Astronomy from 4 Perspectives: the Dark Universe

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Solutions: The planet of the Petit Prince

1. Gravity on the planet of the Petit prince

The Petit Prince by A. de Saint-Exupéry lives on a planet which, according to images, is roughly $R \simeq 1$ m in size and because Saint-Exupéry does not provide any other information, has a value of the surface gravity $g = 9.81$ m/s^2 similar to Earth. But in comparison to Earth where the gradient of the acceleration is almost zero, it is much stronger on the planet of the Petit Prince. Recall that $G = 6.6 \times 10^{-11}$ in SI.

(a) The relation between the mass and the density of the planet is $M = (4\pi/3)R^3$. The surface gravity is tied to the mass by $g = GM/R^2$. Substituting one in the other one can solve to find a density $\rho \simeq 3.5 \times 10^{10}$ kg m^{-3}. This is quite close to the density of a White Dwarf.

(b) The orbital period and velocity can be obtained by equation the gravitational acceleration to the centrifugal acceleration: $GM/(R+1\mathrm{m})^2 = \Omega^2(R+1\mathrm{m})$. This gives $\Omega \simeq 1$ s^{-1} corresponding to a period $P \simeq 6$ sec, and an orbital speed $V = \Omega(R+1\mathrm{m}) \simeq 4$ m s^{-1}. Yes the Petit Prince can throw an object fast enough to put it into orbital motion.

(c) The escape speed is given by equating the specific kinetic energy $V^2/2$ to the potential energy GM/R, and this gives a typical value $V \simeq 4$ m s^{-1}. This is too much for a kid to jump.

(d) Given that the maximum period is 6 sec (computed above, and our day corresponds to 86400 sec then there will be at most 14000 sunsets/sunrises.

2. Devices on the planet of the Petit prince

Imagine that Saint-Exupéry brings simple mechanical systems with him, and find out if they behave differently because of the strong gradient $\partial g/\partial r$ in the gravitational acceleration g.

(a) What's the relation between the oscillation period T of a pendulum clock as a function of height h? Would the oscillation period be independent from the amplitude?

(b) Saint-Exupéry and the Petit Prince have a glass of orange juice with an ice cube. The Petit Prince's ice cube swims higher or not above the surface of the juice compared to Saint-Exupéry's?

3. Relativity on the planet of the Petit Prince

Are there relativistic effects of gravity on the planet of the Petit Prince?

(a) What is the tidal gravitational acceleration between the head and the feet of the Petit Prince? Please compute the difference

$$\Delta g = \frac{GM}{R^2} - \frac{GM}{(R+1)^2} \tag{I}$$

(b) What is the gravitational time dilation between the head and the feet of the Petit Prince? Please use the formula

$$\Delta\tau = \sqrt{1 + 2\frac{\Phi}{c^2}}\,\Delta t \tag{II}$$

and approximate the potential as homogeneous, $\Phi = g\Delta r$.