Do We Have Consensus?

Another example of simplification of Boolean expressions

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The Consensus Theorem

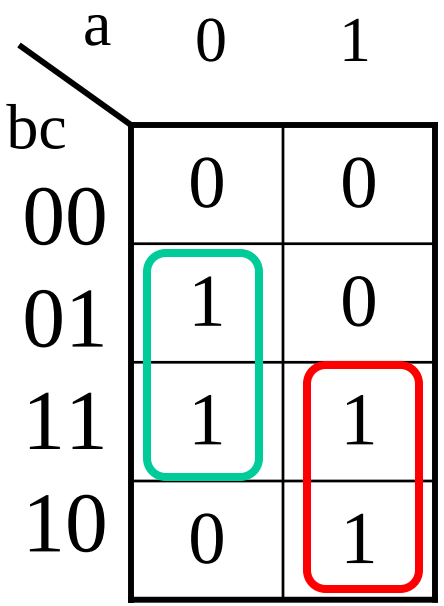
Given an expression of the form:

F_1 = ab + a'c + bc reduce F_1 into its minimized form. First, let's ask the question: Can F_1 be minimized. The simplest way to answer that question is to use the Karnaugh Map method.

The Karnaugh Map Method

$F_1 = ab + a'c + bc$

a	b	c	F1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1



$F_2 = ab + a'c$

Analysis

$F_1 = ab + a'c + bc$ $F_2 = ab + a'c$

F_1 was reduced by eliminating the term bc from the equation. Now that we know that F_1 could be reduced, how could we have arrived at the same minimized F_2 equation if we had used algebraic simplification instead of the Karnaugh Map method?

Notice something about F_1. It has two terms in which a variable and it's complement appear. The variable is 'a'. Let's remove the 'a' and its complement from the two terms in question and AND them together. We get the term "bc" (which is the third term in the F_1 equation). The term "bc" is labeled the "Consensus Term" and it is redundant. This is the term that must be eliminated to yield F_2!

Simplification Process

$F_1 = ab + a'c + bc$

Law of complementarity, $a+a' = 1$

$= ab + a'c + bc(a + a')$

$= ab + a'c + abc + a'bc$ Combine these terms

$= ab(1 + c) + a'c(1 + b)$ Factor and apply identities

$F_2 = ab + a'c$ Done