

SVD - Definition

$$\mathbf{A}_{[m \times n]} = \mathbf{U}_{[m \times r]} \Sigma_{[r \times r]} (\mathbf{V}_{[n \times r]})^{\mathsf{T}}$$

- A: Input data matrix
 - \blacksquare m x n matrix (e.g., m documents, n terms)
- U: Left singular vectors
 - \blacksquare $m \times r$ matrix (m documents, r concepts)
- Σ : Singular values
 - r x r diagonal matrix (strength of each 'concept') (r: rank of the matrix A)
- V: Right singular vectors
 - \blacksquare *n* x *r* matrix (*n* terms, *r* concepts)

SVD - Properties

- It is **always** possible to decompose a real matrix \boldsymbol{A} into $\boldsymbol{A} = \boldsymbol{U} \boldsymbol{\Sigma} \boldsymbol{V}^{\mathsf{T}}$, where
- U, Σ , V: unique
- *U*, *V*: column orthonormal
 - $\blacksquare U^T U = I; V^T V = I (I: identity matrix)$
 - (Columns are orthogonal unit vectors)
- Σ: diagonal
 - Entries (**singular values**) are positive, and sorted in decreasing order $(\sigma_1 \ge \sigma_2 \ge ... \ge 0)$

• $A = U \Sigma V^T$ - example: Users to Movies

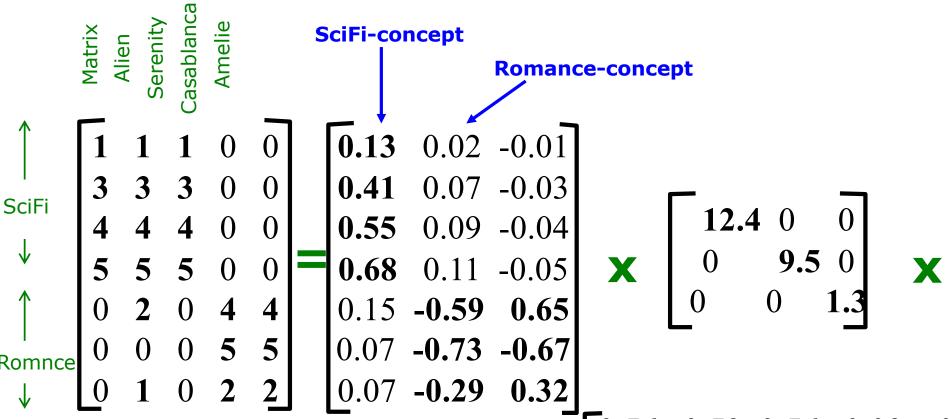
	Matrix	Alien	Casablan	Amelie								
	1	1	1	0	0	0.13		-0.01				
SCIFI	3 4	3 4	3 4	0	0	0.41 0.55	0.07	-0.03 -0.04		12.4	0	0
↓ ↑	5	5 2	5 0	0 4	0 4	0.68 0.15	0.11 -0.59	-0.05 0.65	X	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	9.5 0	0 1.3
Romnce	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	0	0	5	5	0.07	-0.73	-0.67		L		
\downarrow		1	0	2	2	0.07	-0.29	0.32		0.70	0.76	. 0

 0.56
 0.59
 0.56
 0.09
 0.09

 0.12
 -0.02
 0.12
 -0.69
 -0.69

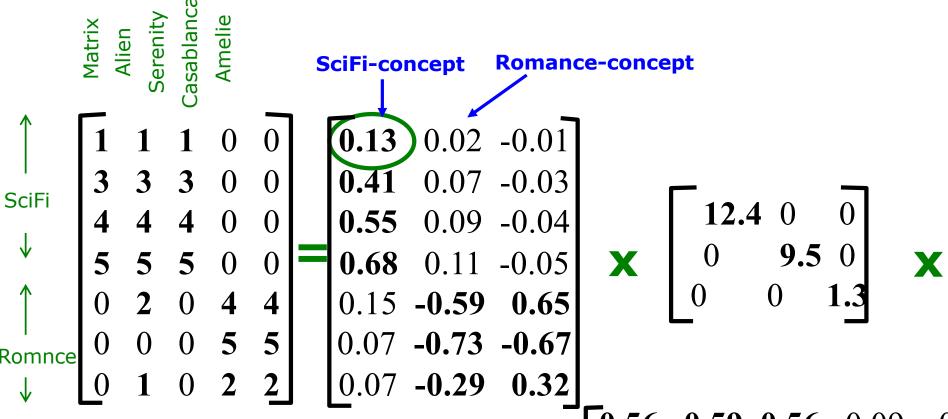
 0.40
 -0.80
 0.40
 0.09
 0.09

• A = U Σ V^T - example: Users to Movies

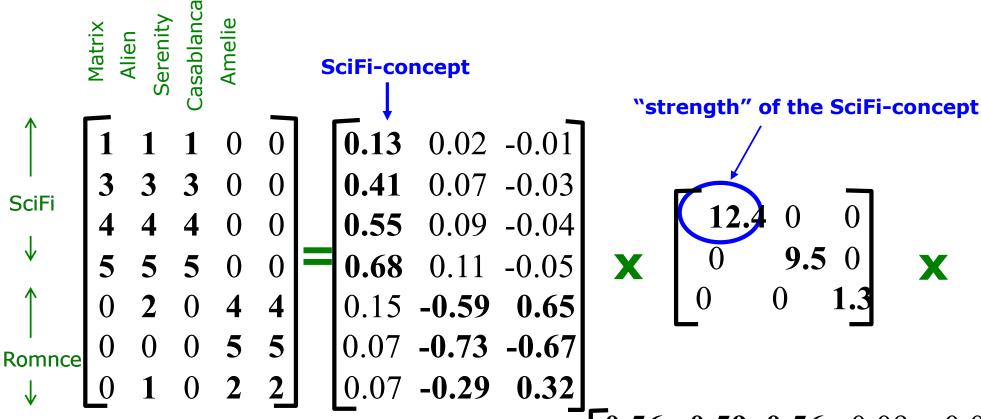


• $A = U \Sigma V^T$ - example:

U is "user-to-concept" similarity matrix



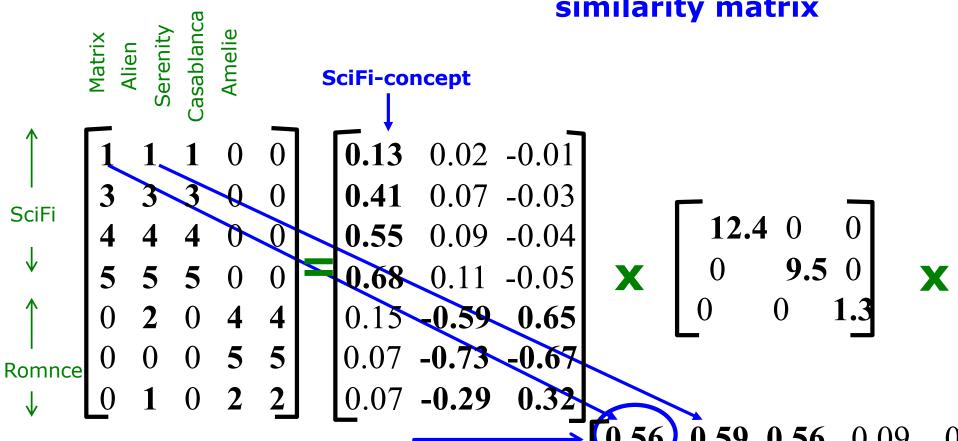
\bullet A = U Σ V^T - example:



$$\begin{bmatrix} \mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -\mathbf{0.69} & -\mathbf{0.69} \\ 0.40 & -\mathbf{0.80} & 0.40 & 0.09 & 0.09 \end{bmatrix}$$

• $A = U \Sigma V^T$ - example:

V is "movie-to-concept" similarity matrix



SciFi-concept

 $\begin{bmatrix} \mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\ 0.12 & -0.02 & 0.12 & -\mathbf{0.69} & -\mathbf{0.69} \\ 0.40 & -\mathbf{0.80} & 0.40 & 0.09 & 0.09 \end{bmatrix}$