Electronic Supplementary Material

Pseudocode for the Multi–Harmonic Balance Method (MHBM)

Algorithm S1: Static Configuration (Summary)

Input: cable length L, weight W, stiffnesses EA and EI, node count N, anchor coordinates $(x_{\text{hang}}, z_{\text{hang}})$, and fairlead coordinates $(x_{\text{end}}, z_{\text{end}})$.

Solve a catenary system such that an inextensible cable passes through anchor and fairlead, obtaining parameters (c_1, c_2, T_0, S_ℓ).

Discretize the arc coordinate $S_0 \in [0, L]$ and compute approximate static positions (x_i, z_i).

Estimate static angles ϕ_i^{approx} from finite differences.

Construct initial guesses for strain and angle fields.

Solve the finite-difference static equilibrium system to obtain e_i^0 and ϕ_i^0.

Define static tension $T_i^0 = EA\, e_i^0$.

Algorithm S2: Multi-Harmonic Balance Method (MHBM)

Harmonic representation

Choose the number of harmonics N_h and represent any periodic quantity $q(t)$ by

$$q(t) \approx \frac{q_0}{\sqrt{2}} + \sum_{h=1}^{N_h} \left(q_h^{\cos} \cos(h\omega t) + q_h^{\sin} \sin(h\omega t) \right).$$

Set $N_{\text{h,eff}} = 2N_h + 1$.

Unknown vector and index map

Build the global unknown vector X containing all Fourier coefficients for u, v, e, ϕ at all nodes.

Build an index map linking each coefficient in X to a node and variable.

Initial guess X_0

Use static fairlead angle ϕ_N^0 to project the imposed fairlead motion into tangential and normal components.

Initialise harmonic coefficients for fairlead u, v to match imposed motion.

Initialise strain and angle coefficients from static fields plus small harmonic perturbations.

Residual assembly

For each collocation time point:

- Reconstruct $u_i(t)$, $v_i(t)$, $e_i(t)$ and $\phi_i(t)$ from X.
- Compute spatial derivatives and curvature by finite differences.
- Evaluate tension and bending forces using EA, EI, and (e_i^0, ϕ_i^0).
- Compute fluid-relative velocities and hydrodynamic drag forces.
- Enforce the semi-discrete equations of motion and boundary conditions.

Collect all equations into the global residual vector $R(X)$.

Nonlinear solve

Solve the nonlinear system $R(X) = 0$ using Newton or Levenberg–Marquardt to obtain X_{sol}.

Reconstruction and fairlead tension

For a dense set of times t over one period, reconstruct $u(t)$, $v(t)$, $e(t)$, $\phi(t)$ at the fairlead.

Compute total angle $\phi_{\text{tot}}(t)$ and total strain.

Compute fairlead tension as

$$T_{\text{fairlead}}(t) = T^0_{\text{fairlead}} + EA\, e_{\text{dyn}}(t).$$

Flowchart of the MHBM Workflow