Building Characteristics

A Worked Example for Flip-Flop Characteristic Equations

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The Characteristic Equation

- The Characteristic Equation is a mathematical representation of the behavior of the flip-flop
- One of the major issues in designing logic with flip-flops is understanding how to use the characteristic equation

The JKFF

Let's take a look at the JKFF. Its characteristic equation is:

$$Q^+ = JQ' + K'Q$$

Q^+ is the next output state of the JKFF

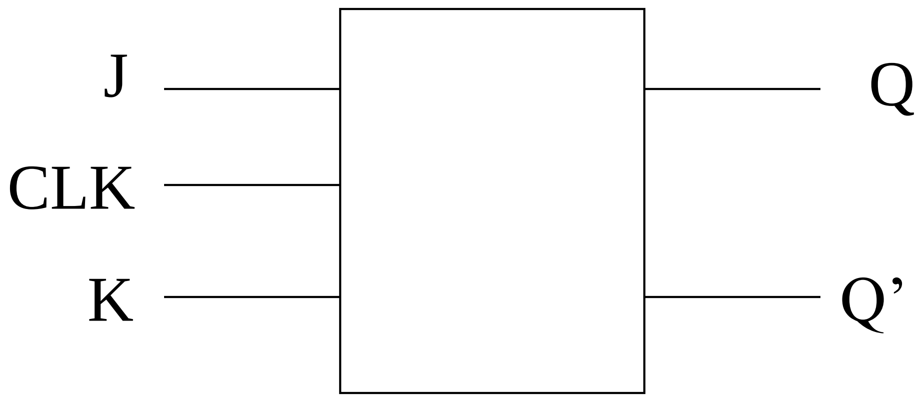
Q is the current output state of the JKFF

J and K are the current input values to the JKFF

Let's take a look at the JKFF and see how its characteristic equation defines its behavior …

The JKFF

Let’s examine the physical properties of the JKFF.



Based on the above figure, we can determine how to use the characteristic equations. The input and output terminals on the JKFF correspond directly to the terms in the characteristic equation.

Characteristic Equation and Complex Logic Designs

- How do we use the characteristic equation in the design of counters or sequence detectors?
- How do we adapt the characteristic equation to help our design process?
- Great questions!!! Let's take a closer look…

Counter Design using JKFFs

- Let’s design a counter that follows the counting sequence: 0-2-4-1-3-0
- Right away, we observe that we have 5 unique states.
- We require 3 digits to cover the 5 states identified in the counting sequence
- This means that 3 JKFFs (A, B, & C) are required.

Special Observation*

Since we have dubbed our 3 JKFFs A, B, and C, respectively, using the JKFF characteristic equation: $Q^+ = JQ' + K'Q$, the current state Q and the next state Q^+ will be replaced by the name of the JKFF, respectively.

For example, the 'A' JKFF characteristic equation will be written as follows:

$$A^+ = JA' + K'A$$

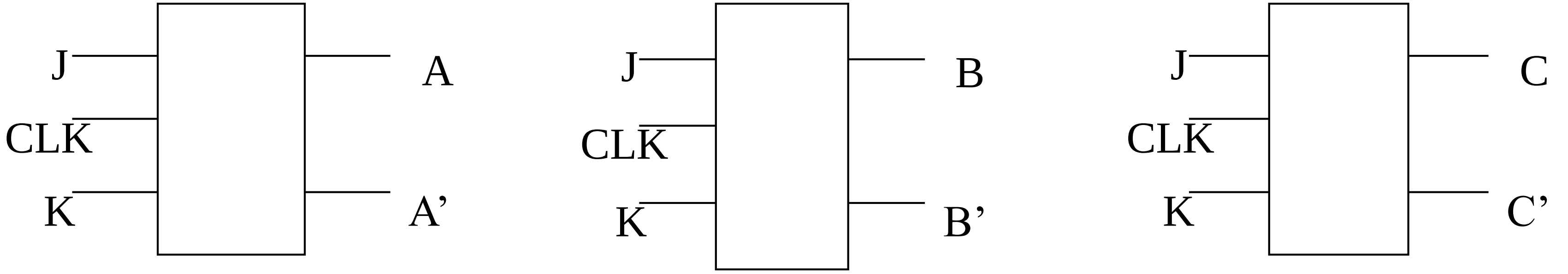
This convention follows for JKFFs B and C, respectively. With this in mind, the following represent the characteristic equations for the B abd C JKFFs respectively:

$$B^+ = JB' + K'B$$

$$C^+ = JC' + K'C$$

Special Observation*

As the characteristic equations are changed to model the naming conventions for our JKFFs in the counter design, the graphical representation changes accordingly.



Step 1: Create a State Table

- The state table captures all of the 5 states in the counter design
- The values are recorded in the order of the required count sequence
- The table contains the present state (P.S.) values, the Next State (N.S.) values and the input values (J and K) for each of the 3 JKFFs (A, B, and C) used in the design
- The J and K input values determine how each of the JKFFs transition from their respective Present States to their respective Next States

The State Table

Present State			Next State			Inputs to JKFFs					
A	B	C	A^+	B^+	C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	0	1	1	0	0	1
0	1	0	1	0	0	1	0	0	1	0	1
1	0	0	0	0	1	0	1	0	1	1	0
0	0	1	0	1	1	0	1	1	0	1	0
0	1	1	0	0	0	0	1	0	1	0	1

Step 2: Populate K-maps; A

Present State			Next State			Inputs to JKFFs					
A	B	C	A^+	B^+	C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	0	1	1	0	0	1
0	1	0	1	0	0	1	0	0	1	0	1
1	0	0	0	0	1	0	1	0	1	1	0
0	0	1	0	1	1	0	1	1	0	1	0
0	1	1	0	0	0	0	1	0	1	0	1

$J_{A\text{-Map}}$

BC \ A	0	1
00	0	0
01	0	X
11	0	X
10	1	X

$$J_A = BC'$$

$K_{A\text{-Map}}$

BC \ A	0	1
00	1	1
01	1	X
11	1	X
10	0	X

$$K_A = B' + C$$

Step 2: Populate K-maps; B

Present State			Next State			Inputs to JKFFs					
A	B	C	A^+	B^+	C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	0	1	1	0	0	1
0	1	0	1	0	0	1	0	0	1	0	1
1	0	0	0	0	1	0	1	0	1	1	0
0	0	1	0	1	1	0	1	1	0	1	0
0	1	1	0	0	0	0	1	0	1	0	1

$J_{B\text{-Map}}$

BC \ A	0	1
00	1	0
01	1	X
11	0	X
10	0	X

$J_B = A'B'$

$K_{B\text{-Map}}$

BC \ A	0	1
00	0	1
01	0	X
11	1	X
10	1	X

$K_B = A + B$

Step 2: Populate K-maps; C

Present State			Next State			Inputs to JKFFs					
A	B	C	A^+	B^+	C^+	J_A	K_A	J_B	K_B	J_C	K_C
0	0	0	0	1	0	0	1	1	0	0	1
0	1	0	1	0	0	1	0	0	1	0	1
1	0	0	0	0	1	0	1	0	1	1	0
0	0	1	0	1	1	0	1	1	0	1	0
0	1	1	0	0	0	0	1	0	1	0	1

J_C-Map

BC \ A	0	1
00	0	1
01	1	X
11	0	X
10	0	X

K_C-Map

BC \ A	0	1
00	1	0
01	0	X
11	1	X
10	1	X

$J_C = A + B'C$

$K_C = A'C' + B$

Step 3: Realization of Counter Logic