

# Subsetix: Sparse 2D Geometry on GPU

From Set Algebra to AMR Simulation

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# Outline

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- Why Kokkos?
- Development Methodology

# I. Context: GPU & Kokkos

# GPU Architecture – Massively Parallel

## Execution Hierarchy

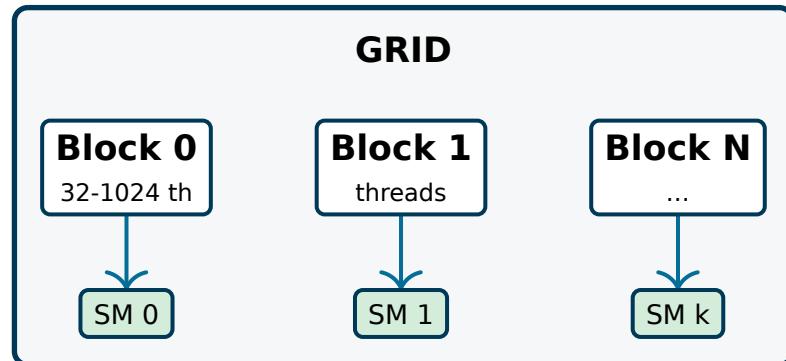


- **Warp** = 32 threads in **lockstep** (SIMT)
- **SM** = autonomous compute unit
- Multiple warps/SM → latency hiding

## For Our Project

- **1 thread** = processes 1 Y row (or 1 cell)
- Thousands of rows → **saturate the GPU**

## Execution Model



## B200 vs EPYC 9965

	GPU B200	CPU EPYC 9965
Bandwidth	<b>8 TB/s</b>	576 GB/s
FP32	<b>80 TFlops</b>	14 TFlops

GPU: **14x more bandwidth** than CPU → ideal for large sparse meshes

# Kokkos — Performance Portability

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## The Problem

- CUDA = NVIDIA only
- OpenMP = CPU only (limited GPU)
- HIP = AMD only
- Rewrite for each platform?

## The Solution: Kokkos

```
// 1. COUNT – unknown result size
parallel_for(num_rows, KOKKOS_LAMBDA(int r) {
    counts[r] = count_intervals(r);
});

// 2. SCAN – compute offsets
exclusive_scan(counts, row_ptr);
// 3. FILL – parallel write
parallel_for(num_rows, KOKKOS_LAMBDA(int r) {
    fill_intervals(r, &out[row_ptr[r]]);
});
```

## CUDA vs Kokkos

### Native CUDA

```
// Allocation
double* d_data;
cudaMalloc(&d_data, n*8);

// Copy Host → Device
cudaMemcpy(d_data, h_data,
           n*8, HostToDevice);

// Kernel
kernel<<<B,T>>>(d_data, n);

// Copy Device → Host
cudaMemcpy(h_data, d_data,
           n*8, DeviceToHost);

// Free
cudaFree(d_data);
```

### Kokkos

```
// Allocation + auto mirror
View<double*> data("d", n);
auto h_data = create_mirror_view(data);

// Copy Host → Device
deep_copy(data, h_data);

// Parallel (CPU or GPU)
parallel_for(n, KOKKOS_LAMBDA(int i){
    data(i) = compute(i);
});

// Copy Device → Host
deep_copy(h_data, data);

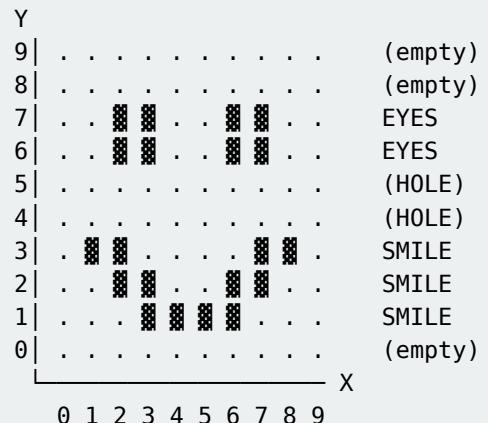
// Automatic cleanup (RAII)
```

**Single source code** → compiles for OpenMP, CUDA, HIP, SYCL, Serial — specializable if needed

## II. Sparse Representation

# Example: 2D Sparse Mesh with Intervals

“Smiley” Geometry :-)



## CSR Representation

```
// 5 rows, HOLE Y=4,5
row_keys = [1, 2, 3, 6, 7] // skips 4,5!
num_rows = 5

// Rows with 1 or 2 intervals
row_ptr = [0, 1, 3, 5, 7, 9]

intervals = [
    {3, 7},           // Y=1: smile bottom
    {2, 4}, {6, 8}, // Y=2: smile thick
    {1, 3}, {7, 9}, // Y=3: smile corners
    {2, 4}, {6, 8}, // Y=6: EYES bottom
    {2, 4}, {6, 8}, // Y=7: EYES top
]
num_intervals = 9

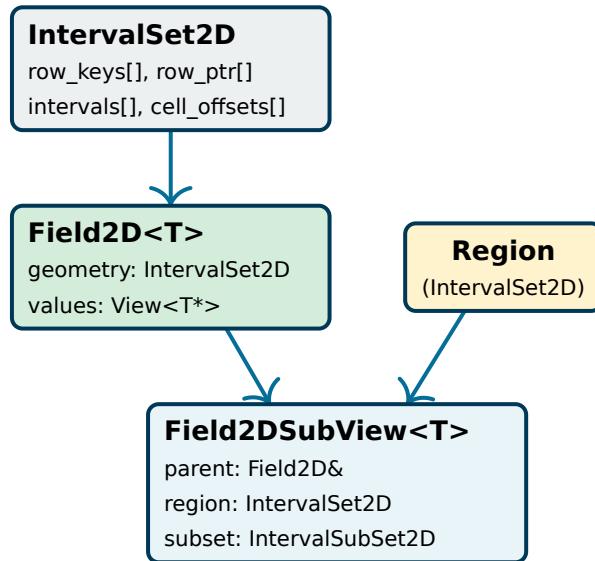
cell_offsets = [0,4,6,8,10,12,14,16,18,20]
total_cells = 20
```

**Hole Y=4,5:** row\_keys jumps from 3 to 6

# **III. Data Structures**

# Overview — Device Structures

## Core Types



## IntervalSubSet2D

```
struct IntervalSubSet2D {
    IntervalSet2D parent;           // ref
    interval_indices[];            // which intervals
    x_begin[], x_end[];            // restricted range
    row_indices[];                 // Y coords
};
```

## SubView: Lazy Intersection

```
// Region = any IntervalSet2D
IntervalSet2DDevice left_bc = make_box_device({0,2,0,ny});
Field2DSubViewDevice<T> sub = make_subview(field, left_bc);

// First op: computes field.geo n region
fill_subview_device(sub, T_inlet, &ctx);

// Time loop: reuses cached intersection
for (int step = 0; step < nsteps; ++step) {
    fill_subview_device(sub, T_inlet); // fast
}
```

## SubView Operations

- fill\_subview\_device(sub, val)
- scale\_subview\_device(sub, alpha)
- copy\_subview\_device(dst, src)
- apply\_stencil\_on\_subview\_device(...)

**Lazy:** intersection computed on first use

**Cached:** reused for subsequent operations

# IntervalSet2D — Complete CSR Structure

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## C++ Definition

```
template<class MemorySpace>
struct IntervalSet2D {
    // Y coordinates of non-empty rows
    View<RowKey2D*> row_keys; // [num_rows]

    // Index into intervals[] for each row
    View<size_t*> row_ptr; // [num_rows + 1]

    // All intervals (contiguous)
    View<Interval*> intervals; // [num_intervals]

    // Linear cell offsets
    View<size_t*> cell_offsets; // [num_intervals]

    size_t total_cells;
    int num_rows;
    int num_intervals;
};
```

## Invariants

- row\_keys sorted by increasing Y
- Intervals sorted by X within each row
- No overlap between intervals
- $\text{row\_ptr}[r+1] - \text{row\_ptr}[r] = \text{nb intervals row } r$

**Template MemorySpace:** Device or Host

# Field2D – Field on Sparse Geometry

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## Definition

Associates a **value** with each sparse cell

```
template<class T, class MemorySpace>
struct Field2D {
    IntervalSet2D geometry; // Geometry ref
    View<T*> values;      // [total_cells]
};
```

## Memory Layout

Geometry:					
values[]:	[v0 v1   v2 v3   v4 v5 v6]				
	↑	↑	↑		
offsets:	0	2	4		

**Contiguous** values → cache-friendly

## Cell Access

```
// O(1) - when interval index known
T val = field.at(interval_idx, x);

// O(log R + log I) - by coordinates
// (binary search on Y, then X)
bool ok = accessor.try_get(x, y, val);
```

## Usage

```
Field2DDevice<double> rho(fluid_geo);
fill_field_device(rho, 1.0);
auto rho_host = to_host(rho); // I/O
```

# SubSet – Targeted Region Operations

## Structure

```
struct IntervalSubSet2D {  
    IntervalSet2D parent; // ref to Field geo  
    interval_indices[]; // which intervals  
    x_begin[], x_end[]; // restricted range  
    row_indices[]; // Y row in parent  
    num_entries;  
};
```

## Usage

```
// Build subset (intersection)  
build_interval_subset(  
    field.geometry, mask, subset, &ctx);  
  
// Operations on subset only  
fill_on_subset(field, subset, 0.0);  
  
// Iteration: O(1) access per entry  
for (e = 0; e < num_entries; ++e) {  
    int iv = interval_indices[e];  
    for (x = x_begin[e]; x < x_end[e]; ++x)  
        field.at(iv, x) = ...; // O(1)  
}
```

## 1D Example: Intersection

Parent:	[==A==]	[==B==]	[==C==]			
idx:	0	1	2			
	0	8	12	18	22	30

Mask:	[=====M=====]	
	5	25

SubSet:	[=]	[==B==]	[=]			
entry:	0	1	2			
	5	8	12	18	22	25

## SubSet = references to Parent

entry		interval_idx		x_begin		x_end
0		0 (A)		5		8
1		1 (B)		12		18
2		2 (C)		22		25

No data copy — just indices + bounds

# Field2DSubView – View on Field + Region

## Structure

```
struct Field2DSubView<T> {
    Field2D<T> parent;          // ref to field
    IntervalSet2D region;        // where to operate
    IntervalSubSet2D subset;     // lazy intersection
};
```

## Lazy Pattern

```
// 1. Create (no computation)
auto sub = make_subview(field, region);
// sub.subset is empty

// 2. First op with ctx → triggers build
fill_subview_device(sub, 0.0, &ctx);
// sub.subset = field.geo n region

// 3. Next ops reuse cached subset
scale_subview_device(sub, 2.0); // fast!
fill_subview_device(sub, 1.0); // fast!
```

## Memory Mapping



■ = skipped   ■ = accessed by SubSet

## Access Formula

values[ offset[idx] + (x - interval.begin) ]

O(1) per cell — no coordinate lookup

# Workspace & AMR Support

## UnifiedCsrWorkspace

Pool of reusable buffers

```
struct UnifiedCsrWorkspace {
    View<int*> int_bufs_[5];
    View<size_t*> size_t_bufs_[2];
    View<RowKey2D*> row_key_bufs_[2];
    View<Interval*> interval_buf_0;

    auto get_int_buf(int id, size_t n) {
        if (n > int_bufs_[id].extent(0))
            Kokkos::resize(int_bufs_[id], n*1.5);
        return subview(int_bufs_[id], {0,n});
    }
};
```

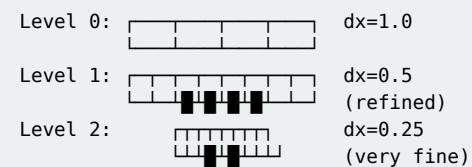
**Avoids** repeated GPU allocations  
Crucial for chained operations

## MultilevelGeo (AMR)

Multi-resolution grids

```
template<class MemorySpace>
struct MultilevelGeo {
    double origin_x, origin_y;
    double root_dx, root_dy;
    int num_active_levels;
    Array<GeoView, 16> levels;

    double dx_at(int level) {
        return root_dx / (1 << level);
    }
};
```



# IV. Algorithms

# Set Algebra — Binary Operations

## CsrSetAlgebraContext

```
struct CsrSetAlgebraContext {  
    UnifiedCsrWorkspace workspace;  
    // Pool of reusable GPU buffers:  
    // - int_bufs_[5], size_t_bufs_[2]  
    // - row_key_bufs_[2], interval_buf_  
    // Auto-grows on demand, never shrinks  
};
```



Same ctx reused → **zero allocations** after warmup

## Complete Example

```
CsrSetAlgebraContext ctx; // create once  
  
auto domain = make_box_device({0, 400, 0, 160});  
auto obstacle = make_disk_device({80, 80, 20});  
  
auto fluid = allocate_interval_set_device(  
    domain.num_rows,  
    domain.num_intervals + obstacle.num_intervals);  
  
set_difference_device(domain, obstacle, fluid, ctx);
```

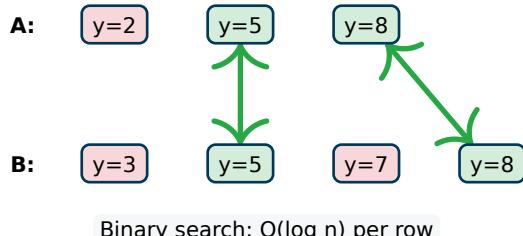
## Chaining with Buffer Reuse

```
CsrSetAlgebraContext ctx;  
  
// Pre-allocate output buffers ONCE  
auto set1 = allocate_interval_set_device(512, 2048);  
auto set2 = allocate_interval_set_device(512, 2048);  
  
// Compute: set1 = A ∪ B  
set_union_device(A, B, set1, ctx);  
  
// Compute: set2 = set1 \ C  
set_difference_device(set1, C, set2, ctx);  
  
// ... use set2 (e.g., create Field2D on it) ...  
  
// Later: reuse same buffers!  
set_intersection_device(D, E, set1, ctx); // set1 reused  
set_union_device(set1, F, set2, ctx); // set2 reused
```

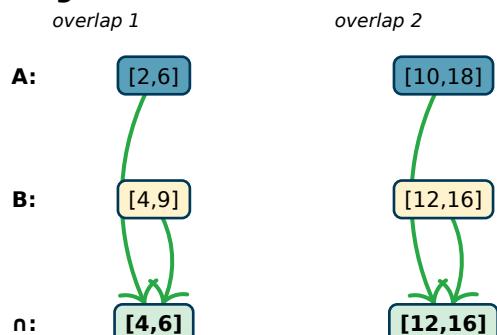
**Allocate once** → reuse for entire simulation  
**ctx + set1 + set2**: zero GPU malloc in hot loop

# Intersection — How It Works

## Phase 1: Row Mapping



## Phase 2: Interval Merge



$\max(2,4)=4$ ,  $\min(6,9)=6$    |    $\max(10,12)=12$ ,  $\min(18,16)=16$

$O(n+m)$  linear merge

## GPU Pattern



### 1. COUNT

```
row_counts[i] = count_intersect(A[i], B[i])
```

### 2. SCAN

```
row_ptr = exclusive_scan(row_counts)
```

### 3. FILL

```
fill_intersect(A[i], B[i], out, row_ptr[i])
```

Same pattern for  $\cup$ ,  $\setminus$ ,  $\oplus$

# Field Operations

## Basic Operations

```
// Algebra & reductions  
field_add_device(a, b, result);  
T dot = field_dot_device(a, b);  
  
// 5-point stencil (W, C, E, S, N)  
apply_csr_stencil_on_set_device(  
    dst, src, region,  
    KOKKOS_LAMBDA(CsrStencilPoint p) {  
        return 0.25 * (p.west + p.east  
                      + p.south + p.north);  
    });
```

## AMR: Restrict & Prolong

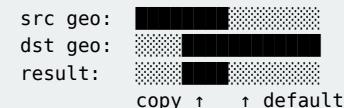
```
// Fine → Coarse (average 4 cells)  
restrict_field_device(fine, coarse);  
  
// Coarse → Fine (interpolation)  
prolong_field_device(coarse, fine);
```

## Threshold: Field → Geometry

```
// Select cells where |value| > epsilon  
IntervalSet2DDevice active =  
    threshold_field(field, epsilon);  
// Use case: detect shock, refine there
```

## Remap: Change Geometry

```
// Project src onto dst geometry  
// (overlap → copy, else → default)  
remap_field_device(src, dst, default_val);
```



# Mathematical Morphology & AMR

## Dilation / Erosion

```
// N-way union with ±radius offset  
row_n_way_union_impl(rows[], radius, out)  
  
// N-way intersection with shrink  
row_n_way_intersection_impl(rows[], r, out)
```

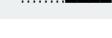
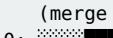
Original:   
Dilate(1):  (+1 sides)  
Erode(1):  (-1 sides)

## 2D Extension

- Consider rows  $y-r$  to  $y+r$
- Merge with N-way operation
- Implicit structuring element (square)

## AMR Operations

```
// Coarsening: fine → coarse  
build_row_coarsen_mapping(fine, ws)  
//  $y_{coarse} = y_{fine} / 2$ , merge X  
  
// Refinement: coarse → fine  
refine_level_up_device(coarse, ws)  
//  $[a,b] \rightarrow [2a, 2b]$ , double Y
```

Fine (level 1):  
Y=3:   
Y=2:   
Y=1:   
Y=0:   
→  
Coarse (level 0):  
Y=1:   
(merge Y=2,3)  
Y=0:   
(merge Y=0,1)

## Field Transfer

```
// Projection fine → coarse (average)  
// Prolongation coarse → fine (interp)  
build_amr_interval_mapping(coarse, fine)
```

# V. Demo

# Mach2 Cylinder — Multi-Level AMR Simulation

## Description

2D compressible flow simulation:

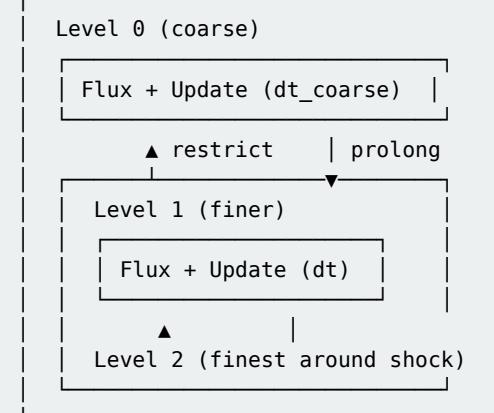
- **Mach 2** supersonic around a cylinder
- 1st order Godunov scheme + Rusanov flux
- **Dynamic AMR**: up to 6 levels

## Subsetix Usage

```
// Fluid geometry = domain - obstacle
auto fluid = set_difference_device(
    make_box_device(domain),
    make_disk_device(cylinder),
    ctx);

// Conserved fields (ρ, ρu, ρv, E)
Field2DDevice<Real> rho(fluid);
Field2DDevice<Real> rhou(fluid);
// ...
```

## AMR Architecture



## Dynamic Refinement

- Indicator: density gradient
- `expand_device()` for guard zones
- Remeshing every N time steps

# Mach2 Cylinder — Results & Visualization

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## Generated Outputs

```
output/
└── fluid_geometry.vtk
└── obstacle_geometry.vtk
└── level_0_density_0000.vtk
└── level_0_density_0050.vtk
└── level_1_density_0050.vtk
└── level_2_density_0050.vtk
└── ...
...
```

## Execution Command

```
./mach2_cylinder \
--nx 400 --ny 160 \
--radius 20 \
--mach-inlet 2.0 \
--max-steps 1000 \
--output-stride 50 \
--amr
```

## Observed Phenomena

- **Bow shock** in front of the cylinder
- Subsonic zone in the wake
- **Von Kármán** vortex street
- Automatic refinement near the shock

## Key Technical Points

- CSR stencil: `apply_csr_stencil_on_set_device()`
- Struct-of-Arrays for cache efficiency
- `prolong_guard_from_coarse()`: interpolation
- `restrict_fine_to_coarse()`: conservation
- Multi-level VTK export for ParaView

**Sparse:** computation only on fluid cells!

# Live Demo

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## Live Demo

### Construction

- Box, Disk, Bitmap
- Difference (obstacle)
- CSR display

### Operations

- Union / Intersection
- Field algebra
- Stencil

### Mach2

- Launch simulation
- ParaView visualization
- AMR in action

*Live demonstration...*

# Thank You!

Questions?

## Key Points

- CSR interval representation
- Count-Scan-Fill pattern
- Kokkos parallelism (CPU/GPU)
- Workspace for memory reuse
- Multi-level AMR (Mach2)

## Contact

Sébastien DUBOIS  
HPC@Maths Team

Code: [include/subsetix/](#)

Demo: [examples/mach2\\_cylinder/](#)

# Appendices

## Implementation History

Version	Description	Performance	Status
v1	CPU only, Sparse CSR + Workspaces First sequential implementation	Faster than baseline	✓ Stable
v3	CUDA only GPU set algebra Proof of concept	Fastest	✓ PoC validated

## Lessons Learned

- **Tiling** improves locality but greatly increases complexity
- Native CUDA faster but less portable
- Kokkos = best **reliability/portability** tradeoff

## Final Choice: Kokkos

- **Single** code for CPU and GPU
- Simplified maintenance
- Easy testing and verification
- Active ecosystem (Sandia, Trilinos)

## Comparison with Native CUDA

Aspect	CUDA	Kokkos

## Supported Backends

- **OpenMP**: CPU multi-thread
- **CUDA**: NVIDIA GPU
- **HIP**: AMD GPU
- **SYCL**: Intel GPU
- **Serial**: debug and tests

## Benefits for This Project

### 1. Faster Development

Debug on CPU (Serial/OpenMP), deploy on GPU

### 2. Reliable Tests

Same code tested on CPU and GPU  
No hidden “GPU-only” bugs

### 3. Std Algorithms

transform, reduce, scan, copy...  
Familiar API, platform-optimized

### 4. Ecosystem

Trilinos, ArborX, Cabana...  
Sandia National Labs support

## Intensive Use of LLMs

### Models Used

- **Claude Opus 4** (Anthropic)
- **Claude Sonnet 4** (Anthropic)

### Work Pattern

#### 1. PLAN

Architecture and interfaces  
Discussion of alternatives

#### 2. QUESTION

Implementation details  
Edge cases

#### 3. IMPLEMENTATION

Code generation  
Review and iteration

### Observed Benefits

- **Rapid exploration** of designs
- Generated inline documentation
- Automatically suggested tests
- Assisted refactoring

### Points of Attention

- Systematic code verification
- LLMs can hallucinate APIs
- Always compile and test
- Maintain **architectural control**

LLM = **accelerator**, not replacement  
Human expertise remains essential

## Kokkos

- Website: [kokkos.org](http://kokkos.org)
- GitHub: [github.com/kokkos/kokkos](https://github.com/kokkos/kokkos)
- Wiki: [kokkos.org/kokkos-core-wiki](https://kokkos.org/kokkos-core-wiki)

## CUDA

- CUDA Toolkit Documentation
- CUDA C++ Programming Guide

## Visualization

- VTK: [vtk.org](http://vtk.org)
- ParaView: [paraview.org](http://paraview.org)

## Mathematical Morphology

- Serra, J. "Image Analysis and Mathematical Morphology" (1982)
- Soille, P. "Morphological Image Analysis" (2003)

## Source Code

```
include/subsetix/
├── geometry/      # IntervalSet2D
├── field/         # Field2D
└── csr_ops/       # Algorithms
└── multilevel/    # AMR
└── detail/        # Utilities

examples/mach2_cylinder/
└── mach2_cylinder.cpp  # AMR Demo
```