01 -- Tue Oct 3

ECE 447: Control Systems

Prof Burden TA Tim

this week: ☐ HW0 assigned → due Fri Oct 6

☐ week 1 lecture material

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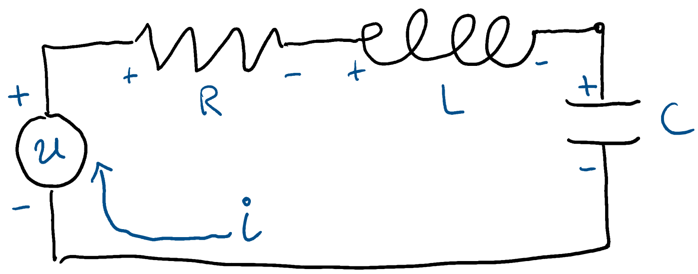
tutorial: apply week 1 lectures to RLC circuit

u - voltage source

R - resistance

L - inductance

C - capacitance



(a) mathematical model

Kirchoff's voltage law: $\sum_{e \in E} v_e = 0 = -v_u + v_R + v_L + v_C$

"lumped element"

lumped element linear model

$$\begin{cases} v_u = u \\ v_R = iR \end{cases}$$

– current i

$v_L = L \frac{di}{dt}$ – change in current $\frac{di}{dt}$

$v_C = \frac{1}{C} q$ – charge q $\qquad \frac{dq}{dt} = i$

(b) differential equation – how physical quantities change in time in

(b) differential equation – how physical quantities change in time in relation to each other

$$v_R + v_L + v_C = iR + L\frac{di}{dt} + \frac{1}{C}q = u$$

$$= \frac{dq}{dt}R + L\frac{d^2q}{dt^2} + \frac{1}{C}q = u$$

$\frac{d}{dt}x = \dot{x}$

$$\left. L\ddot{q} + R\dot{q} + \frac{1}{C}q = u \right\}$$ "time domain" model

(c) transfer function – how input signal transforms to output signal

- recall that $\mathcal{F}(\dot{x}) = s \cdot \mathcal{F}(x)$

$\hookrightarrow$ $\mathcal{F}\left(L\ddot{q} + R\dot{q} + \frac{1}{C}q\right) = Ls^2\hat{q} + Rs\hat{q} + \frac{1}{C}\hat{q} = \hat{u}$

$$\left(Ls^2 + Rs + \frac{1}{C}\right)\hat{q} = \hat{u} \iff \hat{q} = \left(\frac{1}{Ls^2 + Rs + 1/C}\right)\hat{u}$$

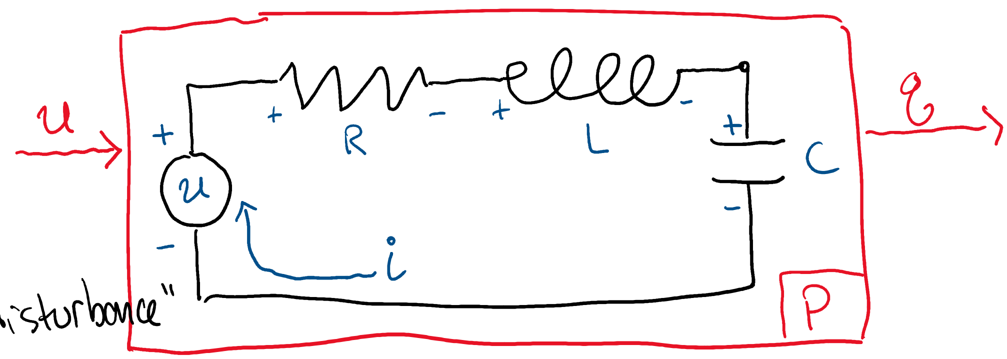
$$= G(s)\,\hat{u}$$

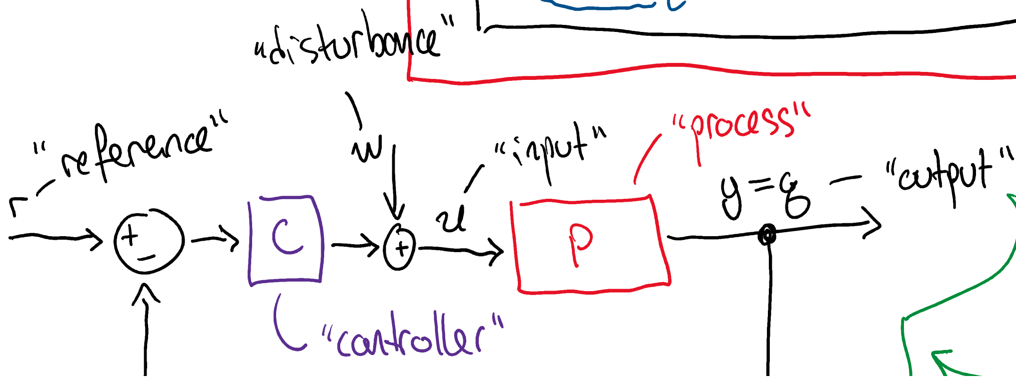
"frequency domain" model

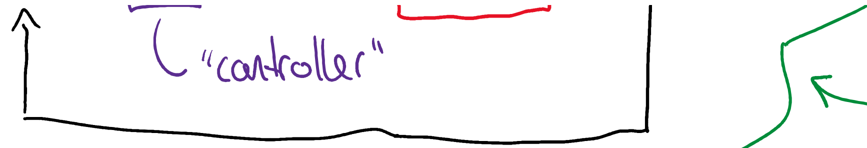
"transfer function"

(d) block diagram

(e) feedback control







* our job: design C so that feedback interconnection behaves the way we want

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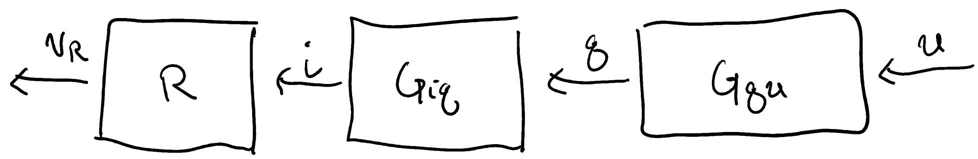
Q: what about using v_R as output from circuit?

A: I'm bad at circuit analysis, so I'll think of it from a systems perspective:



$$q = \left(\frac{1}{Ls^2 + Rs + 1/C} \right) u \qquad v_R = iR \qquad i = \frac{dq}{dt} \rightsquigarrow s \cdot q$$

$$= G_{qu}(s) \cdot u \qquad\qquad = G_{iq}(s) \cdot q$$

$$\frac{\hat{v}_R}{\hat{u}} = R \cdot G_{iq} \cdot G_{qu} \cdot u = \left(\frac{Rs}{Ls^2 + Rs + 1/C} \right) \cdot u$$

01 -- Thu Oct 5

ECE 447: Control Systems

Prof Burden TA Tim

this week:
- ☐ HW0 assigned → due Fri Oct 6
- ☐ week 1 lecture material
- ☐ computation demo

ECE 447: Control Systems

Prof Burden TA Tim

this week:
- ☑ HW1 assigned → due Fri Oct 13
- ☐ week 2 lecture material
- ☐ tutorial on
- ☐ computational demo of

announcements:
- ☑ Zoom recordings
- ☑ Canvas Discussions
- ☑ new TA OH location

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tutorial: quadrotor

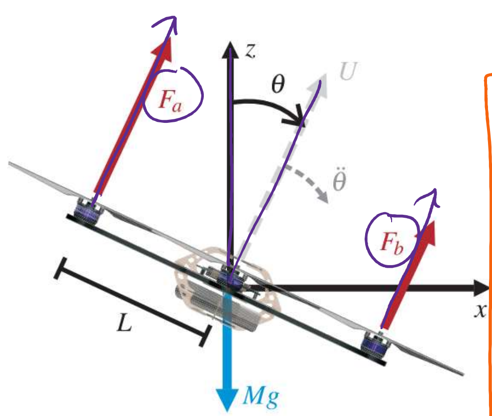
A Simple Learning Strategy for High-Speed Quadrocopter Multi-Flips

Sergei Lupashin, Angela Schöllig, Michael Sherback, Raffaello D'Andrea

M - mass

L - half the width of vehicle

I_{yy} - rotational inertia



$$
\begin{aligned}
M\ddot{z} &= (F_a + F_b + F_c + F_d)\cos\theta - Mg \qquad (1)\\
M\ddot{x} &= (F_a + F_b + F_c + F_d)\sin\theta \qquad (2)\\
I_{yy}\ddot{\theta} &= L(F_a - F_b) \qquad (3)
\end{aligned}
$$

forces from rotors

$F = F_a + F_b + F_c + F_d$

torque applied by rotors

$\tau = L \cdot (F_a - F_b)$

$\nu = z$ "vertical"

$\eta = x$ "horizontal"

$\eta = x$ "horizontal"

* state space model $\quad \eta$ = horizontal $\quad \nu$ = vertical $\quad \theta$ = rotation

$$\text{state} \quad x = \begin{bmatrix} \text{positions} \\ \text{velocities} \end{bmatrix} = \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \eta \\ \nu \\ \theta \\ \dot{\eta} \\ \dot{\nu} \\ \dot{\theta} \end{bmatrix} \in \mathbb{R}^6$$

positions: $q = (\eta, \nu, \theta) \in \mathbb{R}^3$

velocities: $\dot{q} = (\dot{\eta}, \dot{\nu}, \dot{\theta})$

ECE 447: Control Systems

Prof Burden TA Tim

this week: ☐ HW2 assigned → due Fri Oct 20

☐ week 3 lecture material

☐ tutorial

Prof Burden TODO: ☐ HW1 bonus deadline

☐ bonus points in Canvas

☐ post HW by Saturday

☐ homogeneous response coeff's

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tutorial on roots, eigenvalues, and characteristic polynomials

consider an LTI transformation $\xrightarrow{u} \boxed{G(s)} \xrightarrow{y}$

* we know that $u(t) = e^{st}$

yields $$y(t) = \underbrace{\overbrace{G(s)e^{st}}^{\text{"steady-state"}}}_{\text{"particular" response}} + \underbrace{\overbrace{\sum_{k=1}^{n} c_k e^{s_k t}}^{\text{"transient"}}}_{\text{"homogeneous" response}}$$

particular response "homogeneous response

where s_k's are roots of a polynomial

↳ why?

suppose G has representation: $s^n y + a_1 s^{n-1} y + \dots + a_n y$
$= b_1 s^{n-1} u + \dots + b_n u$

↳ then "homogeneous response" corresponds to solutions of the equation $s^n y + a_1 s^{n-1} y + \dots + a_n y = 0$

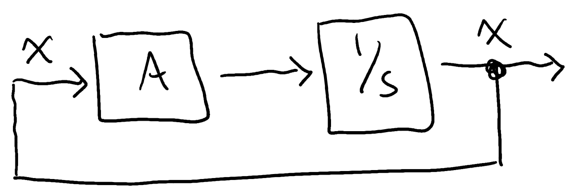
$$\Leftrightarrow (s^n + a_1 s^{n-1} + \dots + a_n) y = 0$$

Q: are there non-zero $y(t)$ that satisfy

A: yes! but — only of the form $y(t) = e^{s_k t}$ where s_k are roots of $(s^n + a_1 s^{n-1} + \dots + a_n)$

* why is stability governed by eigenvalues?

time domain $\dot{x} = Ax$



freq domain $\underset{\in \mathbb{C}}{s}\, \underset{\in \mathbb{R}^n}{x} = \underset{\in \mathbb{R}^{n\times n}}{A} x \Leftrightarrow sx - Ax = 0$

$$\Leftrightarrow (sI - A)x = 0$$

Q: are there nonzero $\underbrace{\text{solutions}}_{\text{i.e. } s \,\&\, x}$ to ?

$\in \mathbb{C}$ $\in \mathbb{R}^n$

ie. s & x

$\in \mathbb{C}$ $\in \mathbb{C}^n$

A: yes! all & only eigval/eigvec pairs s_k & v_k

* s_k's are roots of characteristic polynomial $\det(sI - A)$

correspond to $x_k(t) = e^{s_k t} \cdot v_k$

$$A \cdot (e^{s_k t} v_k) = e^{s_k t} \cdot A \cdot v_k = e^{s_k t} \cdot s_k v_k$$

$$\Rightarrow \frac{d}{dt} x_k = s_k \cdot e^{s_k t} \cdot v_k = A x_k$$

$$\left.\begin{aligned} \dot{x}_1 &= A_1 x_1 + B_1 u_1 \\ \dot{x}_2 &= A_2 x_2 + B_2 u_2 \end{aligned}\right\} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \Rightarrow \dot{x} = \begin{bmatrix} A_1 x_1 + B_1 u_1 \\ A_2 x_2 + B_2 u_2 \end{bmatrix}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \leadsto \dot{x} = A x + B u$$

$$= \begin{bmatrix} A_1 & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_1} & A_2 \end{bmatrix} x + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} u$$

$x_1 \in \mathbb{R}^{n_1}$

$x_2 \in \mathbb{R}^{n_2}$