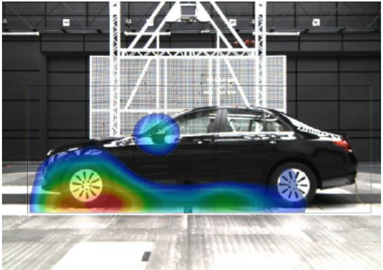
∽ ECL Lecture - Microphone Array Techniques ∼

Sound Source Localization - Beamforming



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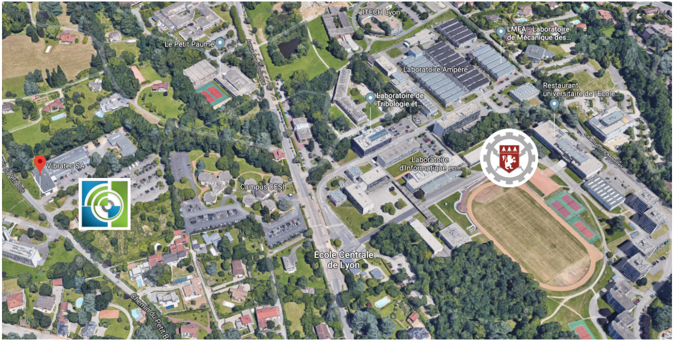
MicrodB

www.microdb.vibratecgroup.com

MicrodB in a nutshell

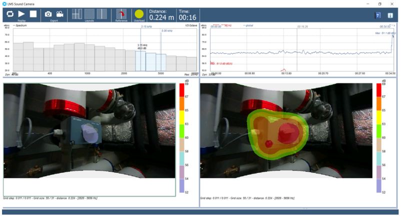
VibraTec	MicrodB	VibraTeam
Acoustic & Vibration Measurement & Simulation	Sound source identification Antennas & Software	Engineering service technical assistance
Since 1986	Since 1994	Since 2005



MicrodB in a nutshell

Industrial products for acoustic imaging

SoundCamera

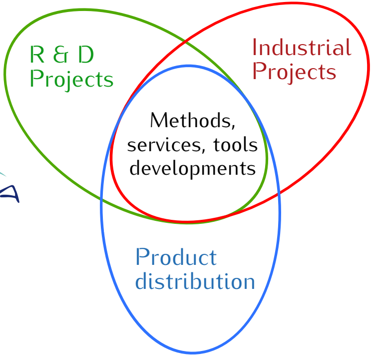


SoundCamera Software

3DCam

MicrodB in a nutshell

From R & D to industrial tool in acoustic imaging



Objectives

This course is dedicated to Sound Source Localization and Quantification using array techniques :

Sound Source Localization (SSL) :

- Acoustic domain aiming at finding the most significative regions of noise emission in an industrial environnement
- The final goal is to reduce the noise emitted by a radiating object under study

Sound Source Quantification (SSQ) :

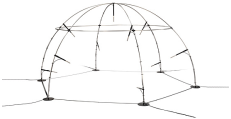
- Additional objective : define the absolute acoustic power of the sources identified in a localization step
- This aims at controlling that sub-elements comply to some acoustics specifications

How to achieve Sound Source Localization & Quantification ?

- Acoustic intensity technique : SSQ, standards
- Hemispheric power measurement : SSQ, standards

- Microphone array techniques : SSL & SSQ, no standard
  - Structure of many microphones in simultaneous recording
  - Signal processing techniques dedicated to draw *source maps*

Sound intensity probe



Hemispheric power measurement

Microphones array

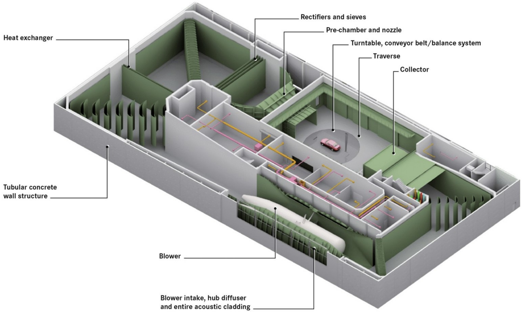
Outline

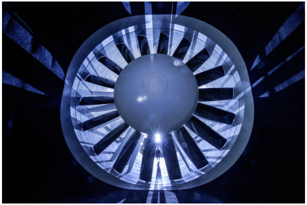
Industrial Needs

Automotive Wind Tunnel

Daimler Facility

- Study and reduce the noise of cars from interior & exterior points of view





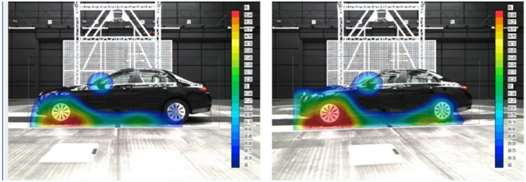
Automotive Wind Tunnel

Array measurements and maps

Daimler aeroacoustic facility

Top and side arrays : ∼ 360 channels

- Several configurations can be visually and quantitatively compared

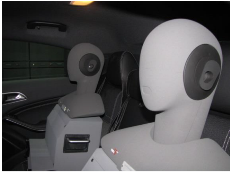


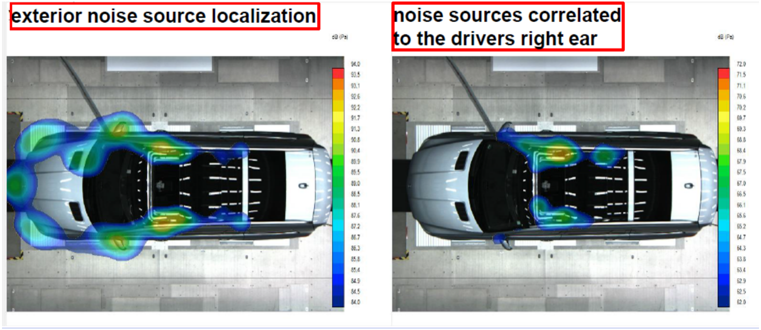
Typical beamforming output maps

Automotive Wind Tunnel

Correlated maps

While arrays are located outside the vehicle, coherence treatments with binaural head microphones can provide information from the interior point of view



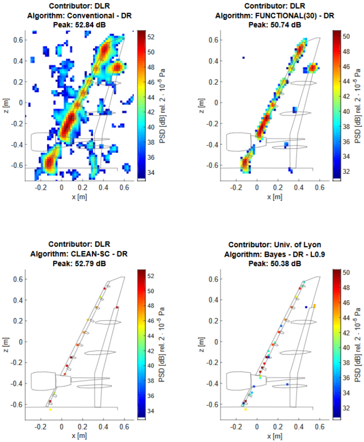


Airframe Noise

DLR Benchmark – Dornier-728 semispan model

Acoustic techniques comparison :

Wind-tunnel test section



PSD source maps at $f = 8496$ Hz

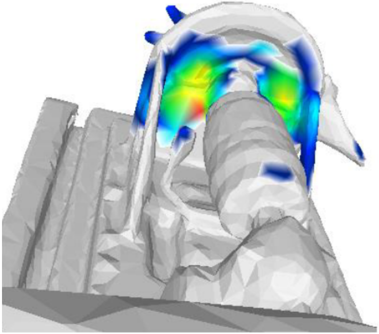
Bahr, C. J. *et al.*, **AIAA**, 2017.

Three-Dimensional Inverse method

Renault Electric motor

Objectives : absolute acoustic sound power per component

Electric motor set-up

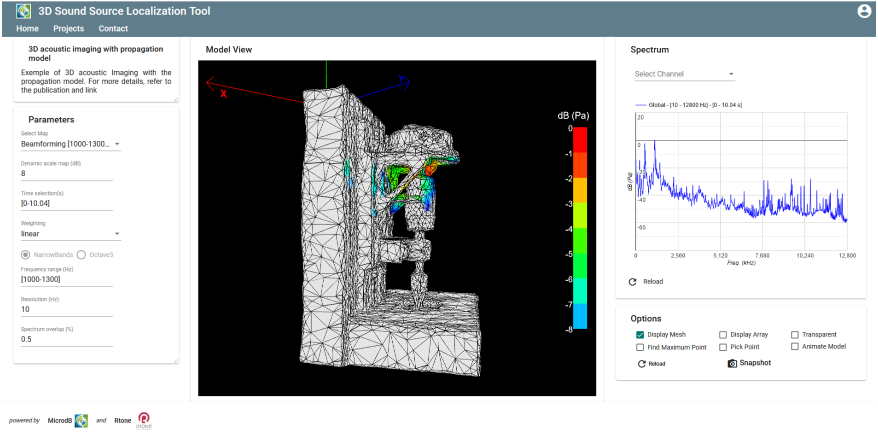


PSD source maps at $f = 1$ kHz

Le Magueresse, T. *et al.*, **BeBeC**, 2020.

Three-Dimensional Inverse method

Renault Electric motor



MicrodB 3D sound source localization web browser viewer

https://saas.microdb.fr/lug/

Three-Dimensional Inverse method

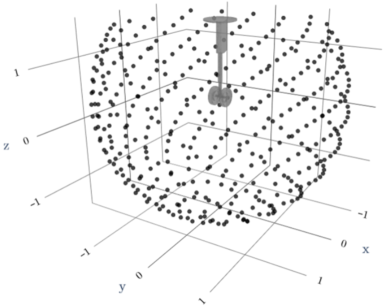
CAA-based acoustic imaging

Landing gear CAA simulation

1. Near field CFD solution
2. FW-H far field propagation to virtual array of microphones
3. Back-propagation on the landing gear 3D mesh

takes into account :

- dipolar aeroacoustic sources
- 3D diffraction
- source correlation



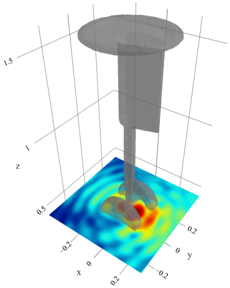
Immersed landing gear and 3D virtual array

Bouley, S. *et al.*, **AIAA**, 2022 (submitted).

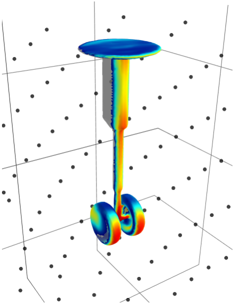
Three-Dimensional Inverse method

CAA-based acoustic imaging

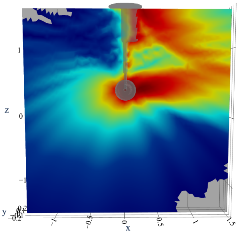
Landing gear CAA simulation



2D cross section acoustic map



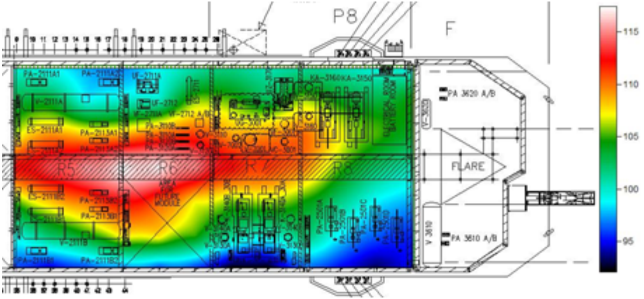
3D mesh back propagation



Repropagation

Overview of Sound Sources Localization Techniques

Acoustic measurement

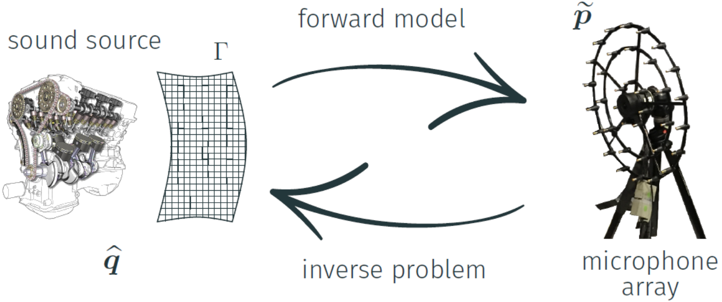


Acoustic map of a factory

The acoustic field is measured with a microphone moved around the radiating device.

- allows to measure directivity of a device
- does not provide sound localization and quantification on the device

Acoustic Imaging



All localization methods require :

- a microphone array
- a source source grid on the radiating device
- a forward model
- a back-propagation method

Classical forward model

Monopole radiation

$$G(\mathbf{r}) = A\,\frac{\exp(i2\pi k\mathbf{r})}{4\pi\mathbf{r}}$$

Tentative Summary

Beamforming $\Leftarrow$ $p = G\,q$ $\Rightarrow$ **Inverse method**

Source dofs processed separately | *Source dofs processed at once*

Data independant
Minimize
$||\mathbf{p} - \mathbf{G}_i q_i||$
↓
Conventional beamforming

Data dependant
$q_i = \mathbf{W}_i \mathbf{p}$
Find $\mathbf{W}_i$ min q_i
Satisfying $\mathbf{W}_i \mathbf{G}_i = 1$
↓
Capon-MVDR
Robust Capon
MUSIC

Tikhonov reg.
Least L_2 norm
Minimize $||\mathbf{q}||$
↓
NAH, SONAH
ESM, HELS
IBEM $\cdots$
↓
Iter. Tikhonov
Least L_p norm
Minimize $||\mathbf{q}||_p$

IRLS, GIB,
Bayesian Focusing

Sparse reg.
Least L_1 norm
Minimize $||\mathbf{q}||_1$
↓
LASSO
BPDN

Sparse heuristics
Minimize $||\mathbf{q}||_0$
↓
Greedy Alg.
CLEAN, RELAX
MP

Deconvolution
Source dofs processed at once
Minimize $||\tilde{\mathbf{q}}^2 - A\mathbf{q}^2||$ *with* $\mathbf{q}^2 > 0$

SEM, DAMAS,
NNLS

↓
sparsity constraints
SC-DAMAS, CLEAN
MP/OMP

Leclère *et al.*, **BeBeC**, 2016.

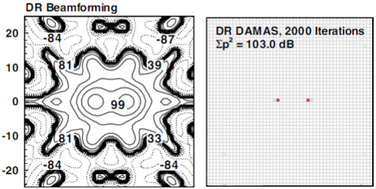
Acoustic Imaging

What distinguish these methods ?

- the inverse problem technique resolution (beamforming, inverse method, iterative technique)
- the forward model

The performances depend on the method choice :

- Computation time (from one second to several hours for one frequency)
- Spatial resolution
- Source quantification accuracy
- Model bias sensitivity



Brooks & Humphrey, **AIAA**, 2006.

Beamforming

Beamforming is the simplest, the most widespread and one of the most robust method for source localization.

Advantages :

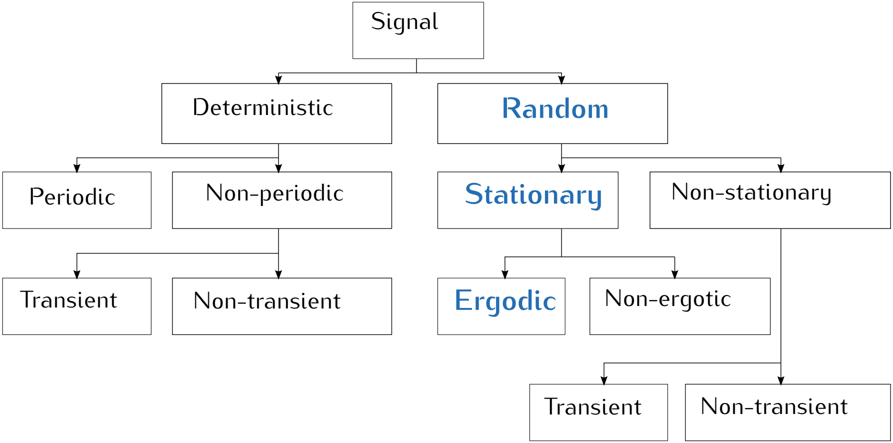
- Versatility : for any geometry of both the array and the calculation grid
- Time or frequency processing
- Robustness
- Implementation simplicity

Drawbacks :

- Non-quantitative if more than one source
- Only decorrelated set of sources considered
- Inaccurate resolution in low frequencies

Signal Processing Prerequisites

Signal Classification



Bendat & Piersol, **Random Data**, 2010.

Basic definitions

A random signal represents one of the many possible observations of a random physical phenomenon that might have occured.

$\Rightarrow$ A time record of a random process cannot be analytically described.

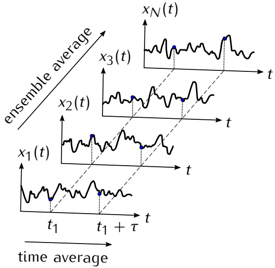
An ***ensemble*** of time records can be studied through its statistical properties :

- mean value at time t : $\mu_x(t)$
- autocorrelation at two time $t + \tau$: $R_{xx}(t, t + \tau)$

If $\mu_x(t) = \mu_x$ and $R_{xx}(t, t + \tau) = R_{xx}(\tau)$, the random process is *weakly* **stationary**.

A **stationary** random process is **ergodic** if the ***ensemble*** and the ***time*** averages produce the same results.

One can draw statistics over time from one signal instead of repeating N times the random process.



Spectral Density Functions

Fundamental tools in signal processing are the spectral density functions :

- Power Spectral Density : $S_{xx}(\omega) \in \mathbb{R}$
- Cross Power Spectral Density : $S_{xy}(\omega) \in \mathbb{C}$

Wiener–Khinchin Theorem

$S_{xx}(\omega) = \mathrm{FT}\{R_{xx}(\tau)\}, \quad S_{xy}(\omega) = \mathrm{FT}\{R_{xy}(\tau)\},$ for ergodic stationary signals

- $|S_{xy}(\omega)|$: coherent energy spread over signals $x(t)$ and $y(t)$ at frequency ω
- $\varphi(S_{xy}(\omega))$: phase relationship between the coherent part of signals $x(t)$ and $y(t)$ at frequency ω

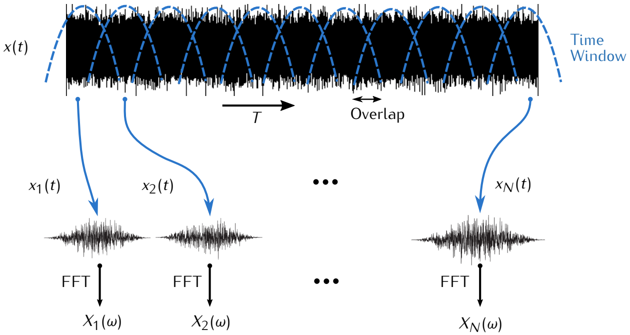
The cross power spectral density (cross-spectrum) reveals information on the acoustic field acquired coherently by distinct microphones.

(C)PSD Estimation

Welch's Method

As signals are time finite and noisy, only a PSD estimation is obtainable. A usual method is the Welch process to compute a periodogram :

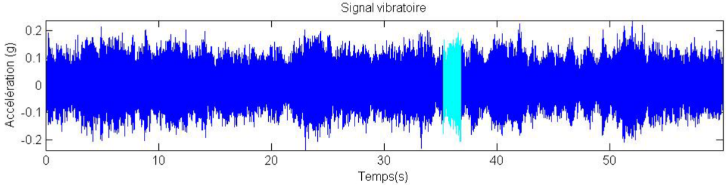
$$\widehat{S}_{xx}(\omega) = \langle X(\omega)X^H(\omega)\rangle = \langle |X(\omega)|^2\rangle, \quad \widehat{S}_{xy}(\omega) = \langle X(\omega)Y^H(\omega)\rangle.$$



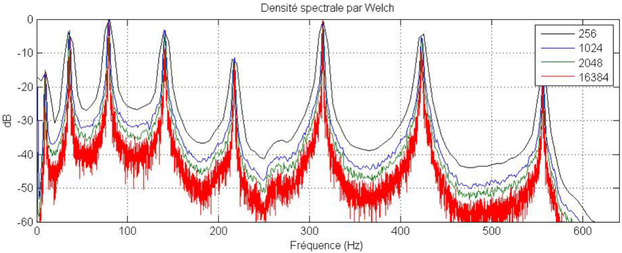
$$\widehat{S}_{xx}(\omega) = \frac{1}{N}\sum_{i=1}^{N} |X_i(\omega)|^2.$$

(C)PSD Estimation

Time block size influence



$$\Delta f = 1/T_{\text{obs}}$$



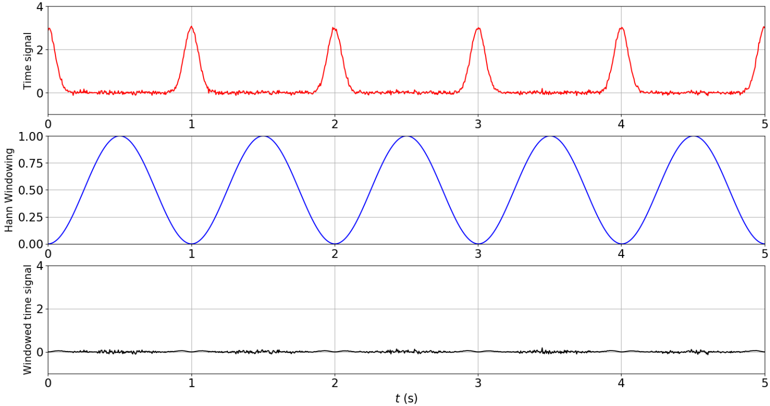
The larger the block size, the less the signal is averaged.

A well-adapted time block size reduces the PSD variance without too much over-estimation.

Ollivier, **Analyse de signaux aléatoires**, 2014.

(C)PSD Estimation

Overlap influence

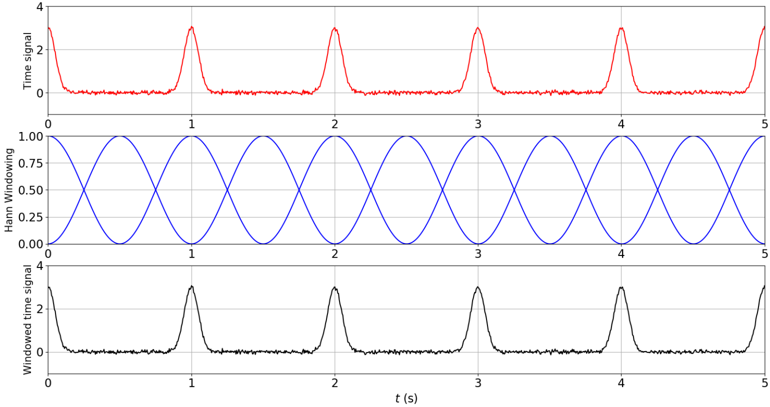


Time signal, *Hann time windowing*, *Windowed time signal*

Necessity to overlap the snapshots to avoid information loss.

(C)PSD Estimation

Overlap influence



Time signal, *Hann time windowing*, *Windowed time signal*

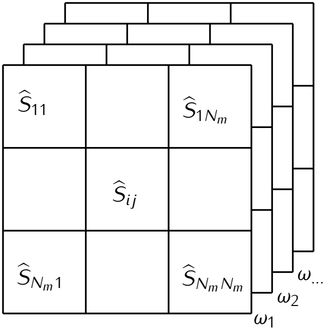
With a well-designed overlap (here 50 %), all the events of the time signal are considered.

(C)PSD Estimation

For the cross power spectral density :

$$\widehat{S}_{xy}(\omega) = \frac{1}{N}\sum_{i=1}^{N} X_i(\omega) Y_i^H(\omega).$$

All the information used in most array techniques is carried by a Cross Spectral Matrix (CSM), noted S_{pp}, which contains the measured data.



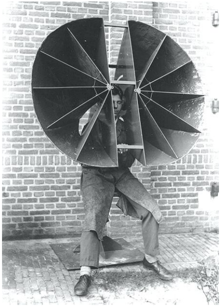
Array processing's goal :

$\Rightarrow$ recover the sources positions from S_{pp} and the geometrical configuration.

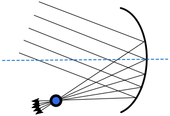
Beamforming

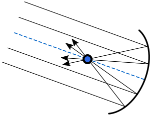
Beamforming

Basic Principle



Acoustic mirror - Royal Air Force, Denge, UK





As for radio waves, parabolic antennas were dedicated to hear environmental noise. The device is either steered in a certain direction or the microphone is moved to the focus point. A ***beam*** is ***formed*** in this direction.

Time Formulation

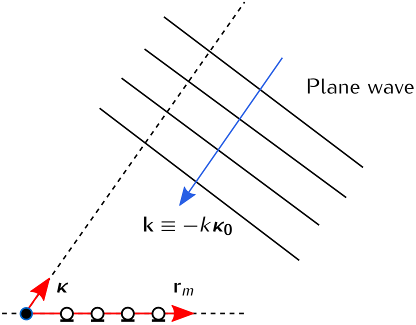
Delay and Sum Beamformer

$$p_m(t) = p_0\left(t - \Delta_m(\boldsymbol{\kappa_0})\right)$$

For a focus angle κ :

1. Compensate the delays measured by the microphones due to the focus direction angle κ
2. Sum the delayed microphone signals

$$b(\boldsymbol{\kappa}, t) = \frac{1}{N_m}\sum_{m=1}^{N_m} p_m\left(t + \Delta_m(\boldsymbol{\kappa})\right),$$



$$b(\boldsymbol{\kappa}, t) = \frac{1}{N_m}\sum_{m=1}^{N_m} p_0\left(t - \frac{(\kappa_0 - \kappa)\cdot \mathbf{r}_m}{c_0}\right)$$

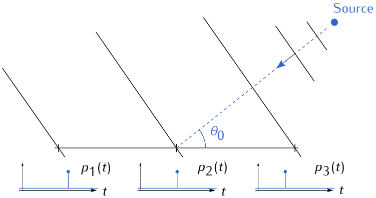
$$\Delta_m(\boldsymbol{\kappa}) = \frac{\boldsymbol{\kappa}\cdot \mathbf{r}_m}{c_0}$$

The beamformer $b(\boldsymbol{\kappa}, t)$ seeks the direction where its value is the highest, when the delayed pressures are summed up ***coherently*** $(\kappa = \kappa_0)$.

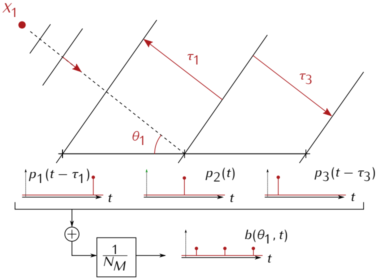
Time Formulation

Delay and Sum Beamformer

For a ground truth angle θ_0 :



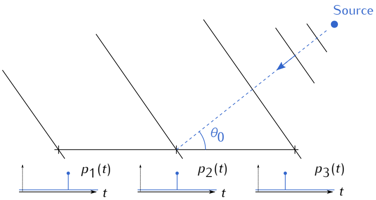
Test a focus angle θ_1 :



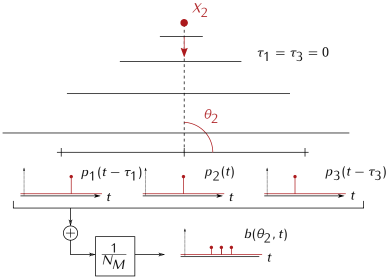
Time Formulation

Delay and Sum Beamformer

For a ground truth angle θ_0 :



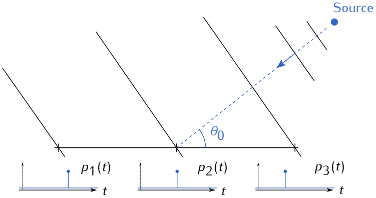
Test a focus angle θ_2 :



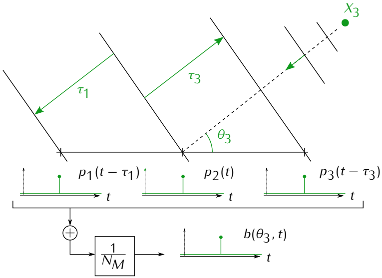
Time Formulation

Delay and Sum Beamformer

For a ground truth angle θ_0 :



Test a focus angle θ_3 :



Time Formulation

Basic propagation model

Far-field / Near-field expressions

Far-Field (Fraunhofer region)

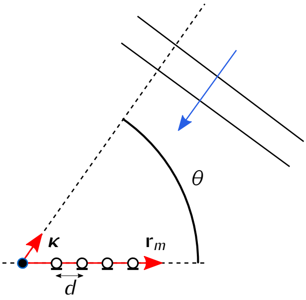
$$\Delta_m = \frac{md}{c_0}\cos\theta_m$$

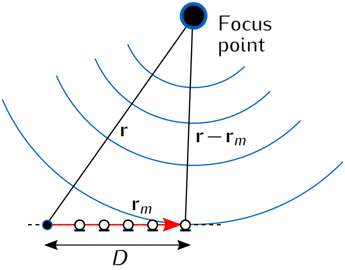
Near-Field (Fresnel region)

$$\Delta_m = \frac{|\mathbf{r}| - r_m(\mathbf{r})}{c_0}$$

Fresnel region :

$$r < \frac{D^2}{\lambda}$$





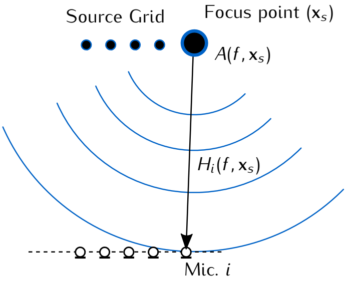
Conventional Beamforming

Frequency Formulation

Source Location

$\mathbf{P} = [P_i(f)] \in \mathbb{C}^{N_m}$: pressure values measured by microphones, indexed i at frequency f for a given snapshot.

Suppose a single source with complex amplitude $A(f)$ for this snapshot and a known transfer function $H_i(f, \mathbf{x}_s)$.



Direct propagation : $\mathbf{P} = A\mathbf{H}$

For a fixed frequency f, we seek the source point $\mathbf{x}_s$ so that $\mathbf{P}(f)$ and $\mathbf{H}(f, \mathbf{x}_s)$ are proportional, i.e. co-linear, done by computing the normalized cross-product between the two (complex) vectors :

$$\gamma^2(\mathbf{x}_s) = \frac{|\mathbf{H}^H(\mathbf{x}_s)\mathbf{P}|^2}{||\mathbf{H}(\mathbf{x}_s)||^2||\mathbf{P}||^2}, \quad \text{coherence indicator.}$$

Elias, **VKI**, 1997.

Coherence Indicator for random sources

For random sources, the pressure is averaged on the N snapshots to drop incoherent noise out.

$$< |\mathbf{H}^H\mathbf{P}|^2 >=< \sum_{i,j=1}^{N_m} H_i^H P_i P_j^H H_j >= \sum_{i,j=1}^{N_m} H_i^H < P_i P_j^H > H_j = \sum_{i,j=1}^{N_m} H_i^H \widehat{S}_{ij} H_j = \mathbf{H}^H \widehat{\mathbf{S}}_{pp}\mathbf{H}$$

$$< ||\mathbf{P}||^2 >=< \sum_{i=1}^{N_m} |P_i|^2 >= \sum_{i=1}^{N_m} < |P_i|^2 >= \sum_{i=1}^{N_m} \widehat{S}_{ii}^2 = \mathrm{Tr}(\widehat{\mathbf{S}}_{pp}), \quad \mathrm{Tr} : \text{matrix trace}$$

The localization function for random sources reads :

$$\boxed{\gamma^2(\mathbf{x}_s) = \frac{\mathbf{H}^H(\mathbf{x}_s)\widehat{\mathbf{S}}_{pp}\mathbf{H}(\mathbf{x}_s)}{||\mathbf{H}(\mathbf{x}_s)||^2\mathrm{Tr}(\widehat{\mathbf{S}}_{pp})}, \quad 0 \leq \gamma^2 \leq 1.}$$

Conventional Beamforming

Source Amplitude

The objective of finding the acoustic source location and its amplitude is a simple minimization problem :

For a given measurement vector $\mathbf{P}$ and a known library of transfer vectors $\mathbf{H}$, find A which minimize the quadratic error :

$$\varepsilon(A) = ||\mathbf{P} - A\mathbf{H}||^2.$$

Solution :

$$\varepsilon(A) = (\mathbf{P} - A\mathbf{H})^H(\mathbf{P} - A\mathbf{H}) = \mathbf{P}^H\mathbf{P} - A\mathbf{P}^H\mathbf{H} - A^H\mathbf{H}^H\mathbf{P} + A^H A\mathbf{H}^H\mathbf{H}.$$

We are free to use A and A^H as independent variables. For a fixed value of $\mathbf{x}_s$:

$$\frac{\partial \varepsilon(\widehat{A})}{\partial A^H} = 0 \Leftrightarrow -\mathbf{H}^H\mathbf{P} + \widehat{A}\mathbf{H}^H\mathbf{H} = 0,$$

$$\widehat{A}(\mathbf{x}_s) = \frac{\mathbf{H}^H(\mathbf{x}_s)\mathbf{P}}{||\mathbf{H}(\mathbf{x}_s)||^2}$$

Conventional Beamforming

Source Amplitude

With this amplitude, the value of the residue ε is :

$$\varepsilon(A) = ||\mathbf{P}||^2 - \frac{|\mathbf{H}^H(\mathbf{x}_s)\mathbf{P}|^2}{||\mathbf{H}(\mathbf{x}_s)||^2}$$

The source location is then where ε is minimum, or for which

$$\gamma^2(\mathbf{x}_s) = 1 - \frac{\varepsilon}{||\mathbf{P}||^2}, \quad \text{is maximum.}$$

For random stationnary sources, the mean squared amplitude reads :

$$\boxed{\sigma^2(\mathbf{x}_s) =< |\widehat{A}(\mathbf{x}_s)|^2 >= \frac{\mathbf{H}^H(\mathbf{x}_s)\widehat{S}_{pp}\mathbf{H}(\mathbf{x}_s)}{||\mathbf{H}(\mathbf{x}_s)||^4}}$$

Beamforming is basically a vector-matrix-vector product between :

- The measured data cross spectral matrix $\widehat{S}_{pp}$
- The model transfer vector $\mathbf{H}$

The principle of Delay and Sum is recovered in $\mathbf{H}^H(\mathbf{x}_s)\mathbf{P}$ through the conjugate operator (time delays) and the vector–vector product (sum).

Conventional Beamforming

Summary

To do it properly, the SSL and SSQ processes should be done in two steps :

- Localize the source : find the maximum position $\mathbf{x}_{\max}$ of the coherence indicator map :

$$\boxed{\gamma^2(\mathbf{x}_s) = \frac{\mathbf{H}^H(\mathbf{x}_s)\widehat{\mathbf{S}}_{pp}\mathbf{H}(\mathbf{x}_s)}{||\mathbf{H}(\mathbf{x}_s)||^2 \mathrm{Tr}(\widehat{\mathbf{S}}_{pp})}, \quad 0 \leq \gamma^2 \leq 1.}$$

- Estimate the source amplitude in $\mathbf{x}_{\max}$:

$$\boxed{\sigma^2(\mathbf{x}_s) = \frac{\mathbf{H}^H(\mathbf{x}_s)\widehat{S}_{pp}\mathbf{H}(\mathbf{x}_s)}{||\mathbf{H}(\mathbf{x}_s)||^4}.}$$

In practice, software and acoustic teams often prefer to look at a single map : $\sigma^2(\mathbf{x}_s)$.

Linear array performances for far-field acoustic wave

2D Configuration

Here a linear array includes N microphones over a length L.

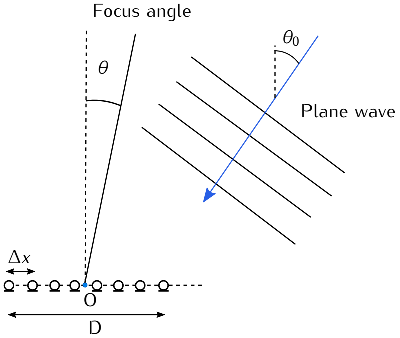
The pressure field due to a plane wave, measured by the microphone n is :

$$p_n = \exp(ikx_n \sin\theta_0).$$

$$H_n(\theta) = \exp(ikx_n \sin\theta).$$

$$\gamma^2(\theta_0, \theta) = \frac{|\mathbf{H}^H(\theta)\mathbf{P}(\theta_0)|^2}{||\mathbf{H}(\theta)||^2||\mathbf{P}(\theta_0)||^2} = \left(\frac{\sin z}{N\sin(z/N)}\right)^2,$$

$$\text{with } z = \pi\frac{L}{\lambda}\frac{N}{N-1}(\sin\theta - \sin\theta_0).$$



Linear array performances for far-field acoustic wave

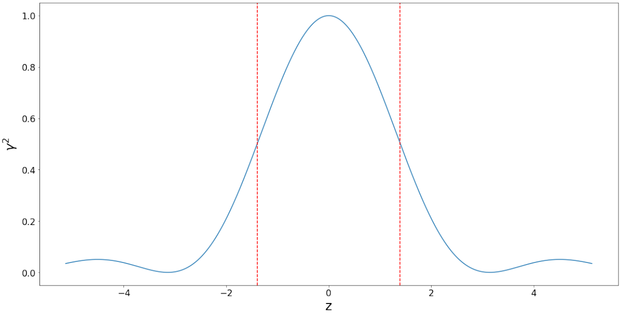
Resolution

Parameters

L = 1 m

N = 10 microphones

f = 500 Hz



Resolution = ability to discriminate two close sources

= overall width of the main lobe 3 dB (50 %) under the maximum.

Linear array performances for far-field acoustic wave

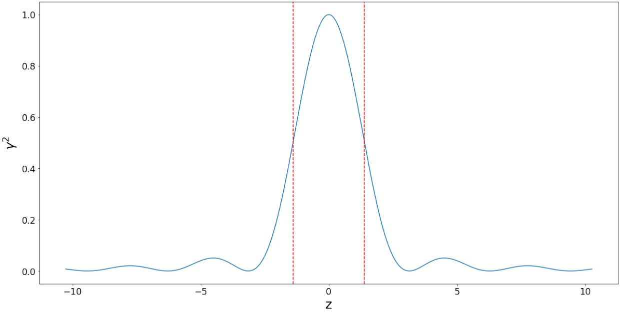
Resolution

Parameters

L = 1 m

N = 10 microphones

f = 1000 Hz



$$\Delta\theta_{3\mathrm{dB}} \simeq 0.9\frac{\lambda}{L\cos\theta_0}$$

The resolution is inversely proportional of the frequency

Linear array performances for far-field acoustic wave

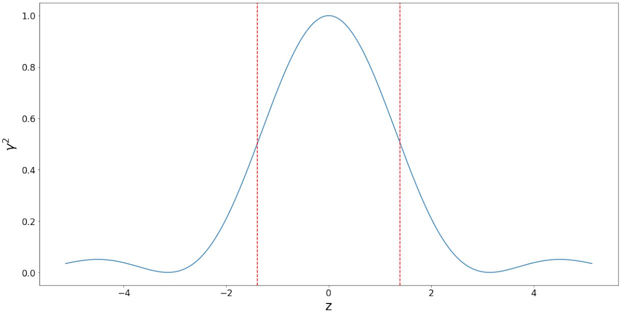
Resolution

Parameters

L = 1 m

N = 10 microphones

f = 500 Hz



Resolution = ability to discriminate two close sources

= overall width of the main lobe 3 dB (50 %) under the maximum.

Linear array performances for far-field acoustic wave

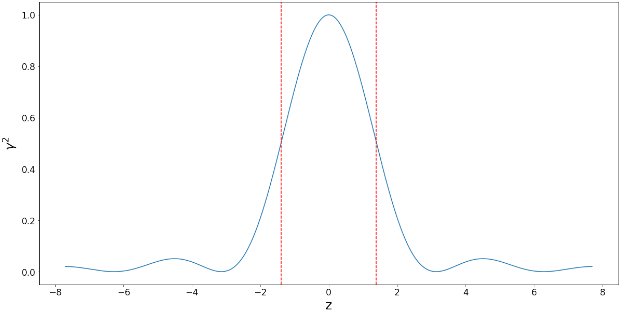
Resolution

Parameters

L = 1.5 m

N = 10 microphones

f = 500 Hz



$$\Delta\theta_{3\mathrm{dB}} \simeq 0.9\frac{\lambda}{L\cos\theta_0}$$

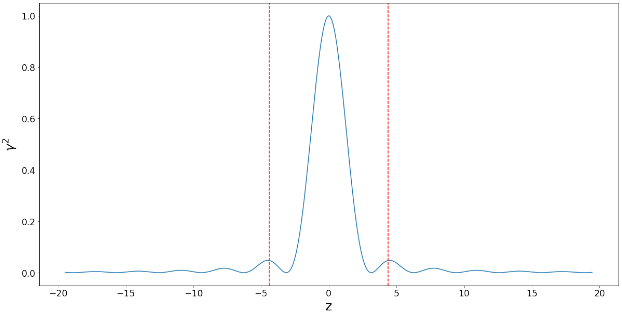
Wide array improves the spatial resolution

Linear array performances for far-field acoustic wave

Dynamics

$\gamma^2(z)$ has side lobes, with a maximum around $z = \frac{3\pi}{2}; \frac{5\pi}{2}; ....$

If a second source of small amplitude is present, it may be hidden by the side lobes of the first one.



Dynamics = Difference between main lobe and the higher side lobe amplitudes

Linear array performances for far-field acoustic wave

Spatial aliasing

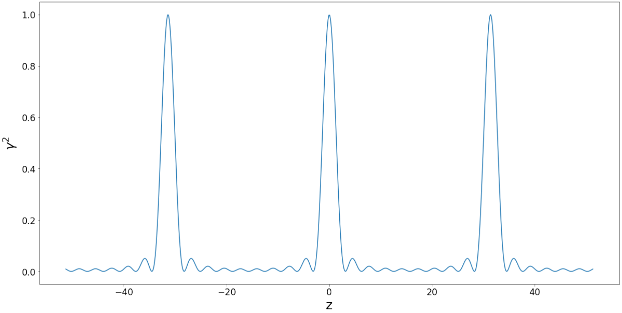
Due to its periodicity, $\gamma^2(z)$ is equal to unity for $\sin\theta = \sin\theta_0 + m\frac{\lambda}{\Delta x}$.

Parameters

`L = 1 m`

`N = 10 microphones`

`f = 5000 Hz`



Ambiguous localisation, the same pressure is measured by all microphones

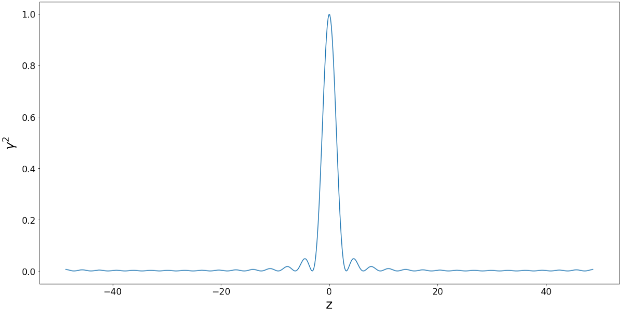
Linear array performances for far-field acoustic wave

Spatial aliasing

Due to its periodicity, $\gamma^2(z)$ is equal to unity for $\sin\theta = \sin\theta_0 + m\frac{\lambda}{\Delta x}$.

Parameters

```
L = 1 m
N = 20 microphones
f = 5000 Hz
```



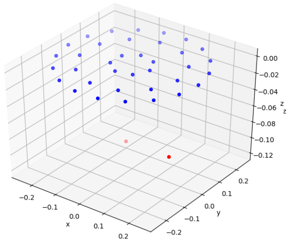
$$\left|\sin\theta_0 + m\frac{\lambda}{\Delta x}\right| > 1, \quad \Delta x < \frac{\lambda}{2}.$$

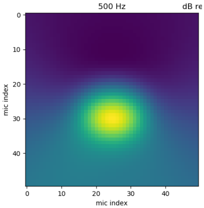
Shannon theorem for spatial sampling

2D Beamforming

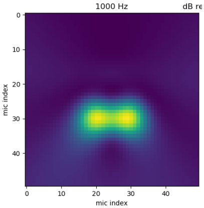
Resolution & Dynamics performances

Circle-Plane array
D = 25 cm
N = 36 microphones
2 sources of same amplitude

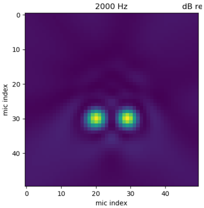




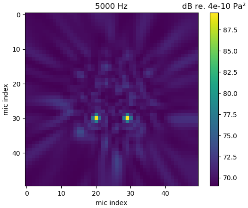
500 Hz



1 kHz



2 kHz

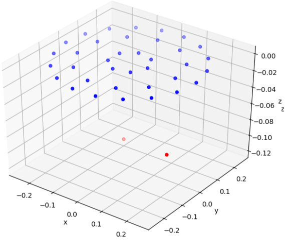


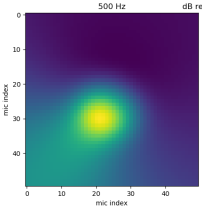
5 kHz

2D Beamforming

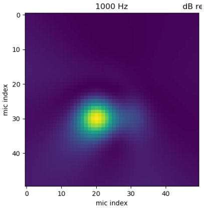
Resolution & Dynamics performances

```
Circle-Plane array
D = 25 cm
N = 36 microphones
2 sources of inegal amplitudes
```

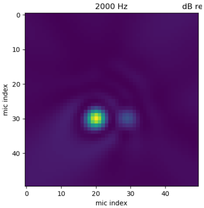




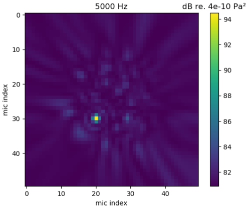
500 Hz



1 kHz

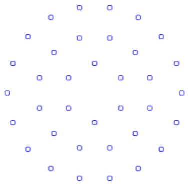


2 kHz

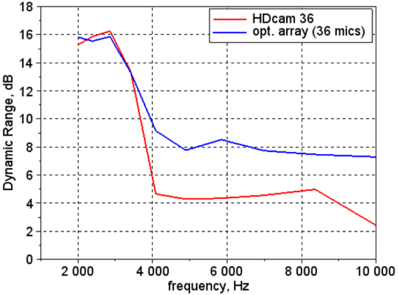


5 kHz

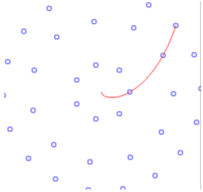
Array Optimization



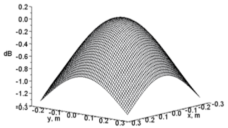
HDcam 36



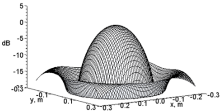
4 dB gain



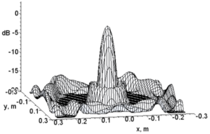
Optimal Array



< 2 *kHz*



3 kHz



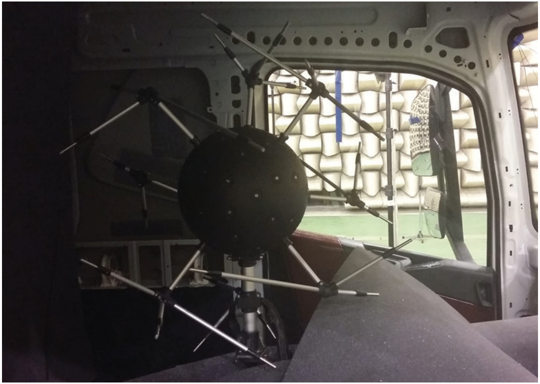
8 kHz

Dynamic optimization = optimal distribution to reduce maximum side lobe level

3D Beamforming

Internal acoustic localization

A 2D plane array cannot distinguish if the acoustic waves are propagating in front of or behind it.



Spherical 3Dcam inside truck cabin

⇒ For internal acoustic localization, a 3D array is required to recover the complete acoustic field

3D Beamforming

Application

Internal acoustic source localization in a car cabin, in a wind tunnel.

Identify noise sources

Test modifications

Evaluate influence of modifications

Enhancement of beamforming method

Deconvolution

Beamforming

Advantages	Drawbacks
Robustness	No quantitative if more than one source
Short computation time	Large resolution for low frequencies
Precise resolution for high frequencies	Presence of side lobes
	Decorrelated sources hypothesis

Deconvolution methods

$\Rightarrow$ Remove the array response (PSF) of the beamforming maps

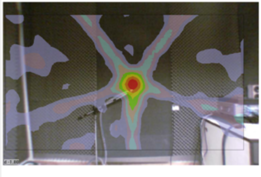
(Methods : DAMAS, CLEAN, CLEAN-SC, CIRA...)

Point Spread Function (PSF) = Response of an imaging system to a point source, system's impulse response

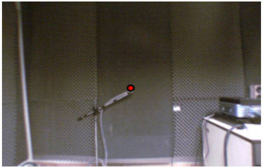
Deconvolution

Main principle

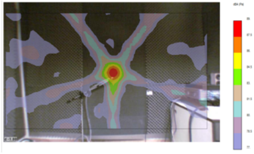
Array



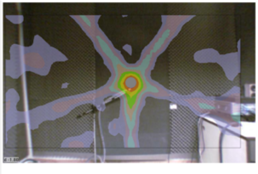
1 – Initial CBF



2 – Maximum selection



3 – CBF from max. only



4 – Source correlated artifacts

5 – Deconvolution result (1-4)

Principle

The side lobes correlated to the maximum sources are iteratively subtracted from the initial map.

How to take into account physical transfer functions ?

In a reverberant environment, the acoustic pressure measured by microphones is modified by diffraction / reflection / scattering phenomena.

Beamformer

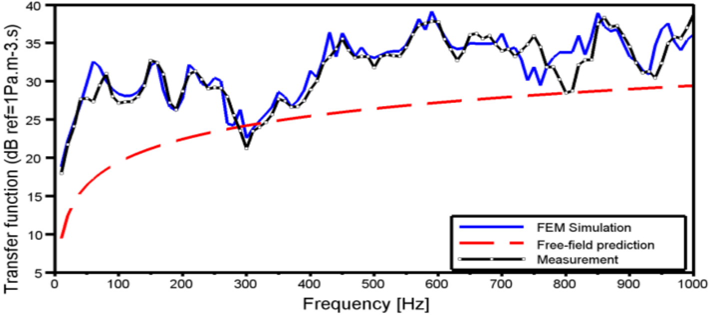
$$\sigma^2(\mathbf{x}_s) = \frac{\mathbf{H}^H(\mathbf{x}_s)\widehat{S}_{pp}\mathbf{H}(\mathbf{x}_s)}{||\mathbf{H}(\mathbf{x}_s)||^4}$$

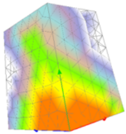
To consider a true environment where the free-fields hypothesis is no longuer appropriate, the transfer function $\mathbf{H}$ must be changed.

How to take into account physical transfer functions ?

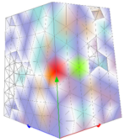
To estimate the acoustic transfert functions, three methods :

- Measurement : needs to place a calibrated source on all the surface, defined as a grid (at least hundreds of points)
- Analytical model : only for simplest case : reflecting plane, sphere, cylindrical or annular duct...
- Numerical simulation : mostly FEM, still costly and time-consuming





Free-fields



Calculated TF

How to take into account source correlation ?

Inverse methods

Naive solution : Moore-Penrose pseudo inverse $G^\dagger$

$$p = Gq \Rightarrow \tilde{q} = G^\dagger p \quad ?$$

- all sources are recovered at once
- correct if G is well-conditioned
- very unstable if not

Need for regularization

- The regularized solution is not solution to $p = Gq$ but to

$$\tilde{\mathbf{q}} = \mathrm{argmin}||\mathbf{p} - \mathbf{G}\mathbf{q}||_2^2 - \lambda^2||\mathbf{q}||_2^2, \quad \tilde{\mathbf{q}} \in \mathbb{C}$$

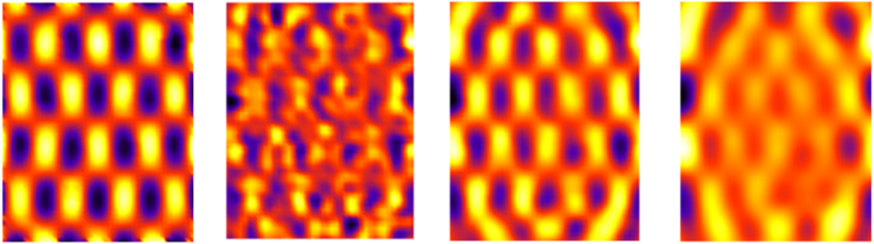
- The regularization parameter λ allows to control the energy of the seeked solution.
- the cross spectral matrix of the sources $S_{qq} = \tilde{\mathbf{q}}\tilde{\mathbf{q}}^H$ contains all cross-correlations between sources

How to take into account source correlation ?

Inverse methods

$$\tilde{\mathbf{q}} = \mathrm{argmin}||\mathbf{p} - \mathbf{G}\mathbf{q}||_2^2 - \lambda^2||\mathbf{q}||_2^2,$$

The regularization parameter λ allows to control the energy of the seeked solution.



1. Reference : vibrating plate with highly correlated sources (modes)
2. $\lambda = \lambda_{\mathrm{opt}}/10$: not enough regularization, unstable solution
3. $\lambda = \lambda_{\mathrm{opt}}$: optimal reconstruction with edge effects
4. $\lambda = 10\lambda_{\mathrm{opt}}$: too much regularization, blurred solution

Conclusion

Conclusion

A leafy tree of methods...

Beamforming $\Leftarrow$ $p = G\, q$ $\Rightarrow$ **Inverse method**

Source dofs processed separately

Data independant
Minimize
$||\mathbf{p} - \mathbf{G}_i q_i||$
↓
Conventional beamforming

Data dependant
$q_i = \mathbf{W}_i \mathbf{p}$
Find $\mathbf{W}_i$ min q_i
Satisfying $\mathbf{W}_i \mathbf{G}_i = 1$
↓
Capon-MVDR
Robust Capon
MUSIC

Source dofs processed at once

Tikhonov reg.
Least L_2 norm
Minimize $||\mathbf{q}||$
↓
NAH, SONAH
ESM, HELS
IBEM $\cdots$
↓
Iter. Tikhonov
Least L_p norm
Minimize $||\mathbf{q}||_p$

IRLS, GIB,
Bayesian Focusing

Sparse reg.
Least L_1 norm
Minimize $||\mathbf{q}||_1$
↓
LASSO
BPDN

Sparse heuristics
Minimize $||\mathbf{q}||_0$
↓
Greedy Alg.
CLEAN, RELAX
MP

Deconvolution
Source dofs processed at once
Minimize $||\tilde{\boldsymbol{q}}^2 - \boldsymbol{A}\boldsymbol{q}^2||$ *with* $\boldsymbol{q}^2 > 0$

SEM, DAMAS,
NNLS

↓
sparsity constraints
SC-DAMAS, CLEAN
MP/OMP

Leclère *et al.*, **BeBeC**, 2016.

Conclusion

Beamforming is the simplest, the most widespread and one of the most robust method for source localization.

It represents an excellent first tool to deal with acoustic imaging issues.

BUT

- No quantitative if more than one source
- Presence of sides lobes
- Uncorrelated sources hypothesis (inexact directivity reconstruction)
- Frequency limitations :
  - High frequency : aliasing due to inter-microphone step (Shannon theorem)
  - Low frequency : resolution (depend on the array size)

All those limitations can be filled by advanced methods, often dedicated to specific experimental configurations.

Any questions ?

Jupyter Notebook / PDF available at : github.com/sbouley

Remarks / questions :

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