CSE548/AMS542 Fall 2018 Analysis of Algorithms

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Due **Oct 19th** 9pm. Each problem, unless specified otherwise, has a maximum of 10 points. Avoid too many details. A succinct and clean proof is the best. You may use the algorithms we covered in class without referring to the details.

Homework 3

1. Textbook [Kleinberg & Tardos] Chapter 5, page 246, problem #3, #5, #6.
2. Solving recurrences. Find the asymptotic order of the following recurrence, represented in big-Θ notation. (Each subproblem is 5 pts; the last problem is an extra credit problem.)
   (a) $A(n) = 4A(\lfloor n/2 \rfloor + 5) + n^2$
   (b) $B(n) = B(n-4) + 1/n + 5/(n^2+6) + 7n^2/(3n^3+8)$
   (c) $C(n) = n + 2\sqrt{n}C(\sqrt{n})$ Hint: take $H(n) = C(n) + n$.
3. **Square of a matrix.** The square of a matrix A is its product with itself, AA.
   (a) Show that 5 multiplications are sufficient to compute the square of a 2×2 matrix. (5pts)
   (b) What is wrong with the following algorithm for computing the square of an $n \times n$ matrix? (5pts)

   > Use a divide-and-conquer approach as in Strassen's algorithm, except that instead of getting 7 subproblems of size $n = 2$, we now get 5 subproblems of size $n = 2$ thanks to part (a). Using the same analysis as in Strassen's algorithm, we can conclude that the algorithm runs in time $O(n^{\log_2 5})$.

   (c) In fact, squaring matrices is no easier than matrix multiplication. In this part, you will show that if $n \times n$ matrices can be squared in time $S(n) = O(n^c)$, $c \geq 2$, then any two $n \times n$ matrices can be multiplied in time $O(n^c)$. (5pts)
      i. Given two $n \times n$ matrices A and B, show that the matrix $AB + BA$ can be computed in time $3S(n) + O(n^2)$.
      ii. Given two $n \times n$ matrices X and Y , define the $2n \times 2n$ matrices A and B as follows:

      $$A = \begin{pmatrix} X & 0 \\ 0 & 0 \end{pmatrix}; \; B = \begin{pmatrix} 0 & Y \\ 0 & 0 \end{pmatrix}$$

      What is $AB + BA$, in terms of X and Y?
      iii. Using (i) and (ii), argue that the product XY can be computed in time $3S(2n)+O(n^2)$. Conclude that matrix multiplication takes time $O(n^c)$.

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