06 Tue Nov 9

ECE 447: Control Systems

Prof Burden TA Tim

this week:
- ☐ HW4x6 assigned → due Fri Nov 19
- ☐ week 6 (& 4) lecture material
- ☐ tutorial

exam 1 results & solution next Tue Nov 14

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state estimation for state-space LTI system

$$u \longrightarrow \boxed{\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}} \xrightarrow{\;y\;}$$

Q: why y?

A: it's often impossible or inconvenient to measure x

ex $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$, $q \in \mathbb{R}^d$, $x \in \mathbb{R}^n = \mathbb{R}^{2d}$ → might be easier/cheaper/safer/more reliable to only measure $y = q = [I_d \;\; 0_{d\times d}]\, x$

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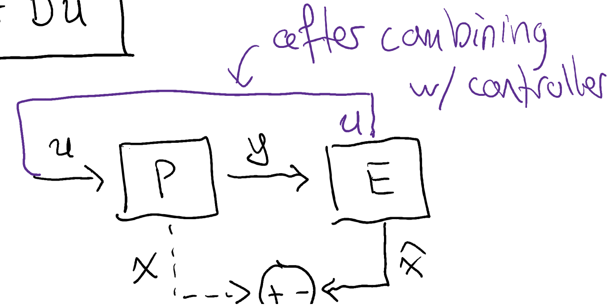
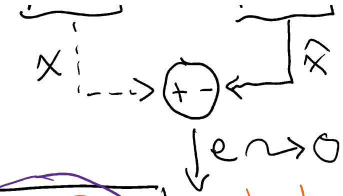
suppose we're given $u \longrightarrow \boxed{\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx + Du\end{aligned}} \xrightarrow{\;y\;}$ $=$ $\xrightarrow{u} \boxed{P} \xrightarrow{y}$

then we want $\xrightarrow{y} \boxed{E} \xrightarrow{\hat{x}}$ s.t.





happy thought: $\xrightarrow{u}\ \boxed{E}\ \xrightarrow{\hat{x}}\ = \boxed{\begin{aligned}\dot{\hat{x}} &= A\hat{x} + Bu - L(y-\hat{y})\\ \hat{y} &= C\hat{x} + Du\end{aligned}}\ \xrightarrow{\hat{x}}\ \xrightarrow{\hat{y}}\ \xrightarrow{u}$

output error

output error feedback (proportional)

Q: what happens to $e = x - \hat{x}$? (want: $e \rightsquigarrow 0$) $= (A-LC)(x-\hat{x})$

A: analyze the dynamics!

$\hat{x} = x - e$

$\dot{e} = \dot{x} - \dot{\hat{x}} = Ax + Bu - A\hat{x} - Bu + L(y - \hat{y})$ 0

let $\tilde{x} = \begin{bmatrix} x \\ e \end{bmatrix}$ so that $\tilde{x} = \bar{A}\tilde{x} + \bar{B}u$

could have chosen $\check{x} = \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$, but $\check{x} = \begin{bmatrix} I & 0 \\ I & -I \end{bmatrix}\begin{bmatrix} x \\ e \end{bmatrix}$

$$\dot{\tilde{x}} = \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} Ax + Bu \\ (A-LC)e \end{bmatrix} = \underbrace{\begin{bmatrix} A & 0 \\ 0 & A-LC \end{bmatrix}}_{=\bar{A}}\begin{bmatrix} x \\ e \end{bmatrix} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{=\bar{B}} u$$