Astronomy from 4 perspectives: the Dark Universe

prepared by: Bucciantini N.

Why galaxies do not expand together with the universe?

This tutorial is aimed to address the question that students might ask: **if the universe (space-time itself) expands, how is that the ruler on my desk, or even my desk itself stays the same?**. The simple answer to this question is that the ruler and the desk are held together by electromagnetic forces among the atoms and molecules, that on the scale of our everyday life (meters, km, cm, g, kg, ton) are much much stronger that the gravitational force. To show this just compute the electrostatic force between two protons at 1 cm distance, and the gravitational force between them also at 1 cm distance. You will see that the electrostatic force is many many orders of magnitude bigger. You will also see that the distance does not matter (try 1m and you get the same ratio between the two forces), because the electrostatic and gravitational forces scale in the same way with the distance.

The clever student at this point might ask: **why the Galaxy does not expands with the universe, given that the Galaxy (the stars within it) is held together only by gravity (there are no charged body in space), so the above argument with the use of electromagnetic forces does not apply?**

To answer we are going to use two concepts: the escape speed and the Hubble expansion. Let us stars with the **escape speed** (if this has already been introduced then skip this part), and we are going to derive it without the use of potential energy (if the student is familiar with the concept of potential and kinetic energy, than the derivation is easier and can be found in standard books).

We are going to use **dimensional arguments** (the famous *apples go with apples and oranges with oranges*). There are 3 possible *dimensions*: length L, to whatever is measured in m, cm, km etc....; mass M, to whatever is measured in g, kg etc...; time T to whatever is measured in sec, min, hr etc... . For example: velocity is measured in m s^{-1} (or km/h etc..) then its dimension is L/T.

Let us consider a mass M at a distance R. Can we derive a typical velocity scale? If there is only gravity the answer is yes. We have 3 quantities available: M which has the dimension of mass (it is measure in kg or g etc...); R which has the dimension of length (it is measured in m cm, light years etc..), and G which is measured in m^3 s^{-2} kg^{-1} and so has the dimension of $L^3T^{-2}M^{-1}$ (if you want to see this recall that $GM/R^2 = g$ has the dimension of an acceleration L/T^2). We are going to search for a combination of these 3 quantities that gives us the same dimension of the velocity.

$$G^aR^bM^c \Rightarrow M^{c-a}L^{3a+b}T^{-2a} \quad \text{must be equal to} \quad LT^{-1} \tag{I}$$

So $c = a$, and $a = 1/2$, $b = -1/2$. Then the typical velocity is:

$$V = G^{1/2}M^{-1/2}R^{-3/2} = \sqrt{GM/R} \tag{II}$$

The clever student will recognize the orbital velocity, and wonder why it is the same. The answer is that the orbital velocity, and the escape velocity are similar (there is just a factor $\sqrt{2}$) and in a gravitational field (if there is just gravity) no matter how you work your problem you always ends up getting similar numbers.

We then pass to **Hubble law** (that the teacher should have already introduced when explaining the expansion of the Universe). This law gives the velocity at which objects in space drift from each other as a function of their distance:

$$V = H_o d \tag{III}$$

Now we make our argument. While the Hubble speed grows with distance, the escape speed drops. So there will be a distance where an object has an Hubble speed equal to the escape speed. Let us compute where this happens for a galaxy. A galaxy has about 10^{11} stars like the Sun each with a mass $M_{\text{Star}} = 10^{30}$ kg, and there will be about 10 times more dark matter. So the total (dark+stars) mass of the Galaxy is $M = 10 \times 10^{11} M_{\text{Star}} = 10^{42}$ kg. The Hubble constan $H_o = 70$ km/s/Mpc and in SI units is 2.2×10^{-18} s^{-1}. Then we solve:

$$\sqrt{GM/R} = H_o R \quad \Rightarrow \quad R_{\text{eq}} = 2.3 \times 10^{22}\ \text{m}\ = 2.3 \text{ million light years} \tag{IV}$$

So everything closer than $R_{\text{eq}} = 2.3 \times 10^{22}$ m $= 2.3$ from the galaxy will be held tied to the galaxy by the galactic gravitation, everything at larger distances will be drifting away due to the expansion of the Universe.

A galaxy has a typical size $\sim$ 100000 ly. So it is smaller than the radius we found above. So a galaxy is held together by its gravity and does not respond (feels) the expansion of the universe. But you do not need to go at very large distances from us to appreciate the expansion of the universe. The Virgo cluster (the closer group of galaxies to us) for example is located at a distance of 100 million light years. So it is moving away from us and feels the expansion of the universe more than the attraction of our Galaxy. This explains why it was possible for Hubble to use just a few nearby galaxies (100 years ago he could only measure the distance of the closest 20-30 galaxies with the instruments available at that time), and get his famous result.