Astronomy from 4 Perspectives: the Dark Universe

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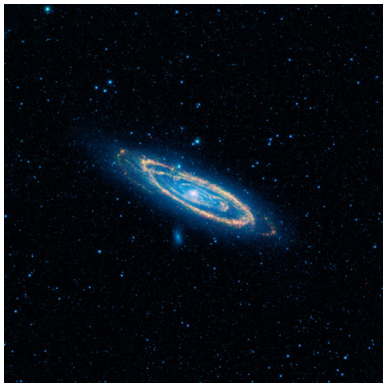
Questions: Dark matter and galaxy rotation curves

1. ***Orientations of galaxies***
   Think about how the galaxy should be orientated to be observed?
   Here are some pictures as example:

(a) Edge on galaxy NGC 7742
Inclination angle $i = 0°$
Image credit:
Hubble Heritage Team
(AURA/STScI/NASA/ESA)



(b) Our galactic neighbour Andromeda as seen in infrared.
Inclination angle $i \approx 13°$
Image credit:
NASA/JPL-Caltech/UCLA

(c) The almost edge-on sombrero galaxy.
Inclination angle $i \approx 90°$
Image credit:
Carsten Frenzl

2. ***Galactic Rotation curves***
   Calculate the radial velocities from the measured wavelengths and plot them over the distance from the galaxy center. use $\lambda_0 = 21.106\,\text{Å}$ and $1\,\text{pc} = 3.1 \cdot 10^{16}$ m.

λ in Å	Radius R in Mpc	v_{rotation} in $\frac{km}{s}$
21.1195	1	
21.1130	2	
21.1173	5	
21.1194	7	
21.1201	10	
21.1208	15	
21.1211	20	
21.1215	22	
21.1213	25	

3. ***Circular obits***
   Derive for circular orbits the formula for the velocity v in dependence of the distance r. Assume a radially symmetric mass distribution.

4. ***Velocity of planets***
   Assuming circular orbits, compute the velocities of the planets in our solar system. Plot the resulting rotation curve v over r.

5. ***Expected Rotation curve***
   Formulate an expectation for the rotation curve of the Milky Way, assuming the mass in the bulge to be $1.6 \cdot 10^{10}\,M_\odot$ and the disk to be $4 \cdot 10^{10}\,M_\odot$

6. ***Observed rotation curve*** The observed rotation curves of spiral galaxies are of the following form: This cannot be explained by visible mass alone. Assuming that dark matter is the source of the

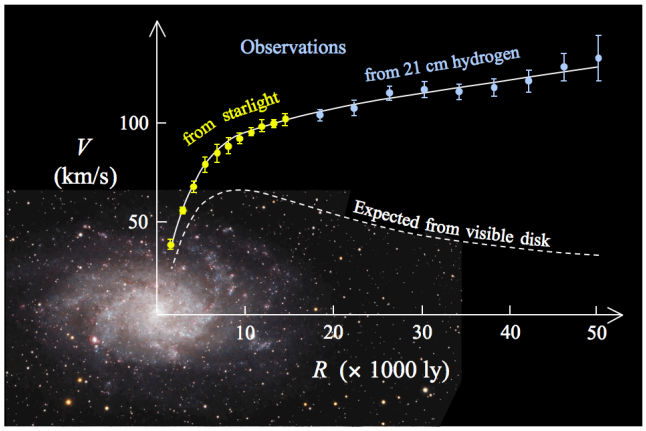


Figure 2: Expected and observed rotation curve of M33.
Image credit: Steffania Deluca

difference between the observed and the predicted rotation curves, please calculate the mass of the dark matter depending on the velocities $v(r)$ and $v_{\text{axis}}(r)$

7. ***Dark matter distribution*** To find out how dark matter is distributed throughout a spiral galaxy, please consider a simple rotation curve consisting of a linear and a constant branch. Assume a spherically symmetric mass distribution of the form

$$\mu(r) \propto r^k$$

For the mass use the formula

$$M(r) = 4\pi \int_{r_0}^{r} \mu(\rho)\,\rho^2 \mathrm{d}\rho$$

   (a) Please calculate the mass of the bulge in dependence of k

   (b) Using the formula for v from Task 3 and the result from a), please determine the exponent k for the bulge. Calculate the mass of the bulge in dependence of r and the complete mass M_B of the bulge.

   (c) To determine k for the halo (v = const.), consider the total mass to be composed of the mass of the bulge M_B and the mass of the halo M_H.

$$M(r) = M_B + M_H(r)$$

   Calculate the mass outside of the bulge with the integral for the mass. Determine the exponent k using the results of b) and Task 3. Find a formula for the mass of the halo in dependence of r.

   (d) Compare the rotation curve of the bulge to the rotation curve of a rigid body.