Number Systems and Conversion Techniques

Comparing Decimal to Binary

- The decimal number system is based on ten unique numbers (0,1,2,3,4,5,6,7,8,9)
- The binary number system is based on two unique numbers (0 and 1)
- Every decimal number may be represented in the binary number system (or binary)

Binary Addition

	1100 (12)		1101 (13)
+	0011 (3)	+	0111 (7)
	------		------
	1111 (15)		10100 (20)

When adding two binary numbers, the rules that apply in the decimal number system apply. When the sum of a column exceeds the base of the number system, a carry is generated. Therefore:

0 + 0 = 0

0 + 1 = 1

1 + 0 = 1

1 + 1 = 0 with a carry of 1

Binary Subtraction

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  1100 (12)      1101 (13)
- 0011 (3)     - 0111 (7)
  ------         ------
  1001 (9)       0110  (6)
```

When subtracting two binary numbers, the rules that apply in the decimal number system apply. When the subtrahend (the number being subtracted) is of a larger value than the number its being subtracted from, a borrow is generated. Therefore:

0 - 0 = 0

0 - 1 = 1 with a borrow of 1

1 - 0 = 1

1 – 1 = 0

A Word about Conventions

Since we are exploring number systems, it is important to recognize the number system in which we are operating. This is done through the radix (sometimes referred to as the “base”) Radix and base are interchangeable terms. The radix is an indicator that is placed as a subscript appended to the end of a number. For example, the number 130_{10} tells the reader that the number is a decimal number 130 because its radix is 10. Likewise, the number 11101_2 is a binary number because its radix is 2.

Examples of Number Systems

Number System	Radix	Valid Number System Digits
Binary	**Base 2**	**{0,1}**
Ternary	**Base 3**	{0,1,2}
Quaternary	**Base** 4	{0,1,2,3}
Quinary	**Base 5**	{0,1,2,3,4}
Senary	**Base 6**	{0,1,2,3,4,5}
Septenary	**Base 7**	{0,1,2,3,4,5,6}
Octary (Octal)	**Base 8**	{0,1,2,3,4,5,6,7}
Nonary	**Base 9**	{0,1,2,3,4,5,6,7,8}
Decimal	***Base 10***	{0,1,2,3,4,5,6,7,8,9}
Hexadecimal	***Base 16***	{0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F}

The Components of Numbers

We often do not think about how numbers are constructed. In this course, however, it is important to gain a better appreciation for number composition. Let's take an example:

The number 150_{10} may be represented as a composite of several terms. If we partition the number into ones, tens, and hundreds components, we get:

0 ones

5 tens

1 hundreds

By adding the components together, we obtain 150. In other words, every number may be represented as a summation of terms (with each term occupying a specific position in the number). Let's write it in a more precise manner.

$$1 \times 10^2 + 5 \times 10^1 + 0 \times 10^0 = 150_{10}$$

The same rules hold for all number systems. This procedure will help us to better understand and evaluate binary numbers.

Let's convert the binary number 1101_2 to its decimal equivalent

$$1101_2 = 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$$

$$= 8 \quad + 4 \quad + 0 \quad + 1$$

$$= 13_{10}$$

Add 1010 and 0011

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  1010
+ 0011
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  1101
```

Converting Binary to Decimal

- Each digit in a binary number may use the 0 or 1 digit only
  - 6 base 10 = 110 base 2
  - Each digit in binary is multiplied by the base raised to the power of the digit location. All the numbers are then summed to get the decimal number.
  - 0*2^0 + 1*2^1 + 1*2^2
  - 0 + 2 + 4 = 6
  - The standard convention in reading binary is right to left

Converting Decimal to Binary

Problem:

Convert 25 base 10 to binary representation.

2	25		
2	12	remainder	1
2	6	remainder	0
2	3	remainder	0
2	1	remainder	1
	0	remainder	1

11001 base 2

Convert decimal number 17 to binary

2 | 17	→	1	lsb – least significant bit
2 | 8	→	0	
2 | 4	→	0	
2 | 2	→	0	
2 | 1	→	1	msb - Most significant bit

Converting Binary to Octal

- This process is a little different because the conversion involves 2 major steps
- Step 1: Convert Binary to Decimal
- Step 2: Convert Decimal to Octal
- Note that when Decimal is not the source or destination number system, it serves as the intermediate number system for all other conversions

Converting Binary to Octal

Step 1: Convert 111011_2 to its Octal equivalent:

$$1x2^5 + 1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 = 59_{10}$$

Step 2: Convert 59_{10} to Octal

$$\begin{array}{r|l} 8 & 59 \\ \hline \end{array} \longrightarrow 3$$

$$\begin{array}{r|l} 8 & 7 \\ \hline & 0 \end{array} \longrightarrow 7$$

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Converting Binary to Hexadecimal (Hex)

Step 1: Convert 111011_2 to its Hex equivalent:

$1x2^5 + 1x2^4 + 1x2^3 + 0x2^2 + 1x2^1 + 1x2^0 = 59_{10}$

Step 2: Convert 59_{10} to Hex

$16 \,\underline{|\; 59} \longrightarrow$ B

$16 \,\underline{|\; 3} \longrightarrow$ 3

0

$3B_{16}$

Check Please!

In order to check that our conversion was performed accurately. Let's convert our Hex result into its Decimal representation to see if the values are identical

$3x16^1 + Bx16^0 =$

$3(16) + 11(1) =$

$48 + 11 = 59_{10}$

The numbers match. That means that the conversion was correct.