

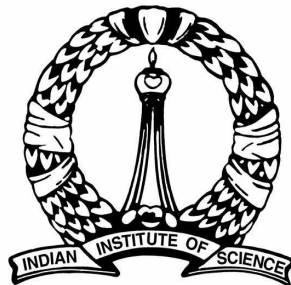
Pseudo Random Phase Precoded Spatial Modulation

A midterm report for the degree of
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in
Telecommunications
by

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Abstract

It is well known that for single-input and single-output narrow band transmission on frequency-flat fading channels, uncoded communication with only receiver channel state information leads to extremely poor reliability performance whereas transmitter CSI allows us approach the reliability of an additive white gaussian noise (AWGN) channel via power control. However with pseudo random phase precoding (PRPP) of modulation symbols prior to temporal multiplexing and joint-detection at the receiver that has polynomial complexity in precoder size we can approach AWGN channel. In this work, we propose a method to achieve higher order diversity in a multi-antenna input and multi-antenna output (MIMO) system with spatial modulation (SM) through pseudo random phase precoding (PRPP) and without any channel side information (CSI) at the transmitter. The PRPP method gives increased reliability for uncoded transmission in MIMO systems without any power control. We propose a novel precoded SM scheme, where both the modulation bits and the antenna index bits are precoded by pseudo random phases. This gives a significant improvement over the traditional scheme of precoding only the modulation bits. As maximum likelihood (ML) detection becomes exponentially complex at large dimensions, we employ local search detector (LSD) for large precoder sizes. Our simulation results show that proposed system achieves better performance than SM.

Contents

Abstract	i
1 Introduction	1
2 Pseudo Random Phase Precoding	3
2.1 PRPP for SISO system	4
2.1.1 Detection of PRPP SISO system :	6
2.1.2 PRPP for MIMO systems :	7
3 System Model	9
3.1 Spatial Modulation without precoding	9
3.2 SM-MIMO with precoded modulation symbols	10
3.3 SM-MIMO with precoded modulation symbols and antenna indices	11
4 Low Complexity Detector	14
5 Results and discussions	16
6 Conclusions	19

List of Figures

2.1	PRPP-SISO system model.	4
2.2	Performance of PRPP on SISO fading channels.	7
2.3	Performance of uncoded single user MIMO ($N_t=2, N_r = 2$,BPSK) systems with PRPP using MMSE-LAS detection algorithm.	8
3.1	performance of uncoded single user SM systems with precoded modulation symbols ($N_t=4, N_r=1$,BPSK and ML detection).	12
3.2	system model.	13
5.1	Average BER performance of PRPP-SM($N_t = 4, N_r = 1$,BPSK) with ML detection.	17
5.2	Average BER performance of PRPP-SM($N_t = 4, N_r = 4$,BPSK) with ML detection.	17
5.3	Average BER performance of PRPP-SM($N_t = 4, N_r = 8$,BPSK) with ML detection.	18
5.4	Average BER performance of PRPP-SM($N_t = 2, N_r = 4$,BPSK) with MMSE-LSD algorithm.	18

Chapter 1

Introduction

Reliable communication over fading and dispersive wireless channels is an important area of research [1]. For a single-input and single-output (SISO) point-to-point link on frequency-flat fading channels, it is well-known that uncoded transmission without channel state information at transmitter (Tx-CSI) has an extremely poor reliability performance [2]. With receiver CSI (Rx-CSI) only, the average uncoded probability of error on SISO fading links decays inversely with the average received signal-to-noise ratio (SNR). This is in contrast with exponential decay on additive white Gaussian noise (AWGN) channel [3]. With Tx-CSI, on the other hand, it is possible to approach the AWGN performance via temporal power control [4], [5]. Spatial diversity, via multiple antennas at the receiver, is an attractive option to improve the reliability of single antenna transmissions [6]. However, due to size and cost constraints, it may not be possible to increase the number of antennas beyond a certain limit. Channel coding, using resources in time and/or frequency, is also a well-known technique to mitigate the effects of fading [7], [8]. The coding gain and the diversity achieved against fading strongly depend on the nature of the channel and the properties of the channel code employed such as code rate, codeword length, constraint length (i.e., free distance), and decoder complexity [7]. We know that pseudo random phase precoding (PRPP) of modulation symbols prior to temporal multiplexing can achieve the performance close to the AWGN channel [9].

Spatial modulation (SM) is a technique in which only one transmit antenna is active at any instant. The active transmit antenna is an added source of information that is exploited by SM to boost the spectral efficiency. In SM, a block of information bits is mapped into a constellation point in the signal domain and antenna index in the spatial domain. At each time instant, only

one transmit antenna of all transmit antennas will be active and the other antennas will remain silent. Therefore the inter channel interference(ICI) at the receiver and the need to synchronize the transmit antennas are completely avoided. Under certain circumstances SM outperforms AWGN channel[10].

In this report, we propose pseudo random phase precoding for spatial modulation which performs better than SM without precoding and outperforms AWGN under certain circumstances.

The rest of the report is organized as follows. Chapter 2 presents brief explanation about PRPP for SISO and MIMO systems. The system model for PRPP for SM is presented in chapter 3. Chapter 4 presents proposed algorithm for low complexity detection. Simulation results are presented in chapter 5. In chapter 6 conclusions and future work are drawn.

Chapter 2

Pseudo Random Phase Precoding

We consider a point-to-point communication system with a single antenna at the transmitter and a single antenna at the receiver. The channel between the transmitter and the receiver is modeled as a frequency flat and slowly varying random variable (r.v) h . The transmitted signal is x with mean $E[x]=0$ and variance $E[|x|^2]=\sigma_x^2$, and the corresponding received signal y is given by

$$y = hx + w \quad (2.1)$$

where w is a zero-mean complex-Gaussian r.v with variance $E[|w|^2]=\sigma_w^2$. With x drawn from a constellation \mathbb{A} with M points, the maximum likelihood detector (MLD) of x has the following well-known form

$$\hat{x} = \arg \min_{x \in \mathbb{A}} ||y - hx||^2 \quad (2.2)$$

However, the main limitation of the above approach is that the reliability, as measured by the average bit error rate (BER), is extremely poor. As an example, with $M = 2$ and $\mathbb{A} = \{-\sigma_x, +\sigma_x\}$

The following notations are used in the report. Vectors are denoted by boldface lowercase letters and matrices are in boldface uppercase letters. $[\cdot]^T$, $[\cdot]^*$, $[\cdot]^\dagger$ denote the transpose, conjugate, hermitian operations. E denotes expectation, $||\cdot||$ is euclidean norm, $diag(\mathbf{y})$ denotes a diagonal matrix with the elements of vector \mathbf{y} along its diagonal, \mathbf{I}_n is an $n \times n$ identity matrix.

with equal probability, the average BER on uncorrelated Rayleigh fading channels (i.e., when h is a zero-mean complex-Gaussian r.v with variance $E[|h|^2]=\sigma_h^2$, is [3]

$$\bar{P}_b = 1/4SNR \quad \text{for high SNR} \quad (2.3)$$

where $SNR = \sigma_x^2 \sigma_h^2 / \sigma_w^2$ is the average received SNR per symbol. This must be contrasted against the performance on an AWGN channel[2] (i.e., when $h=1$) decays exponentially with SNR.

2.1 PRPP for SISO system

The PRPP SISO system model is shown in figure 2.1. The complex-valued information symbol sequence is denoted by $\mathbf{u} = \{u_1, u_2, \dots\}$, where u_i is drawn independently from a constellation \mathbb{A} containing M points with $E[u_i] = 0$ and $E[|u|^2] = \sigma_u^2$. For a given precoder size p , the information symbol vector at the input of the PRPP is denoted by $\mathbf{u} = \{u_1, u_2, \dots, u_p\}^T$. The output of the precoder is denoted by \mathbf{x} and is simply $\mathbf{x} = \mathbf{P}\mathbf{u}$. $\theta_{m,n}$ is pseudo-randomly generated by a seed shared between the transmitter and the receiver. The (m,n) th element of \mathbf{P} is $p_{m,n} = \frac{1}{\sqrt{p}} e^{j\theta_{m,n}}$, $m = 1, \dots, p, n = 1, \dots, p$, where $\theta_{m,n}$ is pseudo-randomly generated by a seed shared between the transmitter and the receiver. The m th element of \mathbf{x} is

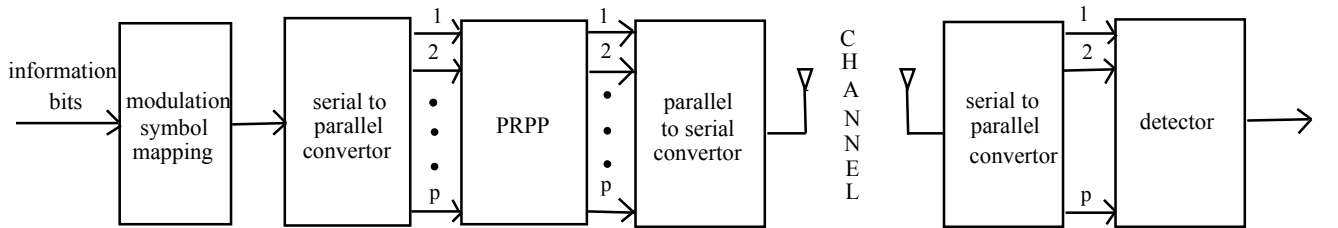


Fig. 2.1 PRPP-SISO system model.

$$x_m = \frac{1}{\sqrt{p}} \sum_{k=1}^p e^{j\theta_{m,n}} u_k \quad (2.4)$$

$\theta_{m,n}$ is uniformly distributed in $[-\pi, +\pi)$, a reasonable assumption for long pseudo sequence,

and due to independence of $\theta_{m,n}$'s and u_i 's we have

$$E[x_m] = \frac{1}{\sqrt{p}} \sum_{k=1}^p E[e^{j\theta_{m,n}}] E[u_k] = 0 \quad (2.5)$$

and

$$\begin{aligned} E[x_m x_l^*] &= \frac{1}{p} \sum_{n=1}^p \sum_{q=1}^p E[e^{j\theta_{m,n}} e^{-j\theta_{l,q}}] E[u_n u_q^*] \\ &= \frac{1}{p} \sigma_s^2 \sum_{n=1}^p \sum_{q=1}^p E[e^{j\theta_{m,n}} e^{-j\theta_{l,q}}] \delta_{n-q} \\ &= \frac{1}{p} \sigma_s^2 \sum_{n=1}^p E[e^{j\theta_{m,n}} e^{-j\theta_{l,n}}] \\ &= \sigma_s^2 \delta_{m-l} \end{aligned} \quad (2.6)$$

where δ_k is Kronecker delta function that evaluates to one when $k = 0$ and zero otherwise. That is, with uncorrelated input sequence, the PRPP produces uncorrelated output sequence. Instead of transmitting the original sequence \mathbf{u} , we transmit the sequence $\mathbf{x} = [x_1, x_2, \dots]$ produced by the pseudo random sequence precoder. Since each x_n goes through flat fading channel, upon collecting complex-valued received symbols over p channel uses, we have

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{w} \\ &= \mathbf{G}\mathbf{u} + \mathbf{w} \end{aligned} \quad (2.7)$$

where $\mathbf{G} = [g_{m,n}]$ is an $p \times p$ matrix whose $(m, n)^{th}$ element $g_{m,n}$ is given by

$$g_{m,n} = \frac{1}{\sqrt{p}} e^{j\theta_{m,n}} h_m \quad (2.8)$$

Clearly, due to the independence of h_m 's and $\theta_{m,n}$'s, we have

$$E[g_{m,n}] = 0 \quad (2.9)$$

and

$$E[g_{m,n} g_{p,q}^*] = \sigma_h^2 \delta_{m-n} \delta_{p-q} \quad (2.10)$$

That is, from above, we conclude that the PRPP applied to p symbols with single-antenna transmission produces an $p \times p$ virtual MIMO system with p virtual spatial streams and uncorrelated channels between each transmit-receive antenna pair.

2.1.1 Detection of PRPP SISO system :

For a PRPP of size p , exact MLD of \mathbf{u} has complexity proportional to M^p , which is prohibitively expensive even for binary modulations ($M = 2$) and a precoder operating on $p = 10$ symbols. Near-ML sphere-decoding algorithms can be applied to detect \mathbf{u} . However, they too have complexity exponential in p . Sub-optimal MIMO detection algorithms such as linear zero-forcing (ZF) and minimum mean-square error (MMSE) receivers have complexity of the order of N^3 (since they require a matrix inversion) but provide a per-symbol diversity of only one [11]. The near-ML local ascent search (LAS) algorithm in [12] essentially performs an p -dimensional search to arrive at a fixed point $\hat{\mathbf{u}}$ that no longer improves the likelihood (conditioned on the channel and the received vector), and has a complexity proportional to p^3 (i.e., comparable to a linear MMSE receiver).

The PRPP transmitter coupled with LAS receiver algorithm and with the linear MMSE solution as the initial vector is referred to as PRPP-MMSE-LAS. The performance of PRPP with MMSE-LAS receiver is shown in Fig 2.2. It compares the average BER performance of the system without PRPP against the PRPP transmission with MMSE-LAS receiver. We set the precoder size $p \in \{50, 400\}$ modulation symbols. The channel is assumed to undergo temporally uncorrelated Rayleigh fading. It is interesting to note that the performance of MMSE-LAS receiver approaches that of the AWGN system. At an bit error rate of 10^{-5} the AWGN system requires an SNR of 9.6 dB whereas the PRPP system with MMSE-LAS requires an SNR of 9.7 dB with 400 symbols.

That is, PRPP system has near-exponential diversity and is within 0.1 dB of the AWGN system performance.

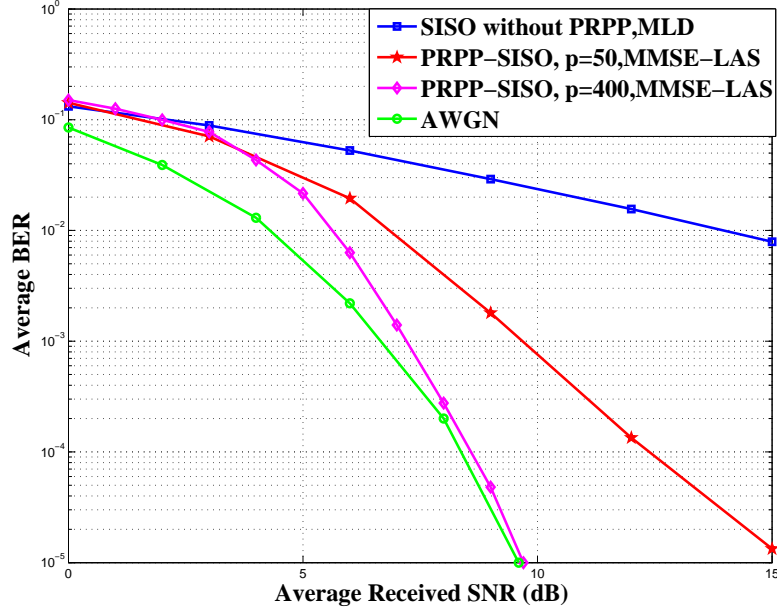


Fig. 2.2 Performance of PRPP on SISO fading channels.

2.1.2 PRPP for MIMO systems :

The pseudo-random phase precoding can be applied over both space (the number of transmit antennas) and time (the number of channel uses) to realize a large dimensional MIMO system with N_t transmit antennas and N_r receive antennas. Denoting the $pN_t \times pN_t$ PRPP by \mathbf{P} , then our PRPP-MIMO system

$$\mathbf{y} = \mathbf{D}\mathbf{P}\mathbf{u} + \mathbf{w} \quad (2.11)$$

where \mathbf{y} is $pN_r \times 1$ received signal vector,

$\mathbf{D} = \text{Diag}\{\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_p\}$ is the $pN_r \times pN_t$ effective channel matrix,

\mathbf{H}_i is $N_r \times N_t$ channel matrix in i th channel use,

$\mathbf{u} = [\mathbf{u}_1^T, \mathbf{u}_2^T, \dots, \mathbf{u}_p^T]^T$ is $pN_t \times 1$ transmitted modulation vector,

\mathbf{u}_i is $N_t \times 1$ modulation vector in i th channel use,

\mathbf{w} is $pN_r \times 1$ vector containing zero-mean complex-Gaussian r.v with variance $=\sigma_w^2$.

We consider binary modulation with $N_t=2, N_r=2$, with spectral efficiency N_t bits per channel use. The number of channel uses p is chosen from $\{50, 100, 150\}$. The BER performance of PRPP-MIMO, averaged over the number of streams and the number of channel uses is shown in Fig. 2.3 as a function of the average received SNR.

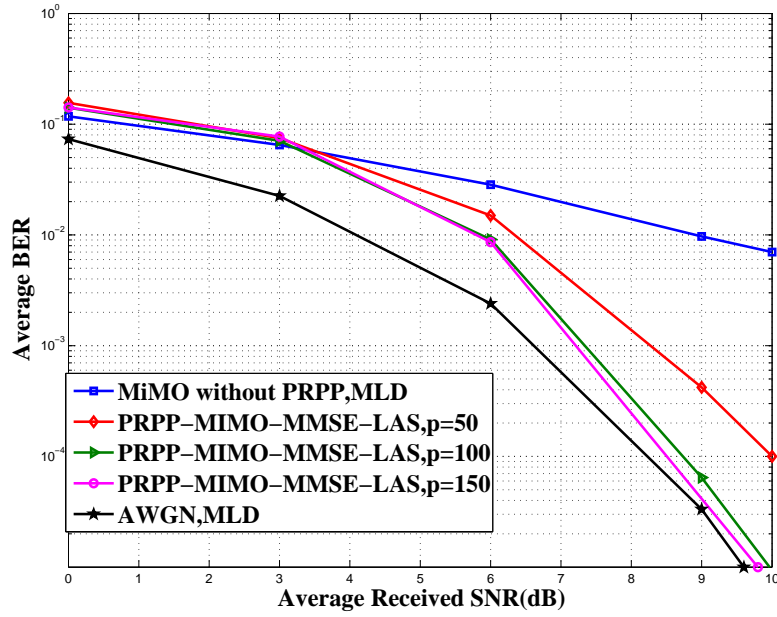


Fig. 2.3 Performance of uncoded single user MIMO ($N_t=2, N_r=2$, BPSK) systems with PRPP using MMSE-LAS detection algorithm.

Chapter 3

System Model

Consider a MIMO system with N_t transmit antennas and N_r receive antennas. Let n_{rf} be the number of transmit RF chains at the transmitter. In case of generalized SM, $n_{rf} < N_t$; for simplicity we assume $n_{rf} = 1$.

3.1 Spatial Modulation without precoding

In a MIMO system with SM, the transmitter has N_t transmit antennas but only one transmit RF chain. In a given channel use, the transmitter selects one of its N_t transmit antennas, and transmits a modulation symbol from the alphabet \mathbb{A} on the selected antenna. The number of bits transmitted per channel use in the modulation symbols is $\lfloor \log_2 |\mathbb{A}| \rfloor$, and the number of bits transmitted per channel use by the index of the transmitting antenna is $\lfloor \log_2 N_t \rfloor$. Therefore, a total of $\lfloor \log_2 |\mathbb{A}| N_t \rfloor$ bits per channel use (bpcu) is achievable in an SM-MIMO system. For example, in a system with $N_t = 2$, 8-QAM, the system throughput is 4 bpcu.

The SM alphabet set for a fixed N_t and \mathbb{A} is given by

$$\begin{aligned} \mathbb{S}_{N_t, \mathbb{A}} &= \{ \mathbf{x}_{j,l} : j = 1, \dots, N_t, \quad l = 1, \dots, |\mathbb{A}| \}, \\ \text{s.t. } \mathbf{x}_{j,l} &= [0, \dots, 0, \underbrace{s_l}_{j\text{th coordinate}}, 0, \dots, 0]^T, \quad x_l \in \mathbb{A}. \end{aligned} \tag{3.1}$$

For example, for $N_t = 2$ and 4-QAM, $\mathbb{S}_{N_t, \mathbb{A}}$ is given by

$$\mathbb{S}_{2,4\text{-QAM}} = \left\{ \begin{bmatrix} +1+j \\ 0 \end{bmatrix}, \begin{bmatrix} +1-j \\ 0 \end{bmatrix}, \begin{bmatrix} -1+j \\ 0 \end{bmatrix}, \begin{bmatrix} -1-j \\ 0 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 \\ +1+j \end{bmatrix}, \begin{bmatrix} 0 \\ +1-j \end{bmatrix}, \begin{bmatrix} 0 \\ -1+j \end{bmatrix}, \begin{bmatrix} 0 \\ -1-j \end{bmatrix} \right\}. \quad (3.2)$$

Let $\mathbf{x} \in \mathbb{S}_{N_t, \mathbb{A}}$ denote the transmit vector. Let $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ denote the channel gain matrix, where $H_{i,j}$ denotes the channel gain from the j th transmit antenna to the i th receive antenna and these channel gains are i.i.d complex Gaussian random variables. The received signal at the i th receive antenna is given by

$$y_i = H_{i,j}x_l + n_i, \quad (3.3)$$

where x_l is the l th symbol in \mathbb{A} , transmitted by the j th antenna, and n_i is the noise which is distributed as $\mathbb{CN}(0, \sigma^2)$. The signal at the receiver can be written in vector form as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (3.4)$$

For this system model, the maximum-likelihood (ML) detection rule is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}_{N_t, \mathbb{A}}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \quad (3.5)$$

3.2 SM-MIMO with precoded modulation symbols

In a transmitter employing precoding, p modulated symbols are accumulated at the transmitter to form the vector $\mathbf{x}_s \in \mathbb{A}^p$ and is precoded using a $p \times p$ matrix \mathbf{P} as $\mathbf{P}\mathbf{x}_s$. The (r, c) th entry of the matrix \mathbf{P} is $\frac{1}{\sqrt{p}}e^{j\theta_{r,c}}$, where the phases $\theta_{r,c}$'s are generated using a pseudo random sequence generator, the seed of this generator is pre-shared among the transmitter and receiver. Let the matrix \mathbf{A} denote the antenna activation pattern, such that $\mathbf{A}\mathbf{x}_s \in \mathbb{S}_{N_t, \mathbb{A}}^p$. The matrix \mathbf{A} constitutes of p submatrices $\mathbf{A} = [\mathbf{A}_{(1)}^T \mathbf{A}_{(2)}^T \cdots \mathbf{A}_{(p)}^T]^T$. The submatrix $\mathbf{A}_{(i)}$ indicates the antenna activated in the i th channel use, $\mathbf{A}_{(i)}$ is constructed as $\mathbf{A}_{(i)} = [\mathbf{0}_{(1)} \cdots \mathbf{0}_{(i-1)} \mathbf{a}_{(i)} \mathbf{0}_{(i+1)} \cdots \mathbf{0}_{(p)}]$, where $\mathbf{0}_{(k)}$ is a $N_t \times 1$ vector of zeroes and $\mathbf{a}_{(i)}$ is a $N_t \times 1$ vector constructed as $[0 \cdots 0 \underbrace{1}_{j\text{th coordinate}} 0 \cdots 0]^T$. For

example, in a system with $N_t = 2$ and $p = 3$, to activate antennas 1, 2 and 1 in three consecutive channel uses, respectively, the matrix \mathbf{A} is given by

$$\begin{bmatrix} \mathbf{A}_{(1)} \\ \mathbf{A}_{(2)} \\ \mathbf{A}_{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ - & - & - \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ - & - & - \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

The matrix \mathbf{A} gives the support of the spatially modulated transmit vector. The signal received at the receiver after p channel uses is given by

$$\begin{aligned} \mathbf{y}_p &= \begin{bmatrix} \mathbf{H}_{(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_{(2)} & \cdots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{H}_{(p)} \end{bmatrix} \mathbf{A} \mathbf{P} \mathbf{x}_s + \mathbf{n}_p \\ &= \mathbf{D} \mathbf{A} \mathbf{P} \mathbf{x}_s + \mathbf{n}_p = \mathbf{G} \mathbf{x}_s + \mathbf{n}_p, \end{aligned} \quad (3.6)$$

where $\mathbf{D} = \text{diag}(\mathbf{H}_{(1)} \mathbf{H}_{(2)} \cdots \mathbf{H}_{(p)})$, $\mathbf{G} = \mathbf{D} \mathbf{A} \mathbf{P}$ and \mathbf{n}_p is the noise vector $[\mathbf{n}_{(1)}^T \mathbf{n}_{(2)}^T \cdots \mathbf{n}_{(p)}^T]^T$. From (2.9) and (2.10), we can see that the entries of the matrix \mathbf{G} is uncorrelated and $\|\mathbf{D} \mathbf{A}\| = \|\mathbf{G}\|$, this creates a $pN_r \times p$ virtual large-MIMO system.

For this system model, the maximum-likelihood (ML) detection rule is given by

$$\{\hat{\mathbf{x}}_s, \hat{\mathbf{A}}\} = \underset{\mathbf{x}_s \in \mathbb{A}^p, \forall \mathbf{A}}{\text{argmin}} \|\mathbf{y}_p - \mathbf{D} \mathbf{A} \mathbf{P} \mathbf{x}_s\|^2, \quad (3.7)$$

3.3 SM-MIMO with precoded modulation symbols and antenna indices

The system described in Sec. 3.2 is the natural method of MIMO precoding; however, on analyzing the performance of the system in Sec. 3.2 (shown in Fig. 3.1), we find that precoding over

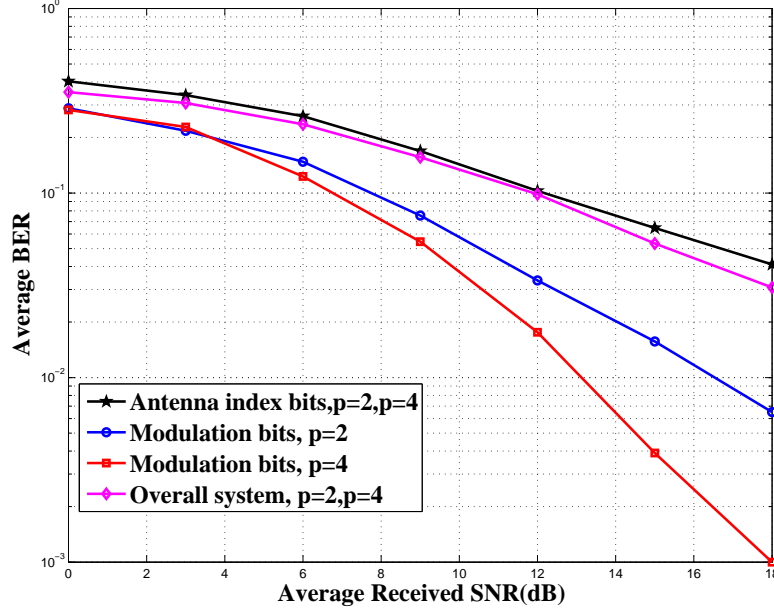


Fig. 3.1 performance of uncoded single user SM systems with precoded modulation symbols ($N_t=4, N_r=1$, BPSK and ML detection).

modulation symbols only is insufficient for achieving higher order diversity in SM-MIMO. From Fig. 3.1 we can see that the probability of error of the antenna index bits is higher than the probability of error of the modulation bits. This motivates us to precode both the modulation symbols and antenna indices in SM-MIMO system to achieve higher order diversity.

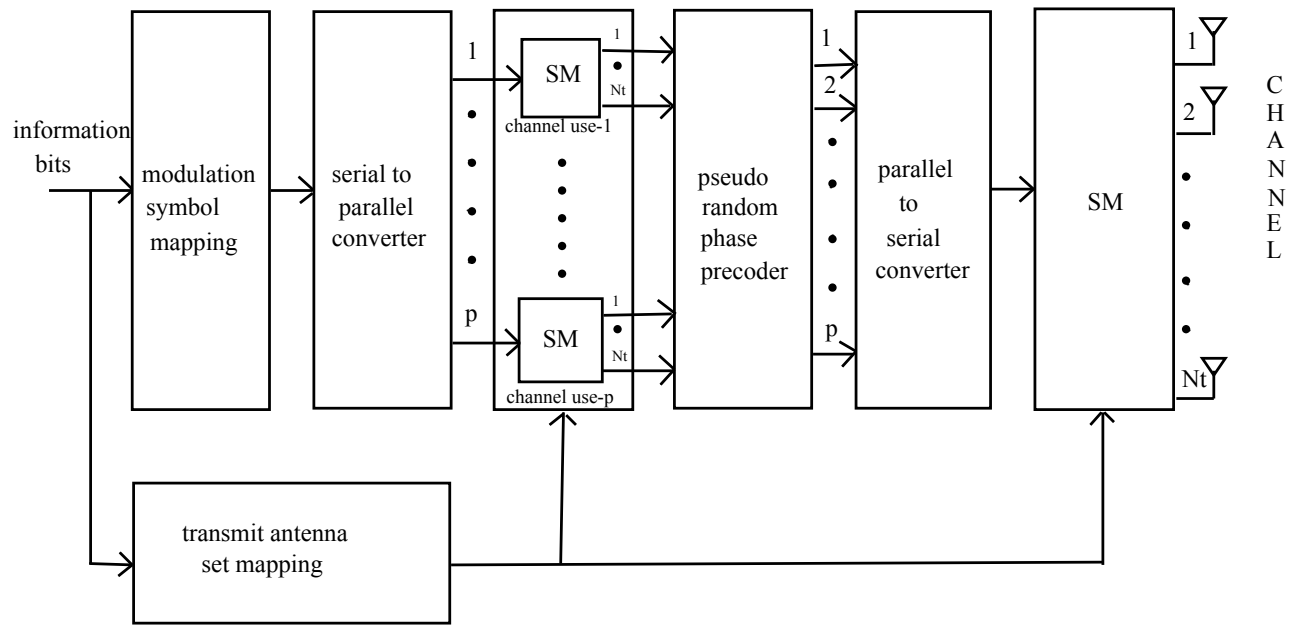
The proposed PRPP transmitter and receiver are illustrated in Figs 3.2a and 3.2b respectively. In this scheme, the signal received at the receiver after p channel uses is given by

$$\mathbf{y}_p = \mathbf{DAPAx}_s + \mathbf{n}_p, \quad (3.8)$$

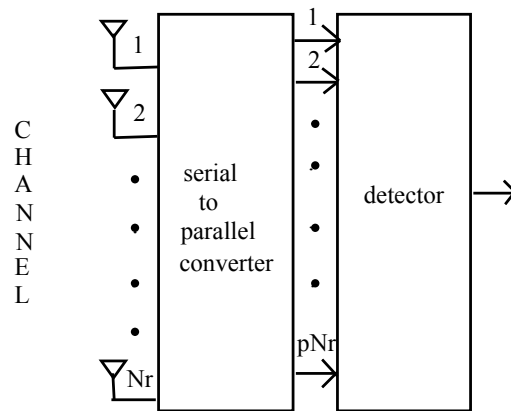
where the precoder matrix \mathbf{P} is a rectangular matrix of dimension $p \times pN_t$ and the entries of \mathbf{P} are generated as described in Sec. 3.2.

For this system model, the maximum-likelihood (ML) detection rule is given by

$$\{\hat{\mathbf{x}}_s, \hat{\mathbf{A}}\} = \underset{\mathbf{x}_s \in \mathbb{A}^p, \forall \mathbf{A}}{\operatorname{argmin}} \|\mathbf{y}_p - \mathbf{DAPAx}_s\|^2, \quad (3.9)$$



(a) PRPP-SM Transmitter.



(b) PRPP-SM Receiver .

Fig. 3.2 system model.

Chapter 4

Low Complexity Detector

From (3.9), it can be seen that the ML detection of the transmitted bits in a precoded SM-MIMO system is exponential in complexity (i.e., $(|\mathbb{A}|N_t)^p$). We propose the use of a local search based detector (LSD) that achieves near-ML detection at large p with a low computational complexity. The local search detector obtains a local minima in terms of the least ML cost among a local neighborhood. The neighbors of a given pair of $\{\mathbf{A}, \mathbf{x}_s\}$, $\mathcal{N}(\mathbf{A}, \mathbf{x}_s)$ is defined as the set of all pairs $\{\mathbf{A}', \mathbf{x}'_s\}$ that satisfies one of the following three conditions,

1. $\mathbf{x}_s = \mathbf{x}'_s$ and $\mathbf{A}_{(i)} \neq \mathbf{A}'_{(i)}$ for exactly a single index i ,
2. $\mathbf{A} = \mathbf{A}'$ and \mathbf{x}_s differs from \mathbf{x}'_s in exactly one entry,
3. $\mathbf{A}_{(i)} \neq \mathbf{A}'_{(i)}$ for exactly a single index i , and $x_s(j) \neq x'_s(j)$ for exactly a single index j .

for example , consider $N_t=2$, $p=2$ and $\mathbb{A} = \{\pm 1\}$ (i.e., $M=2$). then we have

$$\mathcal{N}\left(\left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}\right\}\right) = \left\{ \begin{array}{l} \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} +1 \\ +1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right\} \\ \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} +1 \\ +1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} +1 \\ +1 \end{bmatrix}\right\}, \left\{\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \end{bmatrix}\right\} \end{array} \right\}.$$

The LSD algorithm starts with an initial solution $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$, which is also the current solution. Using the defined neighborhood, the algorithm considers all the neighbors of $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$ and searches for the neighbor with the least ML cost which also has a lesser ML cost than the current solution. If such a neighbor is found, then this neighbor is designated as the current solution. This marks the completion of an iteration of LSD. The iterations are repeated till a local minima is reached (i.e., no neighbor better than the current solution is reachable), the solution corresponding to the local minima is declared as the final output $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\}$. This algorithm is listed in algorithm 1.

Algorithm 1 Listing of the proposed LSD.

- 1: **Input** : $\mathbf{y}, \mathbf{H}, \mathbf{P}$
 - 2: Initial solution : $\{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}, \{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\} = \{\mathbf{A}^{(0)}, \mathbf{x}_s^{(0)}\}$
 - 3: Compute $\mathcal{N}(\hat{\mathbf{A}}, \hat{\mathbf{x}}_s)$
 - 4: $\{\mathbf{A}^c, \mathbf{x}_s^c\} = \underset{\{\mathbf{B}, \mathbf{z}\} \in \mathcal{N}(\hat{\mathbf{A}}, \hat{\mathbf{x}}_s)}{\operatorname{argmin}} \|\mathbf{y}_p - \mathbf{D}\mathbf{B}\mathbf{P}\mathbf{B}\mathbf{z}\|^2$
 - 5: **if** $\|\mathbf{y}_p - \mathbf{D}\mathbf{A}^c\mathbf{P}\mathbf{A}^c\mathbf{x}_s^c\|^2 < \|\mathbf{y}_p - \mathbf{D}\hat{\mathbf{A}}\mathbf{P}\hat{\mathbf{A}}\hat{\mathbf{x}}_s\|^2$ **then**
 - 6: $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\} = \{\mathbf{A}^c, \mathbf{x}_s^c\}$
 - 7: Go to step 3
 - 8: **end if**
 - 9: **Output** : $\{\hat{\mathbf{A}}, \hat{\mathbf{x}}_s\}$
-

Computing the initial solution: We used support recovery based MMSE estimate for obtaining an initial solution to the LSD algorithm. The MMSE estimate is a $pN_t \times 1$ vector given by $\mathbf{v} = (\mathbf{D}^H \mathbf{D} + \sigma^2 \mathbf{I})^{-1} \mathbf{D}^H \mathbf{y}_p$. The vector \mathbf{v} constitutes of p subvectors of size $N_t \times 1$, $\mathbf{v} = [\mathbf{v}_{(1)}^T \mathbf{v}_{(2)}^T \cdots \mathbf{v}_{(p)}^T]^T$. The indices of the elements with the largest amplitude in each $\mathbf{v}_{(i)}$ gives the initial solution for $\mathbf{A}^{(0)}$. The MMSE estimate of \mathbf{x}_s is given by a $p \times 1$ vector given by $\mathbf{z} = (\mathbf{G}^H \mathbf{G} + \sigma^2 \mathbf{I})^{-1} \mathbf{G}^H \mathbf{y}_p$, where $\mathbf{G} = \mathbf{H}\mathbf{A}^{(0)}\mathbf{P}\mathbf{A}^{(0)}$. The hard estimate of \mathbf{x}_s is obtained by mapping each coordinate of \mathbf{z} to nearest symbol in the alphabet in terms of euclidean distance.

Chapter 5

Results and discussions

In this chapter, we present some simulation results on the performance of the proposed PRPP-SM transmission with ML detection and MMSE-LSD. With Figure 5.1 and Figure 5.2 compares the average BER performance of a SM without PRPP and SM with PRPP. In figure 5.1 we set $N_t = 4, N_r = 1$ and using BPSK modulation (i.e. 3 bpcu) with precoder size $p \in \{2, 4, 5\}$. The channel is assumed to undergo temporally uncorrelated Rayleigh fading. We observed that PRPP-SM has performance advantage over the SM without PRPP with ML detection.

Figure 5.2 compares the performance of PRPP-SM with ML detection against the SM without PRPP and SISO AWGN system with 3 bpcu. Here $N_t = 4, N_r = 4$ and BPSK modulation is used with precoder size $p \in \{2, 4\}$. It is interesting to note that the performance of the PRPP-SM is better than the SISO AWGN and SM without PRPP. For smaller precoder size PRPP-SM is inferior to SM without PRPP, as precoder size increases PRPP-SM gets better than SM without PRPP.

Figure 5.3 compares the performance of PRPP-SM with ML detection against SM without PRPP and SISO AWGN system with 3 bpcu. Here $N_t = 4, N_r = 8$ and BPSK modulation is used with precoder size $p \in \{2, 4\}$. It is interesting to note that the performance of the PRPP-SM gets better as the precoder size increases.

Figure 5.4 compares the performance of PRPP-SM with MMSE-LSD detection against SM without PRPP and SISO AWGN system with 2 bpcu. Here $N_t = 2, N_r = 4$ and BPSK modulation is used with precoder size $p \in \{10, 20, 70\}$. It is interesting to note that the performance of the PRPP-SM gets better than SM without PRPP as the precoder size increases.

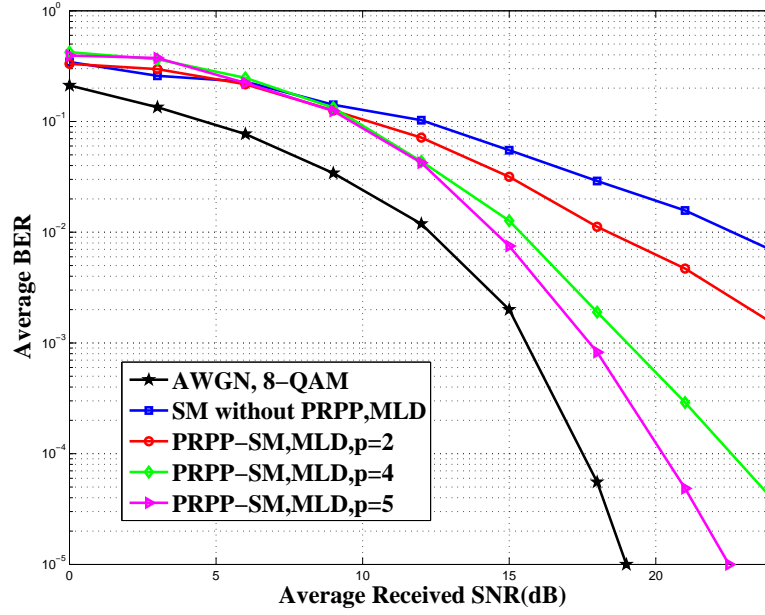


Fig. 5.1 Average BER performance of PRPP-SM($N_t = 4, N_r = 1$, BPSK) with ML detection.

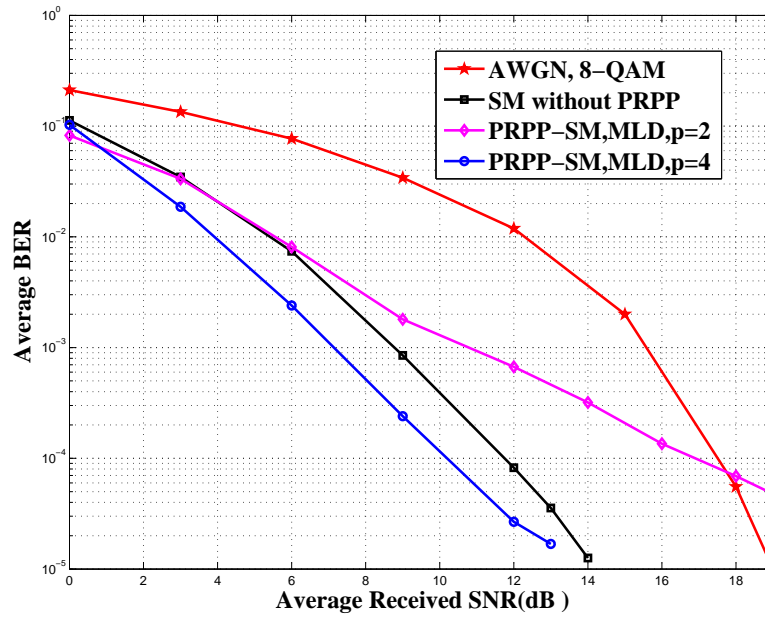


Fig. 5.2 Average BER performance of PRPP-SM($N_t = 4, N_r = 4$, BPSK) with ML detection.

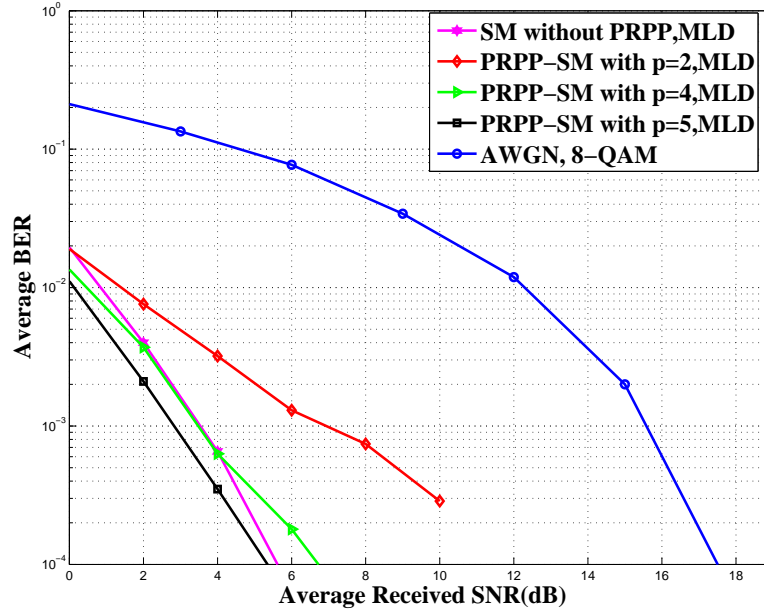


Fig. 5.3 Average BER performance of PRPP-SM($N_t = 4, N_r = 8, \text{BPSK}$) with ML detection.

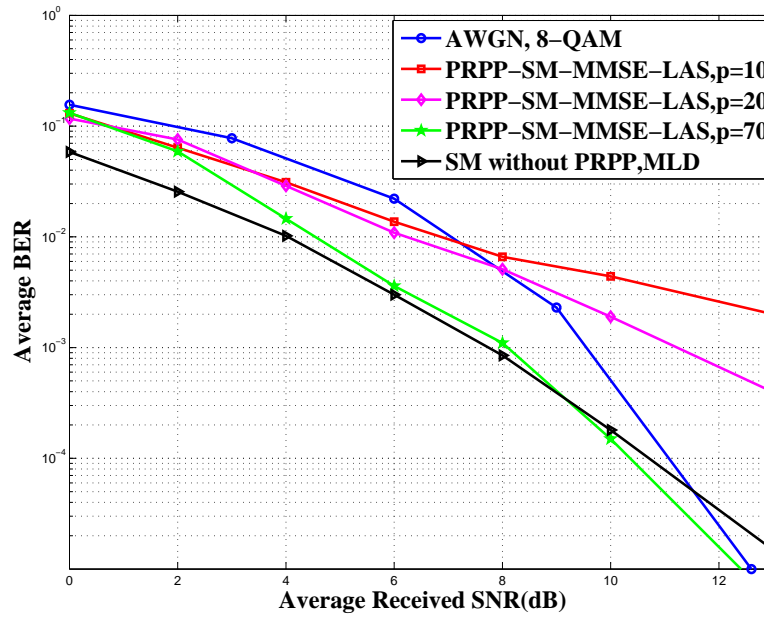


Fig. 5.4 Average BER performance of PRPP-SM($N_t = 2, N_r = 4, \text{BPSK}$) with MMSE-LSD algorithm.

Chapter 6

Conclusions

We proposed a novel PRPP based spatial modulation scheme for uncoded transmissions over fading channels without channel state information at the transmitter. With $N_t = 4, N_r = 4$, BPSK modulation and ML detection, we have demonstrated that the proposed system achieves better performance than SM without precoding. The PRPP introduces a virtual large MIMO channel and hence, the proposed system achieves higher order diversity as the precoder size increases. We assumed channel is independent and identically distributed between channel uses. In future we are working on low complexity detection algorithm to obtain the best achievable performance in SM-MIMO.

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