Optimization Algorithm Overview

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1 Defining Variables

There are two non-binary variables, one for the gates in the quantum circuit and one for the edges in the quantum lattice. The quantum circuit gates are designed by $g(i, i')$, where i is the number of the first qubit and i' is the number of the second qubit. The edges of the quantum lattice are denoted by $e(j, j')$, where j is first lattice node and j' is the adjacent lattice node.

The placement of qubits on the quantum lattice are denoted by the binary decision variable $y_{i,j,t}$, where i is the QUBO variable, j is the lattice node, and t is the time step. For example, $y_{2,3,5}$ is true when the qubit representing the 2^{nd} variable in the QUBO is located at the 3^{rd} node of the quantum lattice at time step 5, and false otherwise. The explicit swap gates are denoted by the binary decision variable $\mathrm{SWAP}_{e,t}$, where e is the edge the swap was done across, and t is the time step that the swap gate was applied. $\mathrm{SWAP}_{e,t}$ is true when there was a swap on the edge e at time step t. Finally, the applied quantum circuit gates are represented by the binary decision variable $Z_{g,e,t}$, where g stands for the quantum circuit gate that was applied, e is the edge the gate was applied across, and t is the time step that the gate was applied. $Z_{g,e,t}$ is true when gate g is applied across lattice edge e at time step t.

2 Constraints

The first constraint is that each qubit must always be assigned to one lattice node. This constraint is enforced by

$$\sum_j y_{i,j,t} = 1 \quad \forall i, \forall t. \tag{1}$$

The second constraint is that each lattice node cannot have more than one qubit assigned to it. This constraint is enforced by

$$\sum_i y_{i,j,t} <= 1 \quad \forall j, \forall t. \tag{2}$$

The third constraint defines the applied gates with

$$Z_{g,e,t} \leq \frac{1}{2}(y_{i,j,t} + y_{i',j,t} + y_{i,j',t} + y_{i',j',t}) \quad \forall g, \forall e, \forall t, \tag{3}$$

which ensures that a gate can only be applied over two qubits if they share an edge.

The fourth constraint is that each gate in the quantum circuit must be applied exactly once. This constraint is enforced by

$$\sum_{e,t} Z_{g,e,t} = 1 \quad \forall g. \tag{4}$$

The fifth constraint governs explicit swap gates and free swap gates from applied gates:

$$y_{i,j,t} \leq y_{i,j,t-1} + \sum_{j' \epsilon N(j)} \left[y_{i,j',t-1} * (SWAP_{e(j,j'),t} + \sum_{g \epsilon i} Z_{g,e(j,j'),t}) \right] \quad \forall i, \forall j, \forall t. \tag{5}$$

Breaking the equation down, the first term is true when the qubit has not changed locations on the lattice from the previous time step. The second term is true when the qubit has changed locations from the previous time step and it either went through a swap gate or an applied gate along the edge connecting its present and former positions.

The sixth constraint is that a qubit can only be part of one gate operation at a time, which is enforced by

$$\sum_{e \epsilon j} \left[\sum_{g} Z_{g,e,t} + SWAP_{e,t} \right] \leq 1 \quad \forall t, \forall j. \tag{6}$$

3 Objective Function

During the algorithm, the goal is to minimize the objective function

$$\sum_{e,t} SWAP_{e,t}, \tag{7}$$

which is the total number of explicit swap gates.