

Rate-compatible Polar Coding and HARQ

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Rate compatible polar codes

- *$\{C_1, C_2 \dots C_N\}$ be a compound channel with $R_1 > R_2 > R_3 \dots > R_N$, and block lengths $n_1 < n_2 \dots < n_N$*
- **Then the set is rate compatible if codeword for C_i can be built by removing $n_j - n_i$ bits from codewords of code $C_j, j > i$**
- Can be constructed by puncturing low rate codes.
- *Polar codes for degraded channels is inherently RCP*

HARQ = ARQ + FEC

Type –I

1. Send **MSG+ED+EC**
2. If recoverable with EC do so.
3. Else use ED to detect error and Retransmit

- Use **Chase combining**
(like MRC) to decode finally

- Effective SNRs for each Tx add.

Type –II

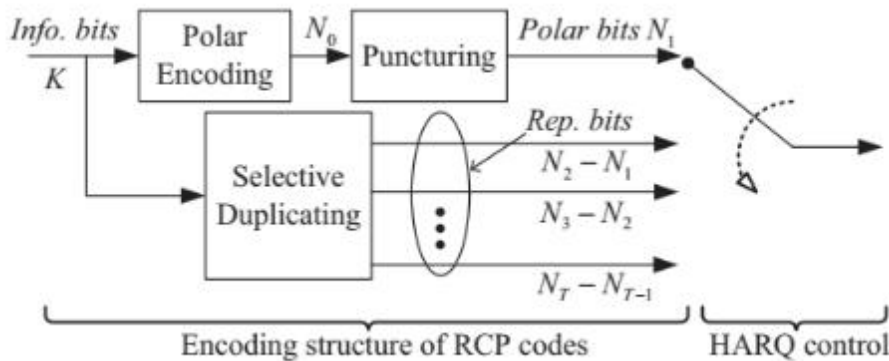
1. Send **MSG+ED**
2. if decodable do so.
3. Else Send **EC incrementally**
4. Decode using all the previous information.
5. Repeat from 3 if not decodable

• **Incremental Redundancy**

- *Data sent on step 3 is RV or redundancy vector, can be chosen appropriately*
- Coding Gain obtained

HARQ for polar codes need a RCP and a HARQ scheme

Scheme 1 *(Kai chen)*



- K information (msg) bits to be sent

RCP

- Use N_0 (block length of a low rate polar code)
- Puncture to create $N_1 < N_2 < N_3 \dots < N_T < N_0$
- T is the maximum number of repeats

RCP Block ($K \leq M \leq N$):

M (polar bits)	N-M repetition bits
K information bits	Chosen from the K bits

Scheme 1 *(Kai chen)*

HARQ

- A Type II ARQ (IR)
- First Transmission N_1 polar bits are sent
- 2nd onwards till 'T'th transmission $N_t - N_{t-1}$ repetitions bits are sent. (N_t, K, M) RCPs constructed on each iteration.
- i.e, The RVs are repetition bits chosen as follows

Selection of repetition bits

1. Choose the most error prone channel among the K bits, repeat this bit.
2. Step 1 increases the reliability, which is re-calculated.
3. Repeat from step 1.

Scheme 2 *(Kai chen , Incremental freezing)*

- A improvement over the previous scheme.
- Uses ***reliability ordering*** to create puncturing patterns, thus the RCP.
- Applicable to degraded channels only.
- RV is repetition bits, choice of repetition bits is based on reliability ordering.
- Extended to general channels by Mondelli , Hassani, using universal polarization (scheme 2a)
- We have implemented this.

Scheme 3_(Tavildar)

RCP

- Uses Subset Polar codes as RCP

Subset Polar code

- Start with (N,K) polar code mother code
- Create (M,K) subset code by puncturing, $N > M$, as follows:
 1. Fix a BLER
 2. puncture 1 bit
 3. estimate BLER
 4. change rate to get fixed BLER
 5. repeat from step 1 till (M,K) reached

Scheme 3_(Tavildar)

HARQ

- Uses Uses Equivalent subset polar code as RV.

Equivalent subset Polar codes

Let $S = \{s_1, s_2, \dots, s_M\}$ and $T = \{t_1, t_2, \dots, t_M\}$

be two $(N \times M, K)$ subset polar codes. We say S is equivalent to T if $s_i = t_i \oplus x \forall i, 1 \leq i \leq M$ for some integer x , where $1 \leq x \leq N$

cont...

Scheme 3_(Tavildar)

Equivalent subset polar codes create redundancy as:

If S, T are equivalent subset polar codes, then channels $W^{(i)}_{N,S}$ and $W^{(i)}_{N,T}$ are equivalent for each $1 \leq i \leq N$.

Where, bit channel equivalence is defined as follows:

Let W_1 and W_2 be two binary input channels with output alphabets Y_1 and Y_2 respectively. We define the two channels are equivalent, $W_1 \sim W_2$, if there exists an invertible function $f : Y_1 \rightarrow Y_2$ such that $W_1(y_1 | x) = W_2(f(y_1) | x)$.

Simulations suggest this beats previous schemes in terms of reliability under joint decoding.

Use of CRC(DLC layer artefact) as ED

- CRC is supposedly used as ED in all mentioned schemes.
- Rateloss is negligible ***but not if short blocklengths are used.***
- Has other adverse effects like complexity due to cross layer cooperation.
- Mukhtar et al. proposed CRC free HARQ with Turbo product codes using inherent ED capability of TPC.

We are trying to get CRC-free HARQ with polar codes

Proposition 1:

- If we can estimate the expected value of the $\text{abs}(\text{LLRs})$ of all the channels from some analytical expression (eg, Z) then ED is **Binary** Hypothesis testing between exactly **2** LLR vectors in each step. Further we can try ML decoding of this received LLR vector using some gaussian approximation of the LLR distribution (which looked like gaussian).

Proposition 2:

Error checking using frozen bits as mentioned here:

<https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4142331>

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