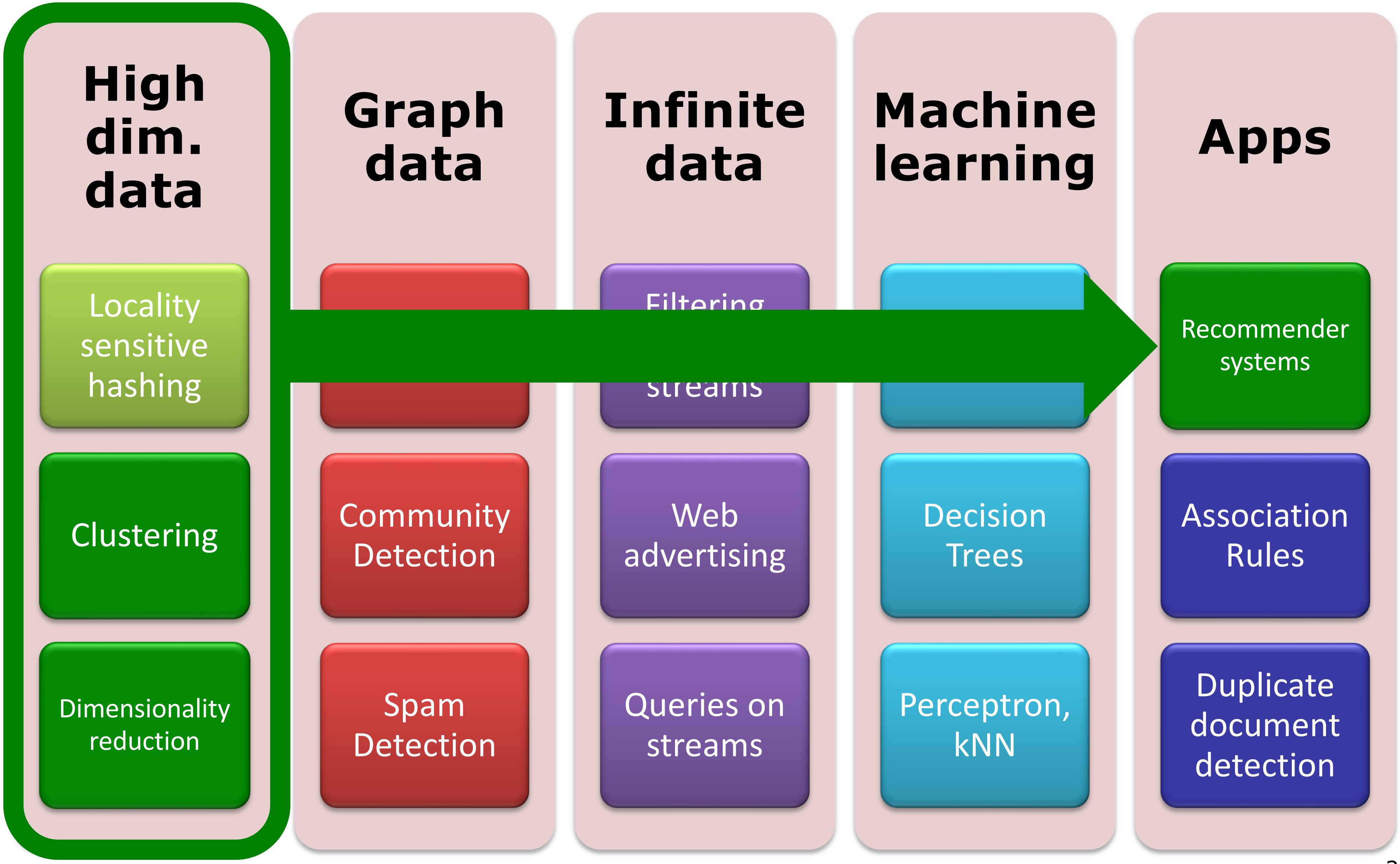
Unsupervised Machine Learning

Recommendation Systems

Prof. Yannis Velegrakis

https://velgias.github.io

High Dimensional Data



High dim. data
- Locality sensitive hashing
- Clustering
- Dimensionality reduction

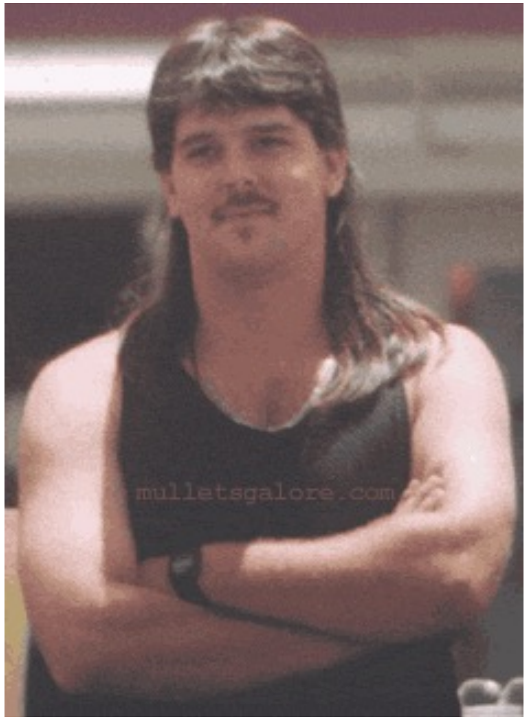
Graph data
- Community Detection
- Spam Detection

Infinite data
- Filtering streams
- Web advertising
- Queries on streams

Machine learning
- Decision Trees
- Perceptron, kNN

Apps
- Recommender systems
- Association Rules
- Duplicate document detection

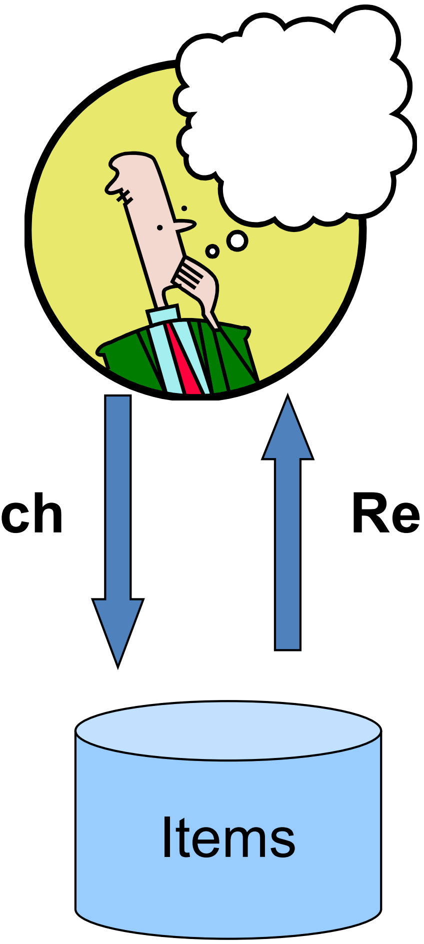
Example: Recommender Systems



- **Customer X**
  - Buys Metallica CD
  - Buys Megadeth CD

- **Customer Y**
  - Does search on Metallica
  - Should we recommend Megadeth ?

Recommendations



Search

Recommendations

Products, web sites, blogs, news items, …

Examples:

amazon.com

PANDORA

StumbleUpon

del.icio.us

NETFLIX

movielens
helping you find the *right* movies

last.fm
the social music revolution

Google News

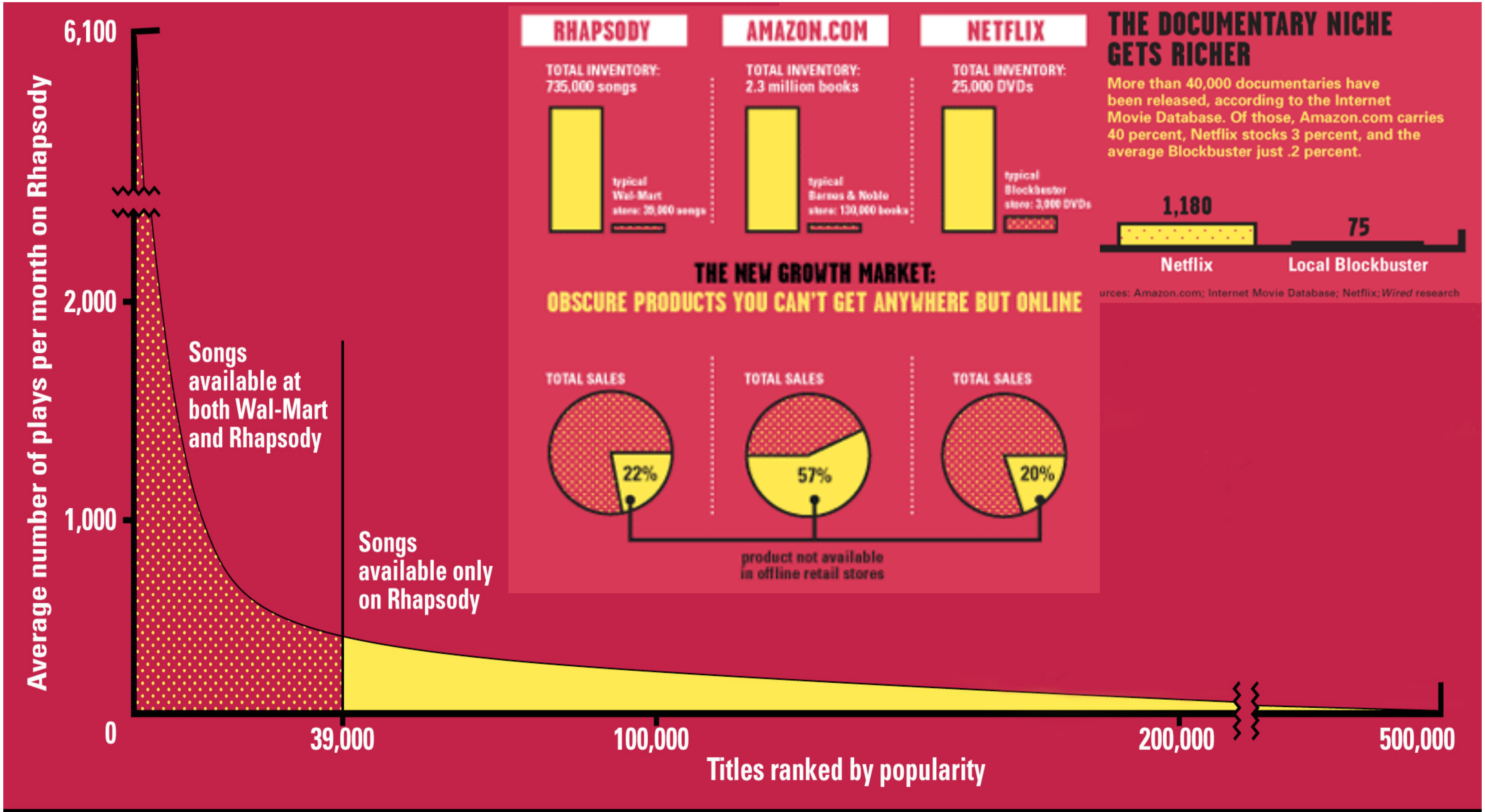
YouTube

XBOX LIVE

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,…
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller: http://www.wired.com/wired/archive/12.10/tail.html

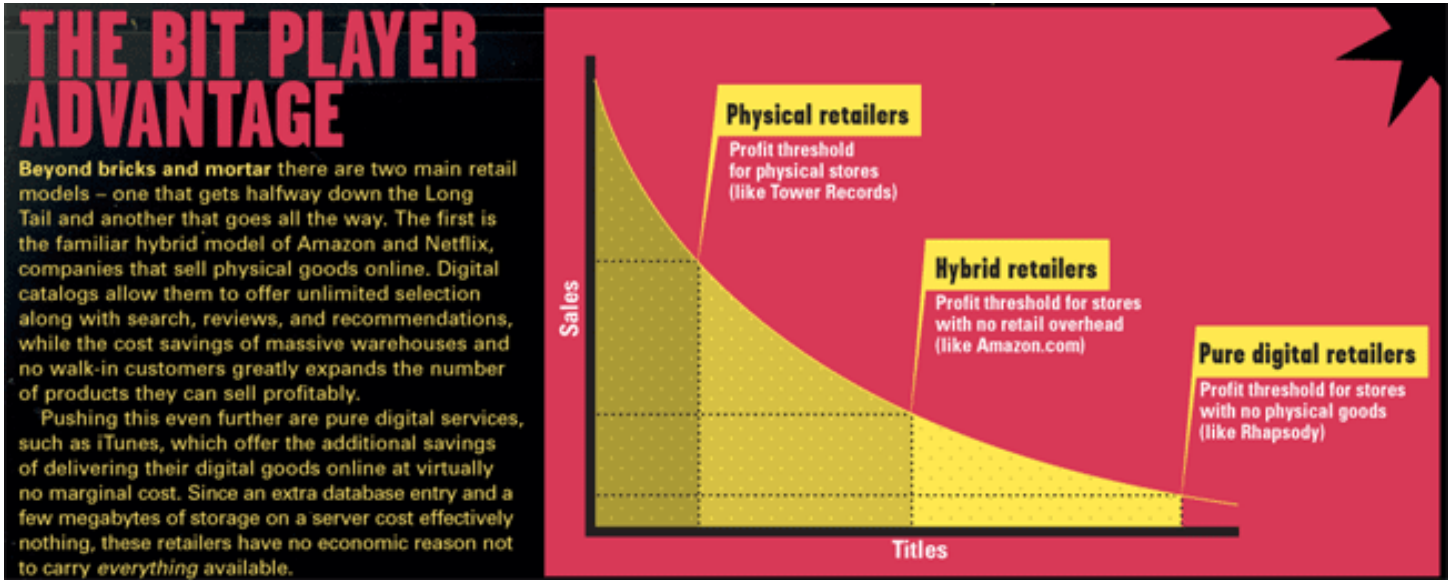
Sidenote: The Long Tail



Sources: Erik Brynjolfsson and Jeffrey Hu, MIT, and Michael Smith, Carnegie Mellon; Barnes & Noble; Netflix; RealNetworks

Source: Chris Anderson (2004)

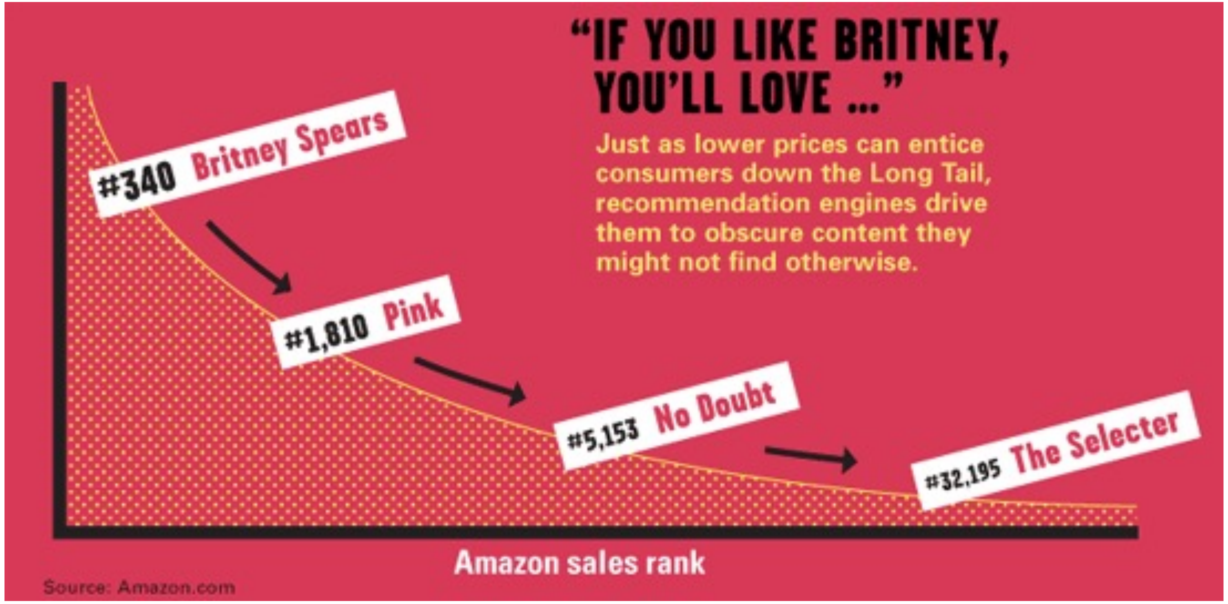
Physical vs. Online



THE BIT PLAYER ADVANTAGE

Beyond bricks and mortar there are two main retail models – one that gets halfway down the Long Tail and another that goes all the way. The first is the familiar hybrid model of Amazon and Netflix, companies that sell physical goods online. Digital catalogs allow them to offer unlimited selection along with search, reviews, and recommendations, while the cost savings of massive warehouses and no walk-in customers greatly expands the number of products they can sell profitably.

Pushing this even further are pure digital services, such as iTunes, which offer the additional savings of delivering their digital goods online at virtually no marginal cost. Since an extra database entry and a few megabytes of storage on a server cost effectively nothing, these retailers have no economic reason not to carry *everything* available.



"IF YOU LIKE BRITNEY, YOU'LL LOVE ..."

Just as lower prices can entice consumers down the Long Tail, recommendation engines drive them to obscure content they might not find otherwise.

Source: Amazon.com

Read http://www.wired.com/wired/archive/12.10/tail.html to learn more!

Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
  - Amazon, Netflix, …

Formal Model

- X = set of **Customers**
- S = set of **Items**

- **Utility function** $u: X \times S \rightarrow R$
  - R = set of ratings
  - R is a totally ordered set
  - e.g., **0-5** stars, real number in **[0,1]**

Utility Matrix

	Avatar	**LOTR**	**Matrix**	**Pirates**
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

- **(1) Gathering "known" ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn’t work well in practice – people can’t be bothered

- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - What about low ratings?

(2) Extrapolating Utilities

- **Key problem:** Utility matrix ***U*** is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history

- **Two approaches to recommender systems:**
  - **1)** Content-based
  - **2)** Collaborative

Content-based Recommender Systems

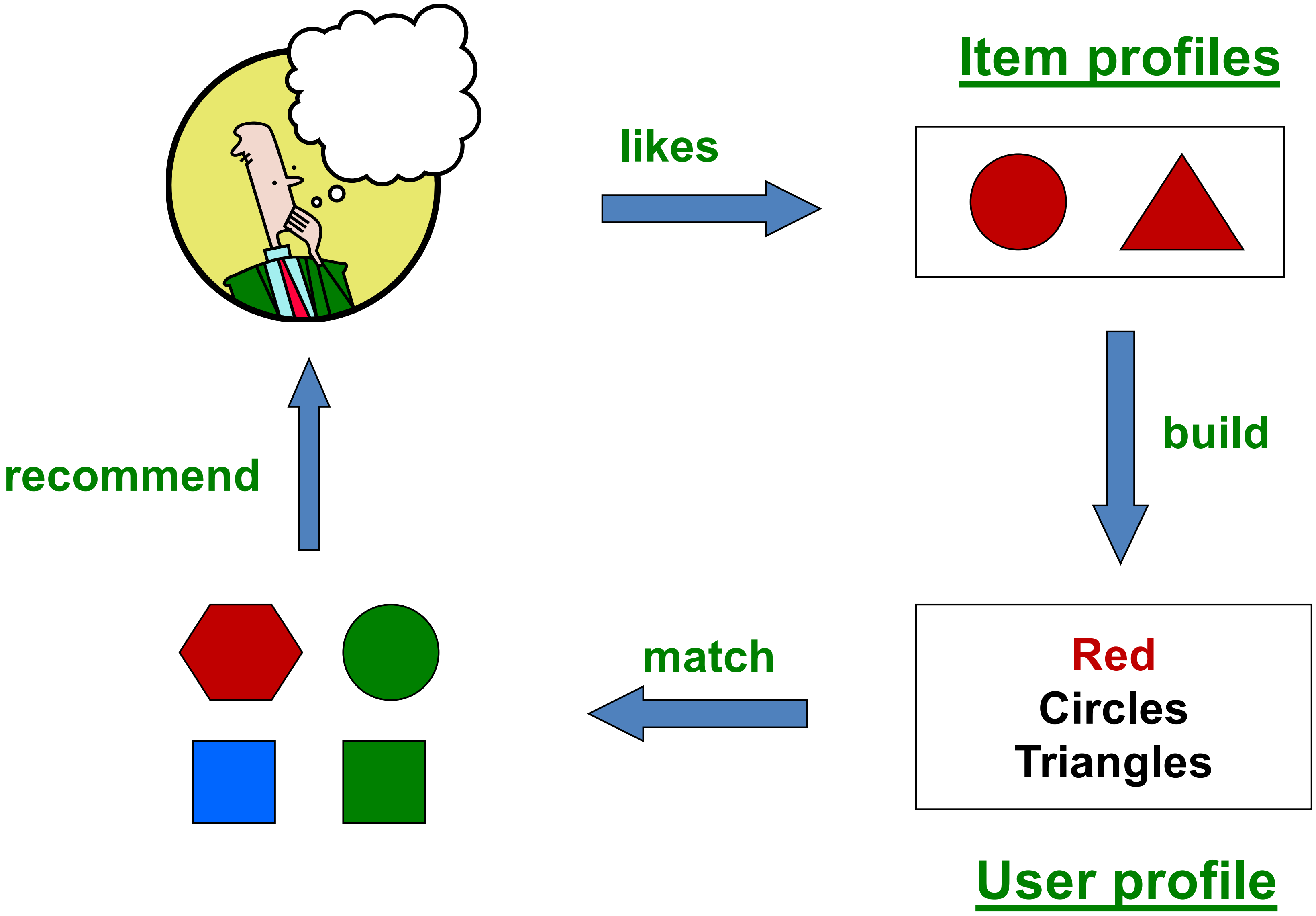
Content-based Recommendations

- **Main idea:** Recommend items to customer ***x*** similar to previous items rated highly by ***x***

Example:

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - Recommend other sites with "similar" content

Plan of Action



Item profiles

likes

build

Red
Circles
Triangles

User profile

match

recommend

Item Profiles

- For each item, create an **item profile**

- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text:** Set of "important" words in document

- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF** (Term frequency * Inverse Doc Frequency)
    - **Term … Feature**
    - **Document … Item**

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) i in doc (item) j

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

Note: we normalize TF to discount for "longer" documents

n_i = number of docs that mention term i

N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest **TF-IDF** scores, together with their scores

User Profiles

- User has rated items with profiles $\mathbf{i}_1,\ldots,\mathbf{i}_n$
- Simple: (weighted) average of rated item profiles
- Variant: Normalize weights using average rating of user
- More sophisticated aggregations possible

Example 1: Boolean Utility Matrix

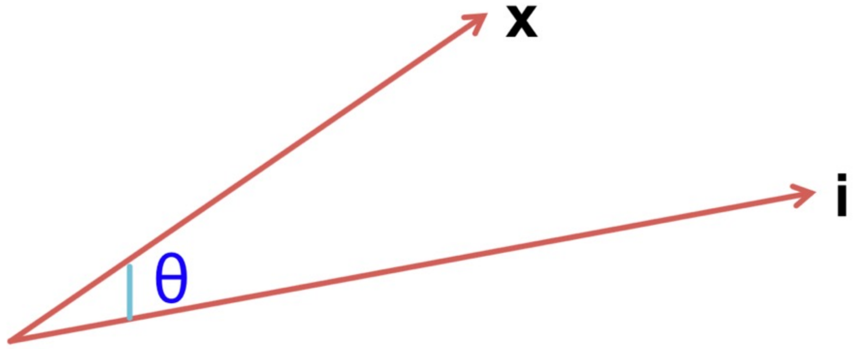
- Items are movies, only feature is “Actor”
    - Item profile: vector with 0 or 1 for each Actor
- Suppose user x has watched 5 movies
    - 2 movies featuring actor A
    - 3 movies featuring actor B
- User profile = mean of item profiles
    - Feature A’s weight = 2/5 = 0.4
    - Feature B’s weight = 3/5 = 0.6

Example 2: Star Ratings

- Same example, 1-5 star ratings
  - Actor A’s movies rated 3 and 5
  - Actor B’s movies rated 1, 2 and 4
- Useful step: Normalize ratings by subtracting user’s mean rating (3)
  - Actor A’s normalized ratings = 0, +2
    - Profile weight = (0 + 2)/2 = 1
  - Actor B’s normalized ratings = -2, -1, +1
    - Profile weight = -2/3

Making Predictions

- User profile **x,** Item profile **i**
- Estimate $U(\mathbf{x},\mathbf{i}) = \cos(\theta) = (\mathbf{x} \cdot \mathbf{i})/(|\mathbf{x}|\,|\mathbf{i}|)$



Technically, the cosine distance is actually the angle θ
And the cosine similarity is the angle 180-θ

For convenience, we use cos(θ) as our similarity measure
and call it the “cosine similarity” in this context.

User Profiles and Prediction

- **User profile possibilities:**
  - Weighted average of rated item profiles
  - **Variation:** weight by difference from average rating for item
  - …
- **Prediction heuristic:**
  - Given user profile ***x*** and item profile ***i***, estimate $u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$

Pros: Content-based Approach

- **+: No need for data on other users**
  - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

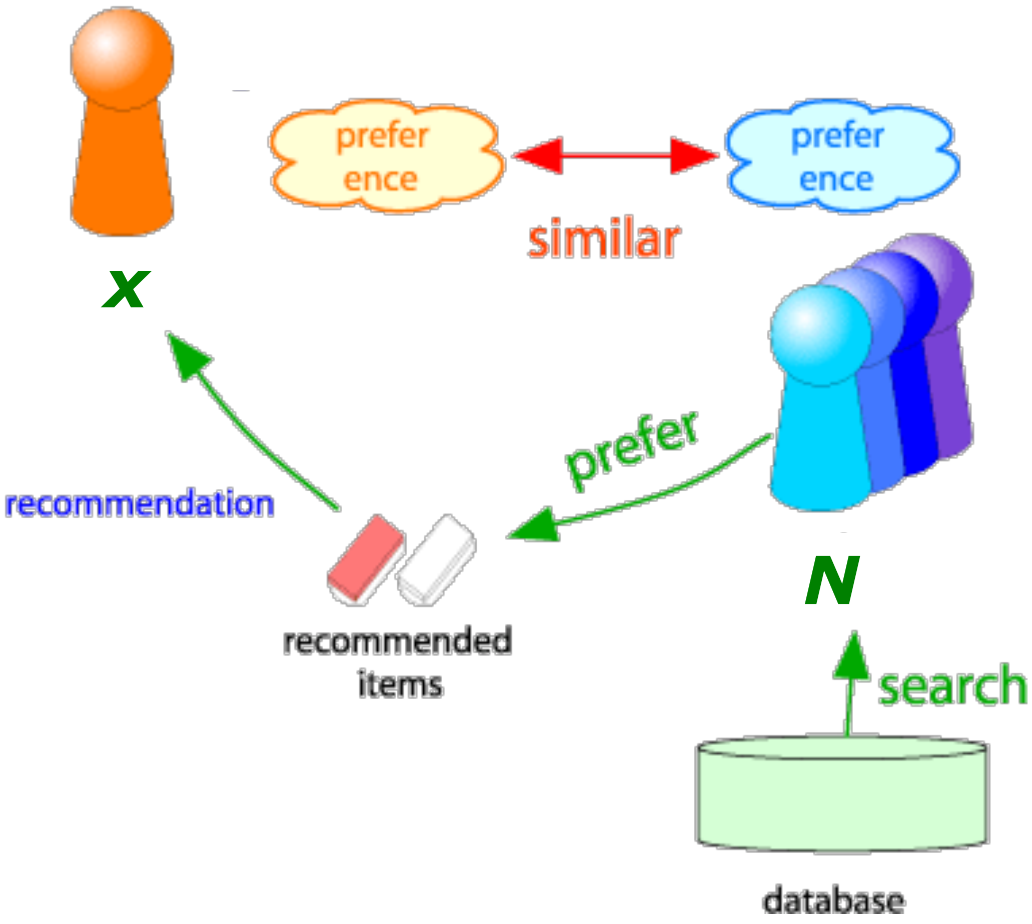
- **–: Finding the appropriate features is hard**
  - E.g., images, movies, music
- **–: Recommendations for new users**
  - **How to build a user profile?**
- **–: Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - **Unable to exploit quality judgments of other users**

Collaborative Filtering

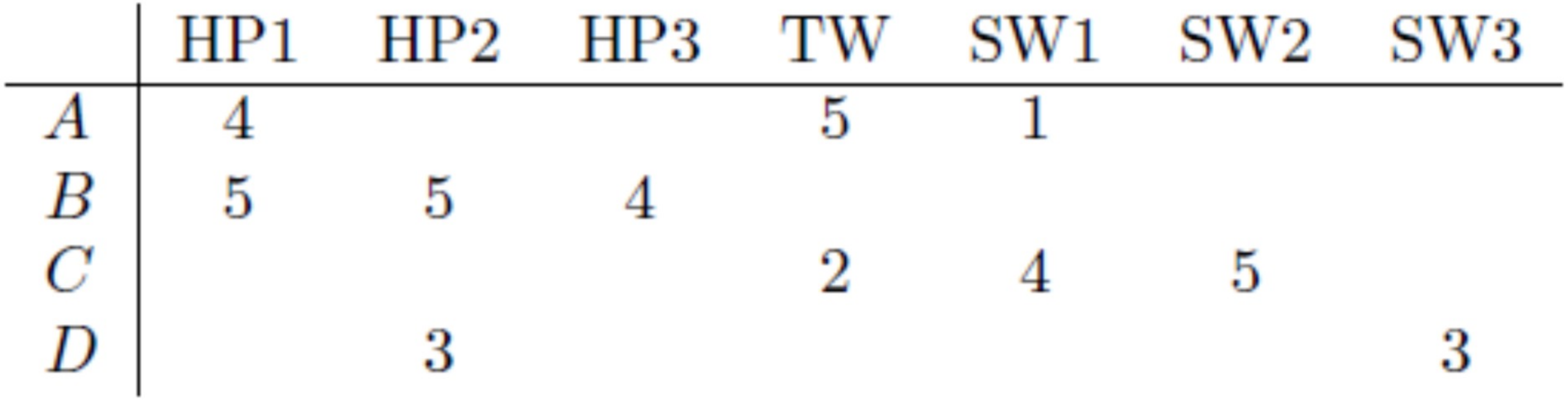
- Harnessing the judgment of other users

Collaborative Filtering

- Consider user ***x***
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



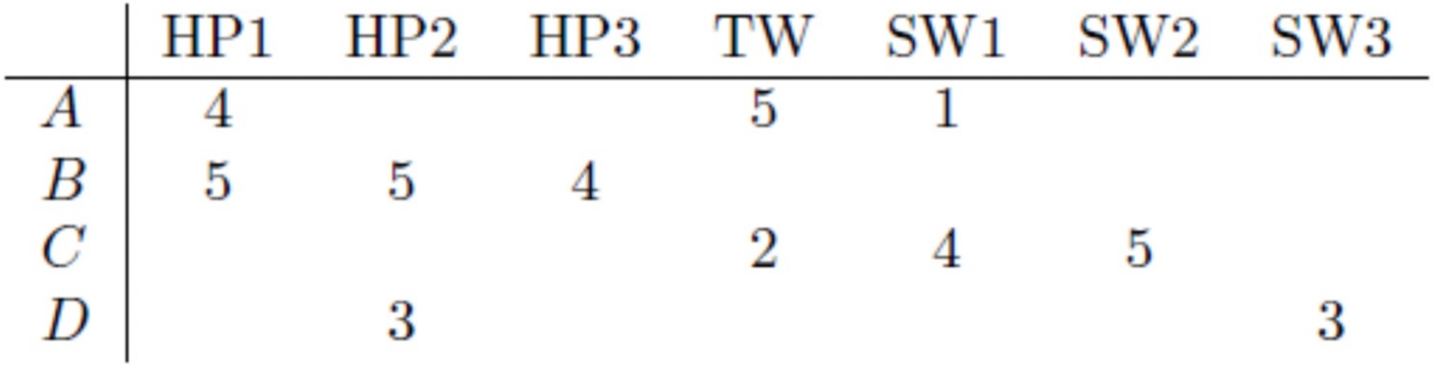
Similar Users (1)



	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- Consider users ***x*** and ***y*** with rating vectors $\boldsymbol{r}_{\boldsymbol{x}}$ and $\boldsymbol{r}_{\boldsymbol{y}}$
- We need a similarity metric sim(***x***, ***y***)
- Capture intuition that sim(A,B) > sim(A,C)

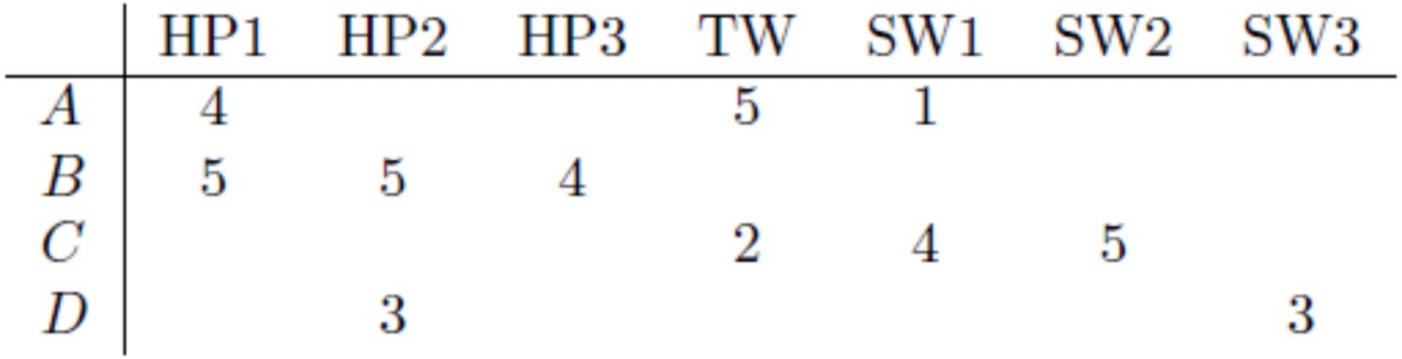
Option 1: Jaccard Similarity



	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- $\mathrm{sim}(A,B) = | r_A \cap r_B | / | r_A \cup r_B |$
- sim(A,B) = 1/5; sim(A,C) = 2/4
  - sim(A,B) < sim(A,C)
- Problem: Ignores rating values!

Option 2: Cosine Similarity



	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- sim(A,B) = cos(r_A, r_B)
- sim(A,B) = 0.38, sim(A,C) = 0.32
  - sim(A,B) < sim(A,C), but not by much
- Problem: treats missing ratings as negative

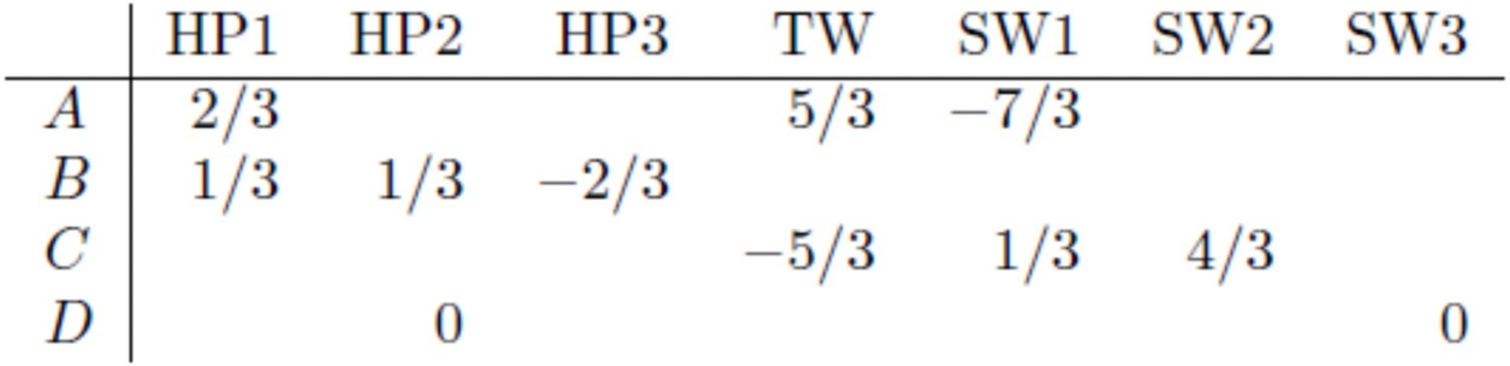
Option 3: Centered Cosine

- Normalize ratings by subtracting row mean

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

Centered Cosine Similarity (2)



	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

- sim(A,B) = cos(r_A, r_B) = 0.09; sim(A,C) = -0.56
  - sim(A,B) > sim(A,C)
- Captures intuition better
  - Missing ratings treated as “average”
  - Handles “tough raters” and “easy raters”
- Also known as Pearson Correlation

Rating Predictions

- Let r_x be the vector of user x's ratings
- Let N be the set of k users most similar to x who have also rated item i
- Prediction for user x and item i

- Option 1: $r_{xi} = 1/k \sum_{y \in N} r_{yi}$
- Option 2: $r_{xi} = \sum_{y \in N} s_{xy} r_{yi} / \sum_{y \in N} s_{xy}$

where $s_{xy} = \text{sim}(x,y)$

Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$\boxed{r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***u*** on item ***j***
N(i;x)… set items rated by ***x*** similar to ***i***

Item-Item CF (|N|=2)

users

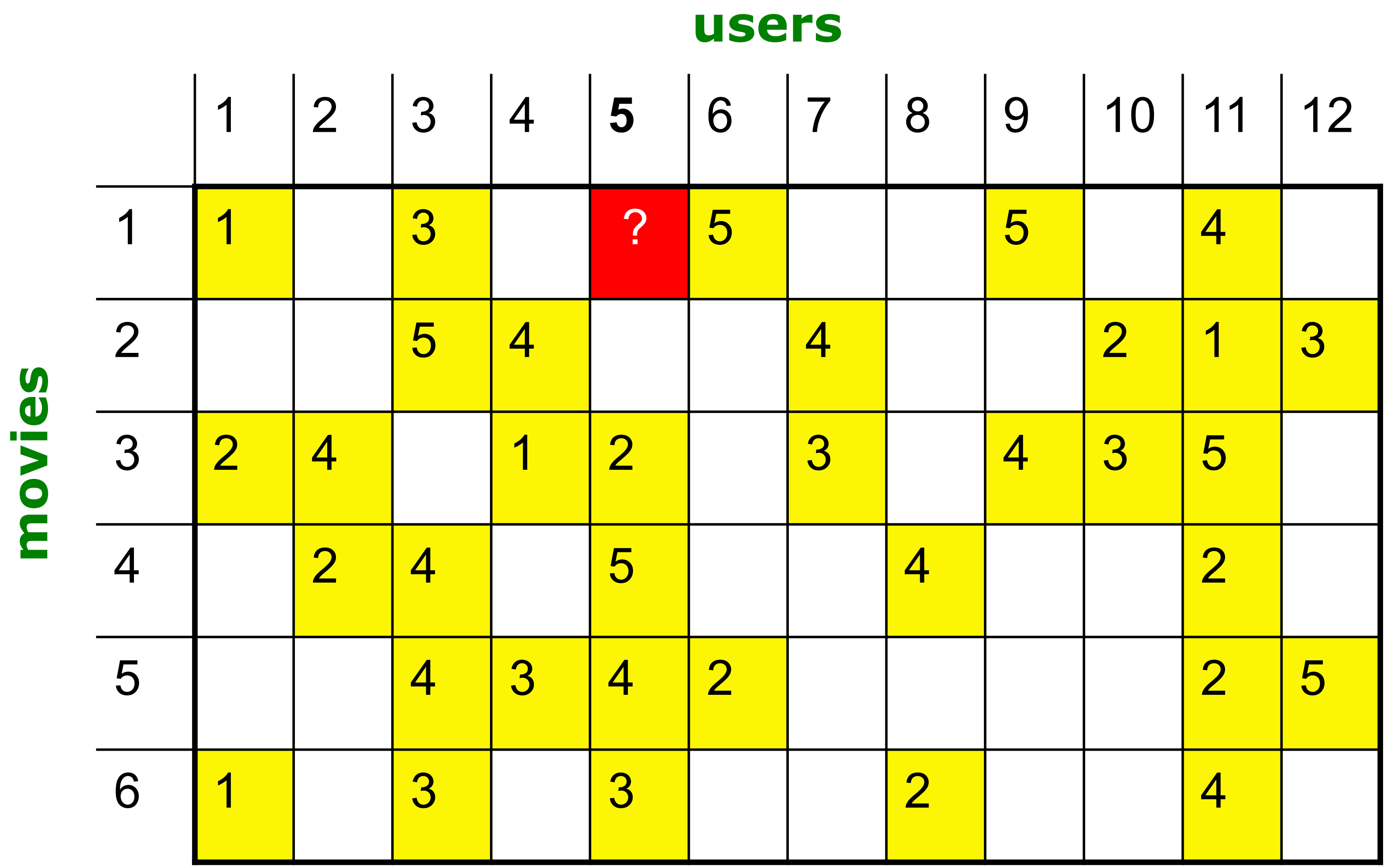
movies

	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3			5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

☐ - unknown rating

🟨 - rating between 1 to 5

Item-Item CF (|N|=2)



users

movies

	1	2	3	4	**5**	6	7	8	9	10	11	12
1	1		3		?	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

🟥 - estimate rating of movie **1** by user **5**

Item-Item CF (|N|=2)

users

movies	1	2	3	4	**5**	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		0.41
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		0.59

Neighbor selection:
Identify movies similar to
movie **1**, **rated by user 5**

Here we use Pearson correlation as similarity:
1) Subtract mean rating m_i from each movie i
$m_1 = (1+3+5+5+4)/5 = \mathbf{3.6}$
row 1: [-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]
2) Compute cosine similarities between rows

Item-Item CF (|N|=2)

users

movies	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		**0.59**

Compute similarity weights:

$s_{1,3}=0.41$, $s_{1,6}=0.59$

Item-Item CF (|N|=2)

users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		**2.6**	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

Predict by taking weighted average:

$$r_{1.5} = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6$$

Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- **In practice, it has been observed that <u>item-item</u> often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes