ECE 447: Control Systems

Prof Burden TA Tim

this week: ☐ HW2 assigned ⟶ due Fri Oct 20

☐ week 3 lecture material

☐ tutorial

Prof Burden TODO: ☐ HW1 bonus deadline

☐ bonus points in Canvas

☐ post HW by Saturday

☐ homogeneous response coeff's

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tutorial on roots, eigenvalues, and characteristic polynomials

consider an LTI transformation $\xrightarrow{u} \boxed{G(s)} \xrightarrow{y}$

* we know that $u(t) = e^{st}$

yields $$y(t) = \overbrace{\underbrace{G(s)e^{st}}_{\text{"particular" response}}}^{\text{"steady-state"}} + \overbrace{\underbrace{\sum_{k=1}^{n} c_k e^{s_k t}}_{\text{"homogeneous" response}}}^{\text{"transient"}}$$

particular response

"homogeneous response

where s_k's are roots of a polynomial

↳ why?

suppose G has representation: $s^n y + a_1 s^{n-1} y + \cdots + a_n y = b_1 s^{n-1} u + \cdots + b_n u$

↳ then "homogeneous response" corresponds to solutions of the equation $s^n y + a_1 s^{n-1} y + \cdots + a_n y = 0$

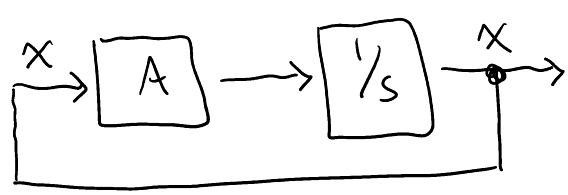
$$\iff (s^n + a_1 s^{n-1} + \cdots + a_n)\, y = 0$$

Q: are there non-zero $y(t)$ that satisfy

A: yes! but – only of the form $y(t) = e^{s_k t}$ where s_k are roots of $(s^n + a_1 s^{n-1} + \cdots + a_n)$

* why is stability governed by eigenvalues?

time domain $\dot{x} = Ax$



freq domain $\underset{\in \mathbb{C}}{s}\, \underset{\in \mathbb{R}^n}{x} = \underset{\in \mathbb{R}^{n\times n}}{A} x \iff sx - Ax = 0$

$$\iff (sI - A)x = 0$$

Q: are there nonzero $\underbrace{\text{solutions}}_{\text{i.e. } s \,\&\, x}$ to ?

$\in \mathbb{C}$ $\quad \in \mathbb{R}^n$

i.e. s & x $\qquad \in \mathbb{C} \qquad \in \mathbb{C}^n$

A: yes! all & only eigval/eigvec pairs s_k & v_k

* s_k's are roots of characteristic polynomial $\det(sI - A)$

→ correspond to $x_k(t) = e^{s_k t} \cdot v_k$ $\qquad A \cdot (e^{s_k t} v_k) = e^{s_k t} \cdot A \cdot v_k = e^{s_k t} \cdot s_k v_k$

$$\Rightarrow \frac{d}{dt} x_k = s_k \cdot e^{s_k t} \cdot v_k = A x_k$$

$$\left.\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ \dot{x}_2 &= A_2 x_2 + B_2 u_2 \end{aligned}\right\} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} A_1 x_1 + B_1 u_1 \\ A_2 x_2 + B_2 u_2 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \rightsquigarrow \dot{x} = A x + B u$$

$x_1 \in \mathbb{R}^{n_1}$

$x_2 \in \mathbb{R}^{n_2}$

$$= \begin{bmatrix} A_1 & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & A_2 \end{bmatrix} x + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u$$