SIMULATION OF TOSSING COINS

EE511-Fall 2019
PROJECT REPORT-#1: COIN FLIPS

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TOOL: MATLAB (version R2018b)

Problem 1: Simulate tossing a fair coin (a Bernoulli trial) 50 times.

1. Count the number of heads.
2. Record the longest run of heads.
3. Generate a histogram for the Bernoulli outcomes.

Experiment: In this experiment of tossing a coin (fair coin or a biased one) , the number of trials is determined by the input parameter Num_of_Trials and property of the coin whether bias or not. The bias parameter of the coin 'p' is passed using a function 'IsHead'. The output of the experiment is number of heads and the longest run of heads observed in the given number of tosses. The Bernoulli outcome is plotted using histogram.

Code:

```
%Problem 1:

Num_of_heads=0; % number of heads in trials
Num_of_Longestrun=0; % maximum number of run in the trails
RunOfHeads=0; % present number of successive heads
Num_of_Trials=50;
results=zeros(1,Num_of_Trials);
for N = 1:Num_of_Trials
    IsHead = flipcoin(0.5);
    if(IsHead)
        results(N) = 1;
        RunOfHeads = RunOfHeads+1;
        Num_of_heads = Num_of_heads+1;% if the run of heads exceed the
longest history, update the records
        if(RunOfHeads > Num_of_Longestrun)
            Num_of_Longestrun = RunOfHeads;
        end
    else
        RunOfHeads = 0;
    end
end
disp('number of flips')
disp(Num_of_Trials)
disp('number of heads')
```

```
disp(Num_of_heads)
disp('longest run of heads')
disp(Num_of_Longestrun)
figure;
histogram(results)
xlabel('0-tail 1-heads')
ylabel('times')
```

flipcoin.m:

```
function [IsHead] = flipcoin(p)
A=rand();
IsHead = A<p;

end
```

Output:

```
>> q1
number of flips
   50

number of heads
   23

longest run of heads
    3
```

Histogram of Bernoulli outcome:

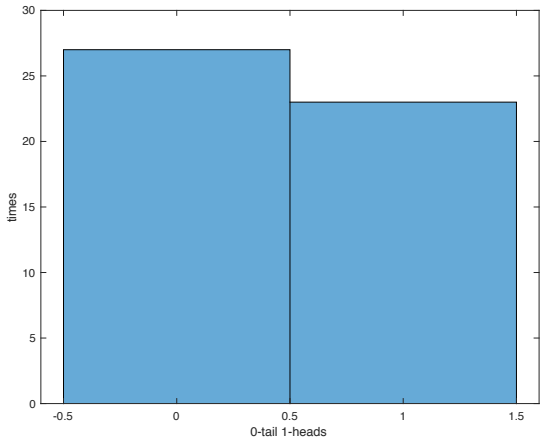

Figure 1: Histogram of Bernoulli Outcome

From figure 1 we observe that the experiment of tossing coin 50 times results in 23 heads and 27 tails. In this particular experiment the longest run of heads is 3. By conducting this experiment with various number of trials we observe that as the number of tosses increases the relative

frequency of both heads and tails converge to around 0.5.

PROBLEM 1a:

Repeat the above experiment 20, 100, 200, and 1000 times.

1. Generate a histogram for each showing the number of heads in 50 flips.
2. Comment on the limit of the histogram.

Experiment: Here we focus on only number of heads in 50 flips by tossing the coin for 20, 100,200 and 1000 times. Here to simulate the trial variable A is used which return an array of integers drawn from the discrete uniform distribution on the given interval. Using the function sum(A(:)==1) ,we determine if the coin has landed as head and count the number of heads.

Code:

```
for i = 1 : 20
A = randi([0:1],[50,1]);
No_of_heads(i)=sum(A(:) == 1); % Total No of heads %
end
subplot(2,2,1); hist(No_of_heads);
xlim([0 50]);
xlabel('Number of heads')
ylabel('Time')
title('20 times')
hold all;

i=1;
for i = 1 : 100
A = randi([0:1],[50,1]);
No_of_heads(i)=sum(A(:) == 1); % Total No of heads %
end
subplot(2,2,2); hist(No_of_heads);
xlim([0 50]);
xlabel('Number of heads')
ylabel('Time')
title('100 times')

i=1;
for i = 1 : 200
A = randi([0:1],[50,1]);
No_of_heads(i)=sum(A(:) == 1); % Total No of heads %
end
subplot(2,2,3); hist(No_of_heads);
xlim([0 50]);
xlabel('Number of heads')
ylabel('Time')
title('200 times')

i=1;
for i = 1 : 1000
A = randi([0:1],[50,1]);
No_of_heads(i)=sum(A(:) == 1) ;% Total No of heads %
end
subplot(2,2,4); hist(No_of_heads);
```

```
xlim([0 50]);
xlabel('Number of heads')
ylabel('Time')
title('1000 times')
```

Output:

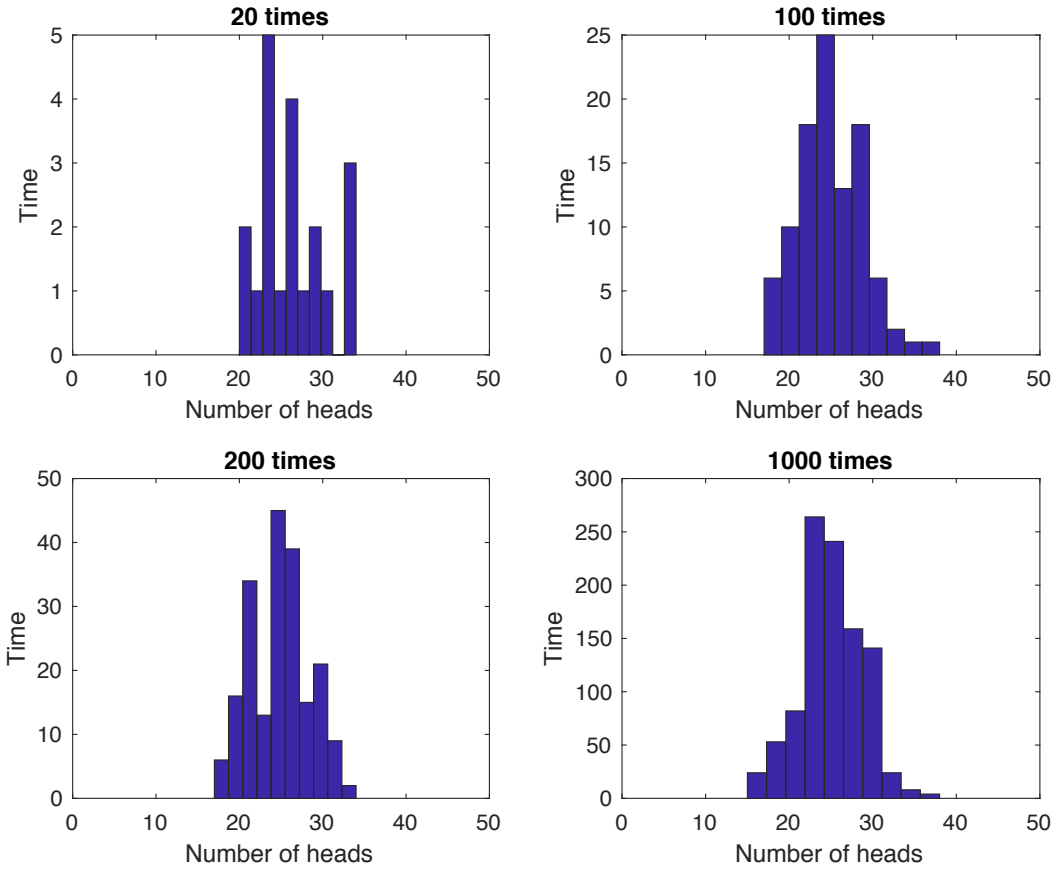

Figure 2: Histogram of number of heads in different number of trilas

Observations:
From the histograms obtained for different number of trials in figure 2, we observe that as the number of trials increase the peak of the histogram is around 25-27 for a fair coin. Also we observe that the peak of histogram declines on both sides as the number of trials are increased.

PROBLEM 2:
Simulate tossing a biased coin 200 times where P(Head)=0.80.

1. Count the number of heads.
2. Record the longest run of heads.
3. Generate a histogram for the Bernoulli outcomes.

Experiment:
It is similar first experiment carried out but with a biased coin of P(HEAD)=0.8. The number of heads and the longest run of heads are recorded. The Bernoulli outcome is plotted using histogram.

Code:

```
%-------------------------------------------------------------------%

%Problem 2:
%Number of Trials=200
%P[heads]=0.8
%-------------------------------------------------------------------%
Num_of_heads=0; %heads in trials
Num_of_Longestrun=0;%maximum # run in the trails
RunOfHeads=0; % present number of successive heads
Num_of_Trials=200;
Num_of_flips_50=50;
Flip_results=zeros(1,200);
%FlipResults = zeros(1,NumOfFlips);

for N = 1:Num_of_Trials
    IsHead = flipcoin(0.8);
    if(IsHead)
        Flip_results(N) = 1;
        RunOfHeads = RunOfHeads+1;
        Num_of_heads = Num_of_heads+1;% if the run of heads exceed the
longest history, update the% records
        if(RunOfHeads > Num_of_Longestrun)
            Num_of_Longestrun = RunOfHeads;
        end
    else
        RunOfHeads = 0;
    end
end
disp('number of flips')
disp(Num_of_Trials)
disp('number of heads')
disp(Num_of_heads)
disp('longest run of heads')
disp(Num_of_Longestrun)
figure;
histogram(Flip_results)
```

Output:
Histogram of number of heads with P(Head)=0.8

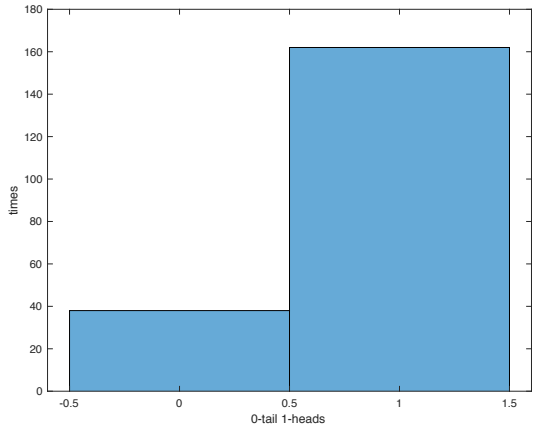

Figure 3: Histogram of number of heads of a biased coin with P(Head)=0.8

Observation:
From figure 3, we observe that the number of heads is 160 which is 80% of the total number of tosses thus we can conclude that the relative frequency of heads has a convergency of 0.8.

PROBLEM 3:
Simulate tossing a fair coin 100 times.
1.Generate a histogram showing the heads run lengths.
Experiment:
Here we are counting the head run lengths when we toss a coin 100 times.

Code:
```
%Simulation of tossing 100 times and plotting histogram of head run
lengths.
close all; clear all; clc;
No_of_heads=0;
length=0;
n=1;
%index = 1;
flag = 0;
for i = 1:100
    IsHead = flipcoin(0.5);
    if(IsHead)
        No_of_heads=No_of_heads+1;
        length=length+1;

    else
        if length>0
         record(n)=length+1;
         n=n+1;

    length=0;
            end
    end
```

```
end
hist(record);
xlabel('Run Lengths');
ylabel('Times')
title('Histogram of Head run lengths')
```

Output:

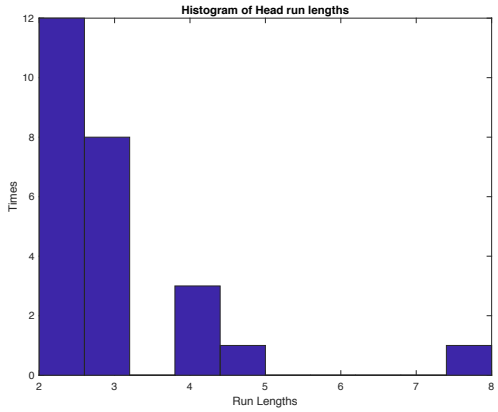

Figure 4: Histogram of Run lengths of heads

Observation:
From figure 4, we observe that in most of trials experimented with we see that the number of head runs is one and they decrease as the number of trials are increased. Also whenever the number of trials are increased the number of heads and tails tend to be almost same for a fair coin hence we don't see longer head runs.

PROBLEM 4:
Simulate tossing a fair coin

1. Count the number of tosses until reaching a user-specified positive number of heads.

Experiment:
Here the user is prompted to provide a positive number of heads and then we count the number of heads in number of tosses until we reach user specified number of heads.

Code:

```
%-------------------------------------------------------------------%
%Simulate tossing a fair coin and count the number of tosses until
%reaching a user?specified positive number of
%heads
%-------------------------------------------------------------------%

prompt='Please provide a positive number of heads: ';
No_of_heads=0;
No_of_tosses=0;
X=input(prompt);
while(No_of_heads<X)
    No_of_tosses=No_of_tosses+1;
    IsHead = flipcoin(0.5);
```

```
      if (IsHead)

       No_of_heads= No_of_heads+1;% Total No of heads %
      end
end
disp('Number of tosses is ')
disp(No_of_tosses)
```

Output:

```
>> q4
Please provide a positive number of heads: 20
Number of tosses is
     44

>> q4
Please provide a positive number of heads: 100
Number of tosses is
    201


>> q4
Please provide a positive number of heads: 500
Number of tosses is
    960

>> q4
Please provide a positive number of heads: 1000
Number of tosses is
         1989
```

Observation:

Here in the above experiment the proportion of heads for the

- First case : p=20/44=0.4545
- Second case : p=100/201=0.497
- Third case. : p=500/960=0.52
- Fourth case : p=1000/1989=0.50

The frequency of heads is observed to be closer to 0.5 as we increase number of tosses of coin.