Recommendation Systems with SVD

SVD - Definition

$$\mathbf{A}_{[m \times n]} = \mathbf{U}_{[m \times r]} \Sigma_{[r \times r]} (\mathbf{V}_{[n \times r]})^T$$

- **A**: **Input data matrix**
  - *m* x *n* matrix (e.g., *m* documents, *n* terms)
- **U**: **Left singular vectors**
  - *m* x *r* matrix (*m* documents, *r* concepts)
- Σ: **Singular values**
  - *r* x *r* diagonal matrix (strength of each 'concept') (*r* : rank of the matrix **A**)
- **V**: **Right singular vectors**
  - *n* x *r* matrix (*n* terms, *r* concepts)

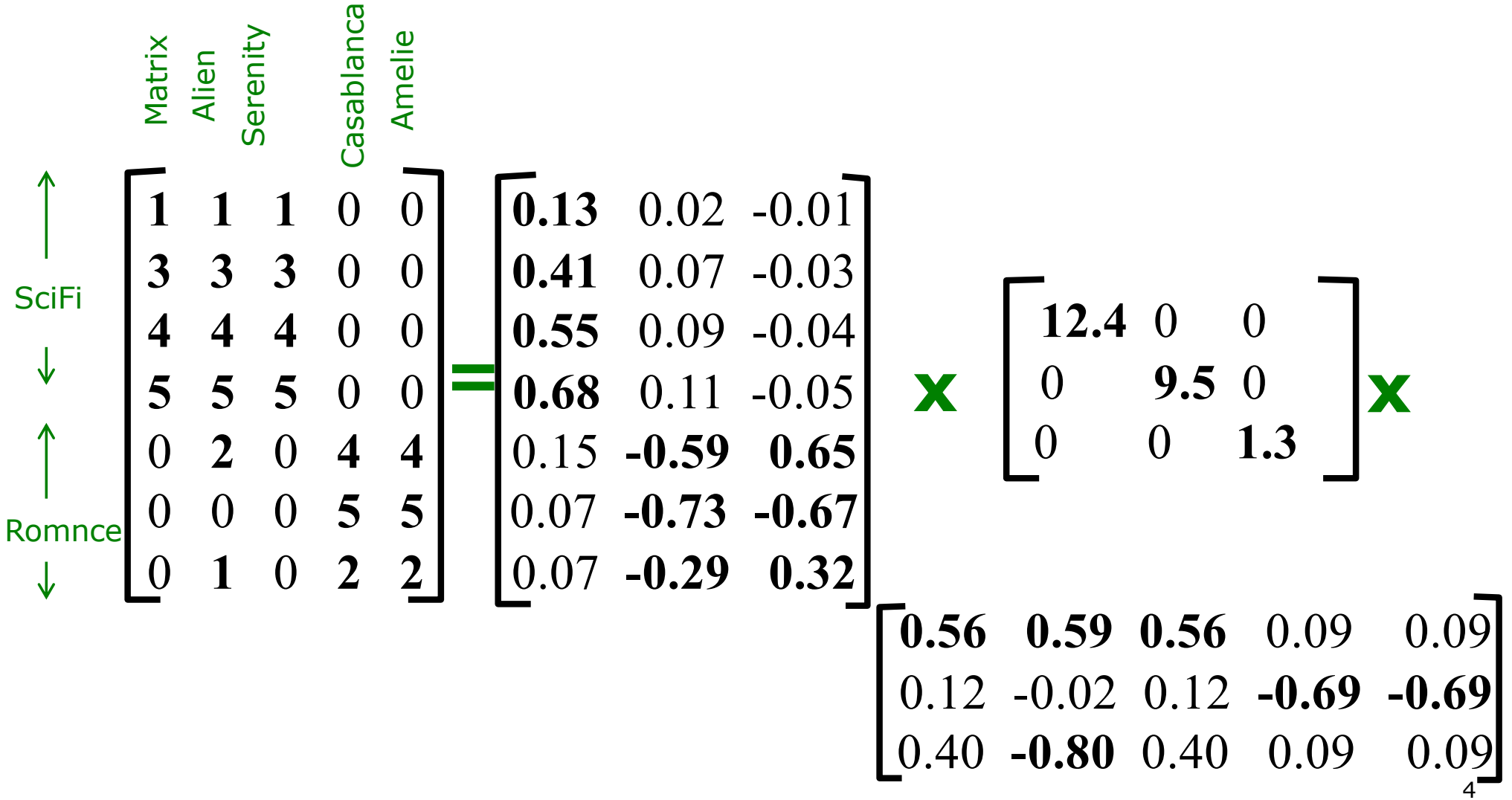
SVD - Properties

It is **always** possible to decompose a real matrix A into $A = U \Sigma V^T$, where

- U, Σ, V: unique
- U, V: column orthonormal
  - $U^T U = I$; $V^T V = I$ (I: identity matrix)
  - (Columns are orthogonal unit vectors)
- Σ: diagonal
  - Entries (**singular values**) are positive, and sorted in decreasing order ($\sigma_1 \geq \sigma_2 \geq \ldots \geq 0$)

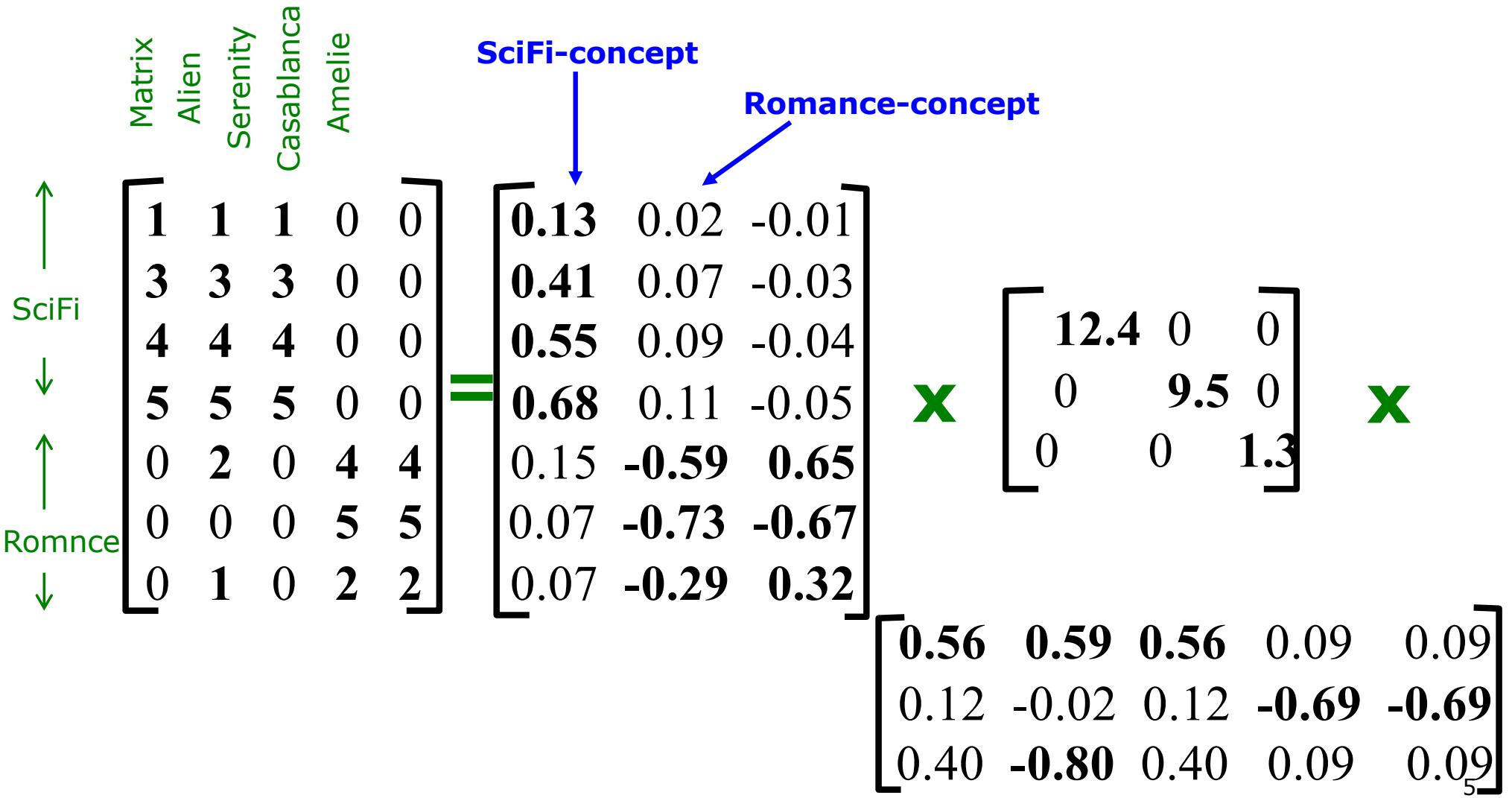
SVD – Example: Users-to-Movies

- **A = U Σ V^T - example: Users to Movies**



SVD – Example: Users-to-Movies

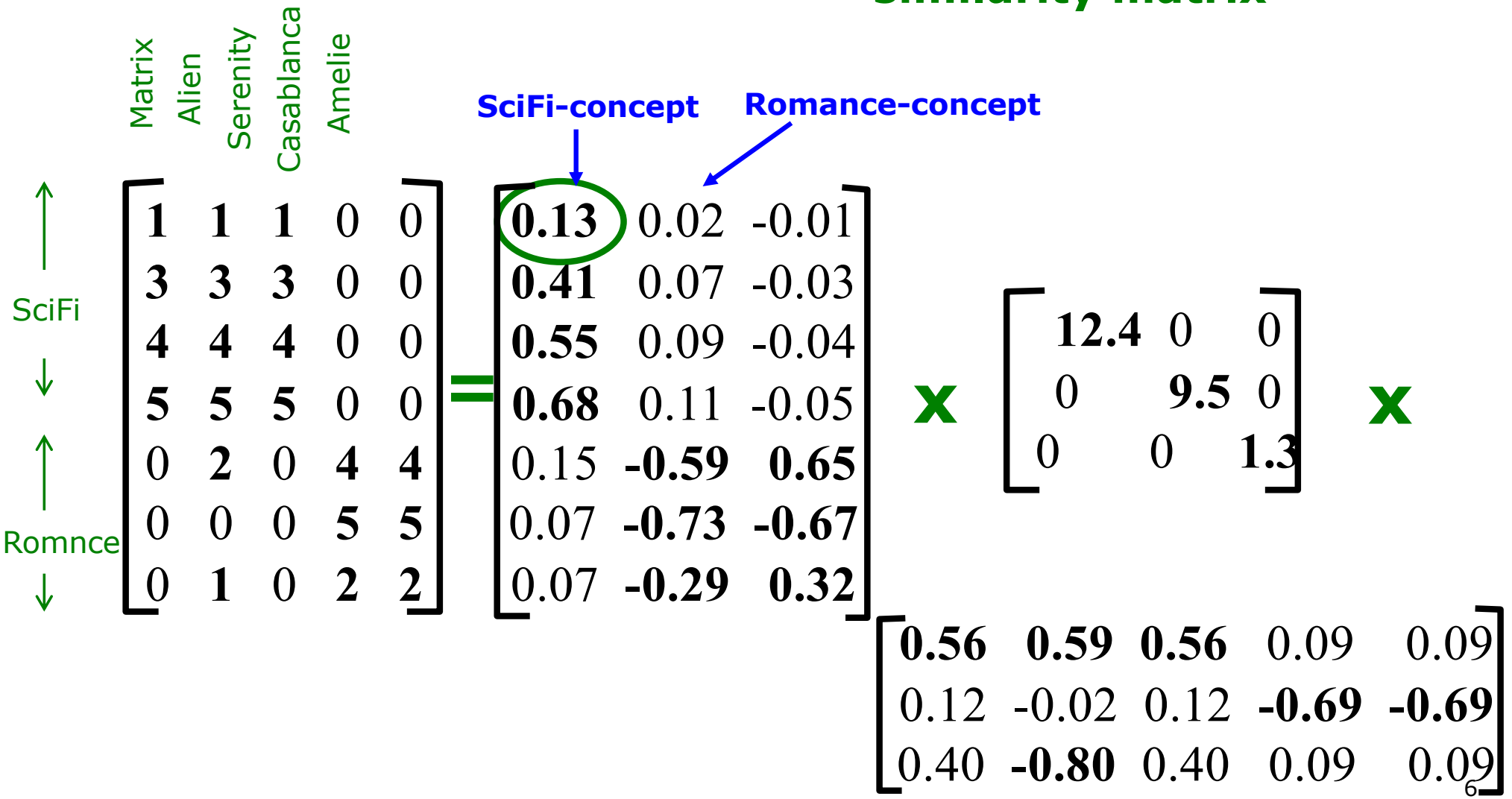
- **A = U Σ V^T - example: Users to Movies**



SVD – Example: Users-to-Movies

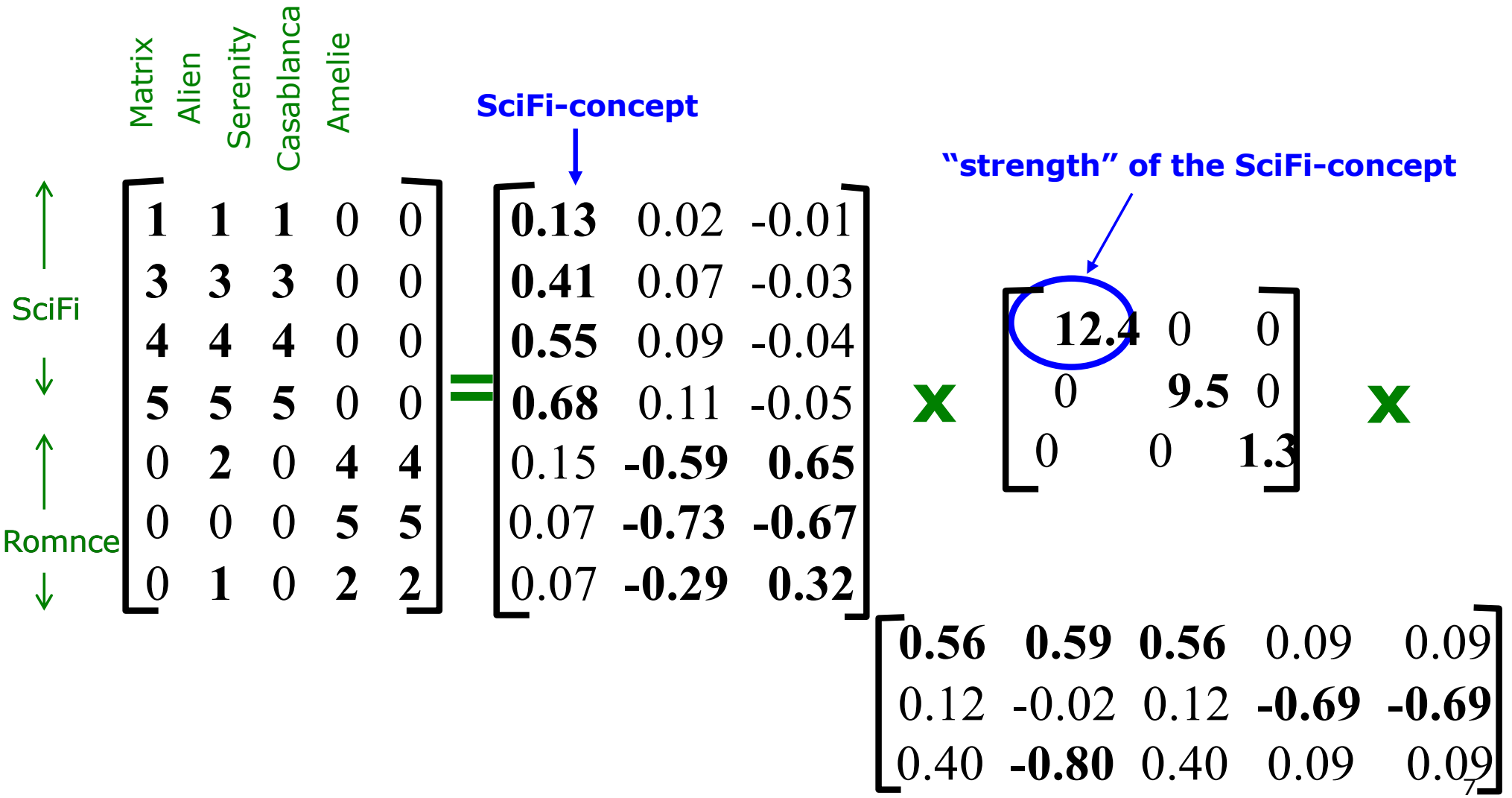
- **$A = U \Sigma V^T$ - example:**

U **is "user-to-concept" similarity matrix**



SVD – Example: Users-to-Movies

- **A = U Σ V^T - example:**



SVD – Example: Users-to-Movies

- **$A = U \Sigma V^T$ - example:**

V **is "movie-to-concept" similarity matrix**