01 -- Tue Oct 3

ECE 447: Control Systems

Prof Burden   TA Tim

this week: ☐ HW0 assigned → due Fri Oct 6

☐ week 1 lecture material

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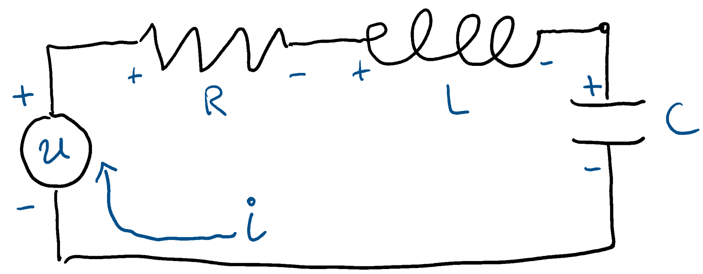
tutorial: apply week 1 lectures to RLC circuit

u - voltage source L - inductance
R - resistance C - capacitance



(a) mathematical model

Kirchoff's voltage law: $\sum_{e \in E} v_e = 0 = -v_u + v_R + v_L + v_C$

e – "lumped element"

lumped element linear model
$$\begin{cases} v_u = u & v_L = L\dfrac{di}{dt} \text{ – change in current } \dfrac{di}{dt} \\ v_R = iR \text{ – current } i & v_C = \dfrac{1}{C} q \text{ – charge } q \quad \dfrac{dq}{dt} = i \end{cases}$$

(b) differential equation – how physical quantities change in time in

(b) differential equation — how physical quantities change in time in relation to each other

$$v_R + v_L + v_C = iR + L\frac{di}{dt} + \frac{1}{C}q = u$$

$$= \frac{dq}{dt}R + L\frac{d^2q}{dt^2} + \frac{1}{C}q = u$$

$\frac{d}{dt}x = \dot{x}$

$$\left. L\ddot{q} + R\dot{q} + \frac{1}{C}q = u \right\}$$ "time domain" model

(c) transfer function — how input signal transforms to output signal

- recall that $\mathcal{F}(\dot{x}) = s \cdot \mathcal{F}(x)$

$\hookrightarrow$ $\mathcal{F}\left(L\ddot{q} + R\dot{q} + \frac{1}{C}q\right) = Ls^2\hat{q} + Rs\hat{q} + \frac{1}{C}\hat{q} = \hat{u}$

$$\left(Ls^2 + Rs + \frac{1}{C}\right)\hat{q} = \hat{u} \Longleftrightarrow \hat{q} = \left(\frac{1}{Ls^2 + Rs + 1/C}\right)\hat{u}$$

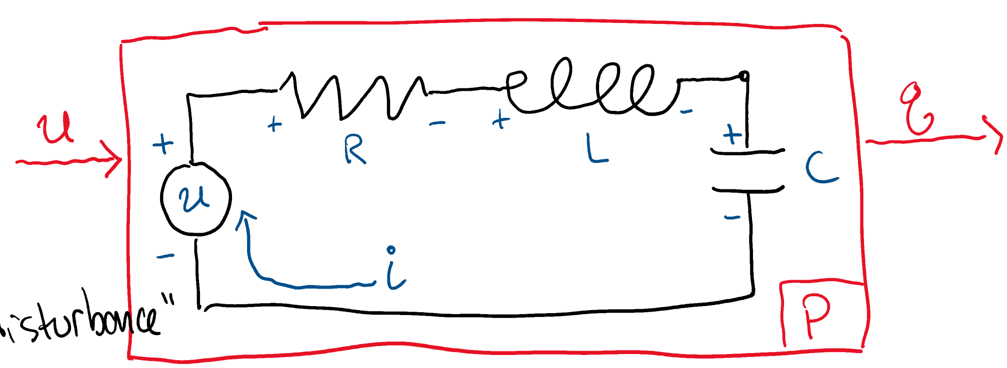
$$= G(s)\,\hat{u}$$

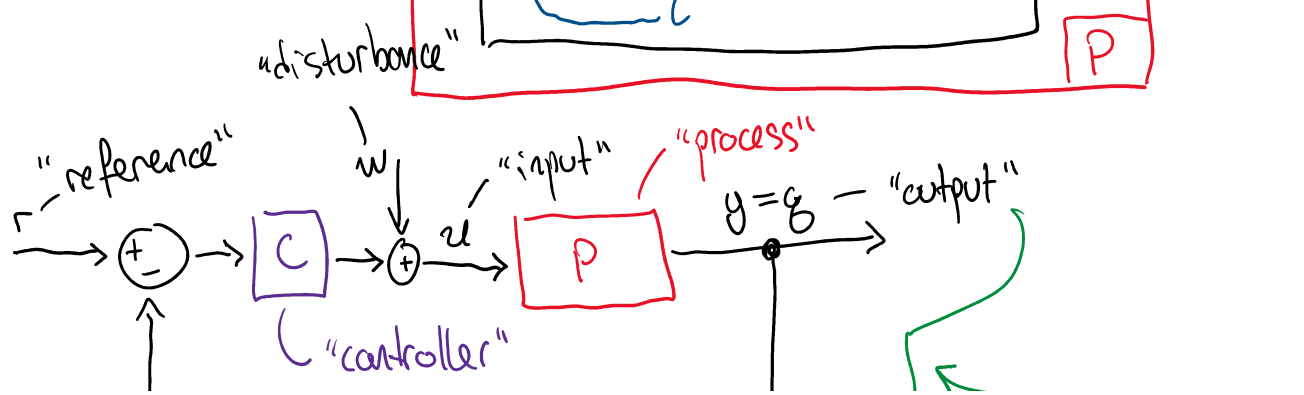
"frequency domain" model

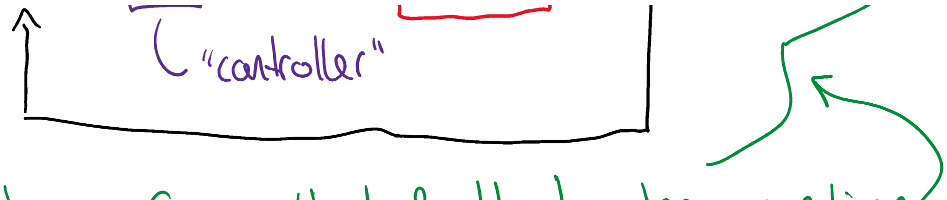
"transfer function"

(d) block diagram

(e) feedback control







* our job: design C so that feedback interconnection behaves the way we want

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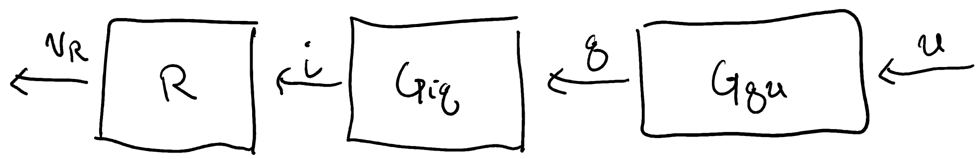
Q: what about using v_R as output from circuit?

A: I'm bad at circuit analysis, so I'll think of it from a systems perspective:



$$q = \left(\frac{1}{Ls^2 + Rs + 1/C}\right) u \qquad v_R = iR \qquad i = \frac{dq}{dt} \rightsquigarrow s\cdot q$$

$$= G_{qu}(s)\cdot u \qquad\qquad = G_{iq}(s)\cdot q$$

$$\frac{\hat{v}_R}{\hat{u}} = R\cdot G_{iq}\cdot G_{qu}\cdot u = \left(\frac{Rs}{Ls^2 + Rs + 1/C}\right)\cdot u$$