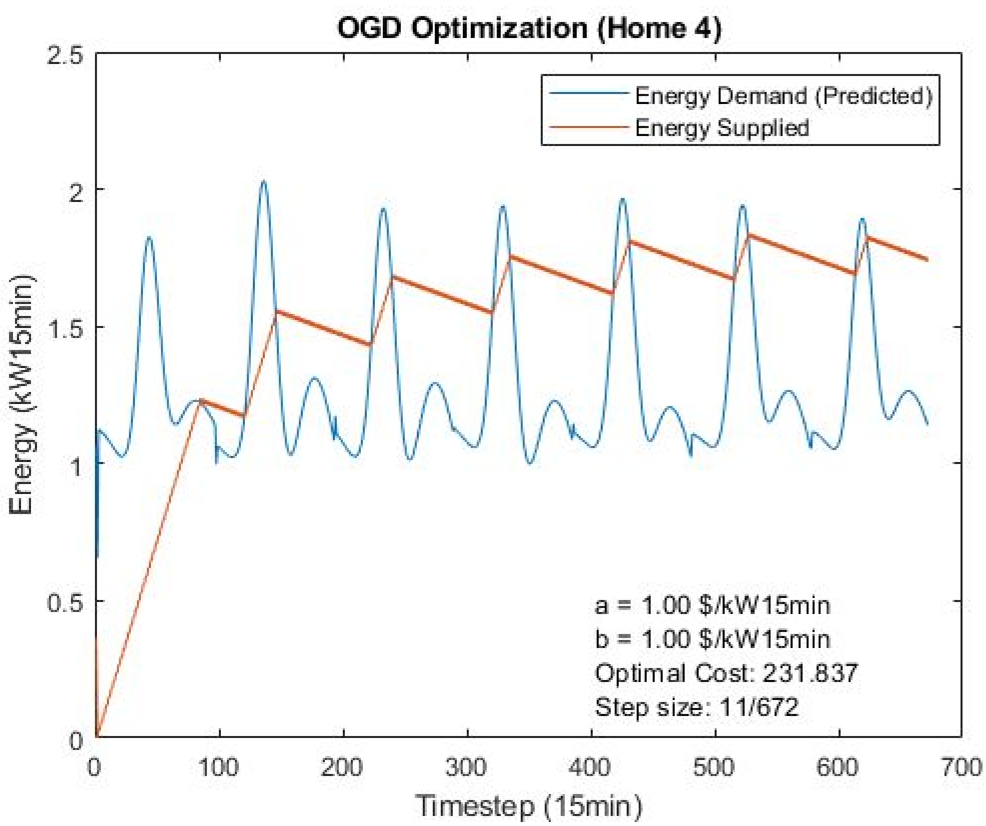

CHC Optimal Window (Home 1)
Objective Function Cost
Optimal Committment = 3
Committment (v)
265
260
255
250
245
240
235
1
2
3
4
5
6
7
8
9
10

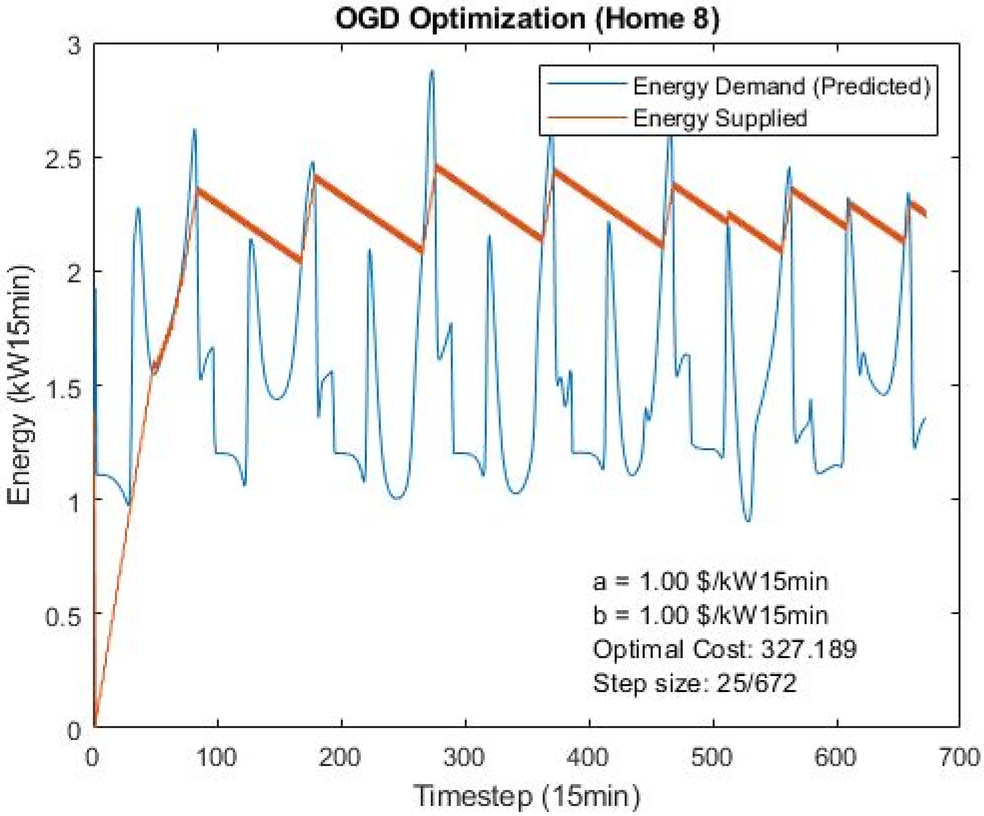

CHC Optimal Window (Home 4)
Objective Function Cost
Optimal Committment = 10
Committment (v)
224
223
222
221
220
219
218
217
1
2
3
4
5
6
7
8
9
10



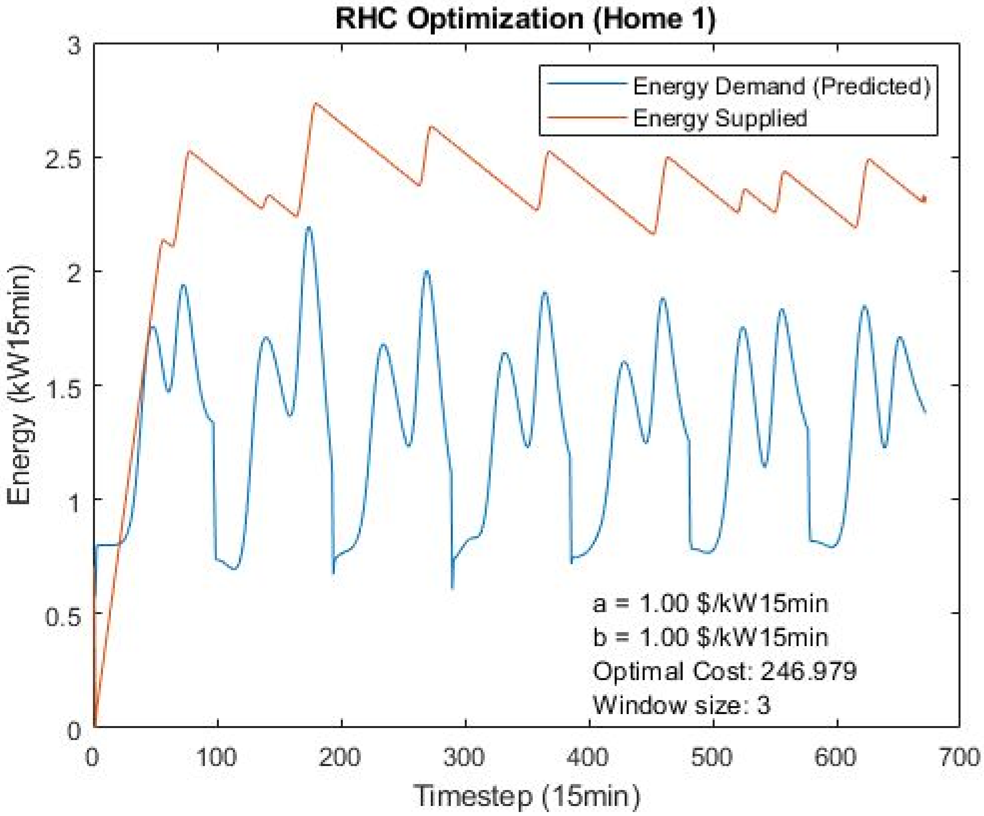
Sensitivity Index: Optimal Value for variables			
Home Number	Home 1	Home 4	Home 8
Step size	22	11	25
Window Size (w)	3	10	9
Commitment Level (v)	3	10	2


OGD Optimization (Home 1)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 285.684
Step size: 22/672
Energy (kW15min)
Timestep (15min)

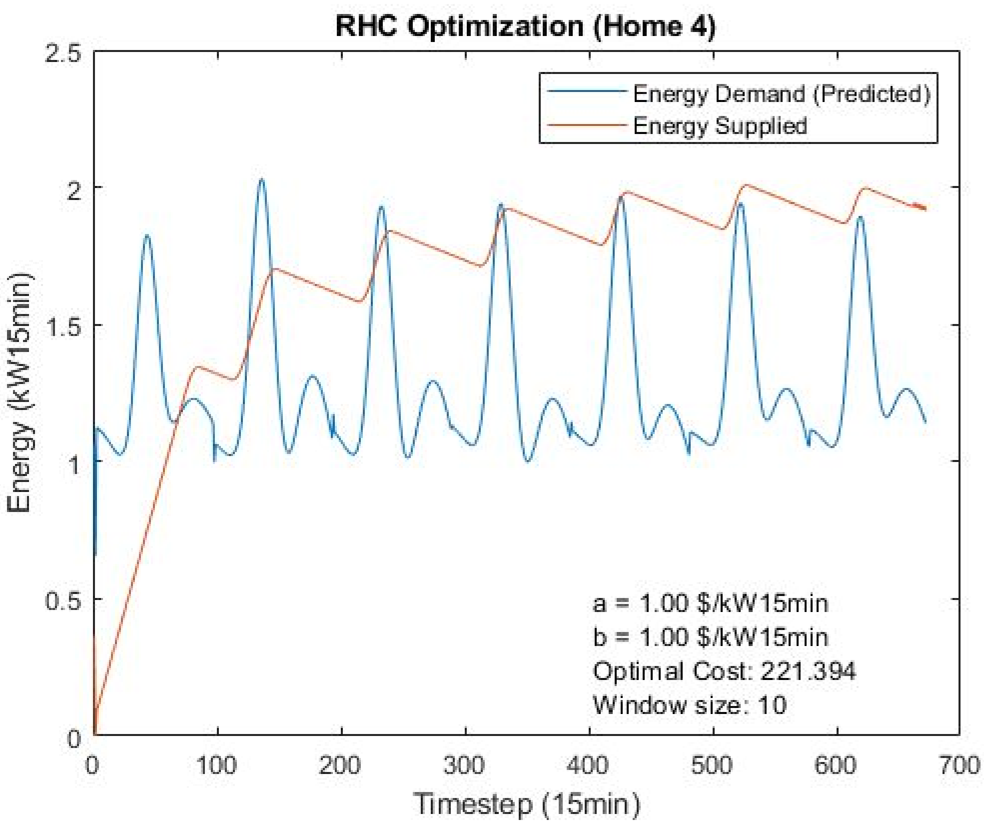

OGD Optimization (Home 4)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 231.837
Step size: 11/672
Energy (kW15min)
Timestep (15min)



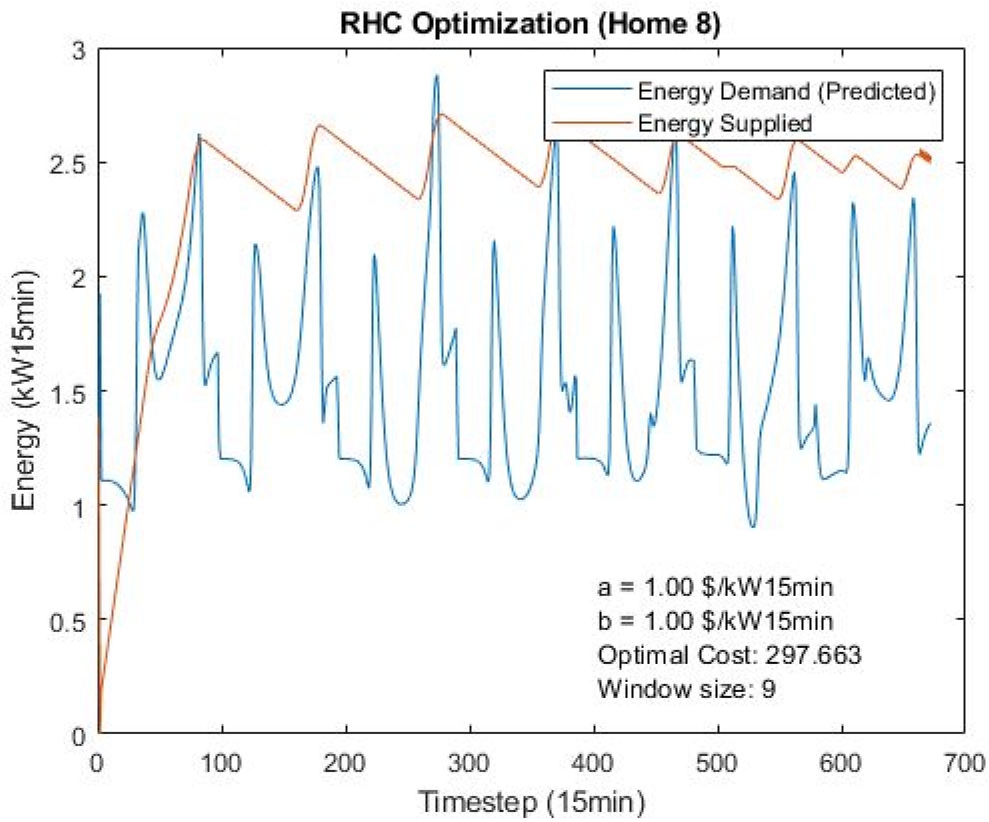
OGD Optimization (Home 8)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 327.189
Step size: 25/672
Energy (kW15min)
Timestep (15min)
0
0.5
1
1.5
2
2.5
3
0
100
200
300
400
500
600
700

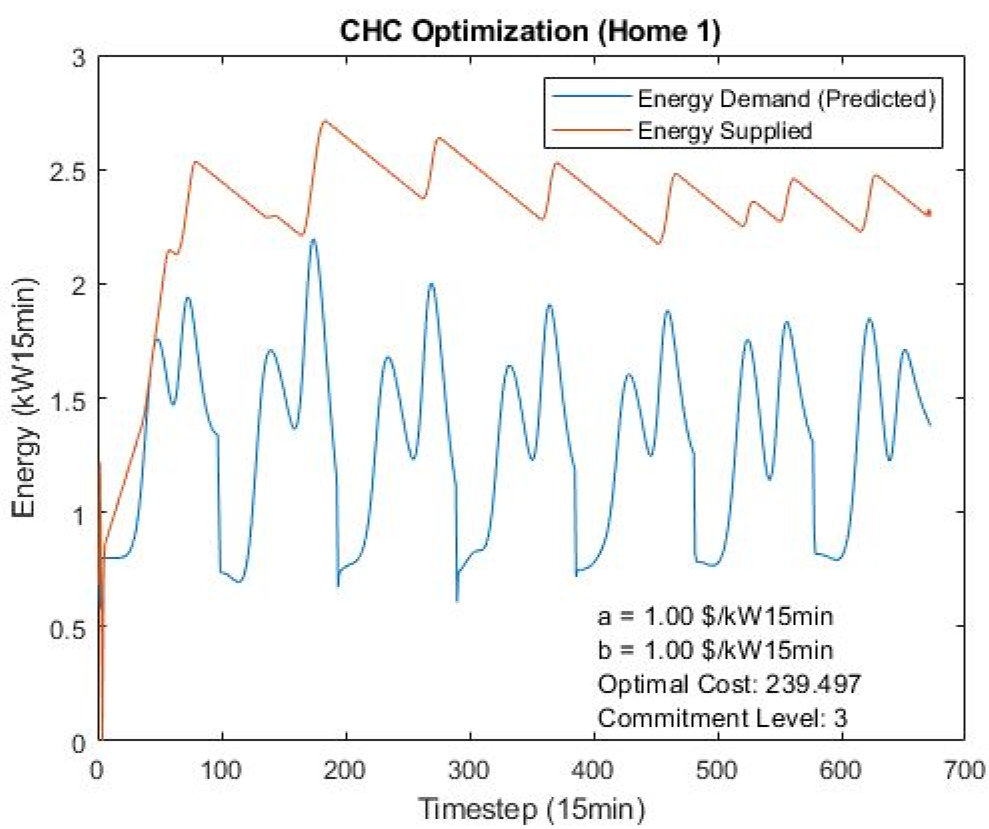


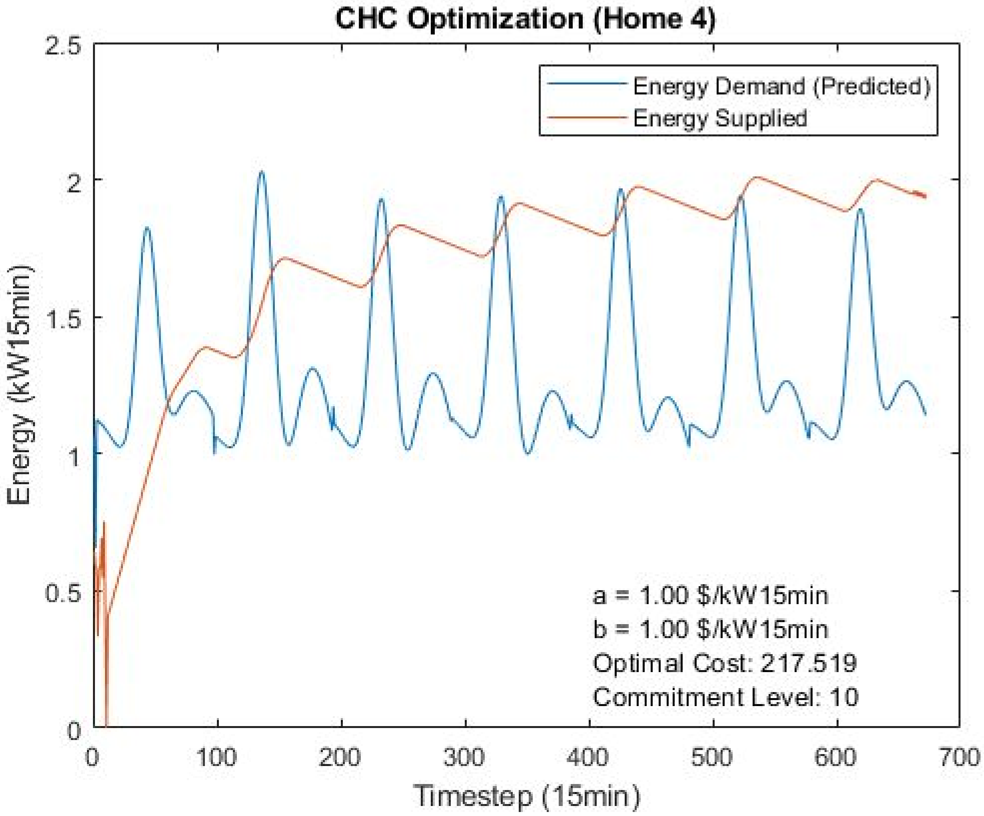
RHC Optimization (Home 1)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 246.979
Window size: 3
Energy (kW15min)
Timestep (15min)

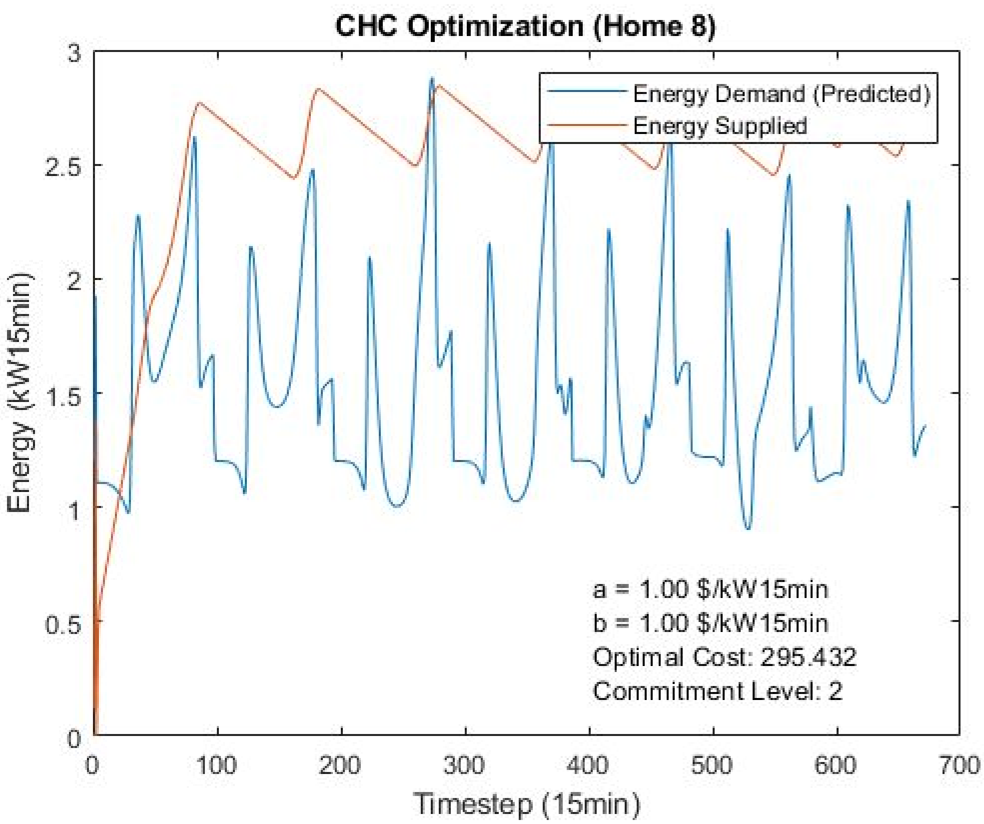


RHC Optimization (Home 4)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 221.394
Window size: 10
Energy (kW15min)
Timestep (15min)


RHC Optimization (Home 8)
Energy Demand (Predicted)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
100
200
300
400
500
600
700
0
0.5
1
1.5
2
2.5
3
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 297.663
Window size: 9


CHC Optimization (Home 1)
Energy Demand (Predicted)
Energy Supplied
Energy (kW15min)
Timestep (15min)
0
100
200
300
400
500
600
700
0
0.5
1
1.5
2
2.5
3
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 239.497
Commitment Level: 3


CHC Optimization (Home 4)
Energy Demand (Predicted)
Energy Supplied
Energy (kW15min)
Timestep (15min)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 217.519
Commitment Level: 10

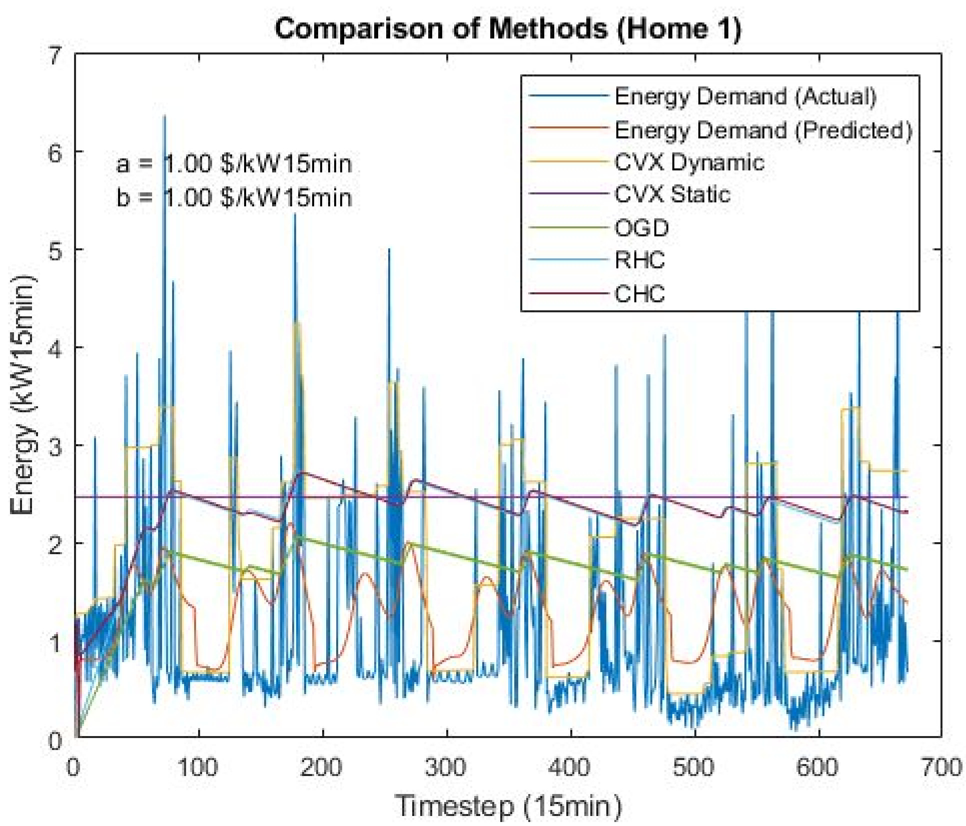

CHC Optimization (Home 8)
Energy Demand (Predicted)
Energy Supplied
Energy (kW15min)
Timestep (15min)
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 295.432
Commitment Level: 2

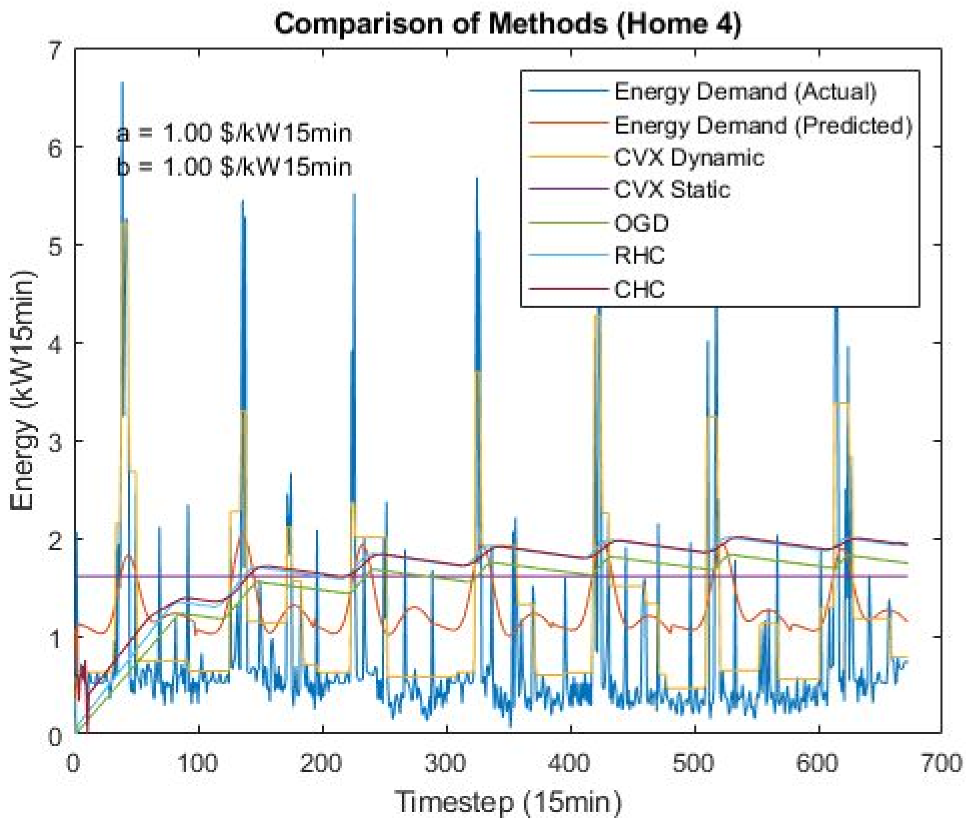
Sensitivity Index: Corresponding Optimal Object Function Costs			
Home Number	Home 1	Home 4	Home 8
OGD	285.6838	231.8369	327.1893
RHC	246.9788	221.3937	297.6634
CHC	239.4974	217.5191	295.4321

These results are consistent with our expectations, as none of them performed as well at the offline dynamic solutions; and each method was marginally more successful than the one it builds off of (OGD > RHC > CHC).

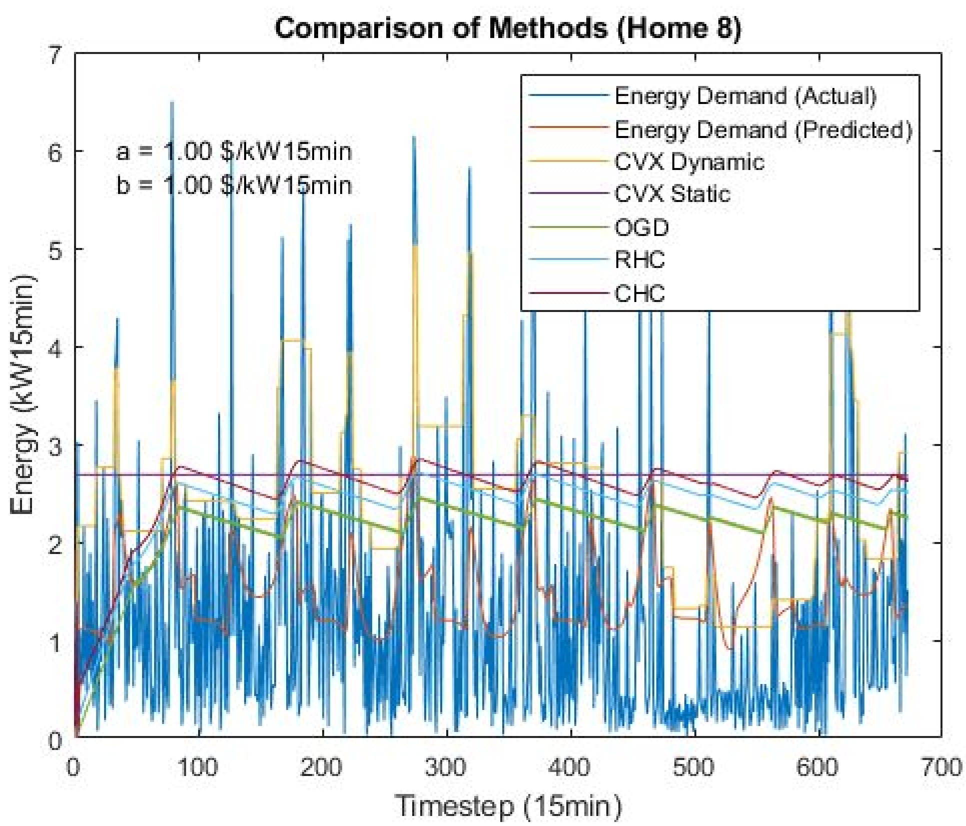
3. Comparison to Offline Dynamic and Static

The three online methods are compared to the offline methods.





Comparison of Methods (Home 4)
Energy Demand (Actual)
Energy Demand (Predicted)
CVX Dynamic
CVX Static
OGD
RHC
CHC
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Energy (kW15min)
Timestep (15min)
0
1
2
3
4
5
6
7
0
100
200
300
400
500
600
700



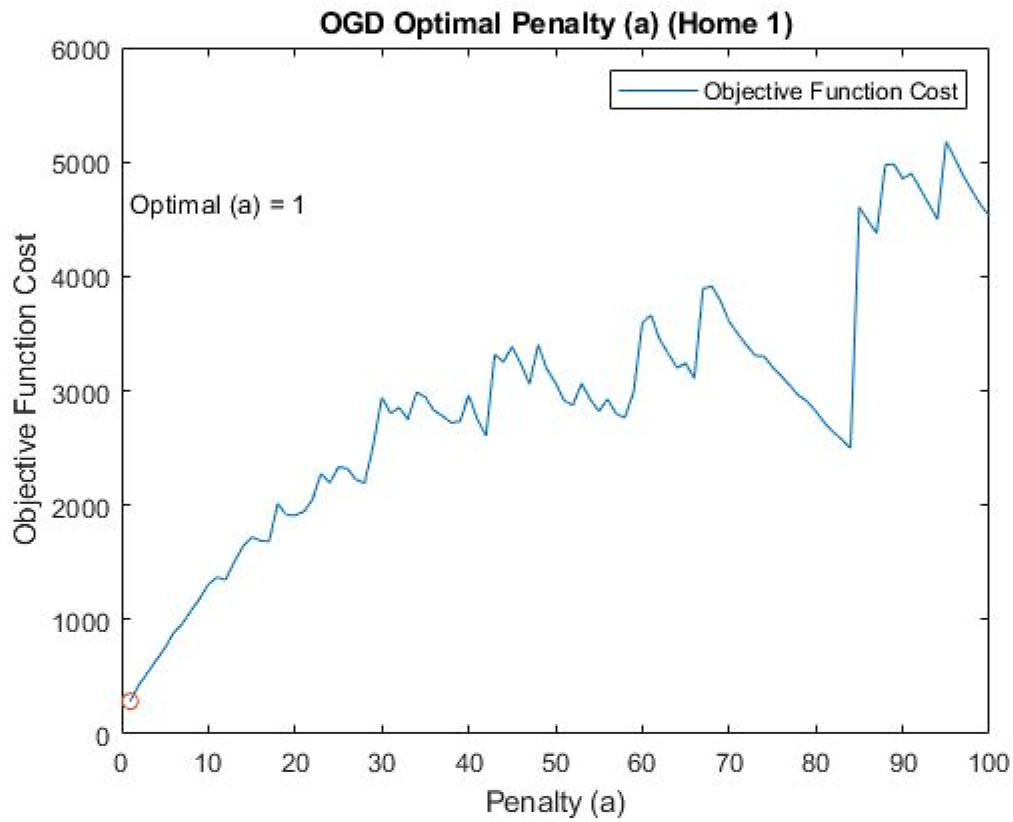
Comparison of Methods (Home 8)
Energy Demand (Actual)
Energy Demand (Predicted)
CVX Dynamic
CVX Static
OGD
RHC
CHC
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Energy (kW15min)
Timestep (15min)
0
1
2
3
4
5
6
7
0
100
200
300
400
500
600
700

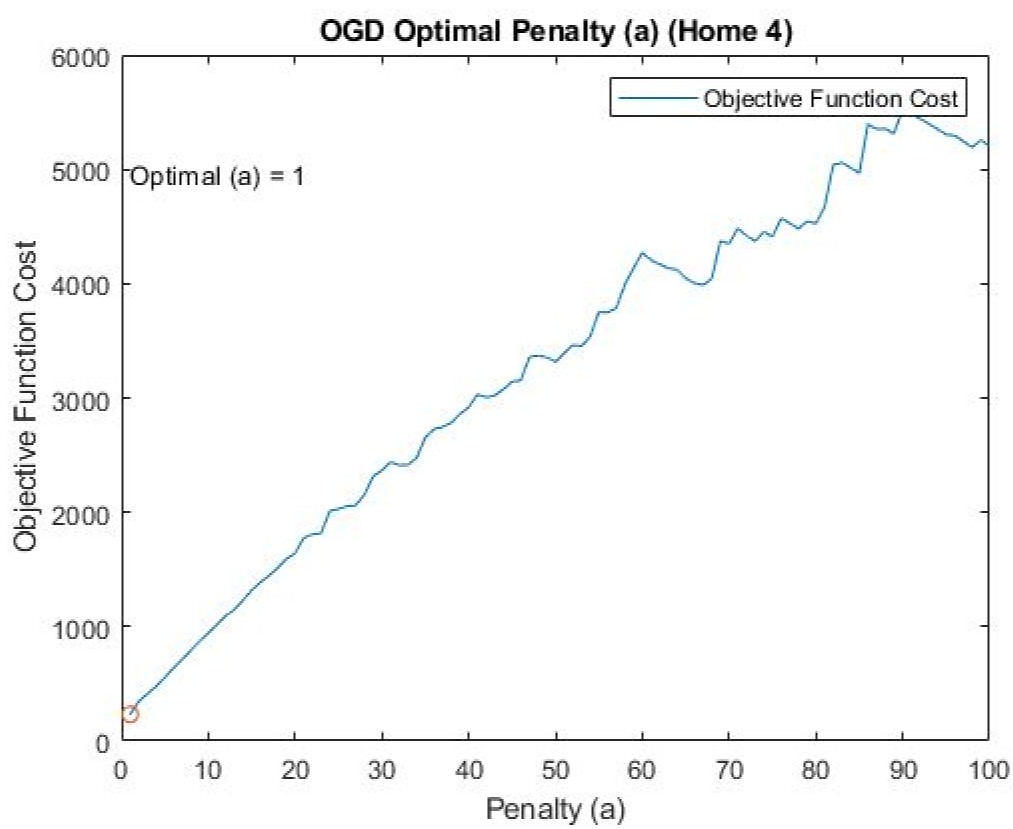
Optimal Cost			
Method	Home Number		
	Home 1	Home 4	Home 8
CVX Dynamic	198.184	166.021	251.024
CVX Static	228.1805	204.7417	275.2412
OGD	285.6838	231.8369	327.1893
RHC	246.9788	221.3937	297.6634
CHC	239.4974	217.5191	295.4321

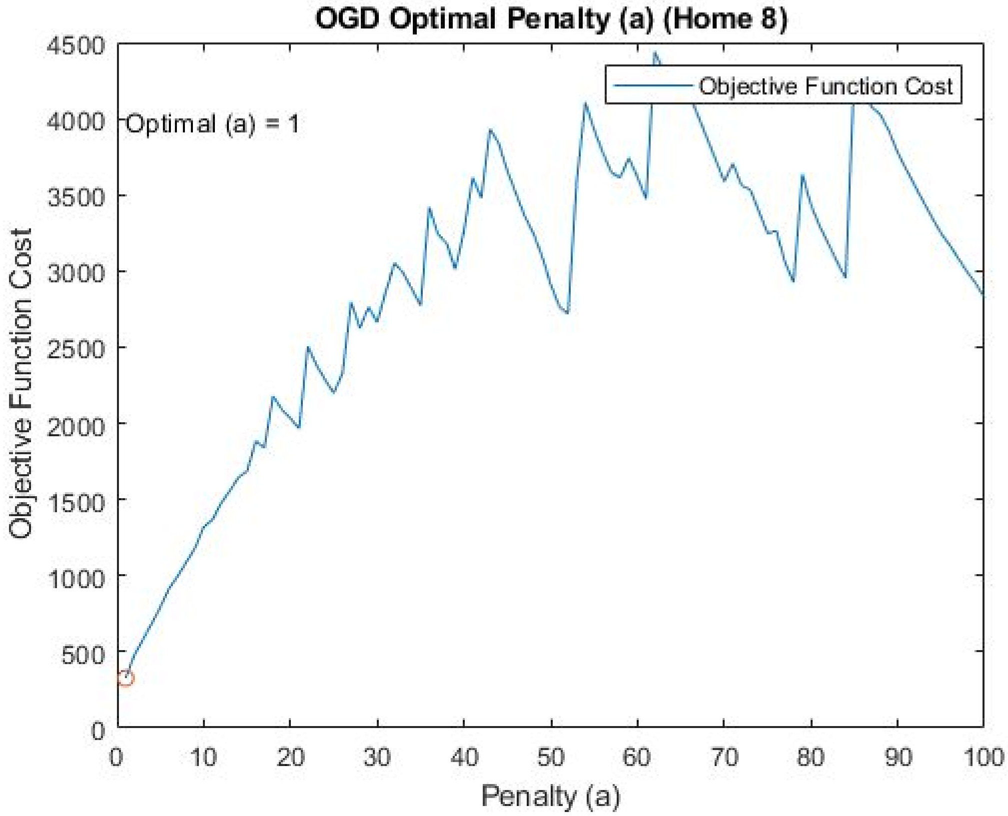
It is consistent that Dynamic CVX should perform the best, as it is an offline solution. CVX Static did not perform as well as the dynamic method, but still better than all online methods. This was unexpected, as we assumed that the dynamic online methods might perform better due to their ability to change values for optimisation. The better performance of CVX static is likely due to the fact that it is working off of observed values, instead of the predicted values.

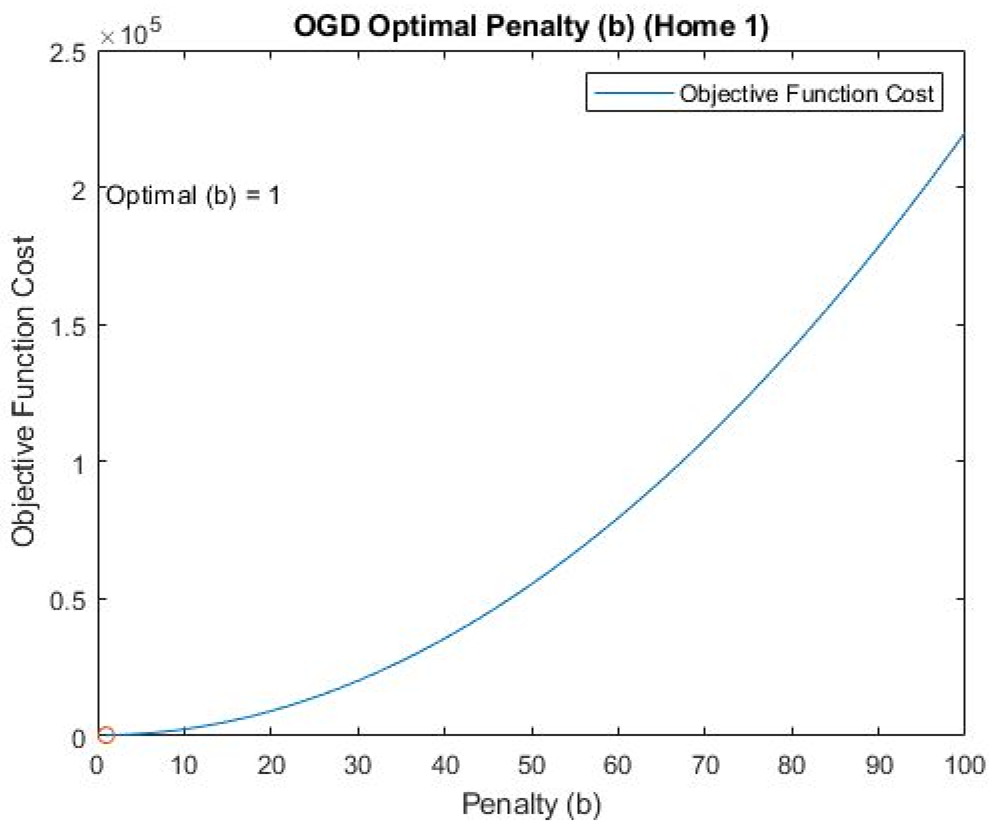
4. Vary values for a and b

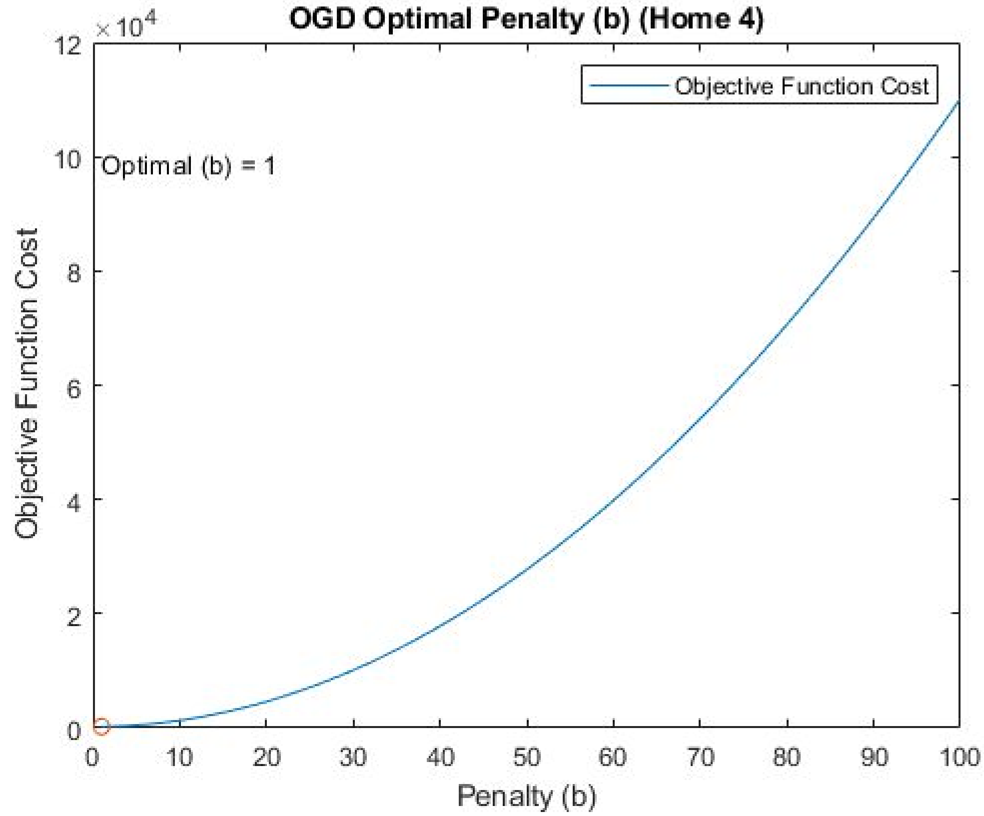
The following section attempts to find optimal penalty values for a and b to reduce objective function cost. These are performed with varied a and fixed b, and vice versa; for homes 1,4,and 8. First is shown OGD with varied a, then varied b, and then RHC with varied a, then varied b.

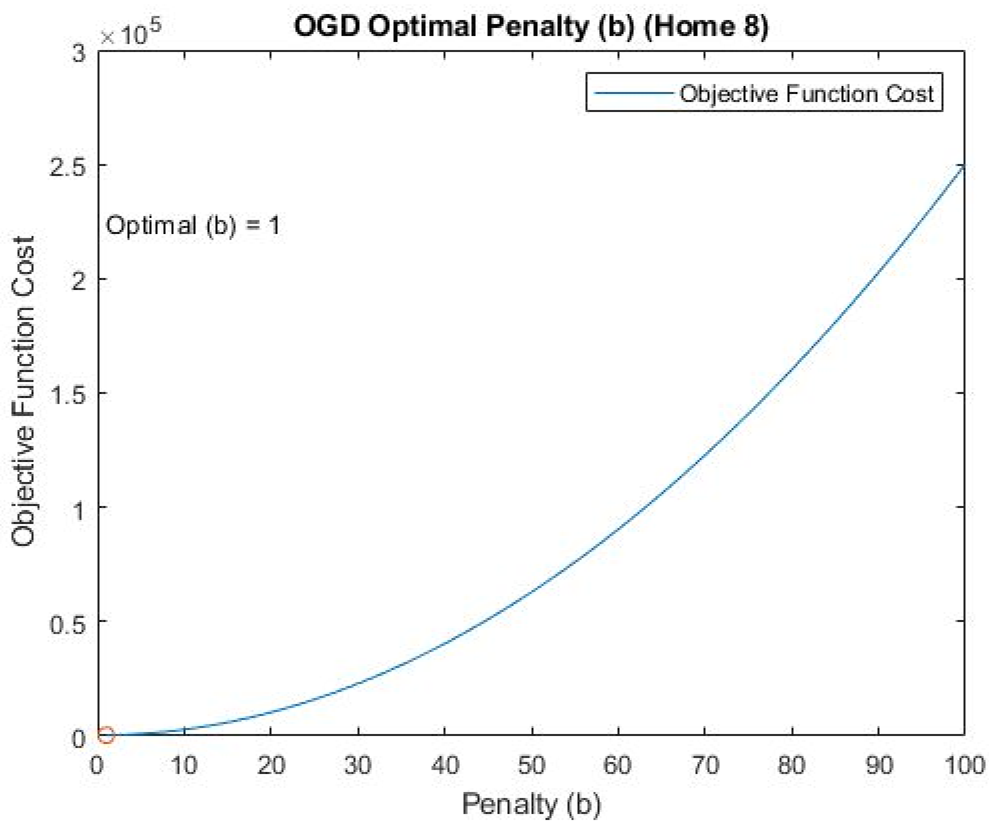

OGD Optimal Penalty (a) (Home 1)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
1000
2000
3000
4000
5000
6000
0
10
20
30
40
50
60
70
80
90
100

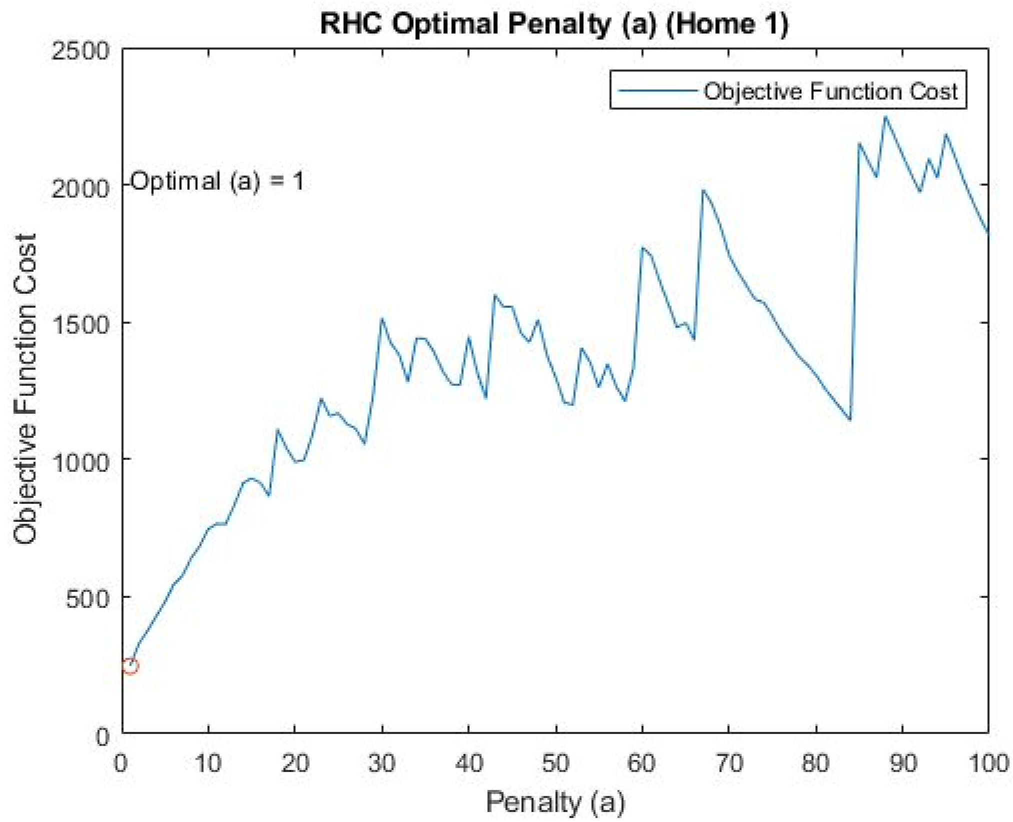

OGD Optimal Penalty (a) (Home 4)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
1000
2000
3000
4000
5000
6000
0
10
20
30
40
50
60
70
80
90
100

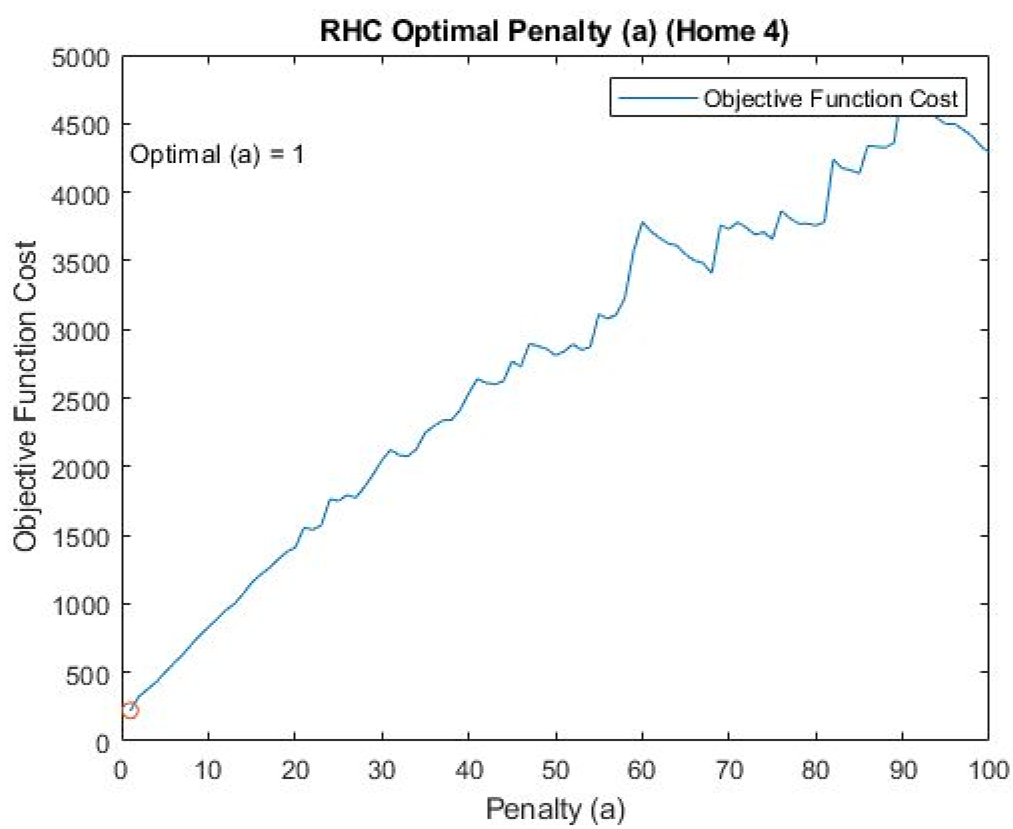

OGD Optimal Penalty (a) (Home 8)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
10
20
30
40
50
60
70
80
90
100

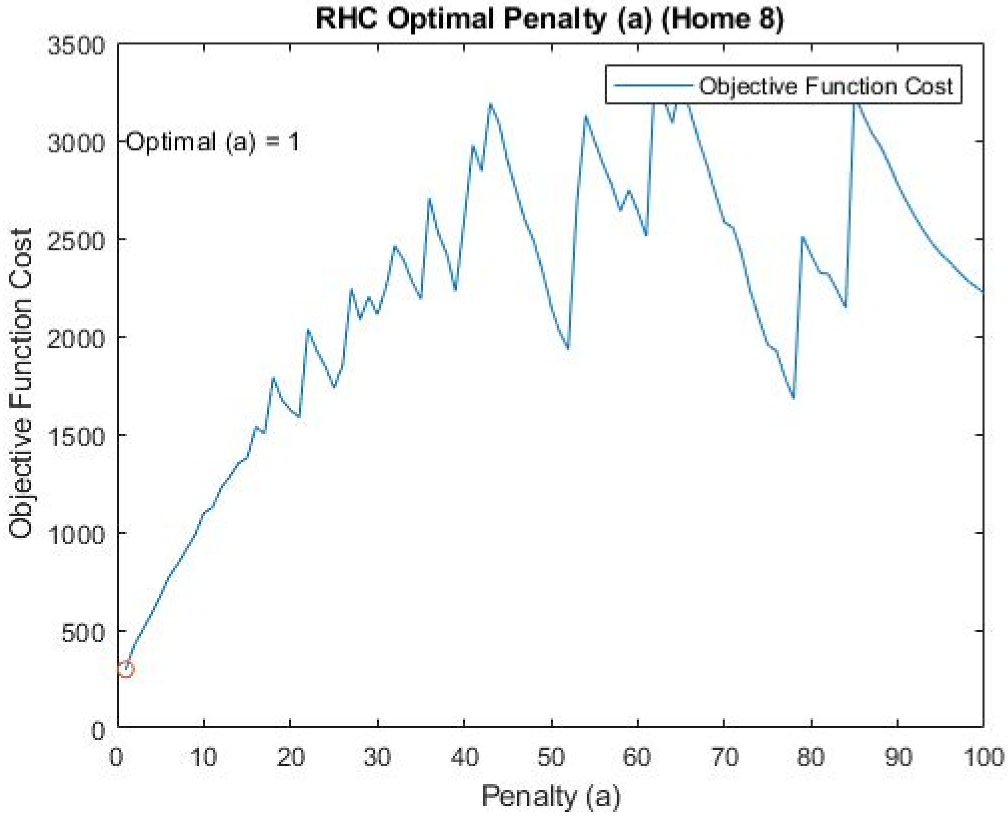

OGD Optimal Penalty (b) (Home 1)
$\times 10^5$
Objective Function Cost
Optimal (b) = 1
Objective Function Cost
Penalty (b)
0
0.5
1
1.5
2
2.5
0
10
20
30
40
50
60
70
80
90
100

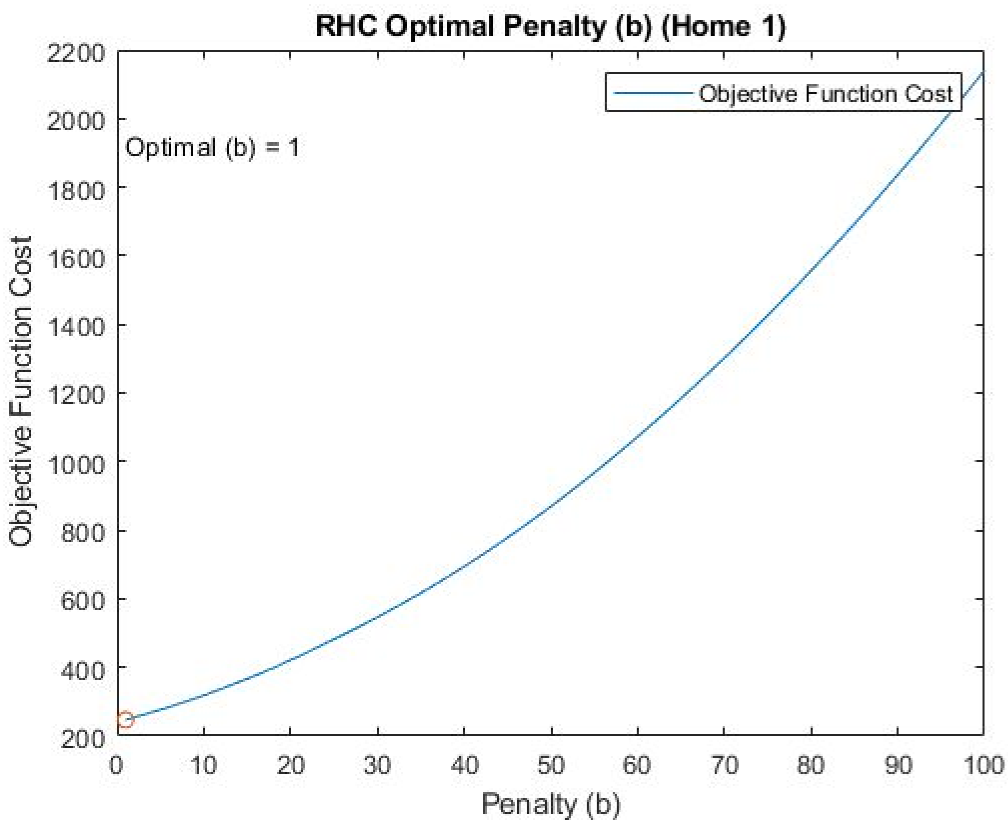

OGD Optimal Penalty (b) (Home 4)
×10^4
12
10
8
6
4
2
0
Objective Function Cost
Optimal (b) = 1
0
10
20
30
40
50
60
70
80
90
100
Penalty (b)

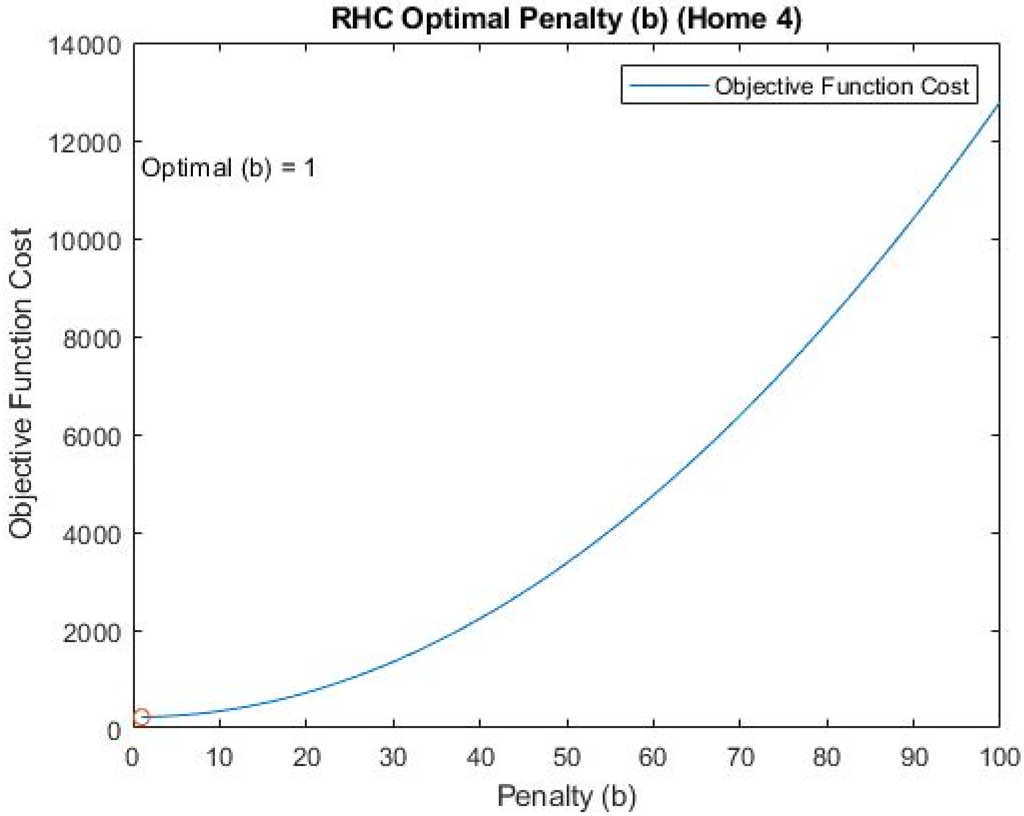

OGD Optimal Penalty (b) (Home 8)
×10^5
3
2.5
2
1.5
1
0.5
0
Objective Function Cost
Optimal (b) = 1
0
10
20
30
40
50
60
70
80
90
100
Penalty (b)

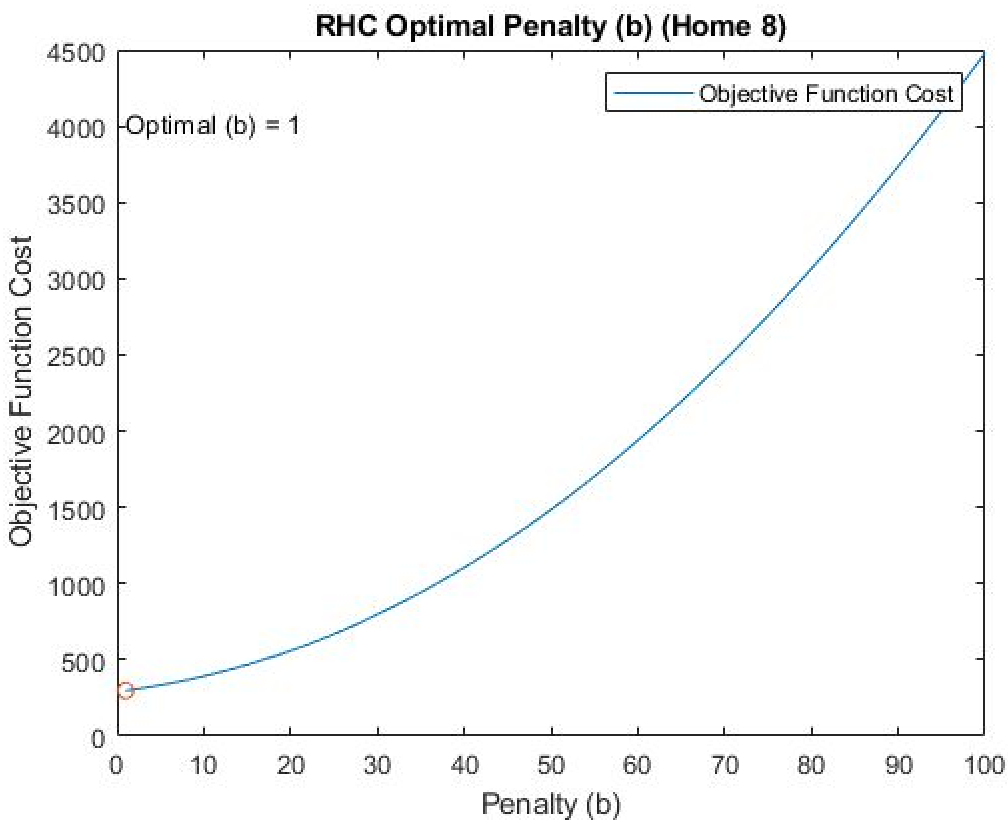

RHC Optimal Penalty (a) (Home 1)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
10
20
30
40
50
60
70
80
90
100
0
500
1000
1500
2000
2500


RHC Optimal Penalty (a) (Home 4)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
10
20
30
40
50
60
70
80
90
100
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000


RHC Optimal Penalty (a) (Home 8)
Objective Function Cost
Optimal (a) = 1
Objective Function Cost
Penalty (a)
0
500
1000
1500
2000
2500
3000
3500
0
10
20
30
40
50
60
70
80
90
100


RHC Optimal Penalty (b) (Home 1)
Objective Function Cost
Optimal (b) = 1
Objective Function Cost
Penalty (b)
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
0
10
20
30
40
50
60
70
80
90
100

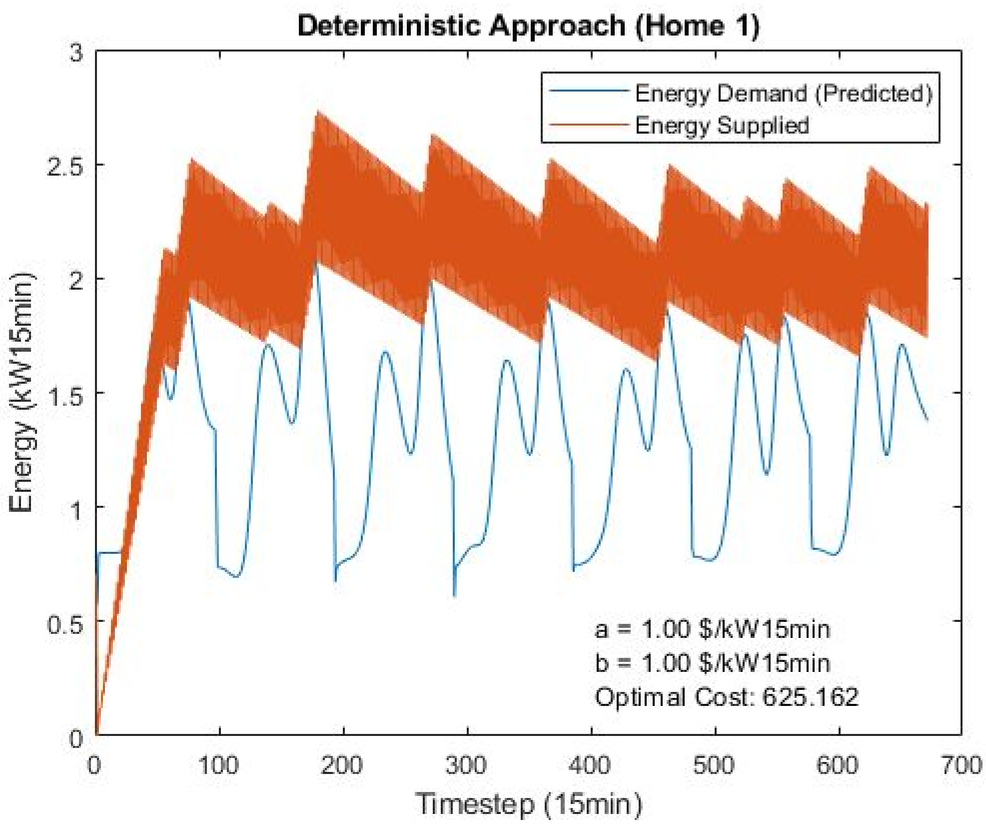

RHC Optimal Penalty (b) (Home 4)
Objective Function Cost
Optimal (b) = 1
Objective Function Cost
Penalty (b)
0
2000
4000
6000
8000
10000
12000
14000
0
10
20
30
40
50
60
70
80
90
100

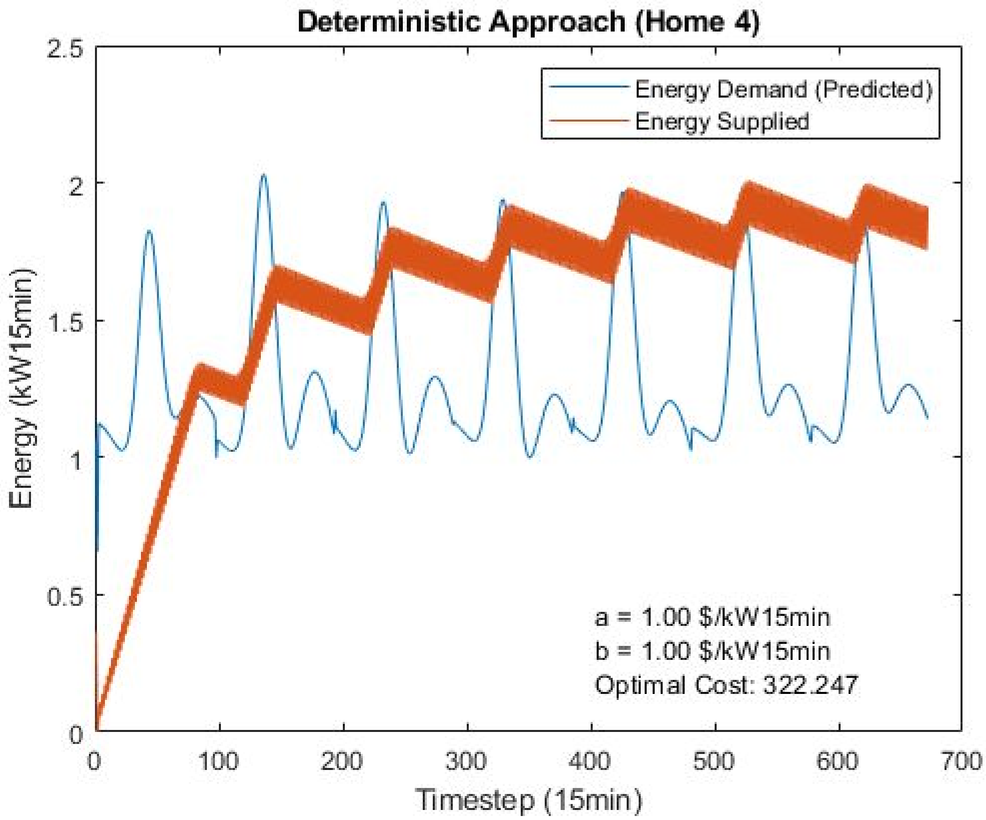

RHC Optimal Penalty (b) (Home 8)
Objective Function Cost
Optimal (b) = 1
Objective Function Cost
Penalty (b)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
0
10
20
30
40
50
60
70
80
90
100

In every case, the optimal values for a and b appear to be $1.00 per kW15min. These are the standard values used for the previous sections, so the results are shown in those figures.

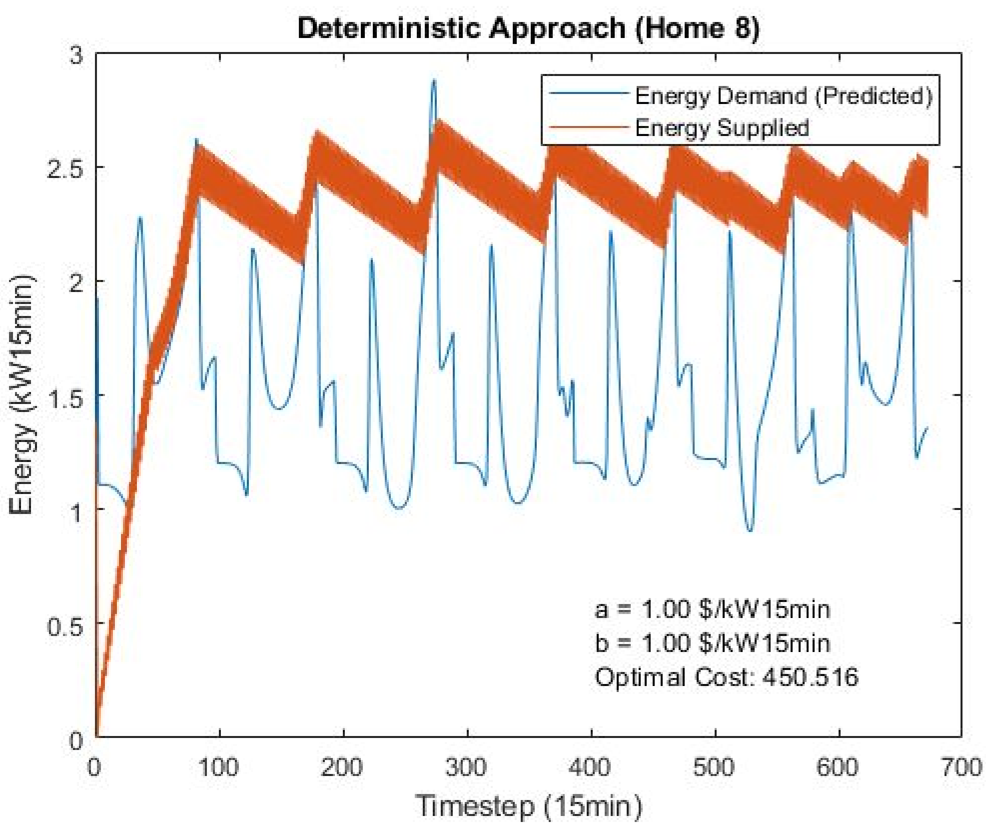
5. Deterministic vs Random Algorithm Selection

We do not expect a combination of algorithms to offer any improvement, as each successive algorithm performs better than the last in every case. However to test whether a deterministic or random approach is viable, we use OGD and RHC. For deterministic, the algorithm simply switches methods with every time step. For the random approach, at every time step a random number generator is used to determine which algorithm should be applied. The results are shown below.

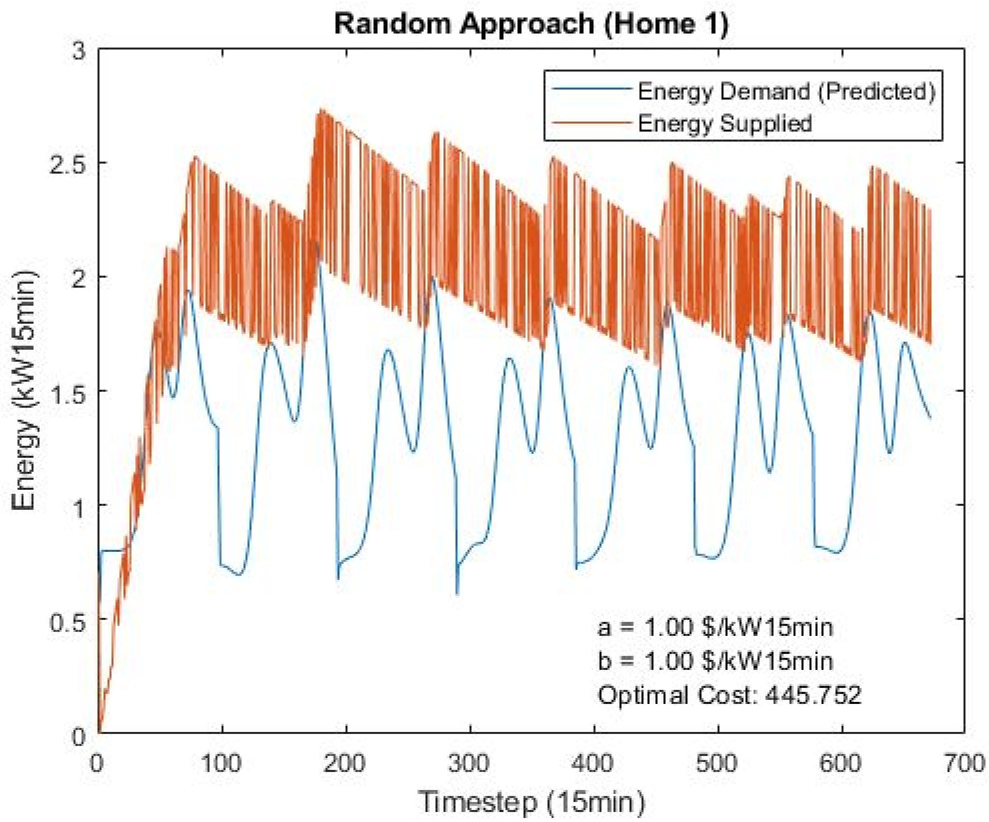




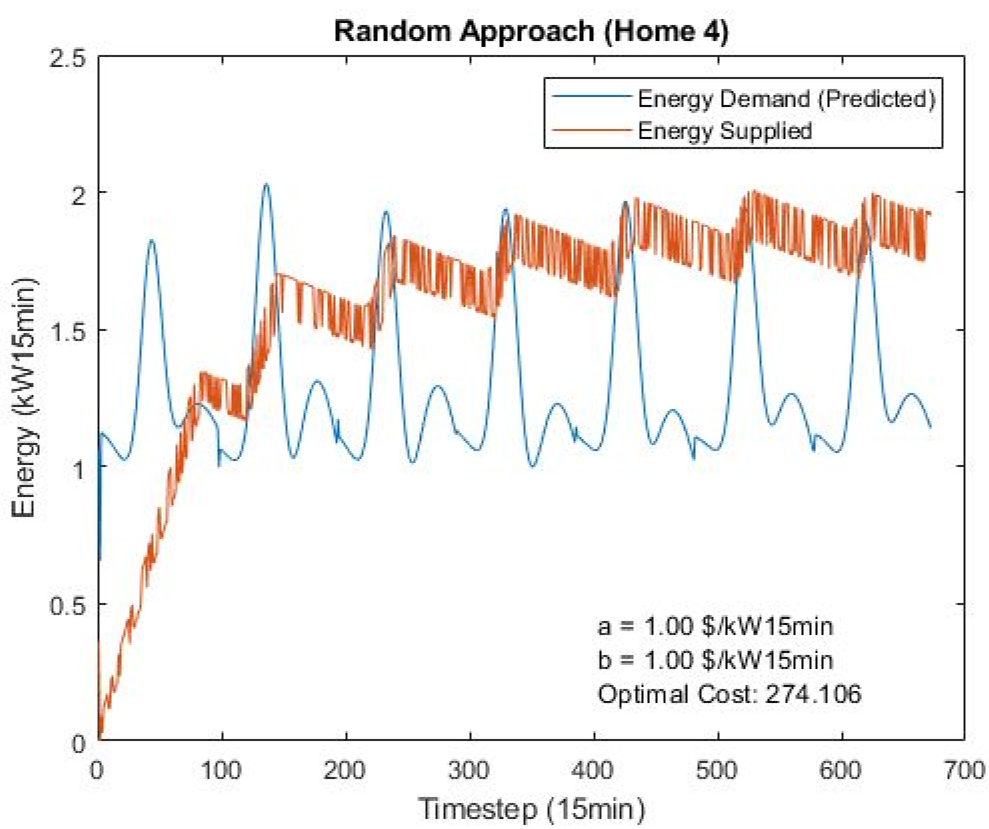
Deterministic Approach (Home 4)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 322.247
Energy (kW15min)
Timestep (15min)



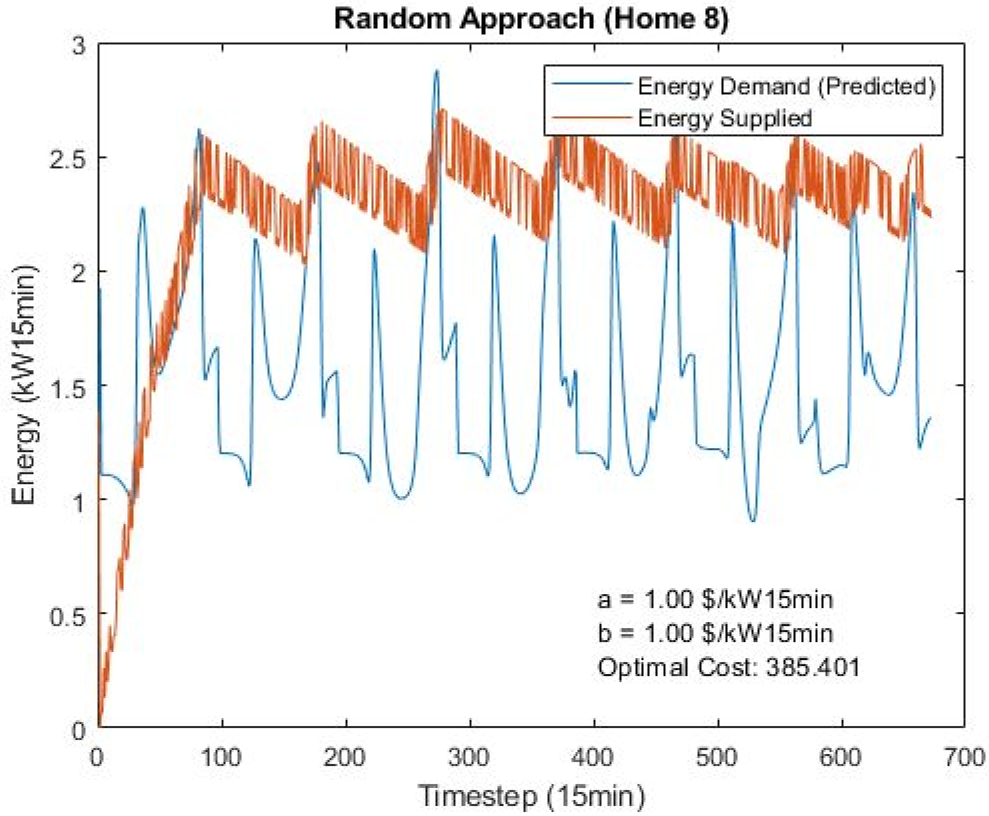
Deterministic Approach (Home 8)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 450.516
Energy (kW15min)
Timestep (15min)



Random Approach (Home 1)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 445.752
Energy (kW15min)
Timestep (15min)



Random Approach (Home 4)
Energy Demand (Predicted)
Energy Supplied
a = 1.00 $/kW15min
b = 1.00 $/kW15min
Optimal Cost: 274.106
Energy (kW15min)
Timestep (15min)



In every case, neither deterministic or random approaches performed better than the individual algorithms on their own. This is due to the massive switching cost incurred by swapping between the different algorithms. A smoother approach may offer better results, one that attempts to average out the changes between each method.