Systèmes robotisés intelligents
Smart Robotic Systems

Locomotion

Gilles TAGNE

Modeling of wheeled mobile robots

Modeling a robot is among other things establishing the differential equations that characterize the evolution of its parameters and variables over time

Modeling of wheeled mobile robots

Goals of modeling:

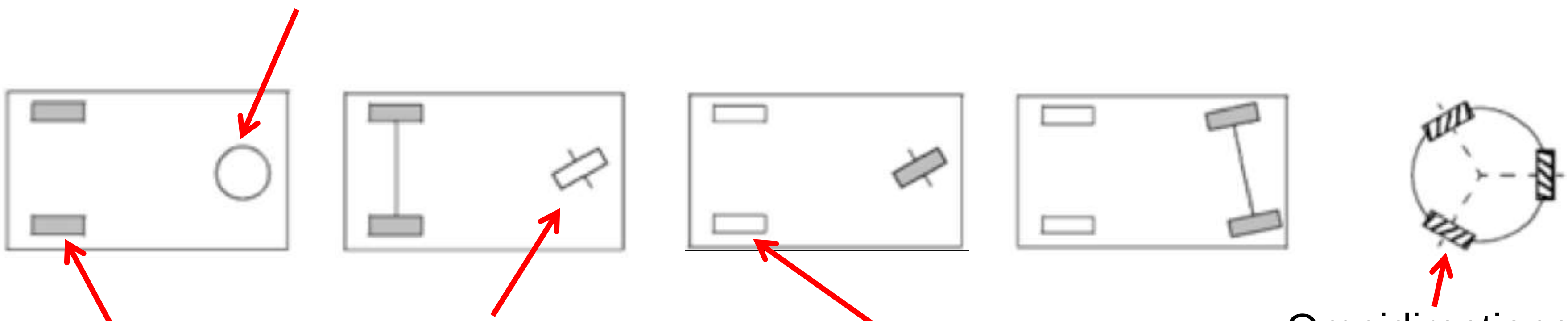
- Describe the behavior of the robot
- Plan its movements (its trajectory)
- Easily control its movements

Challenges:

- Simplicity
- Representativity

Modeling of wheeled mobile robots

Design: main configurations



Modeling of wheeled mobile robots

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Main models:

Wheeled mobile robots: "Unicycle"

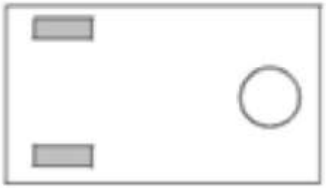
Wheeled mobile robots: "Tricycle"

Wheeled mobile robots: "Car"

Assumptions:

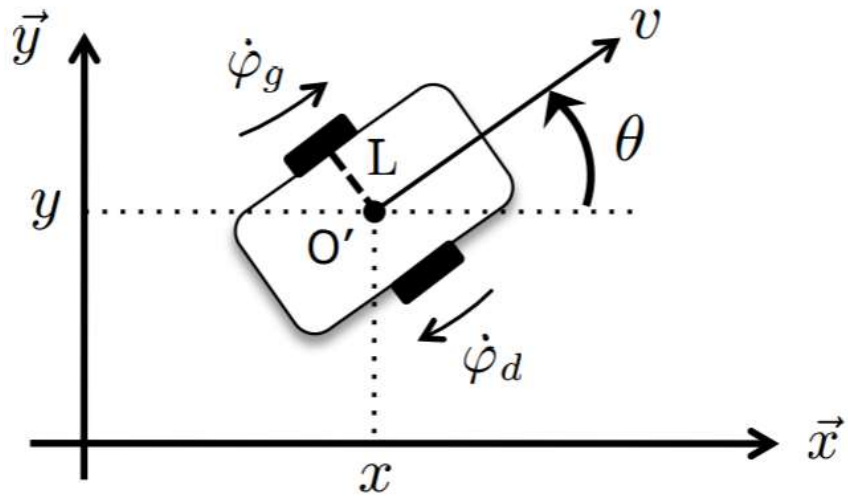
Rolling without sliding

Modeling of wheeled mobile robots: "Char" or "Unicycle"



2 independent drive wheels
1 or more free wheel(s)
Differential driving robot

Modeling of wheeled mobile robots: "Char" or "Unicycle"



$$v = \frac{r(\dot{\varphi}_g - \dot{\varphi}_d)}{2}$$

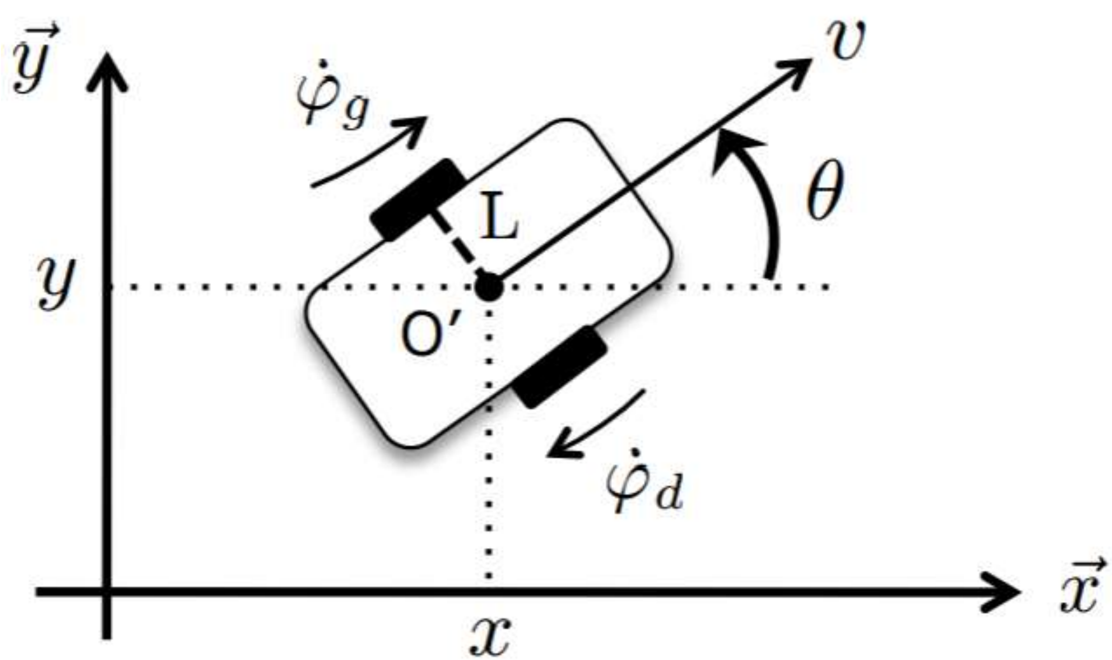
$$\omega = -\frac{r(\dot{\varphi}_g + \dot{\varphi}_d)}{2\mathrm{L}}$$

Typically $v \in [-1, 1], \ \omega \in [-1, 1]$

- $2\mathrm{L}$: Distance between the two wheels [m]
- r : Radius of the wheel [m]
- $\dot{\varphi}_g$: Left wheel rotation speed [rad/s]
- $\dot{\varphi}_d$: Right wheel rotation speed [rad/s]
- v : Longitudinal speed of the robot [m/s]
- ω : Lateral speed of the robot [rad/s]

Modeling of wheeled mobile robots: "Char" or "Unicycle"

Mobile Robot Pose



$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = \omega$$

Modeling of wheeled mobile robots: "Char" or "Unicycle"

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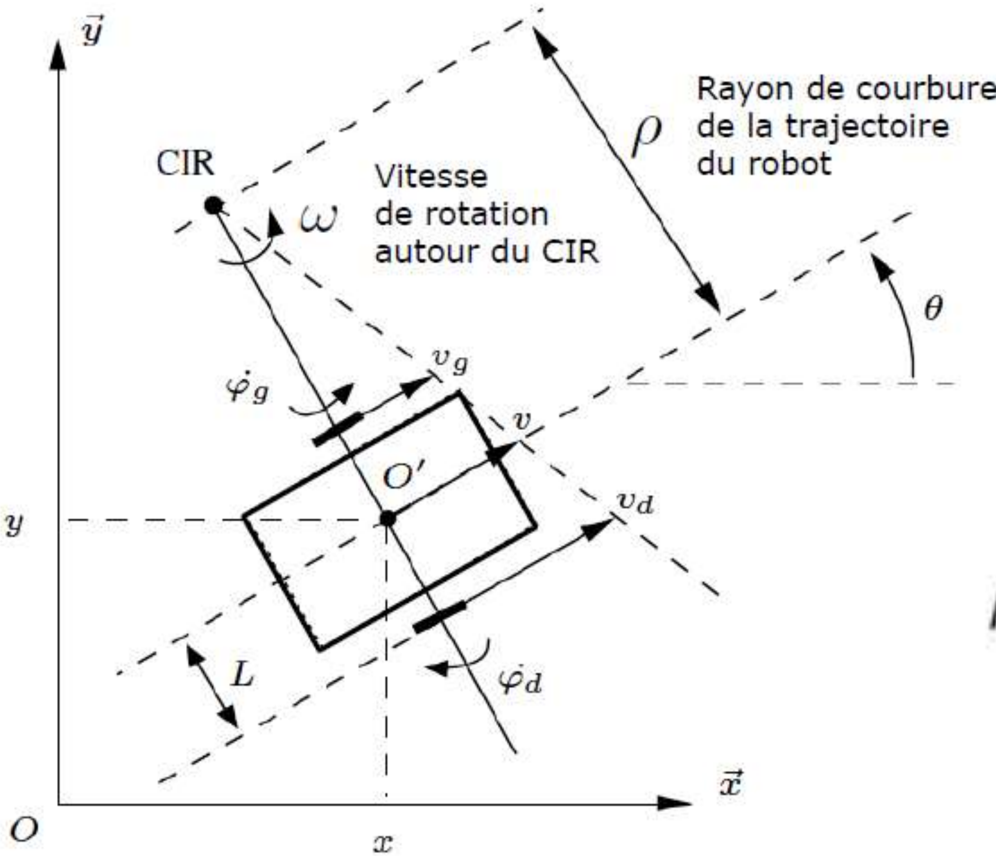
Instant Rotation Center (CIR)
Mobile Robot Curvature

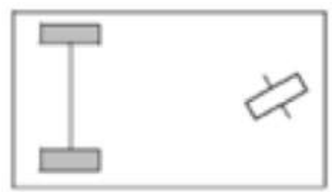


$$v_d = -r\dot{\varphi}_d = (\rho + L)\,\omega$$

$$v_g = r\,\dot{\varphi}_g = (\rho - L)\,\omega$$

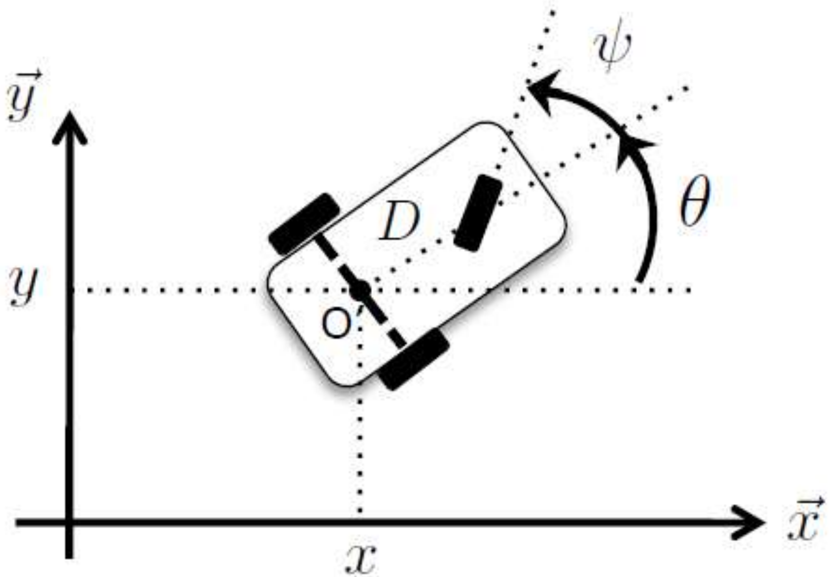
$$\rho = L\left(\frac{\dot{\varphi}_d - \dot{\varphi}_g}{\dot{\varphi}_d + \dot{\varphi}_g}\right)$$

Modeling of wheeled mobile robots: "Tricycle"

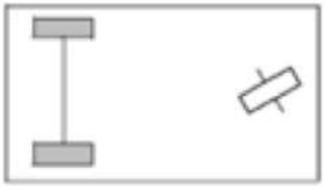


2 fixed wheels on the same axis
1 steerable center wheel

Modeling of wheeled mobile robots: "Tricycle"

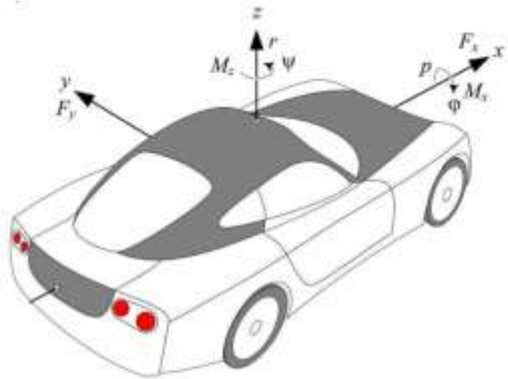


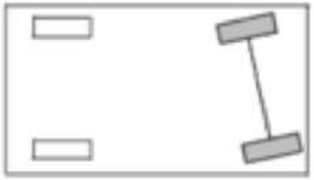
$$\dot{x} = u_1 \cos\theta \cos\psi$$
$$\dot{y} = u_1 \sin\theta \cos\psi$$
$$\dot{\theta} = \frac{u_1 \sin\psi}{D}$$
$$\dot{\psi} = u_2$$



2 fixed wheels on the same axis
1 steerable center wheel

Modeling of wheeled mobile robots: "Car"





Modeling of wheeled mobile robots: "Car"

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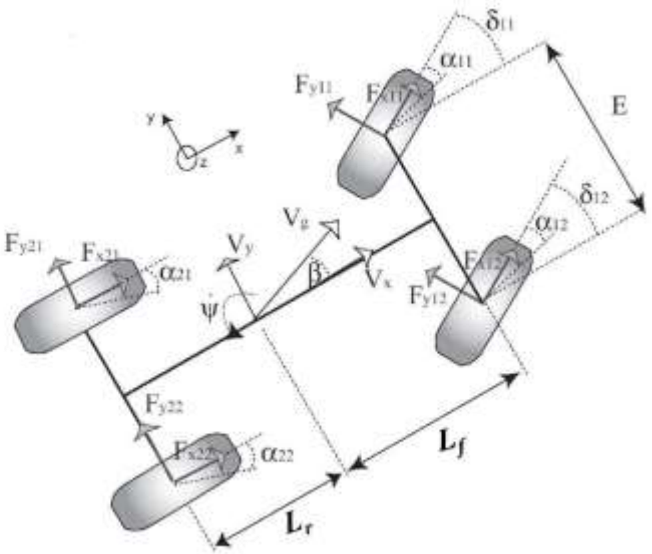
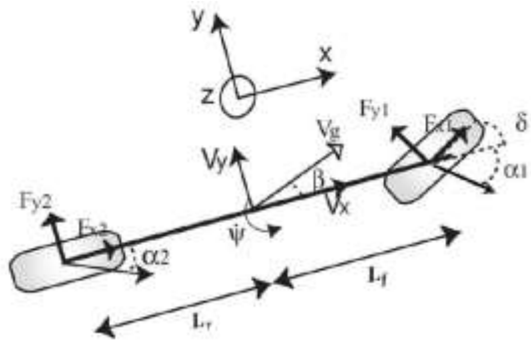
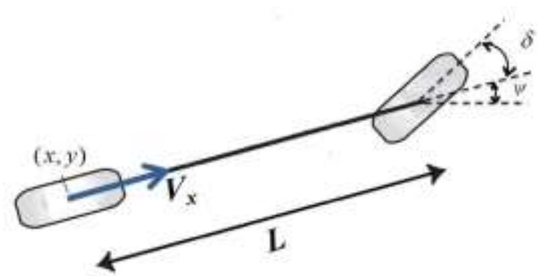


4 wheel model
(full model)



Dynamic bicycle model



Kinematic bicycle model

Modeling wheeled mobile robots: Kinematic bicycle model

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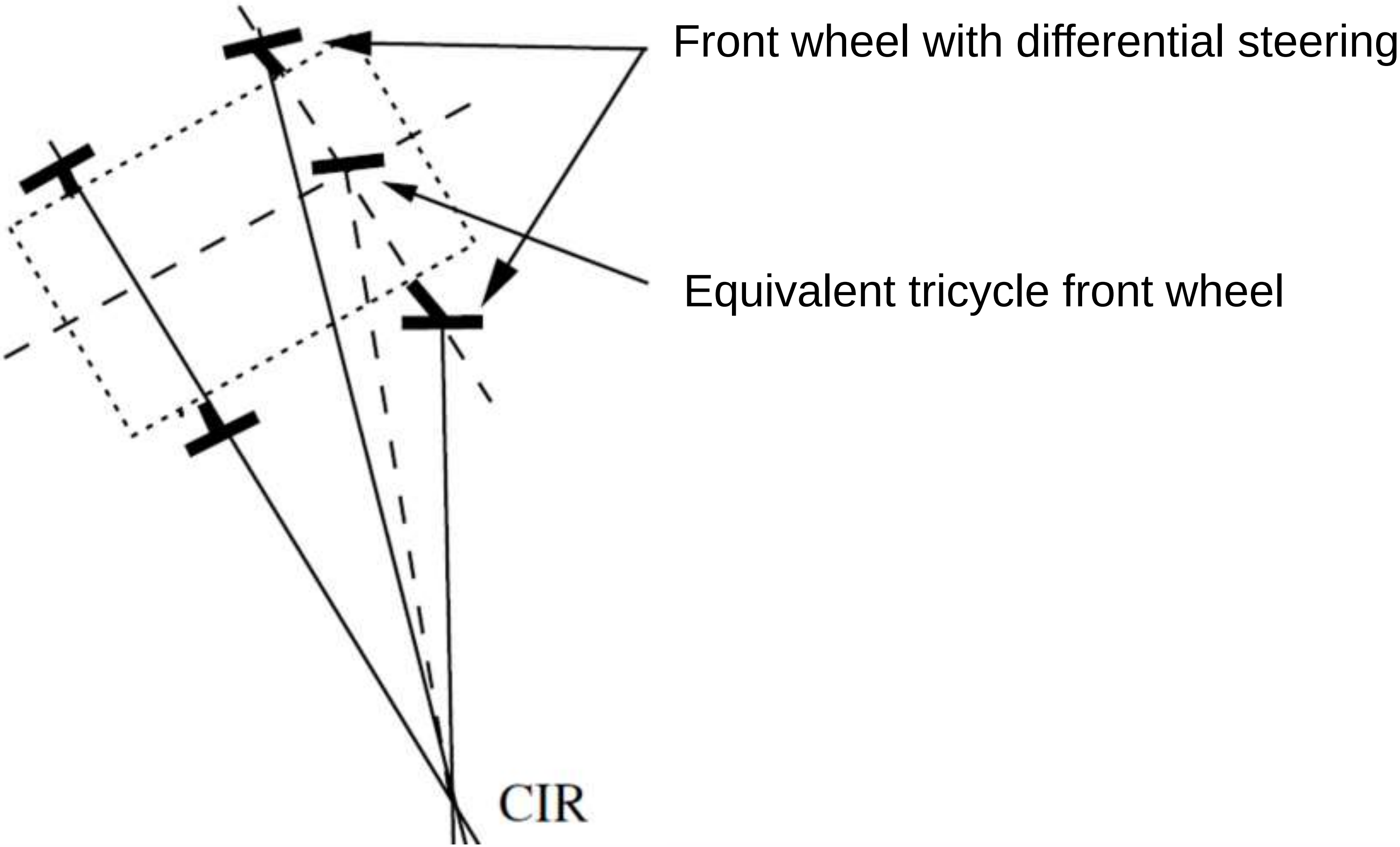

Modeling wheeled mobile robots: Kinematic bicycle model

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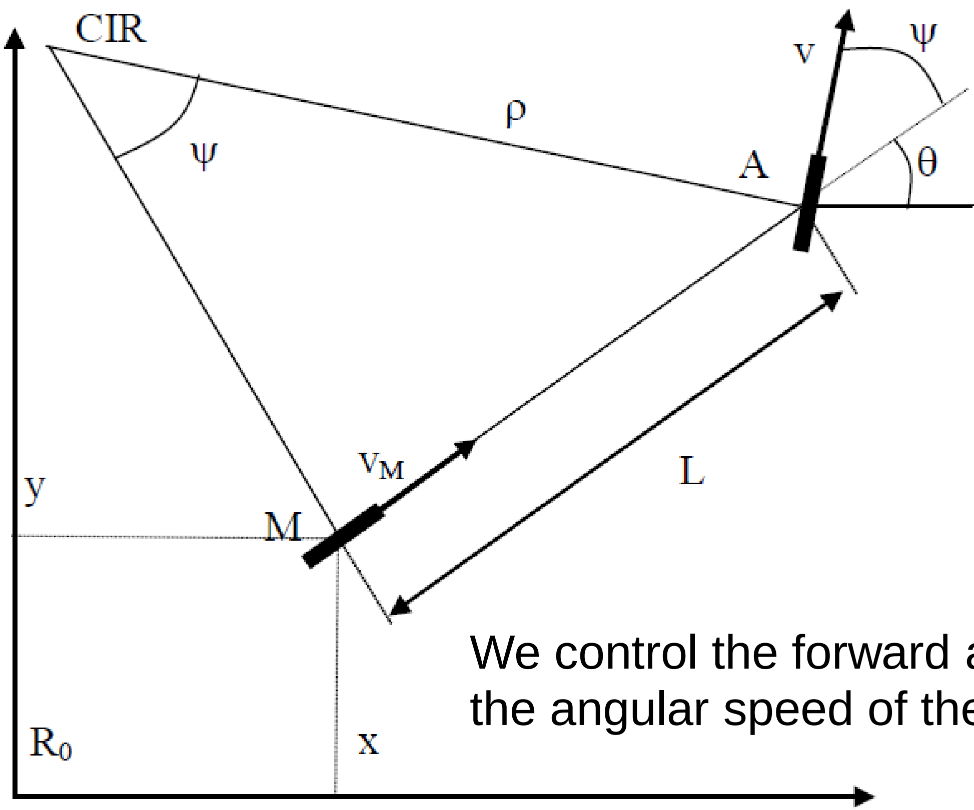


$$\dot{x} = v.\cos(\theta)$$

$$\dot{y} = v.\sin(\theta)$$

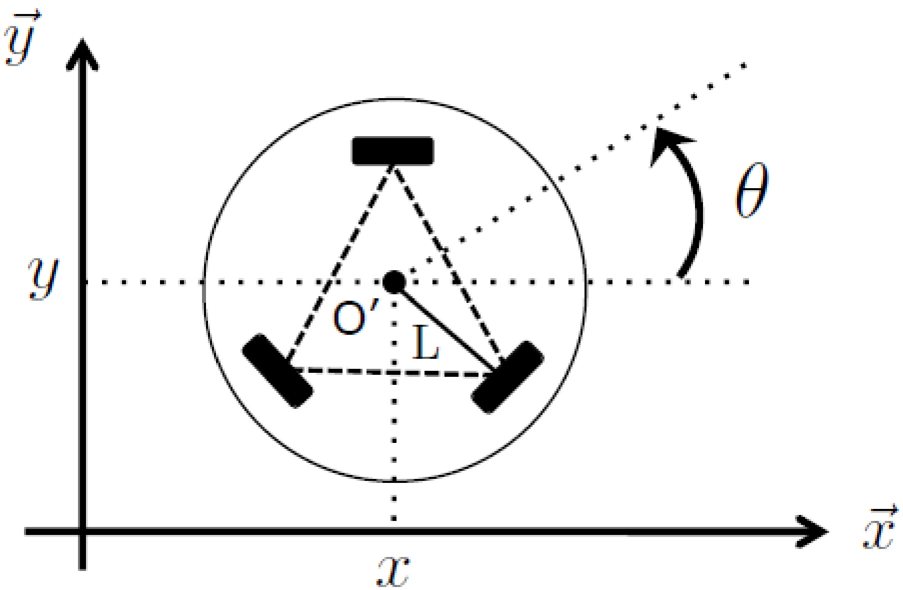
$$\dot{\theta} = \frac{v.\tan(\psi)}{L}$$

$$\dot{v} = u_1$$

$$\dot{\psi} = u_2$$

We control the forward acceleration and the angular speed of the front wheel

Modeling wheeled mobile robots: Omnidirectional robot



$$\dot{x} = u_1$$

$$\dot{y} = u_2$$

$$\dot{\theta} = u_3$$

Swedish wheels or steerable offset wheels

Modeling of aerial mobile robots

- **Aerial vehicles**
  - **Heavier than air**
    - **Fixed wing**
    - **Rotating wing**
  - **Lighter than air**

Dirigeable

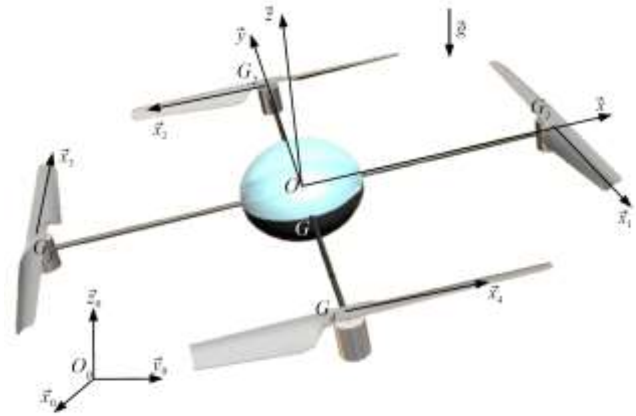
Airplane

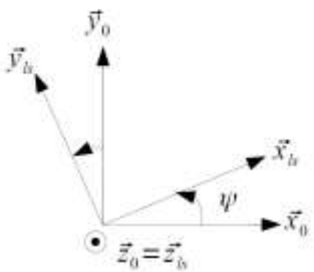
Helicopter

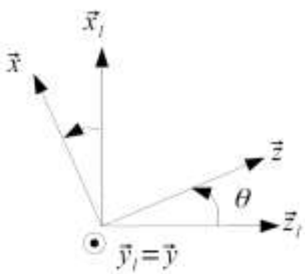
Quadricopter

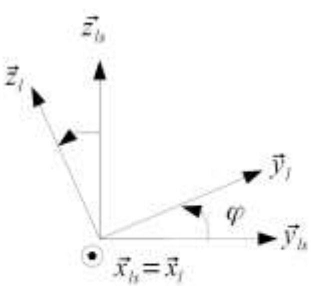
Modeling of aerial mobile robots: Quadricopter

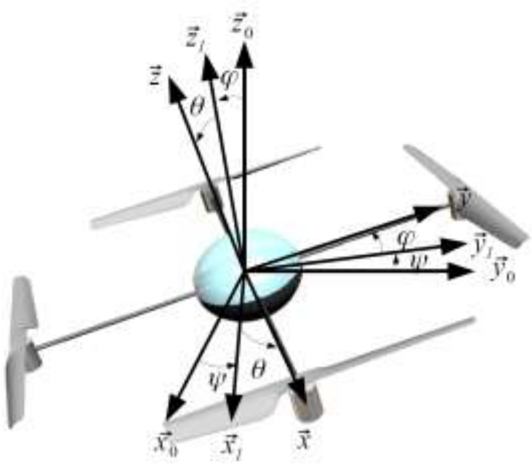
Parrot, AR.drone









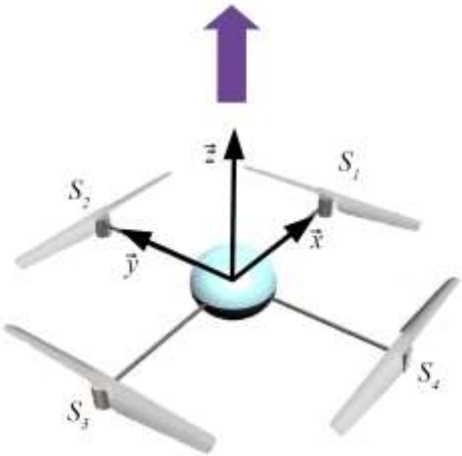


ψ θ φ : Yaw, Roll, and Pitch angles

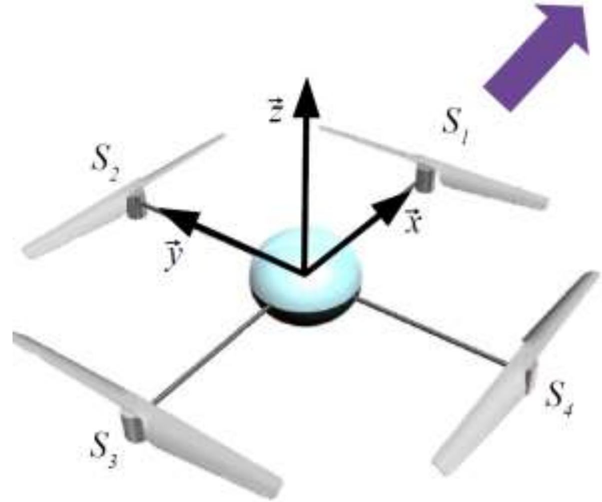
$$\dot{\theta} = q,\ \dot{\varphi} = p \text{ et } \dot{\psi} = r$$

Modeling of aerial mobile robots: Quadricopter

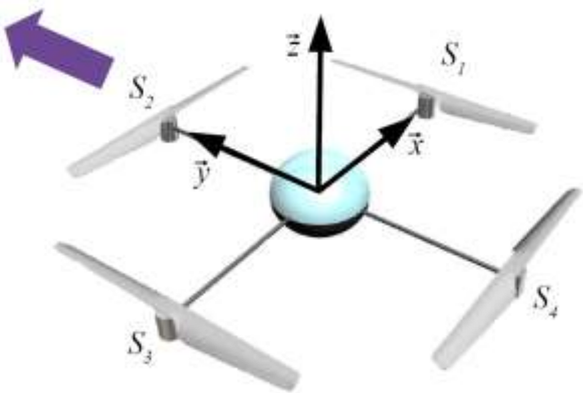
Basic movements of the robot



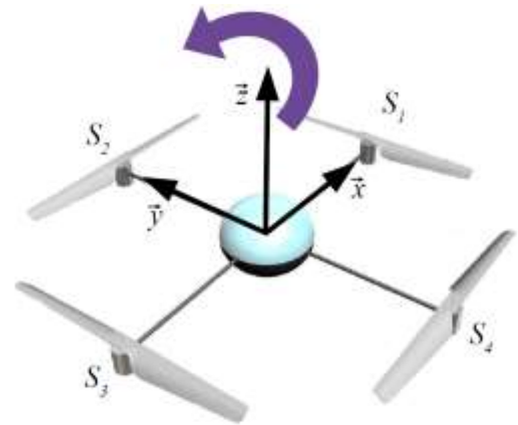
Vertical translation



Longitudinal translation



Lateral translation



Rotation around the vertical axis (yaw)

Modeling of aerial mobile robots: Quadricopter

Dynamics of a propeller

$$u_i = R_m I_i + L_m \frac{dI_i}{dt} + K_m \omega_i$$

$$\Gamma_i = K_m I_i$$

$$J_r \dot{\omega}_i = \Gamma_i - b{\omega_i}^2$$

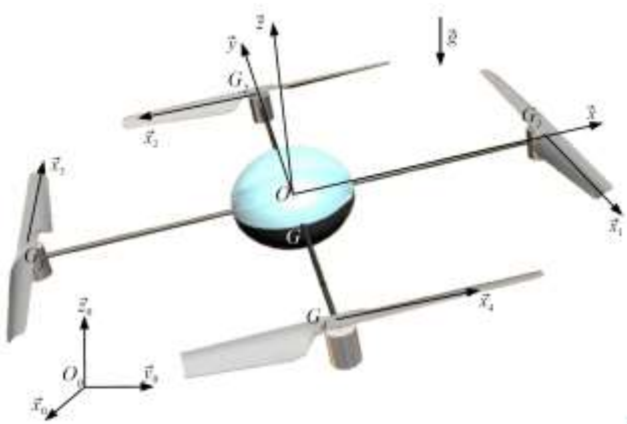
$R_m \;\; L_m \;\; K_m \;\; J_r \;\; b$

Motor constant parameters

Γ_i : Motor couple

ω_i : Angular velocity [rad/s]

u_i : Supply voltage of the motor



$$\Gamma_i = \overline{\Gamma}_i + \widetilde{\Gamma}_i \qquad \omega_i = \overline{\omega} + \widetilde{\omega}$$

$$u_i = \overline{u}_i + \tilde{u}_i$$

$$J_r \dot{\widetilde{\omega}}_i = (\widetilde{u}_i - K_m \widetilde{\omega}_i) \frac{K_m}{R_m} - 2b\overline{\omega}\widetilde{\omega}_i$$

Modeling of aerial mobile robots: Quadricopter

Robot Dynamic model

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Vertical subsystem

$$\dot{w} = (2\varpi \frac{a}{M}) \sum_{i=1}^{i=4} \widetilde{\omega}_i$$

$$J_r \sum_{i=1}^{i=4} \dot{\widetilde{\omega}}_i = \sum_{i=1}^{i=4} \widetilde{\Gamma}_i - 2b\varpi \sum_{i=1}^{i=4} \widetilde{\omega}_i$$

Longitudinal subsystem

$$\dot{u} = g\theta$$

$$I\dot{q} = 2al\varpi(\widetilde{\omega_3} - \widetilde{\omega_1})$$

$$J_r(\dot{\widetilde{\omega}}_3 - \dot{\widetilde{\omega}}_1) = \widetilde{\Gamma}_3 - \widetilde{\Gamma}_1 - 2b\varpi(\widetilde{\omega}_3 - \widetilde{\omega}_1)$$

Lateral Subsystem

$$\dot{v} = -g\varphi$$

$$I\dot{p} = 2al\varpi(\widetilde{\omega}_2 - \widetilde{\omega}_4)$$

$$J_r(\dot{\widetilde{\omega}}_2 - \dot{\widetilde{\omega}}_4) = \widetilde{\Gamma}_2 - \widetilde{\Gamma}_4 - 2b\varpi(\widetilde{\omega}_2 - \widetilde{\omega}_4)$$

Cap Subsystem

$$(J - 4J_r)\dot{r} = -\sum_{i=1}^{i=4} \varepsilon_i \left(J_r \dot{\widetilde{\omega}}_i + 2b\varpi\widetilde{\omega}_i\right)$$

M : The mass of the whole quadrotor

a : Propeller constant parameter

$g = 10\ m.s^{-2}$